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Research Article

Thermodynamics of Modified Cosmic Chaplygin Gas

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We examine the thermodynamic features of an exotic fluid known as modified cosmic Chaplygin gas in the context of homogeneous isotropic universe model. For this purpose, the behavior of physical parameters is discussed that help to analyze nature of the universe. Using specific heat formalism, the validity of third law of thermodynamics is checked. Furthermore, with the help of thermodynamic entities, the thermal equation of state is also discussed. The thermodynamic stability is explored by means of adiabatic, specific heat and isothermal conditions from classical thermodynamics. It is concluded that the considered fluid configuration is thermodynamically stable and expands adiabatically for an appropriate choice of parameters.

1. Introduction

The discovery of accelerated expansion of the universe has unambiguously been proved by a diverse set of high-precision observational data accumulated from various astronomical sources [1–3]. Dark energy (DE) is considered as the root cause behind this tremendous change in cosmic history. It possesses negatively large pressure which violates the strong energy condition ($p+3\rho<0$, where p and ρ are pressure and energy density, respectively) but its complete characteristics are still not known. Planck's observational data reveals that about 68.3% of our universe is filled with this mysterious form of energy while the other cosmic budget includes 4.9% ordinary matter and 26.8% dark matter (DM) [4, 5]. To explore the perplexing nature of DE, there began a search for different candidates that can play their role as an alternative for DE. The simplest candidate is the cosmological constant while other favorable approaches include quintessence, k-essence, and Chaplygin gas (CG) known as DE matter models [6–8].

Chaplygin gas is an intriguing model presented by a Russian physicist Chaplygin as a convenient soluble model to study the lifting force on the wing of an aeroplane in aerodynamics. It efficiently describes the cosmic expansion and elegantly discusses DM and DE in a unified form. The distinct feature of this model is its positive and bounded squared speed of sound (v_s^2) leading to stable results as compared to other fluids with negative pressure. Chaplygin gas acts as an alternative for dust dominated era with small value of the scale factor and tends to cosmic expansion for its large value while primordial universe cannot be discussed in this scenario [9–12]. Despite the fact that it does not meet the strong energy condition, it successfully shows consistency with the observational results accumulated from various cosmic probes such as Hubble space telescope, Wilkinson microwave anisotropy probe, and cosmic background explorer.

To discuss the cosmic history as well as get more accuracy with observational data, several modifications of CG have been presented which are obtained by introducing new parameters in its equation of state (EoS). Bento et al. [13] established the generalization of this model named generalized Chaplygin gas (GCG) which interprets the same evolutionary phases of the universe as CG. Benaoum [14] introduced the modified Chaplygin gas (MCG) which illustrates the radiation dominated era. González-Díaz [15] proposed the generalization of cosmic CG models known as generalized cosmic Chaplygin gas (GCCG) in such a way that it avoids big-rip (singularity at a finite time) which was previously presented in the DE models representing phantom era. This generalization provides stable and physical behavior models even when the vacuum matter configuration

fulfills the phantom energy condition ($p + \rho < 0$, $\rho > 0$, $\omega < -1$, where ω is the EoS parameter). The other proposed CG models include variable Chaplygin gas (VCG), variable modified Chaplygin gas (VMCG), new variable MCG, extended CG, and modified cosmic Chaplygin gas (MCCG) [16–19].

Chaplygin gas models have stimulated many researchers to investigate their thermal stability. Santos et al. [20, 21] explored thermal stability of GCG as well as MCG and deduced that these fluid models verify the third law of thermodynamics along with the adiabatic expansion. Myung [22] proved the third law of thermodynamics for CG model and illustrated that it can represent a unified picture of DM and DE without any phase transition. Kahya and Pourhassan [23] analyzed extended CG model cosmologically as well as thermodynamically and found stable results without any phase transition against density perturbations. They also concluded that all laws of thermodynamics are satisfied for this exotic fluid which came out to be thermodynamically stable at all times. Panigrahi [24] studied thermodynamic behavior of VCG and observed that it is thermodynamically stable throughout the evolution. He also discussed thermal EoS which is an explicit function of temperature only and checked the validity of the third law. Panigrahi and Chatterjee [25] found that current accelerated expansion of the universe can be explained using VMCG model. Sharif and Sarwar [26] explored how GCCG can explain accelerated expansion of the universe by interpreting different physical parameters and showed that the fluid is adiabatically stable.

