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Research Article

The Quantum Description of BF Model in Superspace

Manoj Kumar Dwivedi

Department of Physics, Banaras Hindu University, Varanasi 221005, India

Correspondence should be addressed to Manoj Kumar Dwivedi; manojdwivedi84@gmail.com

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We consider the BRST symmetric four-dimensional BF theory, a topological theory, containing antisymmetric tensor fields in Landau gauge and extend the BRST symmetry by introducing a shift symmetry to it. Within this formulation, the antighost fields corresponding to shift symmetry coincide with antifields of standard field/antifield formulation. Furthermore, we provide a superspace description for the BF model possessing extended BRST and extended anti-BRST transformations.

1. Introduction

Topological gauge field theories (TGFT) which came from mathematics have some peculiar features. The examples of two distinct class of TGFT are topological Yang-Mills theory and Chern-Simons (CS) theory, which are sometimes classified as Witten-type and Schwarz-type, respectively [1]. Besides these two types, there is another Schwarz-type TGFT called topological BF theory, which is an extension of CS theory [2]. The difference between CS theory and BF model is that action of previous theory exists only in odd-dimensions while later one can be defined on manifolds of any dimensions.

In string theory and nonlinear sigma model, four-dimensional antisymmetric (or BF) models [3–8] were introduced some years ago. This model is interesting due to its topological nature [1] and its connection with lower dimensional quantum gravity; for example, three space-time dimensional Einstein-Hilbert models with or without using cosmological constant can be naturally formulated in terms of BF-models [9, 10]. Coupling of an antisymmetric tensor field with the field strength tensor of Yang-Mills is described by these models [11]. Quantization of BF model in Landau gauge has been studied in [11]. Topological BF theory in Landau gauge has a common feature of a large class of topological models [12–16].

On the other hand, the Batalin-Vilkovisky (BV) approach, also known as field/antifield formulation, [17–20] is one of the most powerful quantization algorithms presently available. BV formulation deals with very general gauge theories, including those with open or reducible gauge symmetry algebras. The BV method also address the possible violations of symmetries of the action by quantum effects. The BV formulation (independently introduced by Zinn-Justin [21]) extends the BRST approach [22–36]. In fact, the BRST symmetry [37, 38] is a very important symmetry for gauge theories [26, 39–54]. Besides the covariant description to perform the gauge-fixing in quantum field theory, BV formulation was also applied to other problems like analysing possible deformations of the action and anomalies.

A superspace description for various gauge theories in BV formulation has been studied extensively [55–64]. They have shown that the extended BRST and extended anti-BRST invariant actions of these theories (including some shift symmetry) in BV formulation yield naturally the proper identification of the antifields through equations of motion. The shift symmetry is important and gets relevance, for example, in inflation particularly in supergravity [65] as well as in Standard Model [66]. In usual BV formulation, these antifields can be calculated from the expression of gauge-fixing fermion. We extended BRST formulation and superspace description of the topological gauge (BF) model is still unstudied and we try to discuss these here.

In the present work, we try to generalize the superspace formulation of BV action for BF model. Particularly, we first consider BRST invariant BF model in Landau gauge and extend the BRST symmetry of the theory by including shift symmetry. By doing so, we find that the antighosts of shift symmetry get identified as antifields of standard BV formulation naturally. Further, we discuss a superspace formulation of extended BRST invariant BF model. Here we see that one additional Grassmann coordinate is required if action admits only extended BRST symmetry. However, for both extended BRST and extended anti-BRST invariant BF model two additional Grassmann coordinates are required.

This paper is framed as follows. In Section 2, we discuss the BRST invariant BF model. In Section 3, we study the extended BRST transformation of the model. Further, we describe extended BRST invariant action in superspace in Section 4. The extended anti-BRST symmetry is discussed in Section 5. The superspace formulation of extended BRST and anti-BRST invariant action is given in Section 6. The last section is reserved for concluding remarks.

2. BRST Invariant BF Model

In this section, we discuss the preliminaries of BF model with its BRST invariance. In this view, the BF model in flat $(3+1)$ space-time dimensions is given by the following gauge invariant Lagrangian density [11]:

$$\mathscr{L}_0 = -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma}, \tag{1}$$

where $B^a_{\rho\sigma}$ and $F^a_{\mu\nu}$ are two-form field and field strength tensor for vector field, respectively. In order to remove discrepancy due to gauge symmetry, the gauge-fixing and ghost terms are given by

$$\begin{aligned}\mathscr{L}_{gf+gh} &= b^a\partial^\mu A^a_\mu + \overline{C}^a\partial^\mu\left(D_\mu C\right)^a + h^{a\nu}\left(\partial^\mu B^a_{\mu\nu}\right) + \omega^a\partial\xi^a \\ &\quad + h^a_\mu\left(\partial^\mu e^a\right) + \omega^a\lambda^a + \left(\partial^\mu\overline{\xi}^a_\mu\right)\lambda^a \\ &\quad - \left(\partial^\mu\overline{\phi}^a\right)\left[\left(D_\mu\phi\right)^a + f^{abc}c^b\xi^c_\mu\right] \\ &\quad - \left(\partial^\mu\overline{\xi}^{a\nu}\right)\left[\left(D_\mu\xi_\nu\right)^a - \left(D_\nu\xi_\mu\right)^a + f^{abc}B^b_{\mu\nu}C^c\right] \\ &\quad + \frac{1}{2}f^{abc}\epsilon^{\mu\nu\rho\sigma}\left(\partial_\mu\overline{\xi}^a_\nu\right)\left(\partial_\rho\overline{\xi}^b_\sigma\right)\phi^c,\end{aligned} \tag{2}$$

where fields (C^a, ξ^a_μ), $(\overline{C}^a, \overline{\xi}^a_\mu)$, and (b^a, h^a_μ) are the ghosts, antighosts, and the multipliers fields, respectively, while the fields $\phi^a, \overline{\phi}^a$ and ω^a are taken into account to remove further degeneracy due to the existence of zero modes in the transformations.

The effective Lagrangian density of BF model, $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_{gf+gh}$, possesses the following BRST symmetry:

$$\begin{aligned} sA^a_\mu &= -\left(D_\mu C\right)^a, \\ sC^a &= \frac{1}{2}f^{abc}C^bC^c, \\ s\xi^a_\mu &= \left(D_\mu\phi\right)^a + f^{abc}C^b\xi^c_\mu, \\ sB^a_{\mu\nu} &= -\left(D_\mu\xi_\nu - D_\nu\xi_\mu\right)^a - f^{abc}B^b_{\mu\nu}C^c \\ &\quad + f^{abc}\epsilon_{\mu\nu\rho\sigma}\left(\partial^\rho\overline{\xi}^{b\sigma}\right)\phi^c, \\ s\phi^a &= f^{abc}C^b\phi^c, \\ s\overline{\xi}^a_\mu &= h^a_\mu, \\ s\overline{C}^a &= b^a, \\ s\overline{\phi}^a &= \omega^a, \\ se^a &= \lambda^a, \\ s\left(h^a_\mu, b^a, \omega^a, \lambda^a\right) &= 0. \end{aligned} \tag{3}$$

The gauge-fixing and ghost terms of the effective Lagrangian density are BRST exact and, hence, can be written in terms of BRST variation of gauge-fixing fermion,

$$\begin{aligned}\Psi &= \left(\overline{C}^a\partial^\mu A^a_\mu + \overline{\xi}^{a\mu}\partial^\nu B^a_{\mu\nu} + \overline{\phi}^a\partial^\mu\xi^a_\mu - e^a\omega^a - e^a\partial^\mu\overline{\xi}^a_\mu\right),\end{aligned} \tag{4}$$

as follows:

$$\mathscr{L}_{gf+gh} = s\Psi. \tag{5}$$

In the next section, we would like to study the extended BRST symmetry for the model which incorporates shift symmetry together with original BRST symmetry.

