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Research Article

Higher Dimensional Charged Black Hole Solutions in $f(R)$ Gravitational Theories

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We present, without any assumption, a class of electric and magnetic flat horizon D-dimension solutions for a specific class of $f(R) = R + \alpha R^2$, all of which behave asymptotically as Anti-de-Sitter spacetime. The most interesting property of these solutions is that the higher dimensions black holes, $D > 4$, always have constant electric and magnetic charges in contrast to what is known in the literature. For $D = 4$, we show that the magnetic field participates in the metric on equal foot as the electric field participates. Another interesting result is the fact that the Cauchy horizon is not identical with the event horizon. We use Komar formula to calculate the conserved quantities. We study the singularities and calculate the Hawking temperature and entropy and show that the first law of thermodynamics is always satisfied.

1. Introduction

The most effective gravitational theory in the last century is the theory of general relativity (GR). This theory is a fully accepted one that depicts the macroscopic geometrical properties of spacetime. Using an isotropic and homogeneous symmetry, the field equations of GR give Friedmann equations which depict the evolution of the universe with radiation and then matter dominated epochs. However, recent observations indicate that our universe goes through a phase of accelerated expansion [1–3]. This fact cannot be explained in the frame of GR using ordinary matter as a source. Another issue that GR cannot explain is the cosmological era which is known as inflation [4]. This phase of the universe is believed to have occurred before the radiation era which could relax some issues of cosmology like horizon, flatness singularities, and so on [5]. Moreover, using baryonic matter, GR is not able to discuss the observed density limited by the fitting of the standard Λ cold dark matter (ΛCDM) of the Wilkinson Microwave Anisotropy Probe results for 7 years of observations data (WMAP7) [6], the recent measurements of Baryon Acoustic Oscillations (BAO) [7], and the Hubble constant H_0 [8]. Therefore, GR needs to impose an extra component known as the dark matter (DM) which constitutes about 23% of the energy content of the universe [7]. In spite of the fact that there are many possible roots of such component [9–18], DM is assumed in a form of thermal relics which naturally freeze-out with the right abundance in the extensions of the standard model of particles [19–31]. Coming experiments enable us to distinguish between large number of candidates and model by direct and indirect detection prepared for their search [32–34], or even at large hadron collider (LHC) where they could be produced [35–42].

Another puzzling issue is the one of the accelerated expansion of our universe. Many explanations have been setup to demonstrate such phenomena. Among these explanations is the one which assumes the validity of GR and suggests the presence of extra fluid called dark energy (DE). The equation of state of DE takes the form $p = \omega\rho$ (where p, ρ, and ω are pressure, density, and a dimensionless parameter,

resp.) with $\omega < -1/3$ to create an accelerated cosmic expansion era [43–45]. There is another model which could explain the DE which includes the cosmological constant in the field equations of GR and assumes the equation of state to have the form $\omega = -1$. However, such model suffers from the discrepancy which comes from the fact that if we postulate the cosmological constant to represent the quantum vacuum energy then its value has higher orders of magnitude than those of observations [46]. It has been shown that, in the Palatini formalism of $f(R) = R - \alpha^2/3R + R^2/3\beta$, α and β are dimension parameters, the R^2 term cannot lead to an early time inflation [47].

To overcome the problem of acceleration a modification of GR has been considered by modifying Einstein-Hilbert action [48–62]. Some examples of these modifications are as follows:

(i) Brans-Dicke theory whose interaction is considered by GR tensor and scalar field [63]

(ii) String theory which include Gauss-Bonnet term [64]

(iii) Lovelock theories which are of second derivatives at most [65].

(iv) The $f(T)$ gravitational theory whose field equations are of second order [66–80]

(v) The one known as $f(R)$ theories which we focused the present study on [81]

It is shown that $f(R)$ gravitational theories are able to describe the whole story of cosmology starting from inflation to the present accelerated expansion epoch [82]. Many applications of $f(R)$ gravitational theories have been carried out [83, 84]. Also local tests on $f(R)$ have been achieved to constrain $f(R)$ theories [85–88]. To study modified theories of gravity one requires to assure or reject their validity by deriving solutions that could investigate the evolution of the universe [89, 90] and the occurrence of GR astrophysical prediction. There are many BH derived in the frame of $f(R)$ by assuming a constant scalar curvature, $R = R_0$ [91–93]. It is the aim of the present study to abandon this condition $R = R_0$ in a class of $f(R) = R + \alpha R^2$ and try to derive D-dimension solutions for a flat horizon metric spacetime.

The arrangements of this study are as follows: In Section 2, gradients elements of Maxwell-$f(R)$ gravitational theory are presented. In Section 3, a metric spacetime with one unknown and two unknown functions is presented and applied to the charged field equation of $f(R)$. Exact classes of charged black holes are derived in Section 3. In Section 4, the relevant physics of these classes is discussed by calculating the singularities. In Section 5, we calculate the conserved charges related to each class by using Komar method. In Section 6, we calculate the thermodynamical quantities like Hawking temperature and entropy, and so on. Also in this section, we have shown that the first law of thermodynamics is always satisfied for all the solutions derived in this study. The main results are discussed in the final section.

2. Fundamentals of Maxwell-$f(R)$ Gravitational Theories

The Lagrangian of $f(R)$ theory has the form

$$\mathscr{L} := \mathscr{L}_g + \mathscr{L}_{em}, \tag{1}$$

with $\mathscr{L}_g$ representing the gravitational Lagrangian given by

$$\mathscr{L}_g := \frac{1}{2\kappa}\int d^D x\sqrt{-g}\left(f(R) - \Lambda\right), \tag{2}$$

with Λ being the cosmological constant and κ being the D-dimensional gravitational constant defined as $\kappa = 2(D-3)\Omega_{D-1}G_D$, where G_D is the Newton's constant in D-dimensions (the units for the D-dimensional gravitational constant are $G_D = G_4 L_{D-4}$, where G_4 is the gravitational constant in 4-spacetime dimensions and L is a unit of length). Here Ω_{D-1} refers to the volume of $(D-1)$-dimensional unit sphere that is defined as

$$\Omega_{D-1} = \frac{2\pi^{(D-1)/2}}{\Gamma\left((D-1)/2\right)}, \tag{3}$$

where Γ is the gamma function of the argument (for $D = 4$, one gets $2(D-3)\Omega_{D-1} = 8\pi$), R is the Ricci scalar of the spacetime, g is the determinant of the metric, and $f(R)$ is the analytic function of the considered theory. In this study L_{em} refers to the action of Maxwell field that is defined as

$$L_{em} := -\frac{1}{2}F\wedge{}^{\star}F, \quad \text{where } F = dA, \tag{4}$$

with $A = A_\mu dx^\mu$ being the 1-form electromagnetic potential [94].