Here, we investigate thermodynamic stability of MCCG model in the background of isotropic and homogeneous universe model. In Section 2, we discuss the behavior of physical parameters such as pressure and EoS as well as deceleration parameters and analyze the stability using speed of sound. Section 3 deals with the thermodynamic stability of MCCG. The results are summarized in the last section.

2. Physical Parameters for MCCG

In this section, we discuss the behavior of MCCG in the background of FRW universe model for different physical parameters and examine its stability through squared speed of sound. The line element for FRW universe model is given by

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad (1)$$

where $a(t)$ is the scale factor. The EoS for MCCG is defined as

$$P = A\rho - \rho^{-\alpha}\left[\left(\rho^{\alpha+1} - C\right)^{-\gamma} + C\right], \quad 0 < \alpha \leq 1,\ -b < \gamma < 0,\ b \neq 1, \quad (2)$$

where $C = Z/(\gamma + 1) - 1$ (Z is an arbitrary constant) and A is a positive constant. This EoS reduces to GCCG as $A \longrightarrow 0$ [26] while GCG is recovered in the limit $A \longrightarrow 0$ along with $\gamma \longrightarrow 0$ [13]. The energy density of the fluid configuration is given by

$$\rho = \frac{U}{V}, \quad (3)$$

where U and V represent the internal energy and volume, respectively. Classical thermodynamics provides a useful relationship among the quantities U, V, and P in the form

$$\left(\frac{dU}{dV}\right) = -P. \quad (4)$$

Using (2) in the above expression, we obtain

$$\frac{dU}{dV} + \frac{AU}{V} = \frac{V^\alpha}{U^\alpha}\left[C + \left(\frac{U^{\alpha+1}}{V^{\alpha+1}} - C\right)^{-\gamma}\right], \quad (5)$$

which is a nonlinear ordinary differential equation. Its solution is given by

$$U \approx V\left[\frac{(d/V)^{(\alpha+1)(A+1)}(A+1) + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}}\right]^{1/(\alpha+1)}, \quad (6)$$

where we have used the binomial expansion up to first order and d is an integration constant which is either universal constant or a function of entropy (S). The above equation can also be written as

$$U = V\left[\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}}\right]^{1/(\alpha+1)}, \quad (7)$$

where $\varepsilon = d(A+1)^{1/M}$, $M = (\alpha+1)(A+1)$, and $A+\gamma(-C)^{-\gamma-1} \neq -1$. It is clearly observed that internal energy of the fluid can only be discussed when γ is a whole number between the above-mentioned range for positive values of C. Using (3) and (7), the energy density of MCCG becomes

$$\rho = \left[\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}}\right]^{1/(\alpha+1)}. \quad (8)$$

In the following, we use this equation to discuss different physical parameters.

2.1. Pressure. The pressure of MCCG in terms of V can be obtained using (2) and (8) as

$$P = A\left(\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}}\right)^{1/(\alpha+1)} - \left(\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}}\right)^{-\alpha/(\alpha+1)} \times \left[C + \left(\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma}}{A + 1 + \gamma(-C)^{-\gamma-1}} - C\right)^{-\gamma}\right]. \quad (9)$$

The graphical analysis of this equation is shown in Figure 1 for different values of Z with $A = 2$. The positive and

negative behavior of pressure correspond to decelerated and accelerated eras of the universe, respectively. For $Z = 0.01$, it is observed that the accelerating universe at small volume tends to dust dominated universe at large volume. We note that the decelerating universe tends to accelerate as volume increases for $Z = -5, -7$ whereas accelerated phase is obtained only for $Z = -8$. The same behavior of pressure is observed for different values of the parameter A.

2.2. EoS Parameter. Here we discuss the effective EoS parameter of MCCG. Using (8) and (9), we have

$$\omega = \frac{P}{\rho} = A - \frac{C + \left(\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma}\right) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{-\gamma}}{\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma}\right) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right)}. \tag{10}$$

We study the following two extremal cases for volume to analyze the behavior of above equation.

(i) For small volume $V \ll \varepsilon$, the above equation reduces to

$$P \approx A\rho, \tag{11}$$

which is a barotropic EoS. In this case, ω will depend entirely on the value of A.

