

## Research Article

# A Multiquark Approach to Excited Hadrons and Regge Trajectories

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We propose a novel approach to construction of hadron spectroscopy. The case of light nonstrange mesons is considered. By assumption, all such mesons above 1 GeV appear due to creation of constituent quark-antiquark pairs inside  $\pi$  or  $\rho(\omega)$  mesons. These spin-singlet or triplet pairs dictate the quantum numbers of formed resonance. The resulting classification of light mesons turns out to be in a better agreement with the experimental observations than the standard quark model classification. It is argued that the total energy of quark components should be proportional to the hadron mass squared rather than the linear mass. As a byproduct a certain relation expressing the constituent quark mass via the gluon and quark condensate is put forward. We show that our approach leads to an effective mass counting scheme for meson spectrum and results in the linear Regge and radial Regge trajectories by construction. An experimental observation of these trajectories might thus serve as evidence not for string but for multiquark structure of highly excited hadrons.

## 1. Introduction

The regularities in the masses of atomic nuclei tell us that nuclei certainly consists of some building blocks, the nucleons, and the mass of nuclei is just proportional to the number of nucleons. The masses squared of light mesons also reveal many regularities [1, 2] but they are usually attributed to the properties of strong interactions acting between valent quarks. In the present paper, we will try to develop an alternative point of view on the masses of light hadrons: within the presented scheme, they could be also viewed as originating from some effective “building blocks”.

Historically the main basic framework for description of hadrons is the quark model. This model had a great success in making order in numerous hadronic zoo. In particular, all mesons within this picture are composed of a quark and antiquark having the relative angular momentum  $L$  which dictates the spatial and charge parities for the quark-antiquark systems,  $P = (-1)^{L+1}$  and  $C = (-1)^{L+S}$ , where  $S$  is the total quark spin. By now we know that the ensuing classification of mesons has many phenomenological flaws (see, e.g., the corresponding reviews in the Particle Data [3]).

For instance, it does not explain the scalar mesons below 1 GeV for which a tetraquark structure is often assumed [4]. A question arises why do not we see many light multiquark hadrons above 1 GeV? Among other questions one can mention the experimental absence of many predicted states (e.g., with  $J^{PC} = 2^{-}$ ) and, on the other hand, observation of many not predicted states (e.g., too rich scalar sector and exotic  $\pi_1$ ).

There are some deep theoretical questions as well. The light hadrons represent highly relativistic quantum systems in which  $L$  is not a conserved quantity. The quark angular momentum  $L$  is nevertheless a standard ingredient in constructing dynamical models for light hadrons. Another question concerns the observable quantities like hadron masses: they must be renorminvariant in the field-theoretical sense. Stated simply, they must represent some constants independent of energy scale. The quark masses, for instance, are not such constants since they have anomalous dimension described by QCD. A relation of calculated observables to the renormalization invariance is obscure in many known phenomenological models of hadrons.

The quark model represents a concept rather than a model. A real model appears only when a definite interaction between quarks is postulated. Any such model should be able to explain general features of observed hadron spectrum. In the light hadrons, perhaps the most spectacular general feature is the observation of approximately linear Regge and radial trajectories of the kind:

$$m_{J,n}^2 = aJ + a'n + b, \quad J, n = 0, 1, 2, \dots \quad (1)$$

Here  $J$  denotes the spin,  $n$  is the radial quantum number (enumerating the daughter Regge trajectories), and  $a$ ,  $a'$ , and  $b$  are the slope and intercept parameters. There is no general agreement on the position of different states on the trajectories (1) (see, e.g., [1, 2, 5–20]) but the existence of these trajectories seems to be certain for many phenomenologists [12–27]. There are also some indications on a similar behavior in the heavy quark sector [28–32]. The nonrelativistic and partially relativistic potential models cannot explain the recurrences (1) in a natural way. Usually the observation of linear trajectories (1) is interpreted as an evidence for string picture of mesons. In spite of many attempts (see, e.g., [33–40]), however, a satisfactory quantized hadron string has not been constructed. Among typical flaws of this approach one can mention the absence of spontaneous chiral symmetry breaking, rapidly growing (with mass) size of meson excitations that is not supported experimentally, unclear role of higher Fock components in the hadron wavefunction.

