Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 8728564, 7 pages
https://doi.org/10.1155/2018/8728564

Research Article

Quantum Gravity Effect on the Tunneling Particles from 2 + 1-Dimensional New-Type Black Hole

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Received 13 October 2017; Revised 10 January 2018; Accepted 24 January 2018; Published 6 March 2018

Academic Editor: Elias C. Vagenas

We investigate the generalized uncertainty principle (GUP) effect on the Hawking temperature for the 2 + 1-dimensional new-type black hole by using the quantum tunneling method for both the spin-1/2 Dirac and the spin-0 scalar particles. In computation of the GUP correction for the Hawking temperature of the black hole, we modified Dirac and Klein-Gordon equations. We observed that the modified Hawking temperature of the black hole depends not only on the black hole properties, but also on the graviton mass and the intrinsic properties of the tunneling particle, such as total angular momentum, energy, and mass. Also, we see that the Hawking temperature was found to be probed by these particles in different manners. The modified Hawking temperature for the scalar particle seems low compared with its standard Hawking temperature. Also, we find that the modified Hawking temperature of the black hole caused by Dirac particle's tunneling is raised by the total angular momentum of the particle. It is diminishable by the energy and mass of the particle and graviton mass as well. These intrinsic properties of the particle, except total angular momentum for the Dirac particle, and graviton mass may cause screening for the black hole radiation.

1. Introduction

Black hole radiation is theoretically very important phenomenon for researchers who attempt to merge the gravitation with the thermodynamics and the quantum mechanics [1–8]. With the formulation of the quantum field theory in curved space-time based on the framework of the standard Heisenberg uncertainty principle, it was proved that a black hole can emit particles that are created by the quantum vacuum fluctuation near its outer horizon [6–8]. Since then, many alternative methods have been proposed to derive the black hole radiation known as Hawking radiation in the literature. For instance, the semiclassical method, based on quantum tunneling process of a particle across the outer horizon of a black hole from inside to outside, can be used to derive the Hawking radiation. The method implies two different approaches to compute the imaginary part of the action (S), which is the classically forbidden trajectory of a particle across the outer horizon: the null geodesic [9–12] and the Hamilton-Jacobi [13–15]. In both approaches, the tunneling probability of a particle from a black hole, Γ, is defined in terms of the classical action, as $\Gamma = e^{-(2/\hbar)ImS}$ [9–15]. By using the semiclassical method, lots of studies about the Hawking radiation of a black hole as quantum tunneling process of a point-like particle have been carried out in the literature [16–36]. On the other hand, the above-mentioned studies on this method provide no specific information on type of the particle that is tunneled from a black hole. That is because the Hawking radiation does not depend on the intrinsic properties, such as mass, total (orbital + spin) angular momentum, energy, and charge of the tunneling point-like particle.

The existence of a minimal observable length which can be identified by the order of the Planck scale is a characteristic of the candidate theories of quantum gravity, such as string theory, loop quantum gravity, and noncommutative geometry [37–41]. This length leads us to a generalized uncertainty principle (GUP) instead of the standard Heisenberg uncertainty principle, because a particle is not a point-like particle in the context of these candidate theories anymore. Therefore, the uncertainty on the momentum of a particle increases and thus the standard Heisenberg uncertainty principle can be generalized as follows:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\beta(\Delta p)^2\right], \quad (1)$$

where $\beta = \beta_0/M_p^2$, M_p^2 is the Planck mass, and β_0 is the dimensionless parameter [42–45]. The commutation relations between a particle position, x, and momentum, p, are modified as follows:

$$\left[x_i, p_j\right] = i\hbar\delta_{ij}\left[1+\beta p^2\right], \quad (2)$$

where x_i and p_j represent the modified position and momentum operators, respectively, and their definitions are as follows:

$$\begin{aligned} x_i &= x_{0i}, \\ p_j &= p_{0j}\left(1+\beta p_0^2\right). \end{aligned} \quad (3)$$