3. Extended BRST Invariant Lagrangian Density

The advantage of studying the extended BRST transformations for BF model in BV formulation is that antifields get identification naturally. We begin with shifting all the fields from their original value as follows:

$$\begin{aligned} B^a_{\mu\nu} &\longrightarrow B^a_{\mu\nu} - \widetilde{B}^a_{\mu\nu}, \\ A^a_\mu &\longrightarrow A^a_\mu - \widetilde{A}^a_\mu, \\ C^a &\longrightarrow C^a - \widetilde{C}^a, \\ \overline{C}^a &\longrightarrow \overline{C}^a - \widetilde{\overline{C}}^a, \\ b^a &\longrightarrow b^a - \widetilde{b}^a, \end{aligned}$$

$$
\begin{aligned}
\xi_\mu^a &\longrightarrow \xi_\mu^a - \tilde{\xi}_\mu^a, \\
\overline{\xi}_\mu^a &\longrightarrow \overline{\xi}_\mu^a - \widetilde{\overline{\xi}}_\mu^a, \\
\phi^a &\longrightarrow \phi^a - \tilde{\phi}^a, \\
\overline{\phi}^a &\longrightarrow \overline{\phi}^a - \widetilde{\overline{\phi}}^a, \\
h_\mu^a &\longrightarrow h_\mu^a - \tilde{h}_\mu^a, \\
e^a &\longrightarrow e^a - \tilde{e}^a, \\
\omega^a &\longrightarrow \omega^a - \tilde{\omega}^a, \\
\lambda^a &\longrightarrow \lambda^a - \tilde{\lambda}^a.
\end{aligned} \tag{6}
$$

The effective Lagrangian density of BF model also gets shifted under such shifting of fields, respectively. This is given by

$$
\begin{aligned}
\widetilde{\mathscr{L}} = \mathscr{L}\Big(& A_\mu^a - \tilde{A}_\mu^a, C^a - \tilde{C}^a, \overline{C}^a - \widetilde{\overline{C}}^a, b^a - \tilde{b}^a, \xi_\mu^a \\
& - \tilde{\xi}_\mu^a, \overline{\xi}_\mu^a - \widetilde{\overline{\xi}}_\mu^a, \phi^a - \tilde{\phi}^a, \overline{\phi}^a - \widetilde{\overline{\phi}}^a, h_\mu^a - \tilde{h}_\mu^a, e^a - \tilde{e}^a, \omega^a \\
& - \tilde{\omega}^a, \lambda^a - \tilde{\lambda}^a \Big).
\end{aligned} \tag{7}
$$

The shifted Lagrangian density is invariant under BRST transformation together with a shift symmetry transformation, jointly known as extended BRST transformation. The extended BRST symmetry transformations under which Lagrangian density of BF model is invariant are written by

$$
\begin{aligned}
sA_\mu^a &= \psi_\mu^a, \\
s\tilde{A}_\mu^a &= \psi_\mu^a - \left(D_\mu - \tilde{D}_\mu\right)\left(C - \tilde{C}\right)^a, \\
sC^a &= \epsilon^a, \\
s\tilde{C}^a &= \epsilon^a - \frac{1}{2} f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi_\mu^a - \tilde{\xi}_\mu^a\right), \\
s\widetilde{\overline{C}}^a &= \overline{\epsilon}^a - \left(b - \tilde{b}\right)^a, \\
sb^a &= \chi^a, \\
s\tilde{b}^a &= \chi^a, \\
s\phi^a &= M^a, \\
s\tilde{\phi}^a &= M^a - f^{abc}\left(C^b - \tilde{C}^b\right)\left(\phi^c - \tilde{\phi}^c\right), \\
s\overline{\phi}^a &= \overline{M}^a, \\
s\widetilde{\overline{\phi}}^a &= M^a - \left(\omega^a - \tilde{\omega}^a\right), \\
se^a &= N^a, \\
s\tilde{e}^a &= N^a - \left(\lambda^a - \tilde{\lambda}^a\right), \\
s\xi_\mu^a &= L_\mu^a, \\
s\overline{C}^a &= \overline{\epsilon}^a, \\
s\tilde{\xi}_\mu^a &= L_\mu^a - \Big[\left(D_\mu - \tilde{D}_\mu\right)\left(\phi - \tilde{\phi}\right)^a \\
&\quad + f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi_\mu^c - \tilde{\xi}_\mu^c\right)\Big], \\
s\overline{\xi}_\mu^a &= \overline{L}_\mu^a, \\
s\widetilde{\overline{\xi}}_\mu^a &= \overline{L}_\mu^a - \left(h_\mu^a - \tilde{h}_\mu^a\right),
\end{aligned} \tag{8}
$$

where $\psi_\mu^a, \epsilon^a, \overline{\epsilon}^a, \chi^a, M^a, \overline{M}^a, N^a, L_\mu^a$, and $\overline{L}_\mu^a$ are the ghost fields corresponding to shift symmetry for $A_\mu^a, C^a, \overline{C}^a, b^a$, $\phi^a, \overline{\phi}^a, e^a, \xi_\mu^a$ and $\overline{\xi}_\mu^a$, respectively. The nilpotency of extended BRST symmetry (8) leads to the BRST transformation for the following ghost fields:

$$
\begin{aligned}
s\psi_\mu^a &= 0, \\
s\epsilon^a &= 0, \\
s\overline{\epsilon}^a &= 0, \\
s\chi^a &= 0, \\
sM^a &= 0, \\
s\overline{M}^a &= 0, \\
sN^a &= 0, \\
sL_\mu^a &= 0, \\
s\overline{L}_\mu^a &= 0.
\end{aligned} \tag{9}
$$