The purpose of the present work is to propose a novel realization of quark model concept, a realization that leads to a natural (and alternative to hadron strings) explanation of Regge recurrences (1) and that is potentially free of typical shortcomings inherent to string, potential, and some other approaches.

## 2. Masses and Classification of Light Mesons

In hadron physics, the pion is known to be the most important and best studied meson both experimentally and theoretically. Concerning the theoretical aspect, the pion is the only hadron (along with  $K$  and  $\eta$ ) for which we know a model-independent mass formula: the Gell-Mann–Oakes–Renner (GOR) relation [41],

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{f_\pi^2} (m_u + m_d) = \Lambda \cdot 2m_q, \quad (2)$$

where we set  $m_u = m_d \doteq m_q$  and  $\Lambda \doteq -\langle \bar{q}q \rangle / f_\pi^2$ . We will use the standard values for the quark condensate and masses of current quarks at the scale of the pion mass from the QCD sum rules [42, 43] and Chiral Perturbation Theory (ChPT) [44],  $\langle \bar{q}q \rangle = -(250 \text{ MeV})^3$ ,  $m_u + m_d = 11 \text{ MeV}$ . Using  $f_\pi = 92.4 \text{ MeV}$  (the value of weak pion decay constant), relation (2) yields  $m_\pi = 140 \text{ MeV}$  and  $\Lambda = 1830 \text{ MeV}$ . The famous relation (2) was derived in various approaches assuming the spontaneous chiral symmetry breaking (CSB) in strong interactions.

The renormalization invariance of pion mass follows from the renorminvariance of operator  $m_q \bar{q}q$  in QCD Lagrangian.

Masses of other mesons must be also renorminvariant. On the other hand, the light nonstrange mesons can be viewed, in one way or another, as some quantum excitations of pion. We will assume that the masses of these mesons can be represented as

$$m_h^2 = \Lambda (E_h + 2m_q) = \Lambda E_h + m_\pi^2. \quad (3)$$

Below we will motivate a proportionality of the energy parameter  $E_h$  to the renorminvariant gluon condensate,  $E_h \sim \alpha_s \langle G_{\mu\nu}^2 \rangle / \langle \bar{q}q \rangle$ . But first we would like to develop a phenomenological mass counting scheme based on the relation (3) and demonstrate how it describes the meson spectroscopy.

Our second assumption is that the parameter  $E_h$  can be interpreted as an effective energy of some constituents different from the current quarks. The problem is to propose a model for these constituents which should represent some excitations inside the pion. We postulate three basic excitations. The first one appears when one of quarks absorbs a gluon of certain energy  $E_h = E_\rho$ . The spin of excited quark changes its direction to the opposite one, converting the original spin-singlet  $q\bar{q}$ -pair ( $\pi$ -meson) to the spin-triplet one ( $\rho$ -meson). The second kind of excitation emerges due to formation of spin-singlet  $q\bar{q}$ -pair with effective mass  $E_h = E_0$  (the lower index stays for total spin of  $S$ -wave  $q\bar{q}$ -pair). We will call this pair  $Q_0$ . The third basic excitation is the formation of spin-triplet  $q\bar{q}$ -pair with effective mass  $E_h = E_1$ . This pair will be called  $Q_1$ . The formation of constituent pairs  $Q_0$  and  $Q_1$  may be a QCD analogue of formation of para- and orthopositronia from photons. We assume that any excitation inside pion leading to an observable resonance can be represented as a combination of these basic excitations so that the total effective energy  $E_h$  in (3) is just a sum (appropriate number of times) of  $E_\rho$ ,  $E_0$ , and  $E_1$ . Simultaneously this will dictate the quantum numbers of constructed resonance ( $Q_0$  and  $Q_1$  possess  $J^{PC} = 0^{-+}$  and  $1^{-}$ , respectively). In a sense, the introduction of  $Q_0$  and  $Q_1$  is our model for excitations of higher Fock components in the pion wavefunction.