x_{0i} and $p_{0j} = -i\hbar\partial_j$ in (3) are the standard position and momentum operators, respectively, and $p_0^2 = p_{0j}p^{0j}$ [46]. Then, the modified energy expression becomes

$$\widetilde{E} = E\left(1-\beta E^2\right) = E\left[1-\beta\left(p^2+m_0^2\right)\right], \quad (4)$$

for which the energy mass shell condition, $E^2 = p^2 + m_0^2$, is used. From these relations, the square of the momentum operator can be derived as follows:

$$p^2 = p_i p^i \simeq -\hbar^2\left[\partial_i\partial^i - 2\beta\partial_j\partial^j\left(\partial_i\partial^i\right)\right], \quad (5)$$

where the higher order terms of the β parameter are neglected.

The GUP relations are of great help to understand the nature of a black hole since quantum effects are the essential effects near the event horizon of a black hole. Recently, to investigate the quantum effects under the GUP relations, the thermodynamics properties of various black holes have been studied by using the quantum tunneling process of particles with various spins [46–67]. These studies indicate that the modified Hawking radiation depends not only on the black hole's properties but also on the intrinsic properties of the tunneling particle.

The new-type black hole is one of the important results of the New Massive Gravity, which is 2 + 1-dimensional gravity and graviton in this theory has a mass [68]. In the framework of the standard Heisenberg uncertainty principle, the Hawking radiation of the new-type black holes had been studied by using the quantum tunneling process of the scalar, Dirac, and vector boson particles [23, 36]. In these studies, it was shown that the Hawking radiation only depends on the properties of the black hole and is independent of the properties of the tunneling point particles, that is, all these particles tunnel from the black holes in the same way. This indicates that even if an observer in enough or safe (i.e., infinite) distant from a black hole may detect Hawking radiation of the black hole, the observer cannot determine what kind of particles compose the radiation. Therefore, in this study, we will investigate whether the properties of the tunneling particles will affect the Hawking radiation of the black hole by using the quantum tunneling process of both the scalar and Dirac particles in the framework of the GUP.

The organization of this work is as follows: in Section 2, we modify the Dirac equation with respect to the GUP relations. Subsequently, using the modified Dirac equation, we calculate the tunneling probability of the Dirac particle by using the Hamilton-Jacobi method, and, then, we find the modified Hawking temperature of the black hole. In Section 3, the standard Klein-Gordon equation is rewritten under the GUP for the 2 + 1-dimensional new-type black hole. Subsequently, the tunneling probability of the scalar particle from the black hole and the modified Hawking temperature of the black hole are calculated, respectively. In conclusion, we evaluate and summarize the results.

2. Dirac Particle's Tunneling in the New-Type Black Hole

Using the GUP relations, the standard Dirac equation given in [69] can be modified as follows:

$$\begin{aligned} -i\overline{\sigma}^0(x)\partial_0\widetilde{\Psi} = &\left(i\overline{\sigma}^i(x)\partial_i - i\overline{\sigma}^\mu(x)\Gamma_\mu - \frac{m_0}{\hbar}\right) \\ &\cdot\left(1+\beta\hbar^2\partial_j\partial^j - \beta m_0^2\right)\widetilde{\Psi}, \end{aligned} \quad (6)$$

and its explicit form is

$$\begin{aligned} &i\overline{\sigma}^0(x)\partial_0\widetilde{\Psi} + i\overline{\sigma}^i(x)\left(1-\beta m_0^2\right)\partial_i\widetilde{\Psi} \\ &\quad + i\beta\hbar^2\overline{\sigma}^i(x)\partial_i\left(\partial_j\partial^j\widetilde{\Psi}\right) - \frac{m_0}{\hbar}\left(1-\beta m_0^2\right)\widetilde{\Psi} \\ &\quad - m_0\beta\hbar\partial_j\partial^j\widetilde{\Psi} \\ &\quad - i\overline{\sigma}^\mu(x)\Gamma_\mu\left(1+\beta\hbar^2\partial_j\partial^j - \beta m_0^2\right)\widetilde{\Psi} = 0, \end{aligned} \quad (7)$$