In order to make the theory ghost free, we need further antighosts $A_\mu^{\star a}, C^{\star a}, \overline{C}^{\star a}, b^{\star a}, \xi_\mu^{\star a}, \overline{\xi}_\mu^{\star a}, \phi^{\star a}, \overline{\phi}^{\star a}$, and $e^{\star a}$ to be introduced corresponding to the ghost fields $\psi_\mu^a, \epsilon^a, \overline{\epsilon}^a, \chi^a, M^a, \overline{M}^a, N^a, L_\mu^a$, and $\overline{L}_\mu^a$, respectively. The BRST transformations of these antighosts are constructed as follows:

$$
\begin{aligned}
sA_\mu^{\star a} &= -\zeta_\mu^a, \\
sC^{\star a} &= -\sigma^a, \\
s\overline{C}^{\star a} &= -\overline{\sigma}^a, \\
sb^{\star a} &= -\varpi^a, \\
s\phi^{\star a} &= -\upsilon^a, \\
s\overline{\phi}^{\star a} &= -\overline{\upsilon}^a,
\end{aligned}
$$

$$\begin{aligned} se^{\star a} &= -\tau^a, \\ s\xi_\mu^{\star a} &= -\kappa^a, \\ s\bar{\xi}_\mu^{\star a} &= -\bar{\kappa}^a, \end{aligned} \tag{10}$$

where $\zeta_\mu^a, \sigma^a, \bar{\sigma}^a, \varpi^a, \upsilon^a, \tau^a, \kappa^a$, and $\bar{\kappa}^a$ are the Nakanishi-Lautrup type auxiliary fields corresponding to shifted fields $\tilde{A}_\mu^a, \tilde{C}^a, \tilde{\bar{C}}^a, \tilde{b}^a, \tilde{\phi}^a, \tilde{\bar{\phi}}^a, \tilde{e}^a, \tilde{\xi}_\mu^a$, and $\tilde{\bar{\xi}}_\mu^a$ having the following BRST transformations:

$$\begin{aligned} s\zeta_\mu^a &= 0, \\ s\sigma^a &= 0, \\ s\bar{\sigma}^a &= 0, \\ s\varpi^a &= 0, \\ s\upsilon^a &= 0, \\ s\bar{\upsilon}^a &= 0, \\ s\tau^a &= 0, \\ s\kappa^a &= 0, \\ s\bar{\kappa}^a &= 0. \end{aligned} \tag{11}$$

We can recover our original BF model by fixing the shift symmetry in such a way such that effect of all the tilde fields will vanish. We achieve this by adding following gauge-fixed term to the shifted Lagrangian density (7):

$$\begin{aligned} \widetilde{\mathscr{L}}_{gf+gh} = {} & -\zeta^{a\mu}\tilde{A}_\mu^a - A_\mu^{a\star}\left[\psi^{a\mu} - \left(D^\mu - \tilde{D}^\mu\right)\left(C \right.\right. \\ & \left.\left. - \tilde{C}\right)^a\right] - \bar{\sigma}^a\tilde{C}^a + \bar{C}^{a\star}\left[\epsilon^a - \frac{1}{2}f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi_\mu^a \right.\right. \\ & \left.\left. - \tilde{\xi}_\mu^a\right)\right] - \sigma^a\tilde{\bar{C}}^a + C^{a\star}\left[\bar{\epsilon}^a - \left(b^a - \tilde{b}^a\right)\right] - \upsilon^a\tilde{\phi}^a \\ & - \phi^{\star a}\left[M^a - f^{abc}\left(C^b - \tilde{C}^b\right)\left(\phi^c - \tilde{\phi}^c\right)\right] - \bar{\upsilon}^a\tilde{\bar{\phi}}^a \\ & - \bar{\phi}^{\star a}\left(\bar{M}^a - \omega^a + \tilde{\omega}^a\right) - \tau^a\tilde{e}^a - e^\star\left[N^a - \lambda^a + \tilde{\lambda}^a\right] \\ & - \varpi^a\tilde{b}^a - b^{\star a}\chi^a - \kappa^a\tilde{\xi}_\mu^a - \bar{\kappa}^a\tilde{\bar{\xi}}_\mu^a + \xi_\mu^{\star a}\left(L_\mu^a \right. \\ & - \left[\left(D_\mu - \tilde{D}_\mu\right)\left(\phi - \tilde{\phi}\right)^a \right. \\ & \left.\left. + f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi_\mu^c - \tilde{\xi}_\mu^c\right)\right]\right) + \bar{\xi}_\mu^{\star a}\left[\bar{L}_\mu^a - \left(h_\mu^a \right.\right. \\ & \left.\left. - \tilde{h}_\mu^a\right)\right]. \end{aligned} \tag{12}$$

One can easily check that this gauge-fixing Lagrangian density $\widetilde{\mathscr{L}}_{gf+gh}$ also admits the extended BRST invariance. Integrating the auxiliary fields of the above expression, we obtain

$$\begin{aligned} \widetilde{\mathscr{L}}_{gf+gh} = {} & -A_\mu^{a\star}\left[\psi^{a\mu} - \left(D^\mu C\right)^a\right] \\ & + \bar{C}^{a\star}\left[\epsilon^a - \frac{1}{2}f^{abc}C^b\xi_\mu^c\right] + C^{a\star}\left[\bar{\epsilon}^a - b^a\right] \\ & - \phi^{\star a}\left(M^a - f^{abc}C^b\phi^c\right) - \bar{\phi}^{\star a}\left(\bar{M}^a - \omega^a\right) \\ & - e^{\star a}\left[N^a - \lambda^a\right] - b^{\star a}\chi^a \\ & + \xi_\mu^{\star a}\left(L_\mu^a - \left[\left(D_\mu\phi\right)^a + f^{abc}C^b\xi_\mu^c\right]\right) \\ & + \bar{\xi}_\mu^{\star a}\left[\bar{L}_\mu^a - h_\mu^a\right]. \end{aligned} \tag{13}$$

The gauge-fixing and ghost terms of the Lagrangian density are BRST exact and can be expressed in terms of a general gauge-fixing fermion Ψ as

$$\begin{aligned} s\Psi = {} & sA_\mu^a\frac{\delta\Psi}{\delta A_\mu^a} + sC^a\frac{\delta\Psi}{\delta C^a} + s\bar{C}^a\frac{\delta\Psi}{\delta\bar{C}^a} + sb^a\frac{\delta\Psi}{\delta b^a} \\ & + s\xi_\mu^a\frac{\delta\Psi}{\delta\xi_\mu^a} + s\bar{\xi}_\mu^a\frac{\delta\Psi}{\delta\bar{\xi}_\mu^a} + s\phi^a\frac{\delta\Psi}{\delta\phi^a} + s\bar{\phi}^a\frac{\delta\Psi}{\delta\bar{\phi}^a} \\ & + se^a\frac{\delta\Psi}{\delta e^a} \\ = {} & -\frac{\delta\Psi}{\delta A_\mu^a}\psi_\mu^a + \frac{\delta\Psi}{\delta C^a}\epsilon^a + \frac{\delta\Psi}{\delta\bar{C}^a}\bar{\epsilon}^a - \frac{\delta\Psi}{\delta b^a}\chi^a - \frac{\delta\Psi}{\delta\xi_\mu^a}L_\mu^a \\ & - \frac{\delta\Psi}{\delta\bar{\xi}_\mu^a}\bar{L}_\mu^a - \frac{\delta\Psi}{\delta\phi^a}M^a - \frac{\delta\Psi}{\delta\bar{\phi}^a}\bar{M}^a - \frac{\delta\Psi}{\delta e^a}N^a. \end{aligned} \tag{14}$$