By carrying out variations of (2) with respect to the metric tensor $g_{\mu\nu}$ and the vector potential A_μ one can obtain the following field equations of $f(R)$ gravitational theory [95, 96]:

$$\begin{aligned} &R_{\mu\nu}f_R - \frac{1}{2}g_{\mu\nu}f(R) - 2g_{\mu\nu}\Lambda + g_{\mu\nu}\Box f_R - \nabla_\mu\nabla_\nu f_R \\ &\quad = 2\kappa T_{\mu\nu}, \end{aligned} \tag{5}$$

$$\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right) = 0, \tag{6}$$

with $R_{\mu\nu}$ being the Ricci tensor defined by

$$\begin{aligned} R_{\mu\nu} &= R^\rho{}_{\mu\rho\nu} = \partial_\rho\Gamma^\rho{}_{\mu\nu} - \partial_\nu\Gamma^\rho{}_{\mu\rho} + \Gamma^\rho{}_{\rho\beta}\Gamma^\beta{}_{\mu\nu} - \Gamma^\rho{}_{\nu\beta}\Gamma^\beta{}_{\mu\rho} \\ &= 2\Gamma^\rho{}_{\mu[\nu,\rho]} + 2\Gamma^\rho{}_{\beta[\rho}\Gamma^\beta{}_{\nu]\mu}, \end{aligned} \tag{7}$$

where $\Gamma^\rho{}_{\mu\nu}$ is the Christoffel symbols second kind and the square brackets mean

$$A_{[\mu,\nu]} = \frac{1}{2}\left(A_{\mu,\nu} - A_{\nu,\mu}\right). \tag{8}$$

The D'Alembert operator $\Box$ is defined as $\Box = \nabla_\alpha\nabla^\alpha$, where $\nabla_\alpha V^\beta$ is the covariant derivatives of the vector V^β and $f_R = df(R)/dR$. In this study $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} := g_{\rho\sigma}F^{\nu\rho}F_\mu{}^\sigma - \frac{1}{4}\delta_\mu{}^\nu g^{\lambda\rho}g^{\epsilon\sigma}F_{\lambda\epsilon}F_{\rho\sigma}, \tag{9}$$

which is the energy momentum tensor of the electromagnetic field.

The trace of (5) yields

$$Rf_R - 2f(R) - 8\Lambda + 3\Box f_R = 0. \tag{10}$$

Now we are going to discuss some basic property of the above $f(R)$ theories.

2.1. $f(R)$-Gravitational Theories and Their Viable Conditions. The most important conditions and restrictions [81] that are usually put on $f(R)$ gravitational theories to give consistency on both of gravitational and cosmological evolutions are as follows.

(a) The first condition is given by

$$f''(R) \geq 0, \quad \text{when } R \geq f''(R), \tag{11}$$

which represents the stability condition for curvature [97]. Condition (11) represents the existence of a matter dominated era in cosmological evolution. The relevant physics of (11) is that if the constant,

$$G_{\text{eff}} = \frac{G}{f'(R)}, \tag{12}$$

has a defined value, then dG_{eff}/dR is fixed by the sign of $f''(R)$.

(b) The condition $f'(R) > 0$ guarantees the positivity of the effective gravitational constant. This condition from quantum point of view avoids the graviton from becoming a ghost [98, 99].

(c) The condition $f'(R) < 0$ ensures the recovery of GR behavior at early times.

3. Analytic Solution in $f(R)=R+\alpha R^2$

In this section, we are going to apply the field equations of Maxwell-$f(R)$ with cosmological constant to two different metric spacetimes having flat horizon:

3.1. Flat Horizon Metric with One Variable. Let us assume the first metric spacetime possessing one unknown function has the form

$$ds^2 = -a(r)\,dt^2 + \frac{1}{a(r)}dr^2 + r^2\sum_{i=1}^{D-2} d\theta^2{}_i, \tag{13}$$

where $a(r)$ is an unknown function of the radial coordinate, r. Using (13) we get the Ricci scalar in the form

$$R = -\frac{r^2a'' + 2(D-2)ra' + (D-2)(D-3)a}{r^2}, \tag{14}$$

where $a' = da(r)/dr$ and $a'' = d^2a(r)/dr^2$. The nonvanishing components of the Maxwell-$f(R)$ field equations, (5) and (6), when $f(R) = R+\alpha R^2$ take the form (the detailed calculations of the Ricci curvature tensor are given in Appendix B)

$$\begin{aligned}
\aleph_{tt} &= \frac{1}{2r^4}\Big(\left[\alpha r^4 a''^2 - 4r^4\Lambda + 2r^4 q'^2 + 2s^2_{\theta_{D-4}}\right] - (D-2)r^2\left[2r\alpha a'a'' - ra' - a(D-3)\right] - 4\alpha r^2 aa''(D-2)(3D-11) - 4\alpha raa'\left[D^3 - 10D^2 + 24D - 4\right] \\
&\quad - 4\alpha a^2(4D-11) - 2\alpha r^3\left\{a'''\left[ra' + 6a(D-2)\right] + 2raa''''\right\} - 2\alpha r^2 a'^2(D-2)(D-5)\Big) = 0, \\
\aleph_{rr} &= \frac{1}{2r^4}\Big(\left[\alpha r^4 a''^2 - 4r^4\Lambda + 2r^4 q'^2 + 2s^2_{\theta_{D-4}}\right] - (D-2)r^2\left[2r\alpha a'a'' - ra' - a(D-3)\right] - 8\alpha r^2 aa''(D-2)^2 - 4\alpha raa'\left[D^3 - 7D^2 + 9D + 8\right] - 4\alpha a^2(2D-1)(2D-7) \\
&\quad - 2\alpha r^3 a'''\left[ra' + 2a(D-2)\right] - 2\alpha r^2 a'^2(D-2)(D-5)\Big) = 0, \\
\aleph_{\theta_1\theta_1} &= S_{\theta_2\theta_2} = \cdots = S_{\theta_{D-2}\theta_{D-2}} = \frac{1}{2r^4}\Big(r^4\left(\alpha a'' - 1\right)a'' + 4r^4\Lambda + 2r^4 q'^2 + 2s^2_{\theta_{D-4}} + 4\alpha r^3 a'a''(3D-7) \\
&\quad - 2(D-3)r^3 a' - r^2\alpha a'^2(D-2)(2D-9) - 4\alpha r^3\left\{a'''\left[ra' + a(3D-7)\right] + raa''''\right\} \\
&\quad - 2\alpha a\left[8\left\{D^2 - 13D + 34\right\}ra' - a(2D-1)(D-3)(D-6)\right]\Big) = 0,
\end{aligned} \tag{15}$$

where $s_{\theta_{D-4}} = ds(\theta_{D-4})/d\theta_{D-4}$ and $q' = dq(r)/dr$ with $q(r)$ and $s(\theta)$ being two unknown functions related to the electric and magnetic charges of the system and defined from the gauge potential as

$$A \overset{\text{def.}}{=} q(r)\,dt + s(\theta_{D-4})\,d\theta_{D-4}. \tag{16}$$

The solution of the differential equations (15) has the form

$$\begin{aligned}
q(r) &= c_1 + \delta_4{}^4\frac{c_2}{r}, \\
s(\theta_{D-4}) &= c_3 + \delta_4{}^4 c_4\theta_{D-4}, \\
a(r) &= \left(\frac{2r^2\Lambda}{3} + \frac{c_5}{r} + \frac{c_2{}^2 + c_4{}^2}{r^2(1-16\alpha\Lambda)}\right)\delta_4{}^4 + \delta_h{}^h\left[\frac{(D-2)r^{D-1}\left[1 \pm \sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2}\right]}{2D(D-1)(D-4)\alpha r^{D-3}} + \frac{c_6}{r^{D-3}}\right],
\end{aligned} \tag{17}$$

where $h \geq 5$ and c_i, $i = 1 \cdots 6$ are constants. Solution (17) is decomposed into two parts one for $D = 4$ and the other for $D \geq 0$. The reason for this decomposition is the electromagnetic field. In the four dimensions there is a charged solution; however, for $D \geq 5$ there is no charged solution. In fact this is in conflict with the spherically symmetric case [91]. It is important to mention here that solution (17) when $\alpha = 0$ will reduce to the well known AdS/dS solution in the case of $D = 4$. However, in the case of $D > 4$ this solution does not allow the parameter α to be vanishing; therefore, it has no analogue in GR. In the noncharged case and when $D = 4$ solution (17) coincides with what is know in GR. However, when $D > 4$ and as long as α and Λ have no relation between them solution (17) has no analogue in GR but when $\Lambda = 1/16\alpha$ it will coincide with what is known in GR; that is, it behaves as Ads/dS.