(ii) For large volume $V \gg \varepsilon$, (10) takes the form

$$\omega \approx \frac{P}{\rho} = A - \frac{C + \left(\left((C + (-C)^{-\gamma}) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{-\gamma}}{\left((C + (-C)^{-\gamma}) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right)}. \tag{12}$$

For $\omega = 0$, let volume be denoted by V_c, which is given by

$$V_c = \varepsilon \left[\frac{A + \gamma(-C)^{-\gamma-1}}{C + (-C)^{-\gamma}}\right]^{1/M}. \tag{13}$$

The EoS parameter discusses both accelerated and decelerated phases of the universe and successfully describes the phase transitions (dubbed as flip) at a critical value V_c between these cosmic phases. The proper flip occurs when $(\varepsilon/V)^M < 1$ while the inequality $C + (-C)^{-\gamma} \neq 0$ leads to real flip.

Figure 2 shows the behavior of ω for different values of Z. It is found that the value of V_c decreases as Z becomes negatively large. The negative values of Z show the decelerated cosmic phase at small volume undergoing acceleration at large volume while $Z = 1$ demonstrates only the decelerating phase. We observe that, at large volume, the considered values of $Z = 1, 0, -1.15$, and -2 correspond to stiff matter, dust dominated, ΛCDM, and phantom eras, respectively. Thus, the EoS parameter can interpret different evolutionary phases of the universe.

2.3. Deceleration Parameter. The deceleration parameter is given by

$$q = \frac{1}{2} + \frac{3P}{2\rho}. \tag{14}$$

Using (10), this parameter for MCCG takes the form

$$q = \frac{1}{2} + \frac{3}{2}\left[A - \frac{C + \left(\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma}\right) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{-\gamma}}{\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma}\right) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right)}\right]. \tag{15}$$

For small volume, it reduces to

$$q \approx \frac{1}{2} + \frac{3A}{2}, \tag{16}$$

which implies that the universe undergoes deceleration at its early stage since $A > 0$ while for large volume, (15) becomes

$$q \approx \frac{1}{2} + \frac{3}{2}\left[A - \frac{C + \left(\left((C + (-C)^{-\gamma}) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{-\gamma}}{\left((C + (-C)^{-\gamma}) / \left(A + 1 + \gamma(-C)^{-\gamma-1}\right)\right)}\right]. \tag{17}$$

In this case, the flip occurs when deceleration parameter vanishes and the corresponding flip volume (V_f) is given by

$$V_f = \varepsilon \left[\frac{3A + 1 + 3\gamma(-C)^{-\gamma-1}}{C + (-C)^{-\gamma}}\right]^{1/M}, \tag{18}$$

provided that $C + (-C)^{-\gamma} \neq 0$ and the inequality $C + (-C)^{-\gamma} < 3(A + \gamma(-C)^{-\gamma-1}) + 1$ leads to proper flip. Figure 3 shows the evolution of deceleration parameter against volume for different values of Z. At small volume, the universe undergoes deceleration while accelerating behavior is observed at large volume for considered negative values of Z. The flip occurs at $V \approx 1.6$ and 3 for $Z = -2$ and -1.3, respectively. For

$Z = 0$, the deceleration parameter switches from acceleration to deceleration at $V_f \approx 2.2$ while no flip is observed for $Z = 2$.

2.4. Speed of Sound. Here we analyze the stability of MCCG using speed of sound as

$$\begin{aligned} v_s^2 &= \left(\frac{\partial P}{\partial \rho}\right)_S \\ &= A + \frac{\gamma(\alpha+1)}{\left(\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma-1}\right)/\left(A+1+\gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{\gamma+1}} \\ &\quad + \frac{\alpha}{\left(\left((\varepsilon/V)^M + C + (-C)^{-\gamma-1}\right)/\left(A+1+\gamma(-C)^{-\gamma-1}\right)\right)} \times \left[C + \left(\frac{(\varepsilon/V)^M + C + (-C)^{-\gamma-1}}{A+1+\gamma(-C)^{-\gamma-1}} - C\right)^{-\gamma}\right], \end{aligned} \tag{19}$$

whose feasible range is $0 < v_s^2 < 1$. This equation reduces to $v_s^2 = A$ at early universe while, for $V \gg \varepsilon$, we have

$$\begin{aligned} v_s^2 &= A \\ &\quad + \frac{\gamma(\alpha+1)}{\left(\left((C + (-C)^{-\gamma})/\left(A+1+\gamma(-C)^{-\gamma-1}\right)\right) - C\right)^{\gamma+1}} \\ &\quad + \frac{\alpha}{\left((C + (-C)^{-\gamma})/\left(A+1+\gamma(-C)^{-\gamma-1}\right)\right)} \\ &\quad \times \left[C + \left(\frac{C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma}} - C\right)^{-\gamma}\right]. \end{aligned} \tag{20}$$

Figure 4 shows the behavior of squared speed of sound against the positive parameter A for different values of Z. It is observed that the viable ranges for A are $1.8 < A < 2.8$, $3.2 < A < 3.7$, and $5 < A < 5.6$ corresponding to $Z = -0.01$, 4, and 7, respectively. Thus, the stable results are found for the considered values of Z in a particular range of A.