where the $\widetilde{\Psi}$ is the modified Dirac spinor, m_0 is mass of the Dirac particle, $\overline{\sigma}^\mu(x)$ are the space-time-dependent Dirac matrices, and $\Gamma_\mu(x)$ are the spin affine connection for spin-1/2 particle [69]. The space-time background of the new-type black hole is given by

$$ds^2 = L^2\left[f(r)\,dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\phi^2\right], \quad (8)$$

where L is the AdS_3 radius defined as $L^2 = 1/2m^2 = 1/2\Lambda$ where Λ is cosmological constant and m is graviton mass, and $f(r) = (r - r_+)(r - r_-)$ is defined in terms of the outer, r_+, and inner, r_-, horizons radius, respectively. The black hole's horizons are located at $r_\pm = (1/2)(-b \pm \sqrt{b^2 - 4c})$, where b and c are two constant parameters [23, 68]. Using (8), the spinorial affine connections are derived as follows [23]:

$$\begin{aligned} \Gamma_0 &= -\frac{1}{4}f'(r)\,\overline{\sigma}^3\overline{\sigma}^1, \\ \Gamma_1 &= 0, \\ \Gamma_2 &= -\frac{1}{2}\sqrt{f(r)}\overline{\sigma}^1\overline{\sigma}^2. \end{aligned} \quad (9)$$

To calculate the tunneling probability of a Dirac particle from the black hole, we use the following ansatz for the modified wave function:

$$\widetilde{\Psi}(x) = \exp\left(\frac{i}{\hbar}S(t,r,\phi)\right)\begin{pmatrix} A(t,r,\phi) \\ B(t,r,\phi)\end{pmatrix}, \quad (10)$$

where $A(t,r,\phi)$ and $B(t,r,\phi)$ are the functions of space-time. $S(t,r,\phi)$ is the classical action term for particle trajectory. Inserting (9) and (10) in (7), we obtain the following equations for the leading order in $\hbar$ and β as follows:

$$\begin{aligned} &A\left[\frac{1}{L\sqrt{f}}\frac{\partial S}{\partial t} + m_0\left(1-\beta m_0^2\right) + \frac{\beta m_0}{L^2r^2}\left(\frac{\partial S}{\partial \phi}\right)^2 \right. \\ &\left. + \frac{\beta m_0 f}{L^2}\left(\frac{\partial S}{\partial r}\right)^2\right] + B\left[i\frac{\sqrt{f}\left(1-\beta m_0^2\right)}{L}\frac{\partial S}{\partial r}\right. \\ &\left. + \frac{\left(1-\beta m_0^2\right)}{Lr}\frac{\partial S}{\partial \phi} + i\frac{\beta f^{3/2}}{L^3}\left(\frac{\partial S}{\partial r}\right)^3\right] \\ &+ B\left[i\frac{\beta\sqrt{f}}{L^3r^2}\frac{\partial S}{\partial r}\left(\frac{\partial S}{\partial \phi}\right)^2 + \frac{\beta f}{L^3 r}\frac{\partial S}{\partial \phi}\left(\frac{\partial S}{\partial r}\right)^2\right. \\ &\left. + \frac{\beta}{L^3r^3}\left(\frac{\partial S}{\partial \phi}\right)^3\right] = 0, \\ &A\left[-i\frac{\sqrt{f}\left(1-\beta m_0^2\right)}{L}\frac{\partial S}{\partial r} + \frac{\left(1-\beta m_0^2\right)}{Lr}\frac{\partial S}{\partial \phi}\right. \\ &\left. - i\frac{\beta f^{3/2}}{L^3}\left(\frac{\partial S}{\partial r}\right)^3\right] + A\left[-i\frac{\beta\sqrt{f}}{L^3r^2}\frac{\partial S}{\partial r}\left(\frac{\partial S}{\partial \phi}\right)^2\right. \\ &\left. + \frac{\beta f}{L^3r}\frac{\partial S}{\partial \phi}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{\beta}{L^3r^3}\left(\frac{\partial S}{\partial \phi}\right)^3\right] + B\left[\frac{1}{L\sqrt{f}}\frac{\partial S}{\partial t}\right. \\ &\left. - m_0\left(1-\beta m_0^2\right) - \frac{\beta m_0}{L^2r^2}\left(\frac{\partial S}{\partial \phi}\right)^2 - \frac{\beta m_0 f}{L^2}\left(\frac{\partial S}{\partial r}\right)^2\right] \\ &= 0. \end{aligned} \quad (11)$$