After integrating out the auxiliary fields which set the tilde fields to zero, we have the complete effective action for BF model in Landau gauge possessing extended BRST symmetry as

$$\begin{aligned} \mathscr{L}_{\text{eff}} = {} & \mathscr{L}_0 + \mathscr{L}_{gf+gh} + \widetilde{\mathscr{L}}_{gf+gh} \\ = {} & \mathscr{L}_0 + \left(-A_\mu^{\star a} - \frac{\delta\Psi}{\delta A^{\mu a}}\right)\psi^{\mu a} \\ & + \left(\bar{C}^{\star a} + \frac{\delta\Psi}{\delta C^a}\right)\epsilon^a + \left(C^{\star a} + \frac{\delta\Psi}{\delta\bar{C}^a}\right)\bar{\epsilon}^a \\ & - \left(b^{\star a} + \frac{\delta\Psi}{\delta b^a}\right)\chi^a + \left(\xi_\mu^{\star a} + \frac{\delta\Psi}{\delta\xi_\mu^a}\right)L_\mu^a \\ & + \left(\bar{\xi}_\mu^{\star a} + \frac{\delta\Psi}{\delta\bar{\xi}_\mu^a}\right)\bar{L}_\mu^a - \left(\phi^{\star a} + \frac{\delta\Psi}{\delta\phi^a}\right)M^a \end{aligned}$$

$$
\begin{aligned}
&-\left(\overline{\phi}^{\star a}+\frac{\delta\Psi}{\delta\overline{\phi}^{a}}\right)\overline{M}^{a}+\left(-e^{\star a}-\frac{\delta\Psi}{\delta e^{a}}\right)N^{a}\\
&+A_{\mu}^{a\star}\left(D^{\mu}C^{a}\right)^{a}-\frac{\overline{C}^{a\star}}{2}f^{abc}C^{b}\xi_{\mu}^{c}+C^{\star a}b^{a}\\
&+\xi_{\mu}^{\star a}\left[\left(D_{\mu}\phi\right)^{a}+f^{abc}C^{b}\xi_{\mu}^{c}\right]+\phi^{\star a}f^{abc}C^{b}\xi_{\mu}^{c}.
\end{aligned}
\tag{15}
$$

Integrating out the ghost fields associated with shift symmetry, we obtain

$$
\begin{aligned}
A_{\mu}^{a\star}&=-\frac{\delta\Psi}{\delta A^{\mu a}},\\
\overline{C}^{\star a}&=-\frac{\delta\Psi}{\delta C^{a}},\\
C^{\star a}&=-\frac{\delta\Psi}{\delta\overline{C}^{a}},\\
b^{\star a}&=-\frac{\delta\Psi}{\delta b^{a}},\\
\xi_{\mu}^{\star a}&=-\frac{\delta\Psi}{\delta\xi_{\mu}^{\star a}},\\
\overline{\xi}_{\mu}^{\star a}&=-\frac{\delta\Psi}{\delta\overline{\xi}_{\mu}^{\star a}},\\
\phi^{\star a}&=-\frac{\delta\Psi}{\delta\phi^{a}},\\
\overline{\phi}^{\star a}&=-\frac{\delta\Psi}{\delta\overline{\phi}^{a}},\\
e^{\star a}&=-\frac{\delta\Psi}{\delta e^{a}}.
\end{aligned}
\tag{16}
$$

For a particular choice of gauge-fixing fermion Ψ given in (4), anti-ghost fields get following identifications:

$$
\begin{aligned}
A_{\mu}^{a\star}&=\partial_{\mu}\overline{C}^{a},\\
\overline{C}^{a\star}&=0,\\
C^{a\star}&=-\partial_{\mu}A^{\mu a},\\
b^{a\star}&=0,\\
\xi_{\mu}^{\star a}&=\partial_{\mu}\overline{\phi}^{a},\\
\overline{\xi}_{\mu}^{\star a}&=-\partial^{\nu}B_{\mu\nu}^{a}-\partial_{\mu}e^{a},\\
\phi^{\star a}&=0,\\
\overline{\phi}^{\star a}&=-\partial^{\mu}\xi_{\mu}^{a},\\
e^{\star a}&=\omega^{a}+\partial^{\mu}\overline{\xi}_{\mu}^{a}.
\end{aligned}
\tag{17}
$$

It is obvious to see that, with these anti-ghost fields, the expression (15) changes to the original Lagrangian density of the BF model in Landau gauge.

4. Extended BRST Invariant Superspace Description

In this section, the Lagrangian density of BF model which is invariant under the extended BRST transformations only is described in a superspace (x_{μ},θ), where θ is a Grassmann coordinate and x_{μ} is the four-dimensional spect-time coordinates. In order to give superspace description for the extended BRST invariant theory, we first define superfields of the form

$$
\begin{aligned}
&A_{\mu}^{a}(x,\theta)=A_{\mu}^{a}+\theta\psi_{\mu}^{a},\\
&\widetilde{A}_{\mu}^{a}(x,\theta)=\widetilde{A}_{\mu}^{a}+\theta\left[\psi_{\mu}-\left(D_{\mu}-\widetilde{D}_{\mu}\right)\left(C-\widetilde{C}\right)\right]^{a},\\
&\chi^{a}(x,\theta)=C^{a}+\theta\epsilon^{a},\\
&\widetilde{\chi}^{a}(x,\theta)=\widetilde{C}^{a}+\theta\left[\epsilon^{a}-\frac{1}{2}f^{abc}\left(C^{b}-\widetilde{C}^{b}\right)\left(\xi_{\mu}^{c}-\widetilde{\xi}_{\mu}^{c}\right)\right],\\
&\overline{\chi}^{a}(x,\theta)=\overline{C}^{a}+\theta\overline{\epsilon}^{a},\\
&\widetilde{\overline{\chi}}^{a}(x,\theta)=\widetilde{\overline{C}}^{a}+\theta\left[\overline{\epsilon}^{a}-\left(b-\widetilde{b}\right)^{a}\right],\\
&b^{a}(x,\theta)=b^{a}+\theta\chi^{a},\\
&\widetilde{b}^{a}(x,\theta)=\widetilde{b}^{a}+\theta\chi^{a},\\
&\xi_{\mu}^{a}(x,\theta)=\xi_{\mu}^{a}+\theta L_{\mu}^{a},\\
&\widetilde{\xi}_{\mu}^{a}(x,\theta)=\widetilde{\xi}_{\mu}^{a}+\theta\left[L_{\mu}^{a}-\left[\left(D_{\mu}-\widetilde{D}_{\mu}\right)\left(\phi-\widetilde{\phi}\right)^{a}\right.\right.\\
&\qquad\left.\left.+f^{abc}\left(C^{b}-\widetilde{C}^{b}\right)\left(\xi_{\mu}^{c}-\widetilde{\xi}_{\mu}^{c}\right)\right]\right],\\
&\overline{\xi}_{\mu}^{a}(x,\theta)=\overline{\xi}_{\mu}^{a}+\theta\overline{L}_{\mu}^{a},\\
&\widetilde{\overline{\xi}}_{\mu}^{a}(x,\theta)=\widetilde{\overline{\xi}}_{\mu}^{a}+\theta\left[\overline{L}_{\mu}^{a}-\left(h_{\mu}^{a}-\widetilde{h}_{\mu}^{a}\right)\right],\\
&\phi^{a}(x,\theta)=\phi^{a}+\theta M^{a},\\
&\widetilde{\phi}^{a}(x,\theta)=\widetilde{\phi}^{a}+\theta\left[M^{a}-f^{abc}\left(C^{b}-\widetilde{C}^{b}\right)\left(\phi^{c}-\widetilde{\phi}^{c}\right)\right],\\
&\overline{\phi}^{a}(x,\theta)=\overline{\phi}^{a}+\theta\overline{M}^{a},\\
&\widetilde{\overline{\phi}}^{a}(x,\theta)=\widetilde{\overline{\phi}}^{a}+\theta\left[\overline{M}^{a}-\left(\omega^{a}-\widetilde{\omega}^{a}\right)\right],\\
&e^{a}(x,\theta)=e^{a}+\theta N^{a},\\
&\widetilde{e}^{a}(x,\theta)=\widetilde{e}^{a}+\theta\left[N^{a}-\lambda^{a}+\widetilde{\lambda}^{a}\right].
\end{aligned}
\tag{18}
$$