The metric of solutions (17) has the form

$$ds^2 = -\left\{\left(\frac{2r^2\Lambda}{3} + \frac{c_5}{r} + \frac{{c_2}^2 + {c_4}^2}{r^2(1-16\alpha\Lambda)}\right){\delta_4}^4 + {\delta_h}^h\left[\frac{(D-2)r^{D-1}\left[1 \pm \sqrt{1-16D(D-4)\alpha\Lambda/(D-2)^2}\right]}{2D(D-1)(D-4)\alpha r^{D-3}} + \frac{c_6}{r^{D-3}}\right]\right\}dt^2$$
$$+ \frac{dr^2}{\left(2r^2\Lambda/3 + c_5/r + \left({c_2}^2 + {c_4}^2\right)/r^2(1-16\alpha\Lambda)\right){\delta_4}^4 + {\delta_h}^h\left[(D-2)r^{D-1}\left[1 \pm \sqrt{1-16D(D-4)\alpha\Lambda/(D-2)^2}\right]/2D(D-1)(D-4)\alpha r^{D-3} + c_6/r^{D-3}\right]}$$
$$+ r^2\sum_{i=1}^{D-2} d{\theta^2}_i. \tag{18}$$

3.2. Metric with Two Variables. The metric of a flat horizon with two unknown functions has the form

$$ds^2 = -a(r)\,dt^2 + \frac{1}{b(r)}dr^2 + r^2\sum_{i=1}^{D-2} d{\theta^2}_i, \tag{19}$$

where $a(r)$ and $b(r)$ are two unknown functions of the radial coordinate, r. Using (19) we get the Ricci scalar in the form

$$R = -\left(\frac{a'b'}{2a} + \frac{ba''}{a} - \frac{ba'^2}{2a^2} + \frac{(D-2)ba'}{ar} + \frac{(D-2)b'}{r} + \frac{(D-2)(D-3)b}{r^2}\right). \tag{20}$$

Using (19) we get the nonvanishing components of the field equations, (5) and (6), when $f(R) = R + \alpha R^2$ in the form

$$\begin{aligned}
\aleph_{tt} = {} & \frac{1}{8r^4a^4}\Big(36\alpha r^4a^2b^2a''^2 - 2\alpha r^3b\left(aa'^3\left[29rb' + 26(D-2)b\right] + 8rba^3a''''\right) + 8a^3\left[as^2_{\theta_{D-4}} + r^4bq'^2\right] + 49\alpha r^4b^2a'^4 \\
& + \alpha r^2a'^2a^2\left\{2b(D-2)\left[2(5D-18)b + 30rb'\right] + 9r^2b'^2\right\} - 8\alpha a^2br^3\left\{ab'''\left[2(D-2)a + ra'\right]\right. \\
& \left. - 2a'''\left[3rba' - a\left(2(D-2)b + 3rb'\right)\right]\right\} + 4r^2\alpha aa''\left\{rba'\left[a\left\{27rb' + 22(D-2)b\right\} - 29rba'\right]\right. \\
& \left. - a^2\left[16(D-2)rbb' + 3b'^2 + 4(D-2)(D-4)b^2\right]\right\} - 4r^2a^2\alpha b''\left[2(D-2)a^2\left(2(2D-7)b + rb'\right)\right. \\
& \left. + raa'\left[6(D-2)b + rb'\right] - 6r^2ba'^2\right] - 8(D-2)r\alpha a^3a'\left[rb' + 2(D-4)b\right]\left[rb' - b\right] - 4a^3(D-2) \\
& \cdot rab'\left[(3D-10)\alpha rb' - r^2 - \{2(3D-11)(D-4) + 4(3D-10)(D-5)\}\alpha b\right. \\
& \left. + a\left(4r^4\Lambda - (D-2)(D-3)r^2b - 4(4D-11)\alpha b^2\right)\right]\Big) = 0,
\end{aligned}$$

$$\begin{aligned}
\aleph_{rr} = {} & \frac{1}{8r^4a^4}\Big(4\alpha r^4a^2b^2a''^2 + 2\alpha r^3baa'^3\left[3rb' - 4(D-2)b\right] + 8a^3\left[as^2_{\theta_{D-4}} + r^4bq'^2\right] - 7\alpha r^4b^2a'^4 + \alpha r^2a'^2a^2\left\{r^2b'^2\right. \\
& \left. + 4b(D-2)\left[(3D-4)b + rb'\right]\right\} - 8\alpha a^2b^2r^3a'''\left[2a(D-2) + ra'\right] \\
& - 4r^2\alpha ab\left\{a''\left(ra'\left[2a\left\{rb' - 3(D-2)b\right\} - 3rba'\right] + 4(D-2)a^2\left[rb' + (D-2)b\right]\right) + ab''\left[2(D-2)a + ra'\right]^2\right\} \\
& - 4(D-2)ra^3a'\left[2(3D-8)\alpha rbb' - r^2b - 4\alpha b^2 - \alpha r^2b'^2\right] + 4a^3\left\{(D-2)^2\alpha r^2ab'^2\right. \\
& \left. - a\left[4r^4\Lambda + (D-2)\left[4rb\alpha(D-2)(D-4) - (D-3)r^2b - 4(4D-9)\alpha b^2\right]\right]\right\}\Big) = 0,
\end{aligned}$$

$$\begin{aligned}
\aleph_{\theta_1\theta_1} = \aleph_{\theta_2\theta_2} = \cdots = \aleph_{\theta_{D-2}\theta_{D-2}} = \frac{1}{8r^4a^4}\Bigg(& 28\alpha r^4a^2b^2a''^2 - 2\alpha r^3b\left(aa'^3\left[23rb' + 6(3D-7)b\right] + 8rba^3a''''\right) \\
& - 8a^3\left[as^2_{\theta_{D-4}} + r^4bq'^2\right] + 39\alpha r^4b^2a'^4 \\
& + \alpha r^2a'^2a^2\left\{2b(D-2)\left[\{2(D-2)(D-5) + (2D-5)(D-4)\}b + 4(3D-7)rb'\right] + 7r^2b'^2 - \frac{2r^2b}{\alpha}\right\} \\
& - 8\alpha a^2br^3\left\{ab'''\left[\left(2(D-2)a + ra'\right)\right] - a'''\left[5rba' - a\left(2(2D-5)b + 6rb'\right)\right]\right\} \\
& + 4r^2\alpha aa''\left\{rba'\left[a\left\{2(8D-19)b + 22rb'\right\} - 23rba'\right]\right. \\
& \left. - a^2\left[2(9D-22)rbb' + 3r^2b'^2 - 4\left[2(D-3)(D-5) - (D-4)\right]b^2 - \frac{r^2b}{\alpha} - 8r^2a^2bb''\right]\right\} \\
& - 4r^2a^2\alpha b''\left[2(D-2)a^2\left(4(D-4)b + rb'\right) + raa'\left[2(4D-9)b + rb'\right] - 5r^2ba'^2\right] + 2ra^3a'\left[r^3b' + 2(D-3)r^2b\right. \\
& \left. - 4\left[2(D-1)(D-5) + (D-4)(3D-11)\right]\alpha rbb' - 2(3D-7)r^2\alpha b'^2 + 8(5D-19)\alpha b^2\right] \\
& - 4a^3\left[3(D-4)(D-2)\alpha r^2ab'^2 - (D-3)rab'\left[r^2 + 4(2D-1)\alpha b\right]\right. \\
& \left. + a\left(4r^4\Lambda - \left\{(D-4)(D-3)r^2b - 2\left[(D-6)(D-3)(2D-1) - D(D-4)(D-5)\right]\alpha b^2\right\}\right)\right]\Bigg) = 0.
\end{aligned} \tag{21}$$