These two equations have nontrivial solutions for $A(t,r,\phi)$ and $B(t,r,\phi)$ when the determinant of the coefficient matrix is vanished. Accordingly, neglecting of the terms containing higher order of the β parameter provides

$$\begin{aligned} &\beta\left[2m_0^4 - \frac{4f}{L^4r^2}\left(\frac{\partial S}{\partial r}\right)^2\left(\frac{\partial S}{\partial \phi}\right)^2 - \frac{2}{L^4r^4}\left(\frac{\partial S}{\partial \phi}\right)^4\right. \\ &\left. - \frac{2f^2}{L^4}\left(\frac{\partial S}{\partial r}\right)^4\right] + \frac{1}{L^2 f}\left(\frac{\partial S}{\partial t}\right)^2 - \frac{f}{L^2}\left(\frac{\partial S}{\partial r}\right)^2 \\ &- \frac{1}{L^2r}\left(\frac{\partial S}{\partial \phi}\right)^2 - m_0^2 = 0. \end{aligned} \quad (12)$$

Due to the commuting Killing vectors (∂_t) and (∂_ϕ), we can separate $S(t,r,\phi)$, in terms of the variables t, r, and ϕ, as $S(t,r,\phi) = -Et + j\phi + K(r)$, where E and j are the energy and angular momentum of the particle, respectively, and $K(r) = K_0(r) + \beta K_1(r)$ [57]. Using these definitions in (12), the integral of the radial equation, $K(r)$, becomes

$$K_\pm(r) = \pm\int\frac{\sqrt{E^2 - f(r)\left(m_0^2L^2 + j^2/r^2\right)}}{f(r)}\left[1+\beta\chi\right]dr, \quad (13)$$

where χ is an abbreviation and it is

$$\chi = \frac{E^2\left[E^2 - 2m_0^2L^2f(r)\right]}{L^2 f(r)\left[E^2 - f(r)\left(m_0^2L^2 + j^2/r^2\right)\right]}. \quad (14)$$

Then, by integrating the radial equation, $K_\pm(r)$ are obtained as

$$K_\pm(r) = \pm i\pi\frac{E}{r_+ - r_-}\left[1+\beta\Pi\right], \quad (15)$$

where the abbreviation Π is

$$\Pi = \frac{\left(r_+ - r_-\right)^2\left[3L^2m_0^2r_+^2 - j^2\right] + 4E^2r_+^2}{2L^2r_+^2\left(r_+ - r_-\right)^4}. \quad (16)$$

On the other hand, the tunneling probabilities of particles crossing the outer horizon are given by

$$\begin{aligned} P_{\text{out}} &= \exp\left[-\frac{2}{\hbar}ImK_+(r)\right], \\ P_{\text{in}} &= \exp\left[-\frac{2}{\hbar}ImK_-(r)\right]. \end{aligned} \quad (17)$$