The super-antifields in superspace are defined as follows:

$$
\begin{aligned}
\widetilde{A}_{\mu}^{\star a}(x,\theta)&=A_{\mu}^{\star a}-\theta\zeta_{\mu}^{a},\\
\widetilde{\chi}^{\star a}(x,\theta)&=C^{\star a}-\theta\sigma^{a},\\
\widetilde{\overline{\chi}}^{\star a}(x,\theta)&=\overline{C}^{\star a}-\theta\overline{\sigma}^{a},
\end{aligned}
$$

$$\begin{aligned}
\tilde{b}^{\star a}(x,\theta) &= b^{\star a} - \theta\varpi^a,\\
\tilde{\xi}^{\star a}_\mu(x,\theta) &= \xi^{\star a}_\mu - \theta\kappa^a,\\
\widetilde{\overline{\xi}}^{\star a}_\mu(x,\theta) &= \overline{\xi}^{\star a}_\mu - \theta\overline{\kappa}^a,\\
\tilde{\phi}^{\star a}(x,\theta) &= \phi^{\star a} - \theta\upsilon^a,\\
\widetilde{\overline{\phi}}^{\star a}(x,\theta) &= \overline{\phi}^{\star a} - \theta\overline{\upsilon}^a,\\
\tilde{e}^{\star a}(x,\theta) &= e^{\star a} - \theta\tau^a.
\end{aligned} \tag{19}$$

From the above expressions of superfields and super-antifields, we calculate

$$\begin{aligned}
\frac{\delta\left(\tilde{A}^{a\star}_\mu\tilde{A}^{a\mu}\right)}{\delta\theta} &= -A^{a\star}_\mu\left[\psi^{a\mu} - \left(D^\mu - \tilde{D}^\mu\right)\left(C - \tilde{C}\right)^a\right] - \zeta^a_\mu\tilde{A}^{a\mu},\\
\frac{\delta\left(\tilde{\overline{\chi}}^{a\star}\tilde{\chi}^a\right)}{\delta\theta} &= \overline{C}^{a\star}\left[\epsilon^a - \frac{1}{2}f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi^c_\mu - \tilde{\xi}^c_\mu\right)\right] - \overline{\sigma}^a\tilde{C}^a,\\
\frac{\delta\left(\tilde{\overline{\chi}}^a\tilde{\chi}^{a\star}\right)}{\delta\theta} &= -\sigma^a\widetilde{\overline{C}}^a + C^{a\star}\left[\overline{\epsilon}^a - \left(b^a - \tilde{b}^a\right)\right],\\
\frac{\delta\left(\tilde{b}^{a\star}\tilde{b}^a\right)}{\delta\theta} &= -b^{a\star}\chi^a - \varpi^a\tilde{b}^a,\\
\frac{\delta\left(\tilde{\xi}^{a\star}_\mu\tilde{\xi}^{a\mu}\right)}{\delta\theta} &= \xi^{a\star}_\mu\left[L^a_\mu - \left[\left(D^\mu - \tilde{D}^\mu\right)\left(\phi - \tilde{\phi}\right)^a + f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi^c_\mu - \tilde{\xi}^c_\mu\right)\right]\right] - \kappa^a\tilde{\xi}^a_\mu,\\
\frac{\delta\left(\widetilde{\overline{\xi}}^{a\star}_\mu\widetilde{\overline{\xi}}^{a\mu}\right)}{\delta\theta} &= \overline{\xi}^{a\star}_\mu\left[L^{\mu a} - h^{\mu a} + \tilde{h}^{\mu a}\right] - \overline{\kappa}^a\widetilde{\overline{\xi}}^{a\mu},\\
\frac{\delta\left(\tilde{\phi}^{a\star}\tilde{\phi}^a\right)}{\delta\theta} &= -\tilde{\phi}^a\upsilon^a - \phi^{a\star}\left[M^a - f^{abc}\left(C^b - \tilde{C}^b\right)\left(\phi^c - \tilde{\phi}^c\right)\right],\\
\frac{\delta\left(\widetilde{\overline{\phi}}^{a\star}\widetilde{\overline{\phi}}^a\right)}{\delta\theta} &= -\widetilde{\overline{\phi}}^a\overline{\upsilon}^a - \overline{\phi}^{a\star}\left[\overline{M}^a - \omega^a + \tilde{\omega}^a\right],\\
\frac{\delta\left(\tilde{e}^{a\star}\tilde{e}^a\right)}{\delta\theta} &= -e^\star\left[N^a - \lambda^a + \tilde{\lambda}^a\right] - \tilde{e}^a\tau^a.
\end{aligned} \tag{20}$$

Adding all the equations of (20) side by side, we get

$$\begin{aligned}
&\frac{\delta}{\delta\theta}\left(\tilde{A}^{a\star}_\mu\tilde{A}^{a\mu} + \widetilde{\overline{\chi}}^{a\star}\tilde{\chi}^a + \widetilde{\overline{\chi}}^a\tilde{\chi}^{a\star} + \tilde{b}^{a\star}\tilde{b}^a + \tilde{\xi}^{a\star}_\mu\tilde{\xi}^{a\mu} + \widetilde{\overline{\xi}}^{a\star}_\mu\widetilde{\overline{\xi}}^{a\mu} + \tilde{\phi}^{a\star}\tilde{\phi}^a + \widetilde{\overline{\phi}}^{a\star}\widetilde{\overline{\phi}}^a + \tilde{e}^{a\star}\tilde{e}^a\right) = -\zeta^{a\mu}\tilde{A}^a_\mu\\
&- A^{a\star}_\mu\left[\psi^{a\mu} - \left(D^\mu - \tilde{D}^\mu\right)\left(C - \tilde{C}\right)^a\right] - \overline{\sigma}^a\tilde{C}^a + \overline{C}^{a\star}\left[\epsilon^a - \frac{1}{2}f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi^a_\mu - \tilde{\xi}^a_\mu\right)\right] - \sigma^a\widetilde{\overline{C}}^a\\
&+ C^{a\star}\left[\overline{\epsilon}^a - \left(b^a - \tilde{b}^a\right)\right] - \upsilon^a\tilde{\phi}^a - \phi^{\star a}\left[M^a - f^{abc}\left(C^b - \tilde{C}^b\right)\left(\phi^c - \tilde{\phi}^c\right)\right] - \overline{\upsilon}^a\widetilde{\overline{\phi}}^a - \overline{\phi}^{\star a}\left(\overline{M}^a - \omega^a + \tilde{\omega}^a\right) - \tau^a\tilde{e}^a - e^\star\left[N^a - \lambda^a + \tilde{\lambda}^a\right] - \varpi^a\tilde{b}^a\\
&- b^{\star a}\chi^a - \kappa^a\tilde{\xi}^a_\mu - \overline{\kappa}^a\widetilde{\overline{\xi}}^a_\mu + \xi^{\star a}_\mu\left(L^a_\mu - \left[\left(D_\mu - \tilde{D}_\mu\right)\left(\phi - \tilde{\phi}\right)^a + f^{abc}\left(C^b - \tilde{C}^b\right)\left(\xi^c_\mu - \tilde{\xi}^c_\mu\right)\right]\right) + \overline{\xi}^{\star a}_\mu\left[\overline{L}^a_\mu - \left(h^a_\mu - \tilde{h}^a_\mu\right)\right],
\end{aligned} \tag{21}$$