It is convenient to divide the nonstrange meson resonances above 1 GeV into excited pions and excited  $\rho(\omega)$  mesons. The excited  $\rho$  mesons containing  $n$  pairs  $Q_0$  and  $l$  pairs  $Q_1$  will be labeled as  ${}^J\rho_{Q_0^n Q_1^l}$ , where  $J$  means the total spin. According to our mass counting scheme, masses of these excitations are given by the relation

$$m_{\rho_{Q_0^n Q_1^l}}^2 = \Lambda (E_\rho + nE_0 + lE_1) + m_\pi^2. \quad (4)$$

By assumption, the whole system is in the  $S$ -wave state (one can include into this scheme the higher  $L > 0$  waves; we will consider the simplest possibility); hence, at fixed  $n$  and  $l$ , relation (4) defines mass of a set of degenerate states with various spins up to  $J = l+1$ .  $P$  and  $C$  parities obey the standard multiplicative law:  $(P, C) = ((-1)^{n+l+1}, (-1)^{l+1})$ . The total isospin of system of  $n + l$  constituent  $q\bar{q}$  pairs is zero as this system, by assumption, arises from an isosinglet combination of gluons.

The excited pions follow the same principle. The states  ${}^J\pi_{Q_0^i Q_1^j}$  have mass

$$m_{\pi_{Q_0^i Q_1^j}}^2 = \Lambda(nE_0 + lE_1) + m_\pi^2, \quad (5)$$

maximal spin  $J = l$ , and parities  $(P, C) = ((-1)^{n+l+1}, (-1)^l)$ .

Since a simultaneous excitation of two close pairs  $Q_0 Q_0$  does not change neither spin nor parities, it is natural to refer to a meson containing even number  $2k$  of  $Q_0$  and no one  $Q_1$  as the  $k$ -th radial excitation. The radial excitations appear thus with a "period"  $2E_0 \Lambda$ .

Consider a formation of two close vector pairs  $Q_1 Q_1$ . As they are formed from originally massless gluons we must combine their spins as if they were massless particles. This follows from conservation of total spin of original gluons. We recall that the spin of massless gluons has only two projections: two possible helicities. The total helicity of two gluons can be 0 or 2. This rule (in other words, the addition law for spins of spin-1 massless particles is the same as for massive spin -1/2 particles; just the final result must be multiplied by 2) extends to arbitrary number of close  $Q_1$  pairs. It is easy to see now that the excitations of the kind  ${}^J\rho_{Q_1^{2k}}$ ,  $k = 0, 1, 2, \dots$ , will give rise to a degenerate family of resonances with  $J^{PC} = (1, 3, \dots, 2k+1)^{--}$ . The states on the main  $\rho$ -meson Regge trajectory follow thus with a "period"  $2E_1 \Lambda$ . These excitations generate also daughter trajectories. For instance, the additional  $\rho$  mesons ( $J^{PC} = 1^{--}$ ) arise which appear with the same "period"  $2E_1 \Lambda$ . They are different from the "radial"  $\rho$  mesons that appeared with another period  $2E_0 \Lambda$ .

In order to demonstrate how our scheme works in practice let us consider some examples.

The mass of  $\rho$ -meson is  $m_\rho = \sqrt{\Lambda E_\rho + m_\pi^2}$ . This relation fixes  $E_\rho \approx 310$  MeV from the averaged mass of charged  $\rho$  in the hadronic processes,  $m_\rho = 766.5 \pm 1.1$  MeV [3].

${}^1\rho_{Q_0}$  and  ${}^1\omega_{Q_0}$  have quantum numbers  $J^{PC} = 1^{+-}$  and, according to (4), mass  $m = \sqrt{\Lambda(E_\rho + E_0) + m_\pi^2}$ . Fitting to the masses of experimental states  $h_1(1170)$  and  $b_1(1230)$  [3], we obtain the estimate  $E_0 \approx 430 - 510$  MeV.

The state  ${}^0\pi_{Q_0}$  has the quantum numbers of scalar particle,  $J^{PC} = 0^{++}$ . According to (5) its mass is  $m = \sqrt{\Lambda E_0 + m_\pi^2} \approx 900 - 980$  MeV. This estimate is close to the mass of  $a_0(980)$  [3].