The above system of differential equations (21) has the following solution:

(1) $q(r) = c_7$,

$$s(\theta_{D-4}) = c_8 + \delta_4{}^4 c_9\theta_{D-4},$$

$$a(r) = \frac{c_{10}\left(r\left[2r^3\Lambda + 3c_{11}\right](16\alpha\Lambda - 1) - 3c_{12}{}^2\right)}{r^2}\delta_4{}^4 + \delta_h{}^h c_{20} \cdot \frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right] \pm c_{13}}{r^{D-3}},$$

$$b(r) = \frac{r\left[2r^3\Lambda + 3c_{11}\right](16\alpha\Lambda - 1) - 3c_{12}{}^2}{3r^2(16\alpha\Lambda - 1)}\delta_4{}^4 + \delta_h{}^h\left[\frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right]}{2D(D-1)(D-4)\alpha r^{D-3}} \pm \frac{c_{13}}{r^{D-3}}\right],$$

(2) $q(r) = c_{14} + \delta_4{}^4\dfrac{c_{15}}{r}$,

$$s(\theta_{D-4}) = c_{16} + \delta_4{}^4 c_{17}\theta_{D-4},$$

$$a(r) = \frac{c_{15}{}^2\left(2\Lambda r^4 + 3\left(c_{18} + rc_{19}\right)\right)}{3r^2\left(c_{17}{}^2 + c_{18}\left[16\alpha\Lambda - 1\right]\right)}\delta_4{}^4 + \delta_h{}^h c_{20} \cdot \frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right] \pm c_{13}}{r^{D-3}},$$

$$b(r) = \frac{3\left(c_{18} + rc_{19}\right) + 2r^2\Lambda}{3r^2}\delta_4{}^4 + \delta_h{}^h\left[\frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right]}{2D(D-1)(D-4)\alpha r^{D-3}} \pm \frac{c_{13}}{r^{D-3}}\right],$$

(22)

where $h \geq 5$. We must mention here that solution (22) has the same property of solution (17); that is, when $\alpha = 0$ it will reduce to the well known AdS/dS solution in the case of $D = 4$. However, in the case of $D > 4$ this solution does not allow the parameter α to be vanishing; therefore, this solution has no analogue in GR. In the noncharged case and when $D = 4$ and $\alpha \neq 0$ solution (22) coincides with what is known in GR. However, when $D > 4$ and as long as α and Λ have no relation between them solution (22) has no analogue in GR but when $\Lambda = 1/16\alpha$ it will coincide with GR solution that behaves as Ads/dS. The metric spacetimes of solutions (22) have the form

$$
\begin{aligned}
(1)\ ds^2 = &-\left[\frac{{c_{10}}^2\left(32r^4\alpha\Lambda^2+48c_{11}\alpha\Lambda r-2r^4\Lambda+3{c_{12}}^2+3c_{11}r\right)}{r^2}{\delta_4}^4\right.\\
&\left.+{\delta_h}^h c_{20}\frac{(D-2)\,r^{D-1}\left[\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\pm 1\right]\pm c_{13}}{r^{D-3}}\right]dt^2\\
&+\left\{\frac{32r^4\alpha\Lambda^2+48c_{11}\alpha\Lambda r-2r^4\Lambda-3{c_{12}}^2-3c_{11}r}{3r^2(16\alpha\Lambda-1)}{\delta_4}^4\right.\\
&\left.+{\delta_h}^h\left[\frac{(D-2)\,r^{D-1}\left[1\pm\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\right]}{2D(D-1)(D-4)\,\alpha r^{D-3}}\pm\frac{c_{13}}{r^{D-3}}\right]\right\}^{-1}dr^2+r^2\sum_{i=1}^{D-2}d{\theta^2}_i,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
(2)\ ds^2 = &-\left(\frac{{c_{15}}^2\left(2\Lambda r^4+3\left(c_{18}+rc_{19}\right)\right)}{3r^2\left({c_{17}}^2+c_{18}\left[16\alpha\Lambda-1\right]\right)}{\delta_4}^4+{\delta_h}^h c_{20}\frac{(D-2)\,r^{D-1}\left[1\pm\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\right]\pm c_{13}}{r^{D-3}}\right)dt^2\\
&+\left[\frac{3\left(c_{18}+rc_{19}\right)+2r^2\Lambda}{3r^2}{\delta_4}^4+{\delta_h}^h\left\{\frac{(D-2)\,r^{D-1}\left[\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\pm 1\right]}{2D(D-1)(D-4)\,\alpha r^{D-3}}\pm\frac{c_{13}}{r^{D-3}}\right\}\right]^{-1}dr^2\\
&+r^2\sum_{i=1}^{D-2}d{\theta^2}_i,
\end{aligned}
$$

where c_i, $i = 6\cdots 19$ are constants.

4. Physical Properties of the Analytic Solutions

4.1. Metric with One Variable. From (18) we can deduce the following properties.

(i) In case of 4 dimensions we get

$$
\begin{aligned}
ds^2 &= -\left(\frac{2r^2\Lambda}{3}+\frac{c_5}{r}+\frac{{c_2}^2+{c_4}^2}{r^2(1-16\alpha\Lambda)}\right)dt^2\\
&\quad+\frac{dr^2}{\left(2r^2\Lambda/3+c_5/r+\left({c_2}^2+{c_4}^2\right)/r^2(1-16\alpha\Lambda)\right)}\\
&\quad+r^2\left(d{\theta^2}_1+d{\theta^2}_2\right),
\end{aligned}
\tag{24}
$$

from which it is clear that the metric behaves asymptotically as dS/AdS. Equation (24) shows that the effect of the higher dimension curvature is related to the electric field as well as the magnetic field and also

$$\Lambda \neq \frac{1}{16\alpha}. \tag{25}$$

(ii) In case of more than 4 dimensions we get

$$
\begin{aligned}
ds^2 = &-\frac{(D-2)\,r^{D-1}\left[1\pm\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\right]+2D(D-1)(D-4)\,\alpha c_6}{2D(D-1)(D-4)\,\alpha r^{D-3}}dt^2\\
&+\frac{2D(D-1)(D-4)\,\alpha r^{D-3}dr^2}{(D-2)\,r^{D-1}\left[1\pm\sqrt{1-16D(D-4)\,\alpha\Lambda/(D-2)^2}\right]+2D(D-1)(D-4)\,\alpha c_6}+r^2\sum_{i=1}^{D-2}d{\theta^2}_i.
\end{aligned}
\tag{26}
$$

Equation (26) shows that the dimensional parameter α must not be equal to zero; otherwise, we will have a singular metric. Also the spacetime of metric (26) behaves as dS/AdS and when the cosmological constant Λ takes the form

$$\Lambda = \frac{(D-2)^2}{16D(D-4)\alpha}, \tag{27}$$

then, (26) reduces to

$$\begin{aligned} ds^2 = &-\frac{(D-2)r^{D-1} + 2D(D-1)(D-4)\alpha c_6}{2D(D-1)(D-4)\alpha r^{D-3}} dt^2 \\ &+ \frac{2D(D-1)(D-4)\alpha r^{D-3} dr^2}{(D-2)r^{D-1} + 2D(D-1)(D-4)\alpha c_6} \\ &+ r^2 \sum_{i=1}^{D-2} d\theta^2{}_i, \end{aligned} \tag{28}$$

which is asymptotically dS/AdS and cannot reduce to GR.