3. Thermodynamic Stability

In this section, we discuss thermodynamic stability of MCCG during its evolution. The stability conditions are given by the following [27]:

(i) The pressure reduces for both adiabatic as well as isothermal expansions as

$$\begin{aligned} \left(\frac{\partial P}{\partial V}\right)_S &< 0, \\ \left(\frac{\partial P}{\partial V}\right)_T &< 0, \end{aligned} \tag{21}$$

where T represents temperature.

(ii) Specific heat at constant volume (c_V) is positive.

Differentiation of (9) with respect to volume yields

$$\begin{aligned} &\left(\frac{\partial P}{\partial V}\right)_S \\ &= \frac{(A+1)\varepsilon^M}{V^{M+1}A + 1 + \gamma(-C)^{-\gamma-1}} \left(\frac{\varepsilon^M V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}}\right)^{-\alpha/(\alpha+1)} \\ &\quad \times \left[-A(\alpha+1) + \alpha P\left(\frac{\varepsilon^M V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}}\right)^{-1/(\alpha+1)}\right. \\ &\quad \left. - \gamma(\alpha+1)\left(\frac{\varepsilon^M V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}} - C\right)^{-\gamma-1}\right]. \end{aligned} \tag{22}$$

When volume is very small, the above equation reduces to zero while, for large volume, we have the following expression:

$$\begin{aligned} &\left(\frac{\partial P}{\partial V}\right)_S \\ &= \frac{(A+1)\varepsilon^M}{V^{M+1}\left(A+1+\gamma(-C)^{-\gamma-1}\right)} \left(\frac{C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}}\right)^{-\alpha/(\alpha+1)} \\ &\quad \times \left[\alpha P\left(\frac{C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}}\right)^{-1/(\alpha+1)} - A(\alpha+1) - \gamma(\alpha+1)\right. \\ &\quad \left. \times \left(\frac{C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}} - C\right)^{-\gamma-1}\right]. \end{aligned} \tag{23}$$

Figure 5 shows that the adiabatic condition is fulfilled for all the considered values of Z.

To investigate the positivity of specific heat at constant volume, we consider specific heat in terms of temperature and entropy as

$$c_v = T\left(\frac{\partial S}{\partial T}\right)_V, \tag{24}$$

where the temperature of MCCG is obtained from the following relation:

$$T = \frac{\partial U}{\partial S} = \left(\frac{\partial U}{\partial d}\right)\left(\frac{\partial d}{\partial S}\right). \tag{25}$$

Differentiating (6) with respect to d, we have

$$\frac{\partial U}{\partial d} = \frac{d^{M-1}(A+1)^2 U}{d^M(A+1) + CV^M + (-C)^{-\gamma}V^M}. \tag{26}$$

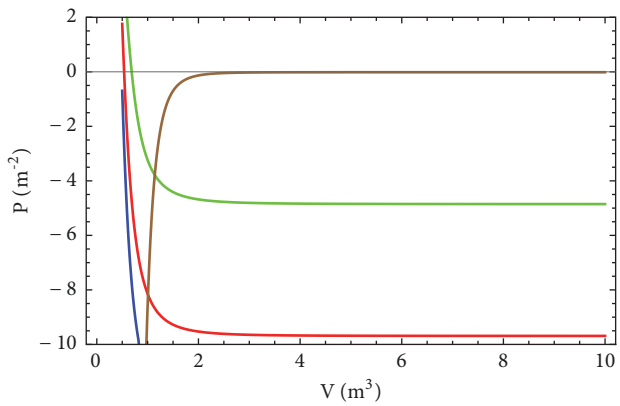

FIGURE 1: Plots of P versus V for $A = 2, \gamma = -2, \alpha = 0.1, d = 1$ with $Z = 0.01$ (brown), -5 (green), -7 (red), and -8 (blue).