Hence, the tunneling probability of the Dirac particle is given by

$$\begin{aligned} \Gamma &= e^{-(2/\hbar)ImS} = \frac{P_{\text{out}}}{P_{\text{in}}} \\ &= \exp\left(-\frac{4\pi E}{\hbar\left(r_+ - r_-\right)}\left[1+\beta\Pi\right]\right), \end{aligned} \quad (18)$$

where $ImS(t,r,\phi) = ImK_+(r) - ImK_-(r)$[70, 71] and $ImK_+(r) = -ImK_-(r)$. Then, the modified Hawking temperature of the Dirac particle, T_H^D, is obtained as

$$T_H^D = \frac{\hbar\left(r_+ - r_-\right)}{4\pi}\frac{1}{\left[1+\beta\Pi_D\right]}, \quad (19)$$

where to find the temperature it used the following relation [72–75]:

$$\Gamma = e^{-(2/\hbar)ImS} = e^{-E/T_H}. \quad (20)$$

If T_H^D is at first expanded in terms of the β powers and second neglected the higher order of the β terms, then the modified Hawking temperature of the black hole is obtained as

$$T_H^D \simeq \frac{\hbar\left(r_+ - r_-\right)}{4\pi}\left[1-\beta\Pi\right]. \quad (21)$$

From this result, we see that the modified Hawking temperature includes not only the mass parameter of the black hole, but also the AdS_3 radius, L, (and, hence, the graviton mass) and the angular momentum, energy, and mass of the tunneled Dirac particle. On the other hand, in the case of $\beta = 0$, the modified Hawking temperature is reduced to the standard temperature obtained by quantum tunneling process of the point particles with spin-0, spin-1/2, and spin-1, respectively [23, 36].

3. Scalar Particle's Tunneling in the New-Type Black Hole

To investigate the quantum gravity effects on the tunneling process of the scalar particles from the black hole, by using the GUP relations, the standard Klein-Gordon equations are modified as follows:

$$\begin{aligned} &-(i\hbar)^2 \partial_t \partial^t \widetilde{\Phi} \\ &\quad = \left[(-i\hbar)^2 \partial_i \partial^i - M_0^2\right]\left[1 - 2\beta\left(-\hbar^2 \partial_i \partial^i + M_0^2\right)\right]\widetilde{\Phi}, \end{aligned} \tag{22}$$

and its explicit form of the modified Klein-Gordon equation is written as follows:

$$\begin{aligned} &\hbar^2 \partial_t \partial^t \widetilde{\Phi} + \hbar^2 \partial_i \partial^i \widetilde{\Phi} + 2\beta\hbar^4 \partial_i \partial^i \left(\partial_i \partial^i \widetilde{\Phi}\right) \\ &\quad + M_0^2 \left(1 - 2\beta M_0^2\right)\widetilde{\Phi} = 0, \end{aligned} \tag{23}$$

where $\widetilde{\Phi}$ and M_0 are the modified wave function and mass of the scalar particle, respectively. Then, the modified Klein-Gordon equation in the new-type black hole background becomes as follows:

$$\begin{aligned} &\frac{\hbar^2}{f}\frac{\partial^2 \widetilde{\Phi}}{\partial t^2} - \frac{\hbar^2}{r^2}\frac{\partial^2 \widetilde{\Phi}}{\partial \phi^2} - 2\beta\hbar^4 f \frac{\partial^2}{\partial r^2}\left[-\frac{f}{L^2}\frac{\partial^2 \widetilde{\Phi}}{\partial r^2}\right] \\ &\quad - \frac{2\beta\hbar^4}{r^2}\frac{\partial^2}{\partial \phi^2}\left[-\frac{1}{L^2 r}\frac{\partial^2 \widetilde{\Phi}}{\partial \phi^2}\right] - \hbar^2 f \frac{\partial^2 \widetilde{\Phi}}{\partial r^2} \\ &\quad + M_0^2 L^2 \left(1 - 2\beta M_0^2\right)\widetilde{\Phi} = 0. \end{aligned} \tag{24}$$

To consider the tunneling radiation of the black hole with (24), we employ the modified wave function of the scalar particle as

$$\widetilde{\Phi}(t, r, \phi) = A \exp\left(\frac{i}{\hbar} S(t, r, \phi)\right), \tag{25}$$

where A is a constant. Substituting (25) into (24) and neglecting the higher order terms of $\hbar$, we get the equation of motion of the scalar particle as

$$\begin{aligned} &\left(\frac{\partial S}{\partial t}\right)^2 - f(r)^2\left(\frac{\partial S}{\partial r}\right)^2 - \frac{f(r)}{r^2}\left(\frac{\partial S}{\partial \phi}\right)^2 - M_0^2 L^2 f(r) \\ &\quad - \beta \frac{2f(r)}{r^4 L^2}\left(\frac{\partial S}{\partial \phi}\right)^4 \\ &\quad + \beta\left[M_0^4 L^2 f(r) - \frac{2f(r)^3}{L^2}\left(\frac{\partial S}{\partial r}\right)^4\right] = 0. \end{aligned} \tag{26}$$