4.2. Metric with Two Variables. From (23), we can show the following.

(iii) In case of 4 dimensions we get

$$\begin{aligned} (1)\ ds^2 &= -\frac{c_{10}{}^2\left[2r^4\Lambda(16\alpha\Lambda - 1) + 48c_{11}\alpha\Lambda r + 3c_{12}{}^2 + 3c_{11}r\right]}{r^2} dt^2 \\ &+ \frac{3r^2(16\alpha\Lambda - 1)}{2r^4\Lambda(16\alpha\Lambda - 1) + 48c_{11}\alpha\Lambda r - 3c_{12}{}^2 - 3c_{11}r} dr^2 \\ &+ r^2\left(d\theta^2{}_1 + d\theta^2{}_2\right). \end{aligned} \tag{29}$$

$$\begin{aligned} (2)\ ds^2 &= -\frac{c_{15}{}^2\left[2\Lambda r^4 + 3(c_{18} + rc_{19})\right]}{3r^2(c_{16}{}^2 + c_{18}[16\alpha\Lambda - 1])} dt^2 \\ &+ \frac{3r^2}{3(c_{18} + rc_{19}) + 2r^4\Lambda} dr^2 + r^2\left(d\theta^2{}_1 + d\theta^2{}_2\right). \end{aligned}$$

First set of equations (29) shows that $\Lambda \neq 1/16\alpha$ and when the constant $c_{10} = 1/\sqrt{3(16\alpha\Lambda - 1)}$ then the metric behaves as dS/AdS. For the second set, it is allowed to put $\Lambda = 1/16\alpha$ and whenthe constant $c_{15} = c_{16}$ then the second equation of (29) behaves as dS/AdS.

(iv) In case of more than 4 dimensions we get from (23) the two sets

$$\begin{aligned} ds^2 = &-\left\{\frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right] \pm c_{13}}{r^{D-3}}\right\} dt^2 \\ &+ \left[\frac{(D-2)r^{D-1}\left[\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2} \pm 1\right]}{2D(D-1)(D-4)\alpha r^{D-3}} \pm \frac{c_{13}}{r^{D-3}}\right]^{-1} dr^2 + r^2 \sum_{i=1}^{D-2} d\theta^2{}_i. \end{aligned} \tag{30}$$

Equation (30) shows that the dimensional parameter α must not be equal to zero; otherwise, we will have a singular metric. The asymptotes of (30) behave as dS/AdS. It is important to stress that metric (30) cannot reduce to that of GR and hence we can say that solution (22) is a new solution. Using condition given by (27) in (30) we get

$$\begin{aligned} (1)\ ds^2 &= -\left\{\frac{\pm(D-2)r^{D-1} \pm c_{13}}{r^{D-3}}\right\} dt^2 \\ &+ \left[\frac{\pm(D-2)r^{D-1}}{2D(D-1)(D-4)\alpha r^{D-3}} \pm \frac{c_{13}}{r^{D-3}}\right]^{-1} dr^2 \\ &+ r^2 \sum_{i=1}^{D-2} d\theta^2{}_i. \end{aligned} \tag{31}$$

We can conclude from the above discussion of the metrics given by (30) or (31) that there is no charged solution for the form of $f(R) = R + \alpha R^2$. Also spacetime metrics of (30) or (31) instruct us that the dimension parameter α must not be equal zero.

4.3. Singularities. Now, let us explain the singularities and the horizons of solutions (17) and (22). For this reason, we have to find at which values of r do the functions $a(r)$ and $b(r)$ turn out to be zero or infinity, due to the fact that singularities may be coordinate ones which are physical singularities. Usually, to study singularities one calculates all the invariants constructed from Riemann tensor and its contractions. The curvature invariants that arise from solution (17), in case of 4 dimensions, have the form

$$R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = \frac{4r^2(1-16\alpha\Lambda)^2\left[8\Lambda^2r^6+9{c_5}^2\right]+24\left({c_2}^2+{c_4}^2\right)\left[6(1-16\alpha\Lambda)+7\left({c_2}^2+{c_4}^2\right)\right]}{3r^8(1-16\alpha\Lambda)^2},$$
$$R^{\mu\nu}R_{\mu\nu} = \frac{4\left[4r^8\Lambda^2(1-16\alpha\Lambda)^2+\left({c_2}^2+{c_4}^2\right)^2\right]}{r^8(1-16\alpha\Lambda)^2}, \quad R=-8\Lambda. \tag{32}$$

And in case of more than 4 dimensions we get

$$R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = \frac{r^4a''+2(D-2)r^2a\prime^2+2(D-2)(D-3)a^2}{r^4},$$
$$R^{\mu\nu}R_{\mu\nu} = \frac{D(D-2)r^2a'^2+2(D-2)r^3a'a''+r^4a''+4(D-2)(D-3)raa'+(D-2)(D-3)^2a^2}{2r^4}, \tag{33}$$
$$R = -\frac{2(D-2)ra'+r^2a''+(D-2)(D-3)a}{r^4}.$$

Equation (32), for the 4 dimensions case, shows the following.

(a) $\Lambda \neq 1/16\alpha$; otherwise, we will have a singularity for both invariants $R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$ and $R^{\mu\nu}R_{\mu\nu}$.

(b) Also (32) tells us that there is a singularity at $r = 0$ which can not be removed for the invariants $R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$ and $R^{\mu\nu}R_{\mu\nu}$.

When $D > 4$ and by using (17), we can show the following.

(c) We have a true singularity at $r = 0$.

(d) From (33), after using (17), we can show that the dimension parameter α must not be equal to zero; otherwise, we get a singularity.

(e) Finally, (33) shows that there is a singularity at

$$c_6 = -\frac{(D-2)r^{(D-1)}}{2D(D-1)(D-4)\alpha}\left[1 \pm\sqrt{1-\frac{16D(D-4)\alpha\Lambda}{(D-2)^2}}\right], \tag{34}$$

and when constraint (27) is used, (34) takes the form

$$c_6 = -\frac{(D-2)r^{(D-1)}}{2D(D-1)(D-4)\alpha}. \tag{35}$$

Repeating the same calculations for solution (22) we get for $D = 4$

$$(1)\ R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = \frac{4r^2(1-16\alpha\Lambda)^2\left[8\Lambda^2r^6+9{c_{11}}^2\right]+9c_{11}{c_{12}}^2r(1-16\alpha\Lambda)+168{c_{11}}^4}{3r^8(1-16\alpha\Lambda)^2},$$
$$(2)\ R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = \frac{36r^2c_{19}+144rc_{19}c_{18}+168{c_{18}}^4+32r^8\Lambda^2}{3r^8},$$
$$(1)\ R^{\mu\nu}R_{\mu\nu} = \frac{4\left[{c_{12}}^4-128r^8\Lambda^3\alpha(1-8\alpha\Lambda)+4r^8\Lambda^2\right]}{r^8(1-16\alpha\Lambda)^2}, \tag{36}$$
$$(2)\ R^{\mu\nu}R_{\mu\nu} = \frac{4\left[{c_{18}}^2+4r^8\Lambda^2\right]}{r^8},$$
$$(1)\ R=-8\Lambda$$
$$(2)\ R=-8\Lambda.$$

We can apply the same discussion applied for solution (17). In case of $D > 4$ we get

$$R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = \frac{f(r)}{r^{2(D-1)}\alpha^2\left((D-2)r^{(D-1)}\left[1 \pm \sqrt{1-16D(D-4)\alpha\Lambda/(D-2)^2}\right] \pm 2D(D-1)(D-4)\alpha c_{13}\right)^2},$$
$$R^{\mu\nu}R_{\mu\nu} = \frac{f_1(r)}{\alpha^2\left((D-2)r^{(D-1)}\left[1 \pm \sqrt{1-16D(D-4)\alpha\Lambda/(D-2)^2}\right] \pm 2D(D-1)(D-4)\alpha c_{13}\right)^2}, \tag{37}$$
$$R = \frac{f_2(r)}{\alpha\left((D-2)r^{(D-1)}\left[1 \pm \sqrt{1-16D(D-4)\alpha\Lambda/(D-2)^2}\right] \pm 2D(D-1)(D-4)\alpha c_{13}\right)},$$

where $f(r)$, $f_1(r)$, and $f_2(r)$ are lengthy functions of r. Using (27) in (37) we get

$$\begin{aligned} &R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} \\ &= \frac{(D-2)^2\left[r^{2(D-1)} + 2D(D-1)^2(D-3)(D-4)^2\alpha^2 {c_{13}}^2\right]}{2D(D-1)(D-4)^2\alpha^2}, \\ &R^{\mu\nu}R_{\mu\nu} = \frac{(D-2)^2}{4D(D-4)^2\alpha^2}, \\ &R = \frac{-(D-2)}{2(D-4)\alpha}, \end{aligned} \tag{38}$$

which indicates that the parameter α must not be equal to zero.