${}^J\rho_{Q_1}$  and  ${}^J\omega_{Q_1}$  have  $J^{PC} = (0, 1, 2)^{++}$ . They should be the series of states  $a_1(1260)$ ,  $f_1(1285)$ ,  $f_2(1270)$ ,  $a_2(1320)$ , and likely  $f_0(1370)$  [3]. Substituting the experimental masses of well measured resonances  $f_1(1285)$  and  $f_2(1270)$  to our relation for their mass,  $m = \sqrt{\Lambda(E_\rho + E_1) + m_\pi^2}$ , we obtain the estimate  $E_1 \approx 570 - 580$  MeV.

${}^1\rho_{Q_0^2}$  and  ${}^1\omega_{Q_0^2}$  have mass  $m = \sqrt{\Lambda(E_\rho + 2E_0) + m_\pi^2} \approx 1470 - 1590$  MeV. They are the first radial excitations of  $\rho$  and  $\omega$  mesons and describe the resonance regions  $\rho(1450)$  and  $\omega(1420)$  [3].

${}^J\rho_{Q_0 Q_1}$  and  ${}^J\omega_{Q_0 Q_1}$  have quantum numbers  $J^{PC} = (0, 1, 2)^{-+}$ , i.e., 3 possible spins and pion parities. These states describe resonances  $\pi$ ,  $\pi_1$ , and  $\pi_2$  above 1 GeV. In contrast to the standard quark model, the  $\pi_1$ -meson is not exotic in our scheme! The relation (5) predicts mass about  $m = \sqrt{\Lambda(E_\rho + E_0 + E_1) + m_\pi^2} \approx 1550 - 1610$  MeV. The possible candidates are  $\pi_1(1600)$  and  $\pi_2(1670)$  [3].

The states  ${}^J\rho_{Q_1^2}$  and  ${}^J\omega_{Q_1^2}$  have  $J^{PC} = (1, 3)^{--}$  as the total spin of the pair  $Q_1 Q_1$  can be 0 or 2. Natural candidates are  $\rho(1700)$  and  $\rho_3(1690)$  (and  $\omega(1650)$  with  $\omega_3(1670)$  for  $\omega$ ) [3]. Our fits yield  $m = \sqrt{\Lambda(E_\rho + 2E_1) + m_\pi^2} \approx 1630 - 1650$  MeV. As in the standard quark model, the exotic states with quantum numbers  $J^{PC} = 0^{--}$  are absent and  $\rho(1700)$  is not the second radial excitation (in the potential quark models,  $\rho(1700)$  (and  $\omega(1650)$ ) is a  $D$ -wave state, while the radial excitations of  $\rho(770)$  should be  $S$ -wave ones) of  $\rho(770)$ . On the other hand, we do not obtain states with  $J^{PC} = 2^{--}$  which are predicted by the standard quark model but were not observed.

### 3. Regge Trajectories

Relations (4) and (5) lead to linear Regge, equidistant daughter Regge, and radial trajectories. Below we give examples for some of them.

The states  ${}^0\pi_{Q_0^{2n}}$ ,  $n = 0, 1, 2, \dots$ , form linear trajectory for the radial excitations of pion,

$$m_\pi^2(n) = 2\Lambda E_0 n + m_\pi^2. \quad (6)$$

The radial excitations of  $\rho(770)$ , the resonances  ${}^1\rho_{Q_0^{2n}}$ , lie on the first radial  $\rho$ -trajectory,

$$m_\rho^2(n)_I = 2\Lambda E_0 \left( n + \frac{E_\rho}{2E_0} \right) + m_\pi^2. \quad (7)$$

The second radial  $\rho$ -trajectory is composed of the states  ${}^1\rho_{Q_1^2 Q_0^{2n}}$ ,

$$m_\rho^2(n)_{II} = 2\Lambda E_0 \left( n + \frac{E_1}{E_0} + \frac{E_\rho}{2E_0} \right) + m_\pi^2. \quad (8)$$

It is evident that the states  ${}^1\rho_{Q_1^{2(k-1)} Q_0^{2n}}$  formally give rise to the  $k$ -th radial  $\rho$ -trajectory. The resonances having structure  ${}^1\rho_{Q_1 Q_0^{2n}}$  form the first radial  $a_1$ -trajectory,

$$m_{a_1}^2(n)_I = 2\Lambda E_0 \left( n + \frac{E_1}{2E_0} + \frac{E_\rho}{2E_0} \right) + m_\pi^2. \quad (9)$$

The expressions for further axial radial trajectories can be easily written. It should be remarked that the standard quark model predicts only two radial  $\rho$ -trajectories ( $S$ - and  $D$ -wave ones) and one  $a_1$ -trajectory (a  $P$ -wave one). The scheme under discussion is much richer.