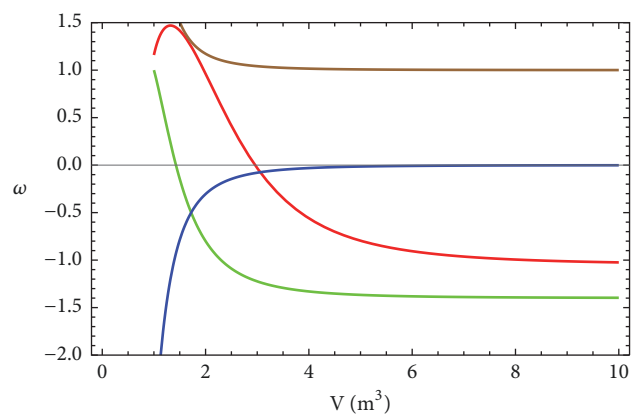

FIGURE 2: Plots of ω versus V for $\gamma = -2, A = 2, \alpha = 0.1, d = 1$ with $Z = -2$ (green), -1.15 (red), 0 (blue), and 1 (brown).

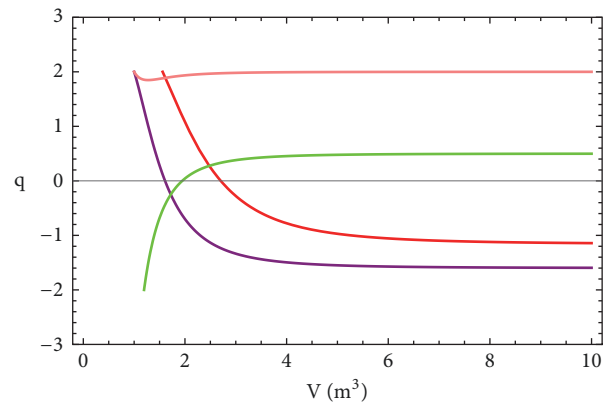

FIGURE 3: Plots of q versus V for $\gamma = -2, A = 2, \alpha = 0.1, d = 1$ with $Z = -2$ (purple), -1.3 (red), 0 (green), and 2 (pink).

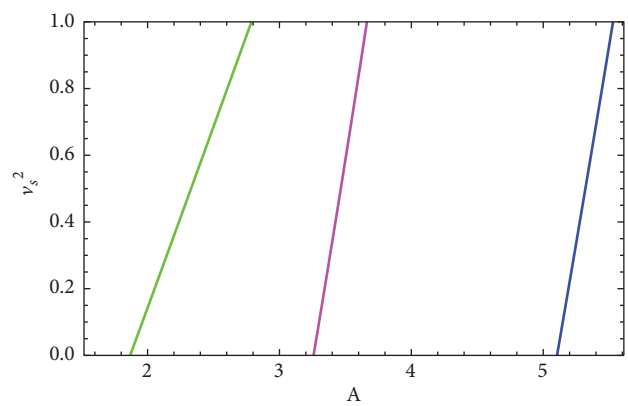

FIGURE 4: Plots of v_s^2 versus A for $\gamma = -2, \alpha = 0.1, d = 1$ with $Z = -0.01$ (green), 4 (magenta), and 7 (blue).

Substituting this relation in (25), the expression of T becomes

$$T = \frac{d^{M-1}(A+1)^2 U}{d^M(A+1) + CV^M + (-C)^{-\gamma} V^M}\left(\frac{\partial d}{\partial S}\right). \tag{27}$$

When d is a universal constant ($\partial d/\partial S = 0$), the temperature vanishes while it varies for CG expansion, so we consider $\partial d/\partial S \neq 0$. Here, we assume the case $\partial d/\partial S > 0$ to have a positive temperature which is cooled down through adiabatic expansion. Using the concept of dimensional analysis, (6) gives

$$[d]^{A+1} = [U, V]^A. \tag{28}$$

Using the relation $[U] = [T][S]$, the above equation becomes

$$[d] = [T]^{1/(A+1)}[S]^{1/(A+1)}[V]^{A/(A+1)}. \tag{29}$$

Taking $d = d(S)$, it follows that

$$d = \left(\tau \nu^A S\right)^{1/(A+1)}, \tag{30}$$

where τ and ν are constants having the dimensions of temperature and volume, respectively. Differentiating (30) with respect to S, we obtain

$$\frac{\partial d}{\partial S} = \frac{1}{A+1}\left(\frac{\tau \nu^A}{S^A}\right)^{1/(A+1)}. \tag{31}$$