Using $S(t, r, \phi) = -Et + j\phi + W(r)$, where E and j are the energy and angular momentum of the particle, respectively, and $W(r) = W_0(r) + \beta W_1(r)$ [57], then, the radial integral, $W(r)$, becomes as follows:

$$\begin{aligned} &W_{\pm}(r) \\ &\quad = \pm \int \frac{\sqrt{E^2 - f(r)\left(M_0^2 L^2 + j^2/r^2\right)}}{f(r)}\left[1 + \beta\Omega\right] dr, \end{aligned} \tag{27}$$

where the abbreviation Ω is

$$\begin{aligned} &\Omega \\ &\quad = \frac{f(r)^2\left(M_0^4 L^4 - j^4/r^4\right) - \left[E^2 - f(r)\left(M_0^2 L^2 + j^2/r^2\right)\right]^2}{L^2 f(r)\left[E^2 - f(r)\left(m_0^2 L^2 + j^2/r^2\right)\right]}. \end{aligned} \tag{28}$$

And, $W_{\pm}(r)$ are computed as

$$W_{\pm}(r) = \pm i\pi \frac{E}{r_+ - r_-}\left[1 + \beta\Sigma\right], \tag{29}$$

with the abbreviation Σ being

$$\Sigma = \frac{\left(r_+ - r_-\right)^2\left[3L^2 M_0^2 r_+^2 + 3j^2\right] + 4E^2 r_+^2}{2L^2 r_+^2\left(r_+ - r_-\right)^4}, \tag{30}$$

where $W_+(r_h)$ is outgoing and $W_-(r_h)$ is incoming solutions of the radial part. Then, using (17), the tunneling probability of the scalar particle is calculated as

$$\Gamma = \exp\left(-\frac{4\pi E}{\hbar\left(r_+ - r_-\right)}\left[1 + \beta\Sigma\right]\right) \tag{31}$$

and, subsequently, using (20), the modified Hawking temperature of the scalar particle, T_H^{KG}, becomes as

$$T_H^{KG} = \frac{\hbar\left(r_+ - r_-\right)}{4\pi}\frac{1}{\left[1 + \beta\Sigma\right]}. \tag{32}$$

Furthermore, neglecting the higher order of the β terms in the expanding form of T_H^{KG} in terms of β, we find the modified Hawking temperature of the black hole as follows:

$$T_H^{KG} \simeq \frac{\hbar\left(r_+ - r_-\right)}{4\pi}\left[1 - \beta\Sigma\right]. \tag{33}$$

This result indicates that the modified Hawking temperature is related not only to the mass parameter of the black hole, but also to the AdS_3 radius, L, (and, hence, to the graviton mass) and angular momentum, energy, and mass of the tunneling scalar particle. Furthermore, as can be seen from (21) and (33), the Hawking temperature probed by a Dirac particle is higher than that of a scalar particle: $T_H^D = T_H^{KG} + \beta(\hbar j^2 m^2/\pi r_+^2(r_+ - r_-))$ for $M_0 = m_0$ and $L^2 = 1/2m^2$. On the other hand, in the case of $\beta = 0$, the modified Hawking temperature reduced to the standard temperature obtained by quantum tunneling process of the point particles with spin-0, spin-1/2, and spin-1, respectively [23, 36].