The spin  $\rho$ -trajectory is composed of the states  ${}^J\rho_{Q_1^{J-1}}$  with  $J = 1, 3, 5, \dots$ . The corresponding masses are

$$m_{\rho_J}^2 = \Lambda E_1 \left( J - 1 + \frac{E_\rho}{E_1} \right) + m_\pi^2. \quad (10)$$

For the even spins,  $J = 2, 4, 6, \dots$ , trajectory (10) describes  $a_J (f_J)$  mesons. The Regge trajectory (10) describes thus states with alternating parities and this agrees with the phenomenology.

The principal  $\rho$ -meson Regge trajectory (10) is accompanied by the daughter trajectories following with the step  $2\Lambda E_1$ . The spin-1  $\rho$ -mesons of the kind  ${}^1\rho_{Q_1^{j-1}}$  are the lowest states on the daughters. For example, the lowest state in (8) is the lowest state on the first daughter. The spectrum of  ${}^1\rho_{Q_1^{j-1}}$  excitations reads

$$m_{\rho_{Q_1^{j-1}}}^2 = 2\Lambda E_1 \left( k + \frac{E_\rho}{2E_1} \right) + m_\pi^2, \quad (11)$$

where  $k = 0, 1, 2, \dots$  enumerates the daughters. Similar relations can be written for the axial and other mesons. An obvious consequence of the emerging spectrum is the degeneracy of spin and daughter radial excitations of the type  $m^2(J, k) \sim J + k$ , which is typical for the Veneziano dual amplitudes [45] and the Nambu–Goto open strings. This kind of degeneracy was observed in the experimental spectrum of light nonstrange mesons [5–11].

The fitted values of  $E_\rho$ ,  $E_0$ , and  $E_1$  are rather close. This allows considering reasonable limits where some of them are equal. The notions of radial and daughter radial trajectories coincide in the limit  $E_0 = E_1$ . In the limit  $E_\rho = E_1$ , the radial vector and axial trajectories are related by

$$m_{a_1}^2(n) = m_\rho^2(n) + m_\rho^2 - m_\pi^2. \quad (12)$$

This relation holds both for radial and for daughter radial trajectories. In the chiral limit,  $m_\pi = 0$ , relation (12) for the ground states,  $n = 0$ , reduces to the old Weinberg relation,  $m_{a_1}^2 = 2m_\rho^2$  [46]. In the most symmetric limit,  $E_\rho = E_0 = E_1$ , the vector and axial radial spectrum in the chiral limit reduce to a very simple form,

$$m_\rho^2(n) = 2m_\rho^2 \left( n + \frac{1}{2} \right), \quad (13)$$

$$m_{a_1}^2(n) = 2m_\rho^2(n + 1).$$

Relations (13) first appeared in the variants of Veneziano amplitude which incorporated the Adler self-consistency condition [45]. This condition (the amplitude of  $\pi\pi$  scattering is zero at zero momentum) incorporates the CSB removing degeneracy between the  $\rho$  and  $a_1$  spectra. Within the QCD sum rules, relations (13) may be interpreted as a large- $N_c$  generalization of the Weinberg relation [47].

We see thus that in certain limits the Regge phenomenology of our approach reproduces various known relations. In addition, it is not excluded that the assumption  $E_\rho = E_0 = E_1$  is compatible with the available data if a global fit is performed. According to the phenomenological analysis in review [2] the averaged slopes of spin and radial trajectories in light nonstrange mesons,  $a$  and  $a'$  in (1), are equal; the reported value is  $a \approx a' \approx 1.14 \text{ GeV}^2$ . This remarkable observation leads to a large degeneracy in the spectrum [5–11, 21–27]. From the given value we have  $E_0 \approx E_1 \approx a/(2\Lambda) \approx$

$1.14/(2 \cdot 1.83) \approx 0.31 \text{ GeV}$  that coincides with our previous estimate for  $E_\rho$ .