The horizons of solutions (17) and (22) are the zeros of the metric g_{rr}; therefore, all the above singularities are far from these horizons. The study of perturbations of solutions (17) and (22) is an important issue to study their stability and then discuss the formulation of weak cosmic censorship. This problem will be discussed elsewhere.

5. Total Conserved Charge

In this section, we are going to study *the conserved quantities of the solutions derived in Section 3*. For this purpose we are going to make a brief review of the geometry used in this calculations. The Lagrangian of Einstein-Cartan theory is defined by [100] (the fundamental entities of this theory Appendix A)

$$S\left(\vartheta^i, R^j{}_k\right) := -\frac{1}{2\kappa}\left(R^{ij}\wedge\eta_{ij} - 2\Lambda\eta\right), \tag{39}$$

where ϑ^i is the one-form coframe and R^{ij} is the two-form curvature tensor. Carrying out variation of (39) with respect to the coframes ϑ^i and R^{ij} lead to [100, 101]

$$\begin{aligned} E_i &:= -\frac{\partial S}{\partial\vartheta^i} = -\frac{1}{2\kappa}\left(R^{jk}\wedge\eta_{ijk} - 2\Lambda\eta_i\right), \\ B_{ij} &:= -\frac{\partial S}{R_{ij}} = \frac{1}{2\kappa}\lambda_{ij}, \end{aligned} \tag{40}$$

which are the canonical energy-momentum and rotational gauge field momentum, respectively. The translational momentum and the spin 2-forms are defined as

$$\begin{aligned} H_i &:= -\frac{\partial S}{\partial T^i} = 0, \\ E_{ij} &:= -\vartheta_{[i}\wedge H_{j]} = 0. \end{aligned} \tag{41}$$

The conserved quantity of the gravitational field has the form [100]

$$\begin{aligned} \jmath[\xi] &= \frac{1}{2\kappa}d\left\{{}^*\left[dk + \xi\rfloor\left(\vartheta^i\wedge T_i\right)\right]\right\}, \\ &\text{where } k = \xi_i\vartheta^i,\ \xi^i = \xi\rfloor\vartheta^i, \end{aligned} \tag{42}$$

with $*$ being the Hodge duality, ξ being a vector field $\xi = \xi^i\partial_i$, and ξ^i being D parameters $\xi^0, \xi^1, \ldots, \xi^{D-1}$. For the solutions having spinless matter or vacuum ones, the torsion is vanishing, T_i, and therefore the total charge of (42) takes the form

$$\mathcal{Q}[\xi] = \frac{1}{2\kappa}\int_{\partial S}{}^*dk. \tag{43}$$

This invariant conserved quantity $\mathcal{Q}[\xi]$ was given before in [102–106].

Now let us apply (43) to solution (17) and calculate the necessary components; we finally get

$$\begin{aligned} \vartheta^0 &= \sqrt{a(r)}dt, \\ \vartheta^1 &= \frac{dr}{\sqrt{a(r)}}, \\ \vartheta^2 &= rd\theta_1, \\ \vartheta^3 &= rd\theta_2, \\ &\vdots \\ \vartheta^{D-1} &= rd\theta_{D-2}. \end{aligned} \tag{44}$$

Using (44) in (42) we get

$$k = \frac{a^2(r)\,\xi_0 dt - \xi_1 dr - r^2 a(r)\left(\xi_2 d\theta_2 + \xi_3 d\theta_2 + \cdots + \xi_{D-2} d\theta_{D-2}\right)}{a(r)}. \tag{45}$$

The total derivative of (45) gives

$$\begin{aligned} dk = {} & \xi_0 a'(r)\,(dr \wedge dt) - 2r\left[\xi_2\,(dr \wedge d\theta_1)\right. \\ & \left. + \xi_3\,(dr \wedge d\theta_2) + \cdots + \xi_{D-1}\,(dr \wedge d\theta_{D-2})\right]. \end{aligned} \tag{46}$$

Calculating the inverse of (46) and using it in (46) and (43) after applying the Hodge-dual to the output of dk, we get conservation of the charge in the form

$$\begin{aligned} \mathcal{Q}\left[\xi_t\right] &= \frac{(D-2)\,\Omega_{D-2} M}{8\pi}, \\ \mathcal{Q}\left[\xi_r\right] &= \mathcal{Q}\left[\xi_{\theta_1}\right] = \cdots = \mathcal{Q}\left[\xi_{\theta_{D-2}}\right] = 0, \end{aligned} \tag{47}$$

where Ω_{D-2} is the volume of the unit $(D-2)$-sphere; we have substitute $c_5 = -2M$ and have used the regularization method, by putting the physical quantities vanishing, and subtract the resulting expression from the original one to remove any divergence term [100] (physical quantities mean the constants c_2, c_4, c_5, and c_6 for solution (17); constants 10, 11, 12, and 13 of first solution of (22); and constants 13, 15, 18, and c_{18} of the second solution of (22)).

Applying the same above techniques we get for the first solution of (22) in case of $D = 4$

$$\begin{aligned} \mathcal{Q}\left[\xi_t\right] &= M\sqrt{1 - 16\alpha\Lambda}, \\ \mathcal{Q}\left[\xi_r\right] &= \mathcal{Q}\left[\xi_{\theta_1}\right] = \cdots = \mathcal{Q}\left[\xi_{\theta_{D-2}}\right] = 0, \end{aligned} \tag{48}$$

where $c_{11} = -2M/\sqrt{c_{10}}$ and for the second one we get

$$\begin{aligned} \mathcal{Q}\left[\xi_t\right] &= \frac{M}{\sqrt{c_{18}\,(1 - 16\alpha\Lambda) - {c_{17}}^2}}, \\ \mathcal{Q}\left[\xi_r\right] &= \mathcal{Q}\left[\xi_{\theta_1}\right] = \cdots = \mathcal{Q}\left[\xi_{\theta_{D-2}}\right] = 0, \end{aligned} \tag{49}$$

where $c_{19} = -2M/c_{15}$.