which is nothing but the gauge-fixed Lagrangian density for shift symmetry $\mathscr{L}_{gf+gh}$ given in (12). Now, one can define the general super-gauge-fixing fermion in superspace as follows:

$$\Phi(x,\theta) = \Psi(x) + \theta(s\Psi), \tag{22}$$

which can further be expressed as

$$\Phi(x,\theta) = \Psi(x) + \theta\left[-\frac{\delta\Psi}{\delta A^a_\mu}\psi^a_\mu + \frac{\delta\Psi}{\delta C^a}\epsilon^a + \frac{\delta\Psi}{\delta\overline{C}^a}\overline{\epsilon}^a - \frac{\delta\Psi}{\delta b^a}\chi^a - \frac{\delta\Psi}{\delta\xi^a_\mu}L^a_\mu - \frac{\delta\Psi}{\delta\overline{\xi}^a_\mu}\overline{L}^a_\mu - \frac{\delta\Psi}{\delta\phi^a}M^a - \frac{\delta\Psi}{\delta\overline{\phi}^a}\overline{M}^a - \frac{\delta\Psi}{\delta e^a}N^a\right]. \tag{23}$$

From this, the original gauge-fixing Lagrangian density can be defined as the left derivation of super-gauge-fixing fermion with respect to θ as $[\delta\Phi(x,\theta)/\delta\theta]$.

Hence, the complete effective action for the BF model in general gauge in the superspace is now given by

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_0 + \frac{\delta}{\delta\theta}\left[\tilde{A}^{a\star}_\mu\tilde{A}^{a\mu} + \widetilde{\overline{\chi}}^{a\star}\tilde{\chi}^a + \widetilde{\overline{\chi}}^a\tilde{\chi}^{a\star} + \tilde{b}^{a\star}\tilde{b}^a + \tilde{\chi}^{a\star}_\mu\tilde{\chi}^{a\mu} + \widetilde{\overline{\chi}}^{a\star}_\mu\widetilde{\overline{\chi}}^{a\mu} + \tilde{\phi}^{a\star}\tilde{\phi}^a + \widetilde{\overline{\phi}}^{a\star}\widetilde{\overline{\phi}}^a + \tilde{e}^{a\star}\tilde{e}^a + \Phi\right]. \tag{24}$$

Next, we will study the extended anti-BRST symmetry for BF model.

5. Extended Anti-BRST Lagrangian Density

In this section, we construct the extended anti-BRST transformation under which the shifted Lagrangian density for BF model remains invariant as follows:

$$
\begin{aligned}
\bar{s}A_\mu^a &= A_\mu^{a\star} + \left(D_\mu - \widetilde{D}_\mu\right)\left(\overline{C} - \widetilde{\overline{C}}\right)^a, \\
\bar{s}\widetilde{A}_\mu^a &= A_\mu^{a\star}, \\
\bar{s}C^a &= C^{a\star} - \frac{1}{2} f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi_\mu^c - \widetilde{\xi}_\mu^c\right), \\
\bar{s}\widetilde{C}^a &= C^{a\star}, \\
\bar{s}\overline{C}^a &= \overline{C}^{a\star} - \left(b^a - \widetilde{b}^a\right), \\
\bar{s}\widetilde{\overline{C}}^a &= \overline{C^{a\star}}, \\
\bar{s}b^a &= b^{a\star} + \chi^a, \\
\bar{s}\widetilde{b}^a &= b^{a\star}, \\
\bar{s}\xi_\mu^a &= \xi_\mu^{a\star} - \left[\left(D_\mu - \widetilde{D}_\mu\right)\left(\phi^a - \widetilde{\phi}^a\right)\right. \\
&\quad \left. + f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi_\mu^c - \widetilde{\xi}_\mu^c\right)\right], \\
\bar{s}\widetilde{\xi}_\mu^a &= \xi_\mu^{a\star}, \\
\bar{s}\overline{\xi}_\mu^a &= \overline{\xi}_\mu^{a\star} - h_\mu^a + \widetilde{h}_\mu^a, \\
\bar{s}\widetilde{\overline{\xi}}_\mu^a &= \overline{\xi}_\mu^{a\star}, \\
\bar{s}\phi^a &= \phi^{a\star} - f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\phi^c - \widetilde{\phi}^c\right), \\
\bar{s}\widetilde{\phi}^a &= \phi^{a\star}, \\
\bar{s}\overline{\phi}^a &= \overline{\phi}^{a\star} - \omega^a + \widetilde{\omega}^a, \\
\bar{s}\widetilde{\overline{\phi}}^a &= \overline{\phi}^{a\star}, \\
\bar{s}e^a &= e^{a\star} - \left(\lambda^a - \widetilde{\lambda}^a\right), \\
\bar{s}\widetilde{e}^a &= e^{a\star}.
\end{aligned}
\tag{25}
$$

The ghost fields associated with the shift symmetry transform under extended anti-BRST symmetry as

$$
\begin{aligned}
\bar{s}\psi_\mu^a &= \zeta_\mu^a, \\
\bar{s}\epsilon^a &= \sigma^a, \\
\bar{s}\overline{\epsilon}^a &= \overline{\sigma}^a, \\
\bar{s}\chi^a &= \varpi^a, \\
\bar{s}L_\mu^a &= \kappa^a, \\
\bar{s}\overline{L}_\mu^a &= \overline{\kappa}^a, \\
\bar{s}M_\mu^a &= \upsilon^a, \\
\bar{s}\overline{M}_\mu^a &= \overline{\upsilon}^a, \\
\bar{s}N_\mu^a &= \tau^a.
\end{aligned}
\tag{26}
$$

The nilpotency of above transformations demands the auxiliary and antighost fields associated with the shift symmetry transform as

$$
\begin{aligned}
\bar{s}\zeta_\mu^a &= 0, \\
\bar{s}A_\mu^{a\star} &= 0, \\
\bar{s}\sigma^a &= 0, \\
\bar{s}C^{a\star} &= 0, \\
\bar{s}\overline{\sigma}^a &= 0, \\
\bar{s}\overline{C}^{a\star} &= 0, \\
\bar{s}\varpi^a &= 0, \\
\bar{s}b^{a\star} &= 0, \\
\bar{s}\kappa^a &= 0, \\
\bar{s}\xi_\mu^{a\star} &= 0, \\
\bar{s}\overline{\kappa}^a &= 0, \\
\bar{s}\overline{\xi}_\mu^{a\star} &= 0, \\
\bar{s}\upsilon^a &= 0, \\
\bar{s}\phi^{a\star} &= 0, \\
\bar{s}\overline{\upsilon}^a &= 0, \\
\bar{s}\overline{\phi}^{a\star} &= 0, \\
\bar{s}\tau^a &= 0, \\
\bar{s}e^{a\star} &= 0.
\end{aligned}
\tag{27}
$$

The gauge-fixing and ghost parts of the effective Lagrangian density are anti-BRST-exact also so it can be expressed as the anti-BRST variation of this gauge-fixing fermion ($\overline{\Psi}$).