Let us clarify further how the excited resonances with identical quantum numbers can have different origin in the proposed scheme. They may represent the radial states, states on daughter Regge trajectories and various “mixed” ones. For instance, the second  $\rho$ -meson excitation with the same parities appears in three forms: the second radial excitation  ${}^1\rho_{Q_0^2}$ , the vector state on the second daughter trajectory  ${}^1\rho_{Q_1^2}$ , and the mixed one  ${}^1\rho_{Q_0^2 Q_1^2}$ . They are degenerate only in the limit  $E_0 = E_1$ . It is likely difficult to detect such a splitting experimentally because of overlapping widths. In reality, one would observe rather a “broad resonance region”.

We note also that placing of the observed “radial” states on a certain trajectory should be made with care: an incorrect interpretation of states leads to a false (or more precisely, introduced by hands) nonlinearity of the trajectory. Take again the  $\rho$  meson as a typical example. The first three radial excitations of  $\rho$  are the states  ${}^1\rho_{Q_0^2}$ ,  ${}^1\rho_{Q_0^2}$ , and  ${}^1\rho_{Q_0^2}$ . They are accompanied by the following states with the quantum numbers of  $\rho$ :  ${}^1\rho_{Q_1^2}$ ,  ${}^1\rho_{Q_1^2 Q_0^2}$ , and  ${}^1\rho_{Q_1^2}$ . Since  $(3/2)E_0 > E_1 > E_0$  in our fits, the sequence of first 7  $\rho$ -mesons is  $\rho$ ,  ${}^1\rho_{Q_0^2}$ ,  ${}^1\rho_{Q_1^2}$ ,  ${}^1\rho_{Q_0^2}$ ,  ${}^1\rho_{Q_1^2 Q_0^2}$ ,  ${}^1\rho_{Q_1^2}$ , and  ${}^1\rho_{Q_0^2}$ . They likely correspond to the vector resonances  $\rho(770)$ ,  $\rho(1450)$ ,  $\rho(1700)$ ,  $\rho(1900)$ ,  $\rho(2000)$ ,  $\rho(2150)$ , and  $\rho(2270)$  [3].

#### 4. A Theoretical Motivation

It is interesting to get a theoretical hint both on the rule  $m_h^2 \sim E_{\rho,0,1}$  underlying the presented mass counting scheme and on the value of energy parameters  $E_{\rho,0,1}$ . The latter point is intriguing by itself since the scale about 0.3 GeV is ubiquitous in the hadron physics; for instance, it often emerges in mass relations between hadrons containing heavy quarks [48].

The mass of a hadron state  $|h\rangle$  can be related to the trace of energy momentum tensor  $\Theta_{\mu\nu}$  in QCD (the sign convention is mostly positive),

$$2m_h^2 = -\langle h | \Theta_\mu^\mu | h \rangle, \quad (14)$$

where  $\Theta_\mu^\mu$  is given by the scale anomaly,

$$\Theta_\mu^\mu = \frac{\beta}{2g_s} G_{\mu\nu}^2 + \sum_{q=u,d,\dots} m_q \bar{q}q. \quad (15)$$

Here  $\beta$  denotes the QCD beta-function and  $g_s$  is the coupling constant. Relation (14) represents trace taken in the following Ward identity known in deep inelastic phenomenology [49]:  $2p_\mu p_\nu = -\langle h(p_\mu) | \Theta_{\mu\nu} | h(p_\nu) \rangle$ . As follows from derivation of the identity (14), the squared mass (not the linear one!) in the l.h.s. appears due to the relativistic invariance. The r.h.s. of (14) is a renorminvariant quantity. One can build two substantially different renormalization invariant operators in QCD; both of these operators are present in (15). The l.h.s. of (14) defines the gravitational mass of a hadron. In reality, the vast majority of hadron resonances probably have no well-defined gravitational (and inertial) mass as they do

not propagate in space: their typical lifetime of the order of  $10^{-23}$ – $10^{-24}$  s does not allow leaving the reaction area of the order of 1 fm which is comparable to their size. They show up only as some structures in the physical observables at certain energy intervals. The resonance mass is commonly associated with the real part of an  $S$ -matrix pole on the second (unphysical) sheet. How this definition is related to the gravitational mass is by far not obvious in a theory with confinement.