In case $D > 4$ we get

$$\begin{aligned} \mathcal{Q}\left[\xi_t\right] &= \frac{(D-2)\,\Omega_{D-2} M\sqrt{\alpha}}{8\pi}, \\ \mathcal{Q}\left[\xi_r\right] &= \mathcal{Q}\left[\xi_{\theta_1}\right] = \cdots = \mathcal{Q}\left[\xi_{\theta_{D-2}}\right] = 0, \end{aligned} \tag{50}$$

where

$$c_{13} = -\frac{2M\sqrt{\alpha D\,(D-3)\,(D-4)}}{\sqrt{c_{15}}}. \tag{51}$$

6. Thermodynamics of Black Holes

In this section, we are going to study the thermodynamical quantities of solutions (17) and (22). The temperature of Hawking of any solution can be derived by requiring the singularity at the horizon to be vanishing in the Euclidean continuation of the black hole solutions. One can obtain the temperature of the outer event horizon at $r = r_h$, for solution (17) in case of $D = 4$ in the form

$$\begin{aligned} T &= \frac{1}{4\pi}\left(\frac{dg_{tt}(\mathrm{r})}{dr}\right)_{r_h} \\ &= \frac{1}{4\pi}\left[2r_h\Lambda + \frac{c_2^2 + c_4^2}{(16\alpha\Lambda - 1)\,{r^3}_h}\right], \end{aligned} \tag{52}$$

and in case of $D > 4$ we get

$$\begin{aligned} T &= \frac{1}{4\pi}\left(\frac{dg_{tt}(r)}{dr}\right)_{r_h} \\ &= \frac{r_h}{4\pi}\left[\frac{(D-2)\left[1 \pm \sqrt{1 - 16D\,(D-4)\,\alpha\Lambda/(D-2)^2}\right]}{2D\,(D-4)\,\alpha}\right]. \end{aligned} \tag{53}$$

For the first solution of (22) in case $D = 4$, we get the Hawking temperature as

$$\begin{aligned} T &= \frac{1}{4\pi}\left(\frac{dg_{tt}(r)}{dr}\right)_{r_h} \\ &= \frac{1}{4\pi}\left[\frac{3c_{10}\left[2\Lambda {r^4}_h\,(1 - 16\alpha\Lambda) + {c_{11}}^2\right]}{{r^3}_h}\right], \end{aligned} \tag{54}$$

and for the second solution of (22) in case of $D = 4$, we get

$$\begin{aligned} T &= \frac{1}{4\pi}\left(\frac{dg_{tt}(r)}{dr}\right)_{r_h} \\ &= \frac{1}{4\pi}\left[\frac{{c_{15}}^2\left[\left(c_{18} - 2\Lambda {r^4}_h\right)\right]}{{r^3}_h\left[c_{18}\,(16\alpha\Lambda - 1) + c_{16}^2\right]}\right]. \end{aligned} \tag{55}$$

In the case of $D > 4$ we get

$$\begin{aligned} T &= \frac{1}{4\pi}\left(\frac{dg_{tt}(r)}{dr}\right)_{r_h} \\ &= \frac{r_h c_{13}\,(D-1)\,(D-2)}{4\pi}\left[\sqrt{1 - \frac{16D\,(D-4)\,\alpha\Lambda}{(D-2)^2}}\right. \\ &\quad \left. \pm 1\right]. \end{aligned} \tag{56}$$

We give a brief discussion of the entropy of black hole in $f(R)$ gravity. For this purpose, we use the arguments presented in [107]. From the Noether method used to calculate the entropy

associated with black holes in $f(R)$ theory that have constant Ricci scalar, one finds [95]

$$S = \frac{A}{4G} f'(R)\Big|_{r=r_h}, \tag{57}$$

where A is the area of the event horizon. Using solution (17) we get for $D = 4$

$$S = (1 - 16\Lambda\alpha)\pi r_h^{\,2}, \tag{58}$$

and for $D > 4$ we get

$$S = \frac{r_h^{\,D-2}\Omega_{D-2}}{4}\left[1 - \frac{(D-2)}{(D-4)}\left\{1 + \sqrt{1 - \frac{16D(D-4)\alpha\Lambda}{(D-2)^2}}\right\}\right]. \tag{59}$$

Using (17) and (27) in (59) we get the entropy in the form

$$S = \frac{r_h^{\,D-2}\Omega_{D-2}}{2(D-4)}. \tag{60}$$

For solution (22) and in case of $D = 4$ we get the same expression of entropy as given by (58) and in case $D > 4$ we get

$$S = \frac{r_h^{\,D-2}\Omega_{D-2}}{4}\{1 + 2\alpha R(r_h)\}. \tag{61}$$

When (27) is used we get the entropy in the form of (60).

Utilizing (52) and (58), the Smarr relation in the extended phase space can be obtained in the case $D = 4$ of solution (17) as

$$M = 2TS + \Phi Q, \quad \text{where } \Phi = \sqrt{\frac{(c_2^{\,2} + c_4^{\,2})(1 - \Lambda\alpha)}{r_h}},\ Q = \sqrt{(c_2^{\,2} + c_4^{\,2})(1 - \Lambda\alpha)}. \tag{62}$$

For the first solution of (22) we get the the Smarr relation in the extended phase as

$$M = 2TS + \Phi Q, \quad \text{where } \Phi = \frac{c_{11}}{r\sqrt{(1 - 2\Lambda\alpha)}},\ Q = \frac{c_{11}}{\sqrt{1 - 2\Lambda\alpha}}, \tag{63}$$

where we have put $c_{10} = 1/3(1-2\Lambda\alpha)^2$. Finally, for the second solution of (22) we get the Smarr relation as

$$M = 2TS + \Phi Q, \quad \text{where } \Phi = \frac{\sqrt{2c_{18}}}{r_h},\ Q = \sqrt{2c_{18}}, \tag{64}$$

where we have put $c_{15} = \sqrt{(c_{18}(1 - 16\alpha\Lambda) - c_{16}^2)/(16\alpha\Lambda - 1)}$.

In the case $D > 4$, we introduce the extended phase space where the cosmological constant is identified as the thermodynamic pressure while the conjugate quantity is regarded as the thermodynamic volume. We adopt the following definition of pressure that is commonly used in the literatures of extended phase space [108]

$$P = -\frac{(D-1)(D-2)\Lambda}{48\pi}. \tag{65}$$

Using solution (17) in the case $D > 4$ we get the Smarr relation in the extended phase as [108]

$$M = \frac{D-2}{D-3}TS - \frac{2}{D-3}VP, \quad \text{where } P \text{ is given by Eq. (65) and } V \text{ has the form}$$
$$V = \frac{3(D-2)r_h^{\,D-1}\Omega_{D-2}\left(2 + (D-2)\sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2}\right)\left(1 + \sqrt{1 - 16D(D-4)\alpha\Lambda/(D-2)^2}\right)}{4\alpha\Lambda D(D-1)(D-4)^2}. \tag{66}$$

It is important to note that Smarr relation given by (66) has no charge term because the higher dimension solution given by Eq. (17) has constant electric and magnetic charges. Using constrains (27) in (66) we get

$$V = \frac{24r_h^{\,D-1}\Omega_{D-2}}{(D-1)(D-2)(D-4)}. \tag{67}$$

Using the same above procedure we get for solution (22) in case of $D > 4$ the the Smarr relation in the extended phase as given by (66) where the volume has the form

$$V = \frac{2r_h^{\,D-1}\Omega_{D-2}}{(D-4)}. \tag{68}$$

7. Main Results and Discussion

In this study, we have presented Maxwell-$f(R)$ gravity in D-dimensions and have checked the flat horizon solutions. We have applied two metrics, the first with one unknown and the second with two unknown functions to Maxwell-$f(R)$ field equations using the special case $f(R) = R + \alpha R^2$. The resulting differential equations are solved analytically without any assumption and general solutions containing

three classes have been obtained. These classes are classified as follows.

(i) For the metric with one unknown, the electric and magnetic charges of this class are not constant for $D = 4$ and are constants for $D > 4$.

(ii) For the metric with two unknowns, we have two sets. The first one has a constant charge and a nontrivial magnetic field for $D = 4$ and for $D > 4$ we have constant electric and magnetic charges.

(iii) For the second set of solutions (22) and for $D = 4$ the electric and magnetic charges are not trivial and are constants for $D > 4$.