6. Extended BRST and Anti-BRST Invariant Superspace

The extended BRST and anti-BRST invariant Lagrangian density for BF model can be written in superspace with the help of two additional Grassmannian coordinates θ and $\overline{\theta}$.

Requiring the field strength to vanish along unphysical directions θ and $\overline{\theta}$ direction, we obtain the following superfields:

$$
\begin{aligned}
&\mathbf{A}^a_\mu\left(x,\theta,\overline{\theta}\right) = A^a_\mu\left(x\right) + \theta\psi^a_\mu + \overline{\theta}\left[A^{a\star}_\mu + \left(D_\mu - \widetilde{D}_\mu\right)\right. \\
&\quad \left.\cdot\left(\overline{C} - \widetilde{\overline{C}}\right)^a\right] + \theta\overline{\theta}\zeta^a_\mu, \\
&\widetilde{\mathbf{A}}^a_\mu\left(x,\theta,\overline{\theta}\right) = \widetilde{A}^a_\mu\left(x\right) + \theta\left[\psi^a_\mu - \left(D_\mu - \widetilde{D}_\mu\right)\left(C - \widetilde{C}\right)^a\right] \\
&\quad + \overline{\theta}A^{a\star}_\mu + \theta\overline{\theta}\zeta^a_\mu, \\
&\mathbf{C}^a\left(x,\theta,\overline{\theta}\right) = C^a\left(x\right) + \theta\epsilon^a + \overline{\theta}\left[C^{a\star} - \frac{1}{2}f^{abc}\left(C^b\right.\right. \\
&\quad \left.\left. - \widetilde{C}^b\right)\left(\xi^c_\mu - \widetilde{\xi}^c_\mu\right)\right] + \theta\overline{\theta}\sigma^a, \\
&\widetilde{\mathbf{C}}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{C}^a\left(x\right) + \theta\left[\epsilon^a - \frac{1}{2}f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi^c_\mu\right.\right. \\
&\quad \left.\left. - \widetilde{\xi}^c_\mu\right)\right] + \overline{\theta}C^{\star a} + \theta\overline{\theta}\sigma^a, \\
&\overline{\mathbf{C}}^a\left(x,\theta,\overline{\theta}\right) = \overline{C}^a\left(x\right) + \theta\overline{\epsilon}^a + \overline{\theta}\left[\overline{C}^{\star a} - \left(b - \widetilde{b}\right)^a\right] \\
&\quad + \theta\overline{\theta}\overline{\sigma}^a, \\
&\widetilde{\overline{\mathbf{C}}}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{\overline{C}}^a\left(x\right) + \theta\left[\overline{\epsilon}^a - \left(b - \widetilde{b}\right)^a\right] + \overline{\theta}\overline{C}^{\star a} \\
&\quad + \theta\overline{\theta}\overline{\sigma}^a, \\
&\mathbf{b}^a\left(x,\theta,\overline{\theta}\right) = b^a\left(x\right) + \theta\chi^a + \overline{\theta}\left(b^{\star a} + \chi^a\right) + \theta\overline{\theta}\varpi^a, \\
&\widetilde{\mathbf{b}}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{b}^a\left(x\right) + \theta\chi^a + \overline{\theta}b^{\star a} + \theta\overline{\theta}\varpi^a, \\
&\xi^a_\mu\left(x,\theta,\overline{\theta}\right) = \xi^a_\mu\left(x\right) + \theta L^a_\mu + \overline{\theta}\left(\xi^{a\star}_\mu\right. \\
&\quad - \left[\left(D_\mu - \widetilde{D}_\mu\right)\left(\phi^a - \widetilde{\phi}^a\right)\right. \\
&\quad \left.\left. + f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi^c_\mu - \widetilde{\xi}^c_\mu\right)\right]\right) + \theta\overline{\theta}\kappa^a, \\
&\widetilde{\xi}^a_\mu\left(x,\theta,\overline{\theta}\right) = \widetilde{\xi}^a_\mu\left(x\right) + \theta\left[L^a_\mu - \left(D_\mu - \widetilde{D}_\mu\right)\left(\phi - \widetilde{\phi}\right)^a\right. \\
&\quad \left. + f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi^c_\mu - \widetilde{\xi}^c_\mu\right)\right] + \overline{\theta}\xi^{a\star}_\mu + \theta\overline{\theta}\kappa^a, \\
&\overline{\xi}^a_\mu\left(x,\theta,\overline{\theta}\right) = \overline{\xi}^a_\mu\left(x\right) + \theta\overline{L}^a_\mu + \overline{\theta}\left(\overline{\xi}^{\star a}_\mu - h^a_\mu + \widetilde{h}^a_\mu\right) \\
&\quad + \theta\overline{\theta}\overline{\kappa}^a, \\
&\widetilde{\overline{\xi}}^a_\mu\left(x,\theta,\overline{\theta}\right) = \widetilde{\overline{\xi}}^a_\mu\left(x\right) + \theta\left(\overline{L}^a_\mu - h^a_\mu + \widetilde{h}^a_\mu\right) + \overline{\theta}\overline{\xi}^{\star a}_\mu \\
&\quad + \theta\overline{\theta}\overline{\kappa}^a, \\
&\phi^a\left(x,\theta,\overline{\theta}\right) = \phi^a\left(x\right) + \theta M^a + \overline{\theta}\left(\phi^{a\star} - f^{abc}\left(C^b\right.\right. \\
&\quad \left.\left. - \widetilde{C}^b\right)\left(\phi^c - \widetilde{\phi}^c\right)\right) + \theta\overline{\theta}\upsilon^a, \\
&\widetilde{\phi}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{\phi}^a\left(x\right) + \theta\left(M^a - f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\phi^c\right.\right. \\
&\quad \left.\left. - \widetilde{\phi}^c\right)\right) + \overline{\theta}\phi^{\star a} + \theta\overline{\theta}\upsilon^a, \\
&\overline{\phi}^a\left(x,\theta,\overline{\theta}\right) = \overline{\phi}^a\left(x\right) + \theta\overline{M}^a + \overline{\theta}\left(\overline{\phi}^{\star a} - \omega^a + \widetilde{\omega}^a\right) \\
&\quad + \theta\overline{\theta}\overline{\upsilon}^a, \\
&\widetilde{\overline{\phi}}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{\overline{\phi}}^a\left(x\right) + \theta\left(\overline{M}^a - \omega^a + \widetilde{\omega}^a\right) + \overline{\theta}\overline{\phi}^{\star a} \\
&\quad + \theta\overline{\theta}\overline{\upsilon}^a, \\
&\mathbf{e}^a\left(x,\theta,\overline{\theta}\right) = e^a\left(x\right) + \theta N^a + \overline{\theta}\left(e^{\star a} - \lambda^a + \widetilde{\lambda}^a\right) \\
&\quad + \theta\overline{\theta}\tau^a, \\
&\widetilde{\mathbf{e}}^a\left(x,\theta,\overline{\theta}\right) = \widetilde{e}^a\left(x\right) + \theta\left(N^a - \lambda^a + \widetilde{\lambda}^a\right) + \overline{\theta}e^{\star a} \\
&\quad + \theta\overline{\theta}\tau^a.
\end{aligned}
\tag{28}
$$