In the sector of light  $u$  and  $d$  quarks, there are only two hadrons with well-defined gravitational mass: the pion and nucleon. The pion mass is given by the GOR relation (2). This relation should somehow follow from the Ward identity (14). An explicit derivation of (2) from (14) would likely give an analytical proof for spontaneous CSB in QCD. Our present aim is to show, at least on a rather intuitive level, how the mass gap may emerge in nonperturbative gluon vacuum and get some quantitative insight. Consider the case of nucleon,  $|h\rangle = |N\rangle$ , for which (14) must yield the proton mass. Let us insert a complete set of vacuum states  $|0\rangle\langle 0|$  from both sides of  $\Theta_\mu^\mu$  in (14); the nucleon mass is then given by

$$m_N^2 = -\frac{1}{2} \langle 0 | N \rangle^2 \left\langle 0 \left| \frac{\beta}{2g_s} G_{\mu\nu}^2 + \sum_{q=u,d} m_q \bar{q}q \right| 0 \right\rangle. \quad (16)$$

This expression suggests that, in the chiral limit,  $m_q = 0$ , the nucleon mass is determined by the vacuum average  $\langle 0 | G_{\mu\nu}^2 | 0 \rangle$ .

From the one-loop QCD beta-function we have  $\beta/2g_s = -(\beta_0/8)(\alpha_s/\pi)$ , where  $\beta_0 = 11 - (2/3)n_f$ ,  $\alpha_s = g_s^2/4\pi$ . Relation (16) can be then rewritten as

$$m_N^2 = \frac{1}{2} \langle 0 | N \rangle^2 \langle \bar{q}q \rangle \left( -\sum_q m_q + \frac{\beta_0}{8} \frac{(\alpha_s/\pi) \langle G_{\mu\nu}^2 \rangle}{\langle \bar{q}q \rangle} \right). \quad (17)$$

Relation (17) demonstrates that the masses of current quarks in nucleon acquire a contribution from nonperturbative gluon vacuum. Since nucleon at rest is interpreted in the quark model as a bound system of three constituent quarks, we can estimate from (17) an effective energy per quark, i.e., the value of constituent quark mass; in the chiral limit, it is just one-third of gluon contribution in (17):

$$M_c \approx -\frac{\beta_0}{24} \frac{(\alpha_s/\pi) \langle G_{\mu\nu}^2 \rangle}{\langle \bar{q}q \rangle}. \quad (18)$$

Substituting into (18) the standard value of gluon condensate from QCD sum rules,  $(\alpha_s/\pi) \langle G_{\mu\nu}^2 \rangle = 0.012(3) \text{ GeV}^4$  [42, 43], and  $\langle \bar{q}q \rangle = -(0.25 \text{ GeV})^3$ ,  $n_f = 2$ , we obtain the estimate  $M_c \approx 310 \text{ MeV}$ . It is a typical estimate for the value of constituent quark mass (strictly speaking, the value of constituent quark mass is very model dependent; as far as we know, it ranges from 220 to 450 MeV in various models. The value  $M_c(p^2) \approx 310 \text{ MeV}$  at small momentum  $p$ , however, proves to be seen in unquenched lattice simulations in the chiral limit (see, e.g., [50]). We can indicate a simple qualitative way leading to this estimate. When one incorporates the vector and axial mesons into the low-energy models describing the

spontaneous CSB in QCD and related physics, e.g., into the Nambu–Jona-Lasinio model [51] or the linear sigma-model, a model-independent relation emerges:  $m_{a_1}^2 = m_\rho^2 + 6M_c^2$ . On the other hand, the idea of CSB was also exploited in the derivation of famous Weinberg relation,  $m_{a_1}^2 = 2m_\rho^2$  [46]. Combining both relations, one has  $M_c = m_\rho/\sqrt{6}$ . Substitution of the value  $m_\rho \approx 766 \text{ MeV}$  used in our fits leads to  $M_c \approx 0.31 \text{ GeV}$ ,  $M_c \approx m_p/3$ , where  $m_p$  is the proton mass.