To understand the physics of these solutions we wrote the metric of each solution and have shown the following.

(a) We have shown that the metric behaves asymptotically as dS/AdS for the first solution for $D = 4$ and it is not allowed to put $\Lambda = 1/16\alpha$. When $D > 4$ the metric also behaves asymptotically as dS/AdS and have shown that the following relation between the cosmological constant and the dimension parameter α holds $\Lambda = (D-2)^2/16D(D-4)\alpha$.

(b) For the metric with two unknowns, when $D = 4$, we have shown that the metric behaves as dS/Ads for both sets. However, for the first solution $\Lambda \neq 1/16\alpha$ but for the second one $\Lambda = 1/16\alpha$.

(c) When $D > 4$, for solution (22), we have shown that the parameter α can not be equal to zero and therefore this solution is new.

We have calculated the singularities of each class by calculating the invariants of curvature. For the first solution we have shown that the invariants of curvature have singularity at $r = 0$ and $\Lambda = 1/16\alpha$. However, when $D > 4$ we have shown that there is singularities at $r = 0$ and $\alpha = 0$. For the first set of (22) the same discussion of solution (17) can be applied for the invariants of curvature. However, for the second set the invariants of curvature have a singularity at $r = 0$ which represents horizon. For $D > 4$ the same discussion of solution (17) can be applied for the two sets.

To understand the meaning of the constants of each solution we have calculated the conserved quantities. In the case of $D = 4$ we have shown that the only nonvanishing physical quantity is the energy. We have shown that the value of energy of solution (17) is not affected by the dimension parameter α while it is affected for solution (22) and reduces to the well known formula of ADM when $\alpha = 0$. When $D > 4$ the energy of solution (17) does not depend on the parameter α while it depends on it for solution (22) and can not be equal to zero.

Finally, we have calculated the thermodynamical quantities like Hawking temperature, entropy, and so on. For solution (17) and when $D = 4$ we have shown that the temperature depends on the electric and magnetic charges and for $D > 4$ the parameter α is not allowed to be zero; otherwise, the temperature will be indefinite. We have shown that the entropy, for solution (17), depends on α and when it is vanishing we return to GR. For this solution, we have shown that the first law of thermodynamics is satisfied for general $D \geq 4$ using the extended phase space [108].

For solution (22) we have shown that temperature will be finite if $\alpha = 0$ and will return to GR after rescaling the constants of integration. For the entropy and when $D = 4$ the same discussion of solution (17) can be applied. The entropy will have a finite value only when the constraint (27) is used. Finally we have shown that the first law of thermodynamics is satisfied for general $D \geq 4$.

Appendix

A. Notation

The indices $i, j, \ldots$ are employed for (co)frame components while $\alpha, \beta, \ldots$ are used for spacetime coordinates. The exterior product is represented by $\wedge$ and the interior product is denoted by $\xi \rfloor \Psi$. The coframe ϑ^i is defined as $\vartheta^i = b^i{}_\mu dx^\mu$ and the frame b_i is defined as $b_i = b_i{}^\mu \partial_\mu$ with $b^i{}_\mu$ and $b_i{}^\mu$ being the covariant and contravariant components of the tetrad field. The volume is defined as $\eta := \vartheta^{\hat{0}} \wedge \vartheta^{\hat{1}} \wedge \vartheta^{\hat{2}} \wedge \vartheta^{\hat{3}}$. Using the interior product one can define

$$\eta_i := b_i \rfloor \eta = \frac{1}{3!}\epsilon_{ijkl}\vartheta^j \wedge \vartheta^k \wedge \vartheta^l, \tag{A.1}$$

with ϵ_{ijkl} being completely antisymmetric.

B. The Nonvanishing Components of the Christoffel Symbols Second Kind and Ricci Curvature Tensor

Using (13) we get the following nonvanishing components of the Christoffel symbols second kind and Ricci curvature tensor:

$$\begin{aligned}
&\Gamma^t{}_{tt} = -\Gamma^r{}_{rr} = \frac{a'}{2a},\\
&\Gamma^t{}_{rt} = \frac{aa'}{2},\\
&\Gamma^{\theta_1}{}_{r\theta_1} = \Gamma^{\theta_2}{}_{r\theta_2} \cdots \Gamma^{\theta_{D-2}}{}_{r\theta_{D-2}} = \frac{1}{r},\\
&\Gamma^r{}_{\theta_1\theta_1} = \Gamma^r{}_{\theta_2\theta_2} \cdots \Gamma^r{}_{\theta_{D-2}\theta_{D-2}} = -ra.\\
&R_{trtr} = \frac{a''}{2},\\
&R_{t\theta_1 t\theta_1} = R_{t\theta_2 t\theta_2} = \cdots = R_{t\theta_{D-2} t\theta_{D-2}} = \frac{raa'}{2},\\
&R_{r\theta_1 r\theta_1} = R_{r\theta_2 r\theta_2} = \cdots = R_{r\theta_{D-2} r\theta_{D-2}} = -\frac{ra'}{2a},\\
&R_{\theta_1\theta_2\theta_1\theta_2} = R_{\theta_1\theta_2\theta_1\theta_2} = \cdots = R_{\theta_1\theta_{D-2}\theta_1\theta_{D-2}} = R_{\theta_2\theta_3\theta_2\theta_3}\\
&\quad = R_{\theta_2\theta_4\theta_2\theta_4} \cdots R_{\theta_2\theta_{D-2}\theta_2\theta_{D-2}} \cdots R_{\theta_{D-3}\theta_{D-2}\theta_{D-3}\theta_{D-2}}\\
&\quad = -r^2 a.
\end{aligned} \tag{B.1}$$

Using (19) we get the following nonvanishing components of the Christoffel symbols second kind and Ricci curvature tensor:

$$\begin{aligned}
&\Gamma^{t}{}_{tt} = \frac{a'}{2a}\\
&\Gamma^{r}{}_{rr} = \frac{b'}{2b},\\
&\Gamma^{t}{}_{rt} = \frac{aa'}{2},\\
&\Gamma^{\theta_1}{}_{r\theta_1} = \Gamma^{\theta_2}{}_{r\theta_2}\cdots\Gamma^{\theta_{D-2}}{}_{r\theta_{D-2}} = \frac{1}{r},\\
&\Gamma^{r}{}_{\theta_1\theta_1} = \Gamma^{r}{}_{\theta_2\theta_2}\cdots\Gamma^{r}{}_{\theta_{D-2}\theta_{D-2}} = -rb.\\
&R_{trtr} = \frac{a''}{2} - \frac{a'^2}{4a} + \frac{a'b'}{4b},\\
&R_{t\theta_1t\theta_1} = R_{t\theta_2t\theta_2} = \cdots = R_{t\theta_{D-2}t\theta_{D-2}} = \frac{rba'}{2},\\
&R_{r\theta_1r\theta_1} = R_{r\theta_2r\theta_2} = \cdots = R_{r\theta_{D-2}r\theta_{D-2}} = -\frac{rb'}{2b},\\
&R_{\theta_1\theta_2\theta_1\theta_2} = R_{\theta_1\theta_2\theta_1\theta_2} = \cdots = R_{\theta_1\theta_{D-2}\theta_1\theta_{D-2}} = R_{\theta_2\theta_3\theta_2\theta_3}\\
&\quad = R_{\theta_2\theta_4\theta_2\theta_4}\cdots R_{\theta_2\theta_{D-2}\theta_2\theta_{D-2}}\cdots R_{\theta_{D-3}\theta_{D-2}\theta_{D-3}\theta_{D-2}}\\
&\quad = -r^2b.
\end{aligned} \tag{B.2}$$

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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