With these expressions of superfields, we can calculate

$$
\begin{aligned}
&-\frac{1}{2}\frac{\partial}{\partial\overline{\theta}}\frac{\partial}{\partial\theta}\left(\widetilde{\mathbf{A}}^a_\mu\widetilde{\mathbf{A}}^{\mu a} + \widetilde{\chi}^a\widetilde{\overline{\chi}}^a + \widetilde{\mathbf{b}}^a\widetilde{\mathbf{b}}^a + \widetilde{\xi}^a_\mu\widetilde{\xi}^{\mu a} + \widetilde{\overline{\xi}}^a_\mu\widetilde{\overline{\xi}}^{\mu a}\right. \\
&\quad \left. + \widetilde{\phi}^a\widetilde{\phi}^a + \widetilde{\overline{\phi}}^a\widetilde{\overline{\phi}}^a + \widetilde{\mathbf{e}}^a\widetilde{\mathbf{e}}^a\right) = -\zeta^{a\mu}\widetilde{A}^a_\mu - A^{a\star}_\mu\left[\psi^{a\mu}\right. \\
&\quad \left. - \left(D^\mu - \widetilde{D}^\mu\right)\left(C - \widetilde{C}\right)^a\right] - \overline{\sigma}^a\widetilde{C}^a + \overline{C}^{a\star}\left[\epsilon^a - \frac{1}{2}\right. \\
&\quad \left.\cdot f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi^a_\mu - \widetilde{\xi}^a_\mu\right)\right] - \sigma^a\widetilde{\overline{C}}^a + C^{a\star}\left[\overline{\epsilon}^a\right. \\
&\quad \left. - \left(b^a - \widetilde{b}^a\right)\right] - \upsilon^a\widetilde{\phi}^a - \phi^{\star a}\left[M^a - f^{abc}\left(C^b - \widetilde{C}^b\right)\right. \\
&\quad \left.\cdot\left(\phi^c - \widetilde{\phi}^c\right)\right] - \overline{\upsilon}^a\widetilde{\overline{\phi}}^a - \overline{\phi}^{\star a}\left(\overline{M}^a - \omega^a + \widetilde{\omega}^a\right) - \tau^a\widetilde{e}^a \\
&\quad - e^\star\left[N^a - \lambda^a + \widetilde{\lambda}^a\right] - \varpi^a\widetilde{b}^a - b^{\star a}\chi^a - \kappa^a\widetilde{\xi}^a_\mu \\
&\quad - \overline{\kappa}^a\widetilde{\overline{\xi}}^a_\mu + \xi^{\star a}_\mu\left(L^a_\mu - \left[\left(D_\mu - \widetilde{D}_\mu\right)\left(\phi - \widetilde{\phi}\right)^a\right.\right. \\
&\quad \left.\left. + f^{abc}\left(C^b - \widetilde{C}^b\right)\left(\xi^c_\mu - \widetilde{\xi}^c_\mu\right)\right]\right) + \overline{\xi}^{\star a}_\mu\left[\overline{L}^a_\mu - \left(h^a_\mu\right.\right. \\
&\quad \left.\left. - \widetilde{h}^a_\mu\right)\right],
\end{aligned}
\tag{29}
$$

which is nothing but the gauge-fixed Lagrangian density for shift symmetry. Being the $\theta\overline{\theta}$ component of a superfield, this Lagrangian density is manifestly invariant under both the extended BRST and the anti-BRST transformations.

Now, we define the general super-gauge-fixing fermion in superspace as

$$
\Phi\left(x,\theta,\overline{\theta}\right) = \Psi\left(x\right) + \theta\left(s\Psi\right) + \overline{\theta}\left(\overline{s}\Psi\right) + \theta\overline{\theta}\left(s\overline{s}\Psi\right), \tag{30}
$$

which yields the original gauge-fixing and ghost part of the effective Lagrangian density upon differentiation as follows: $\mathrm{Tr}[(\partial/\partial\theta)[\delta(\overline{\theta})\Phi(x,\theta,\overline{\theta})]]$.

Therefore, the gauge-fixed Lagrangian density corresponding to BRST and shift symmetries for BF model can now be given as

$$\begin{aligned}\mathscr{L}_{gf+gh}+\widetilde{\mathscr{L}}_{gf+gh}=&-\frac{1}{2}\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta}\left(\widetilde{\mathbf{A}}_{\mu}^{a}\widetilde{\mathbf{A}}^{\mu a}+\widetilde{\chi}^{a}\widetilde{\widetilde{\chi}}^{a}\right.\\&\left.+\widetilde{\mathbf{b}}^{a}\widetilde{\mathbf{b}}^{a}+\widetilde{\xi}_{\mu}^{a}\widetilde{\xi}^{\mu a}+\widetilde{\widetilde{\xi}}_{\mu}^{a}\widetilde{\widetilde{\xi}}^{\mu a}+\widetilde{\phi}^{a}\widetilde{\phi}^{a}+\widetilde{\widetilde{\phi}}^{a}\widetilde{\widetilde{\phi}}^{a}+\widetilde{\mathbf{e}}^{a}\widetilde{\mathbf{e}}^{a}\right)\\&+\frac{\partial}{\partial\theta}\left[s\left(\bar{\theta}\right)\Phi\left(x,\theta,\bar{\theta}\right)\right].\end{aligned}\tag{31}$$

Therefore, we see that the BF model in superspace can be expressed in an elegant manner.

7. Conclusion

The $(3+1)$ dimensional BF model is a subject of great interest due to its topological nature and its intriguing properties. In present work, we have considered $(3+1)$ dimensional BF model in Landau gauge and then we have shifted the Lagrangian to obtain the extended BRST and anti-BRST invariant (including some shift symmetry) BF model in BV formulation. The antifields corresponding to each field naturally arise. Further we have provide the superfield description of BF model in superspace, where we show that the BV action for BF model can be written in a manifestly extended BRST invariant manner in a superspace by considering one additional Grassmann (fermionic) coordinate. However, we need two additional Grassmann coordinates to express both the extended BRST and extended anti-BRST invariant BV actions of BF model in superspace.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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