It is interesting to observe the numerical coincidence  $M_c \approx E_\rho$ . This suggests that the “spin flip” converting pion to  $\rho$  is essentially equivalent to creation of constituent quark. Thus the  $\rho$  meson in our approach may be interpreted as a system of one constituent quark and one current antiquark (or vice versa) which interact with the QCD vacuum. This picture is different from the usual quark model where  $\rho$  consists of two constituent quarks interacting with each other (in view of these speculations one can ask what is the exact relation for the nucleon mass within the given approach? We can propose the following guess. In the case of  $\rho$  meson, we motivated “a rule of transition” from the nonrelativistic quark model to our scheme:  $m_h = \sum M_c + \text{interactions} \rightarrow m_h^2 = \Lambda \cdot (1/2) \sum M_c + m_\pi^2$ . Applying this “rule” to the ground state of nucleon (3 constituent quarks) we arrive at the relation  $m_N^2 = (3/2)\Lambda M_c + m_\pi^2$  that gives  $m_N \approx 0.93 \text{ GeV}$ . The factor  $3/2$  could be also qualitatively interpreted as follows: the creation of 3 constituent quarks and 3 constituent antiquarks (i.e., the creation of  $N\bar{N}$  pair in vacuum) is equivalent to creation of 3 constituent quark-antiquark pairs, so the relation of energies of constituents in nucleon and vector meson is 3 to 2).

The given interpretation can be further substantiated by the observation that the same “spin flip” should convert the nucleon to  $\Delta$  baryon. As this flip “costs”  $\Lambda M_c$  for the hadron mass squared, we should expect the relation  $m_\Delta^2 = m_N^2 + \Lambda M_c$ . It indeed yields  $m_\Delta \approx 1.2 \text{ GeV}$  in a good agreement with experimental data for  $\Delta(1232)$  [3].

Coming back to motivation of our approach, the presented mass counting scheme is based on the assumption that the origin of hadron masses is similar to the case of nucleon mass in (17); the hadron mass squared (not the linear one as in many other approaches!) is proportional to effective energy of hadron constituents. This rule is conjectured from the Ward identity (14).

The second assumption refers to the postulated form of meson constituents; the valent  $q\bar{q}$  pair plus constituent  $q\bar{q}$  pairs which effectively parametrize contributions to hadron mass from strong gluon field. The given choice is a model which works in the meson spectroscopy above and near 1 GeV. Besides the spectroscopy it could explain why the highly excited nonstrange mesons prefer to decay to more than two pions or, say, to have sometimes  $\rho$ -meson in the decay products instead of  $\pi\pi$ -pair [3]; the corresponding possibilities are likely enciphered in the expression for the mass of a resonance as is seen in the examples of Section 2.

It should be mentioned that hadron structure strongly depends on a reference frame. For instance, the proton structure experimentally looks very different if it probed for a proton at rest or for ultrarelativistic proton. It might be that

the hadron model developed in this paper is more appropriate as a picture of light hadrons near the light cone while the traditional quark models refer to the rest frame. In this case there is no contradiction between different approaches.

The constituent  $q\bar{q}$  pairs are not operative degrees of freedom noticeably below 1 GeV. It is interesting to note, however, that the spectroscopy in this sector can be constructed following essentially the same relation (3) if the pseudoscalar pair  $Q_0$  is replaced by the pseudo-goldstone bosons  $\pi$ ,  $K$ ,  $\eta$  [52].

## 5. Summary

We have proposed a novel approach to classification of mesons and description of meson spectroscopy. The approach represents a new realization of the quark model concept, a realization in which highly excited states appear due to multi-quark components in hadron wavefunctions. We constructed a mass counting scheme that allows estimating meson masses by a trivial arithmetics with accuracy comparable to numerical calculations in complicated dynamical models. The given scheme is based on introduction of constituent scalar and vector quark-antiquark pairs. These pairs in excited hadrons, in some sense, bear a superficial resemblance to neutrons and protons in atomic nuclei.

The inclusion of strange quarks into our approach is straightforward. It would be interesting to extend the presented ideas to light baryons and to hadrons with heavier quarks.

The proposed approach is broader than “just another one model” as it provides a new framework for analysis of hadron resonances and can be used as a starting point for construction of essentially new dynamical models in hadron physics.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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