Substituting this value in (27), the temperature of MCCG takes the form

$$T = \frac{d^{M-1}(A+1) U B^{1/(A+1)} S^{-A/(A+1)}}{d^M(A+1) + CV^M + (-C)^{-\gamma} V^M}, \tag{32}$$

where $B = \tau \nu^A$. Using (6) and (30) in the above equation, we have

$$T = \frac{S^\alpha B^{\alpha+1}(A+1)\left[(BS)^{\alpha+1}(A+1) + CV^M + (-C)^{-\gamma} V^M\right]^{-\alpha/(\alpha+1)}}{V^A\left(A+1+\gamma(-C)^{-\gamma-1}\right)^{1/(\alpha+1)}}. \tag{33}$$

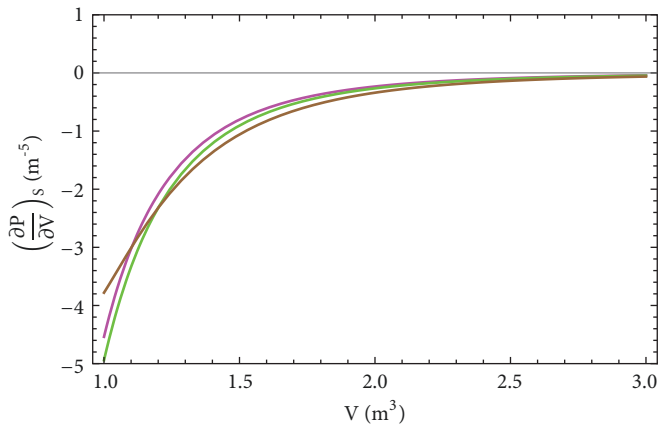

FIGURE 5: Plots of $(\partial P/\partial V)_S$ versus V for $\gamma = -2, A = 2, \alpha = 0.1, d = 1$ with $Z = -15$ (magenta), -7 (green), and -2 (brown).

For a positive definite entropy, we assume $0 < T < \tau$ and $0 < V < \nu$. It is worth mentioning here that when $T = 0$, the entropy vanishes which indicates that the considered fluid obeys third law of thermodynamics (if the temperature of a physical system approaches zero then entropy becomes zero). Differentiating (33) with respect to S, we obtain

$$\begin{aligned}\frac{\partial T}{\partial S} = \alpha Z S^{\alpha} \Big[& S^{-1} \left\{ (A+1)(BS)^{\alpha+1} + CV^{M} \right. \\ & \left. + (-C)^{-\gamma} V^{M} \right\}^{-\alpha/(\alpha+1)} - B^{\alpha+1} S^{\alpha} (A+1) \\ & \cdot \left\{ (A+1)(BS)^{\alpha+1} + CV^{M} \right. \\ & \left. + (-C)^{-\gamma} V^{M} \right\}^{(-2\alpha-1)/(\alpha+1)} \Big],\end{aligned} \tag{34}$$

where $Z = B^{\alpha+1}(A+1)/V^{A}(A+1+\gamma(-C)^{-\gamma-1})^{1/(\alpha+1)}$. Inserting (33) and (34) in (24), it follows that

$$c_V = \frac{S}{\alpha} \left[1 - \frac{(BS)^{\alpha+1}(A+1)}{\left[(BS)^{\alpha+1}(A+1) + CV^{M} + (-C)^{-\gamma} V^{M} \right]} \right]^{-1}. \tag{35}$$

For c_V to be real, $(BS)^{\alpha+1}(A+1) \neq (BS)^{\alpha+1}(A+1) + CV^{M} + (-C)^{-\gamma}V^{M}$ and to be positive, the inequality $(BS)^{\alpha+1}(A+1) < (BS)^{\alpha+1}(A+1) + CV^{M} + (-C)^{-\gamma}V^{M}$ must hold. The graphical analysis of (35) is shown in Figure 6 for different values of Z with $A = 2$. We observe that the positivity of specific heat is obtained for $Z = 6$ in the range $V > 1.5$ while MCCG is thermally stable for both values of $Z = 0$ and -0.5 throughout the evolution. It is also noted that when temperature is zero, thermal capacity vanishes which also assures the validity of third law of thermodynamics.