4. Concluding Remarks

In this study, we investigated the quantum gravity effect on the tunneled both spin-0 scalar and spin-1/2 Dirac particles from new-type black hole in the context of 2 + 1-dimensional New Massive Gravity. For this, at first, using the GUP relations, we modified the Klein-Gordon and Dirac equations that describe the spin-0 scalar and spin-1/2 Dirac particles, respectively. Then, using the Hamilton-Jacobi method, the tunneling probabilities of the these particles are derived, and, subsequently, the corrected Hawking temperature of the black hole is calculated. We find that the modified Hawking temperature not only depends on the black hole's properties but also depends on the emitted particle's mass, energy, and total angular momentum. Also, it is worth mentioning that the modified Hawking temperature depends on mass of the graviton, that is, quantum particle which mediates gravitational radiation in the context of New Massive Gravity. As can be seen from (21), the Hawking temperature of the Dirac particle increases by the total angular momentum of the particle while it decreases by the energy and mass of the particle and the graviton mass.

In addition, we can summarize some important results as follows:

(i) In (33), the modified Hawking temperature of the scalar particle is lower than the standard Hawking temperature.

(ii) However, in (21), as $4E^2r_+^2 + (3m_0^2/2m^2)r_+^2(r_+ - r_-)^2 < j^2(r_+ - r_-)^2$, the modified Hawking temperature of the Dirac particle is higher than the standard Hawking temperature. Furthermore, when $4E^2r_+^2 + (3m_0^2/2m^2)r_+^2(r_+ - r_-)^2 > j^2(r_+ - r_-)^2$, the modified Hawking temperature is lower than the standard Hawking temperature. If $4E^2r_+^2 + (3m_0^2/2m^2)r_+^2(r_+ - r_-)^2 = j^2(r_+ - r_-)^2$, then the GUP effect is canceled, and the Hawking temperature of the Dirac particle reduces to the standard Hawking temperature.

(iii) According to (21) and (33), the modified Hawking temperature of the new-type black hole probed by tunneling Dirac particle is higher than that of scalar particle:

$$T_H^D = T_H^{KG} + \beta \frac{\hbar j^2 m^2}{\pi r_+^2 (r_+ - r_-)}, \tag{34}$$

where we adopt that the mass of the Dirac particle is equivalent to the mass of the scalar particle, that is, $m_0 = M_0$.

(iv) The new-type black hole is classified as six classes according to the signatures of the parameters b and c, and, hence, it exhibits different physical and mathematical properties. For example, it reduced to the static BTZ black hole in the case of $b = 0$ and $c < 0$. In this context, according to tunneling of the scalar and Dirac particles, the modified Hawking temperature of the static BTZ black hole is

$$\begin{aligned} T_H^{KG} &= T_H \left[1 - \alpha m^2 \frac{4\left(\left(3M_0^2/2m^2\right)|c| + 3j^2\right) + 4E^2}{c^2} \right], \\ T_H^D &= T_H \left[1 - \alpha m^2 \frac{4\left(\left(3m_0^2/2m^2\right)|c| - j^2\right) + 4E^2}{c^2} \right], \end{aligned} \tag{35}$$

respectively. Here, $r_+ = -r_- = \sqrt{|c|}$ is used and $T_H = \hbar(\sqrt{|c|}/2\pi)$ is the standard Hawking temperature of the static BTZ black hole in the context of the 2 + 1-dimensional New Massive Gravity theory [23].

(v) In the absence of the quantum gravity effect, that is, $\beta = 0$, the modified Hawking temperature is reduced to the standard temperature obtained by quantum tunneling of the massive spin-0, spin-1/2, and spin-1 point particles [23, 36].

Finally, in the context of GUP, we have seen that the graviton and the tunneling particle masses have an effect decreasing the Hawking temperature in both scalar and Dirac particle tunneling process. On the other hand, the total angular momentum has different effect on the Hawking temperature for a type of tunneling particle. For a scalar particle, it results in decrease in the temperature whereas it provides an increase in the temperature for a Dirac particle. These results show that the intrinsic properties of the particle, except total angular momentum for the Dirac particle, and graviton mass may cause screening for the black hole radiation.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by Akdeniz University, Scientific Research Projects Unit.

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