Finally, we analyze the behavior of considered model through isothermal condition. For this purpose, we assume $P = P(V, T)$ and by solving (9) and (30), we have

$$\begin{aligned}P = A & \left(\frac{(BS)^{\alpha+1}(A+1)V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}} \right)^{1/(\alpha+1)} \\ & - \left(\frac{(BS)^{\alpha+1}(A+1)V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}} \right)^{-\alpha/(\alpha+1)} \\ & \times \Bigg[C \\ & + \left(\frac{(BS)^{\alpha+1}(A+1)V^{-M} + C + (-C)^{-\gamma}}{A+1+\gamma(-C)^{-\gamma-1}} - C \right)^{-\gamma} \Bigg].\end{aligned} \tag{36}$$

The corresponding EoS parameter takes the form

$$\omega = \frac{P}{\rho} = A - \frac{C + \left(\left((BS)^{\alpha+1}(A+1)V^{-M} + C + (-C)^{-\gamma} \right) / \left(A+1+\gamma(-C)^{-\gamma-1} \right) - C \right)^{-\gamma}}{\left(\left((BS)^{\alpha+1}(A+1)V^{-M} + C + (-C)^{-\gamma} \right) / \left(A+1+\gamma(-C)^{-\gamma-1} \right) \right)}. \tag{37}$$

To check the isothermal condition, we should have $\rho = \rho(T)$ and $P = P(T)$. In our case, it is difficult to have a thermal EoS for MCCG as a function of temperature only since (33) is a complicated equation such that the explicit expression for S in terms of T cannot be extracted. For this reason, we are unable to analyze the isothermal condition in this scenario.

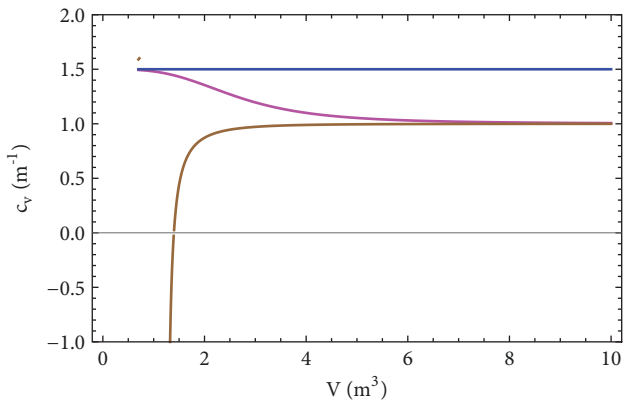

Figure 6: Plots of c_V versus V for $\gamma = -2, A = 2, \alpha = 0.1, \tau = 2.73, S = 1, \nu = 1, d = 1$ with $Z = -0.5$ (magenta), 0 (blue), and 6 (brown).

4. Conclusions

In this paper, we have analyzed thermodynamic stability of MCCG within the framework of FRW universe model. We have examined the expanding evolution of the universe through different physical parameters like pressure, effective EoS, and deceleration parameters as well as speed of sound. The results of these parameters can be summarized as follows:

(i) The consistent behavior of pressure with the evolutionary picture of the universe is obtained for the considered values of Z whereas inconsistent evolution is observed for its positive values (Figure 1).

(ii) The EoS parameter for MCCG depicts that decelerated and accelerated phases of our universe can be discussed for different values of parameter Z (Figure 2). We have also calculated the critical value at $\omega = 0$ and found that its value increases as Z increases from its negative values to zero.

(iii) The evolution of deceleration parameter against volume gives the decelerated universe when $V < V_f$ for negative values of Z while accelerating behavior is observed when $V > V_f$. For positive values of Z, we have found only deceleration while at $Z = 0$, acceleration occurs before the flip (Figure 3).

(iv) We have analyzed the stability of MCCG through speed of sound and obtained stable regions at large volume for the considered values of Z (Figure 4). For VMCG model, the squared speed of sound could be positive or negative [25] whereas for GCCG model, the stable regions do not exist in late universe [26].

Finally, we have investigated thermodynamic stability of considered fluid configuration using adiabatic, isothermal, and specific heat conditions. We have found the validity of adiabatic as well as positivity of specific heat for the considered values of Z (Figures 5 and 6). It is worth mentioning here that third law of thermodynamics is obeyed for MCCG. We conclude that MCCG expands adiabatically and the expansion is thermodynamically stable for a suitable choice of the parameters.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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