

Research Article

Hadron Resonance Gas EoS and the Fluidity of Matter Produced in HIC

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We study the equation of state (EoS) of hot and dense hadron gas by incorporating the excluded volume corrections into the ideal hadron resonance gas (HRG) model. The total hadron mass spectrum of the model is the sum of the discrete mass spectrum consisting of all the experimentally known hadrons and the exponentially rising continuous Hagedorn states. We confront the EoS of the model with lattice quantum chromodynamics (LQCD) results at finite baryon chemical potential. We find that this modified HRG model reproduces the LQCD results up to $T = 160$ MeV at zero as well as finite baryon chemical potential. We further estimate the shear viscosity within the ambit of this model in the context of heavy-ion collision experiments.

1. Introduction

The phase diagram of quantum chromodynamics (QCD) is under intense theoretical investigation especially in the context of heavy-ion collision (HIC) experiments where two heavy nuclei are accelerated to very high energies and then they collide to produce very hot and dense QCD matter. These experiments can probe part of the QCD phase diagram where the most interesting nonperturbative aspects of QCD lie. The most reliable theoretical tool at zero baryon chemical potential is lattice quantum chromodynamics (LQCD). One of the most important prediction of LQCD simulation is that the phase transition from hadronic to quark-gluon-plasma (QGP) is an analytic crossover [1]. However, at finite baryon chemical potential QCD has been plagued with the so-called sign problem and one has to resort to certain approximation schemes to fetch the reliable results [2]. One can adopt an alternative approach where the effective model of QCD can be constructed which not only preserves certain important symmetries of QCD but is tractable at finite chemical potential as well. The hadron resonance gas model is the statistical model of QCD describing the low temperature hadronic phase of quantum chromodynamics. The essential starting point of the model is the Dashen-Ma-Bernstein theorem [3] which allows

us to compute the partition function of the interacting system in terms of scattering matrix. Using this theorem it can be shown that if the dynamics of the system is dominated by narrow-resonance formation it behaves like a noninteracting system [4–6]. Thus the thermodynamics of interacting gas of hadrons through formation of resonances can be well approximated by the noninteracting gas of hadrons and resonances. The HRG model has been very successful in describing the hadron multiplicities in HICs [7–16]. Recently, the interacting HRG model with multicomponent hard-core repulsion has been successful in describing the heavy-ion collision data with the unprecedented accuracy [17–20].

One possible improvement in the HRG model is to include the exponentially rising Hagedorn density of states apart from the known hadrons and resonances included in the form of discrete mass spectrum. This exponential mass spectrum arises in the string picture [21–25] or the glueball picture of the hadrons [26]. A typical form of the continuous mass spectrum is $\rho(m) \sim A_1 m^{-a} e^{A_2 m}$ which satisfies the statistical bootstrap condition [27, 28]. With the proper choice of the parameters A_1 , A_2 and with $a > 5/2$ all the experimentally found hadrons fit in this exponential mass spectrum [29, 30]. Finite temperature LQCD simulations provide strong evidence of the existence of Hagedorn states

in hot and dense matter created in heavy-ion collision experiments. The mass spectrum of these states is found to be of the form $\rho(m) \sim m^{-a} e^{m/T_H}$ (T_H is the Hagedorn temperature) [31].

Another possible improvement in the ideal HRG model is to take into account short range repulsive interactions between hadrons. There are many different ways of incorporating the repulsive interactions in the ideal HRG model without spoiling the thermodynamical consistency of the model [32–34]. We shall consider the excluded volume correction scheme of [32] where the repulsive interactions are accounted for through the excluded volume correction in the ideal gas partition function.

In the past few decades the LQCD results of the equation of state at zero baryon chemical potential have been analyzed within the HRG model and its extensions [35–40]. Recently, in [41] the authors have observed that the hadron resonance gas model with the discrete mass spectrum augmented with the continuous Hagedorn mass spectrum is not sufficient to explain the recent lattice QCD results. Further the excluded volume corrections to the ideal HRG model also fail to do the same. But if both these physical effects, viz., the Hagedorn states as well as excluded volume corrections, are included in the ideal HRG model then the model reproduces the lattice QCD data all the way up to $T = 160$ MeV.

In the context of relativistic heavy-ion collision (RHIC) experiments the shear viscosity coefficient governs the evolution of the nonequilibrium system towards the equilibrium state. In the off-central nuclear collision the spatial anisotropy of the produced matter gets converted into momentum anisotropy and the equilibration of this momentum anisotropy is governed by the shear viscosity coefficient. The produced matter in the fireball after the collision, with quarks and gluons degrees of freedom, behaves like a strongly interacting liquid with very small shear viscosity. Assuming that this liquid of quarks and gluons is in thermal equilibrium, it expands due to pressure difference and cools and finally undergoes a phase transition to hadronic degrees of freedom which finally free stream to the detector. One of the successful descriptions of such an evolution is through dissipative relativistic hydrodynamics [48–56] and transport simulations [57–64]. In the hydrodynamic description of the heavy-ion collision experiments finite but the small ratio of shear viscosity (η) to entropy (s) is necessary to explain the flow data [65, 66]. The smallness of this ratio η/s and its connection to the conjectured Kovtun-Son-Starinets (KSS) bound of $\eta/s = 1/4\pi$ obtained using AdS/CFT correspondence [67] have motivated many theoretical investigations of this ratio to understand and derive it rigorously from a microscopic theory [43–46, 68–79].

In this work we confront the equation of state of the excluded volume HRG model which includes the discrete hadron states and continuum Hagedorn states in the density of states at finite baryon chemical potential. We further attempt to make rough estimates of the shear viscosity coefficient within the ambit of this extended HRG model in the context of heavy-ion collision experiments. Throughout the discussion we shall adopt the Boltzmann approximation

(i.e., Boltzmann classical statistics) since it is a rather excellent approximation in the region of the QCD phase diagram in which we are interested (i.e., $T = 100 - 160$ MeV).

We organize the paper as follows. In Section 2 we briefly describe the thermodynamics of the hadron resonance gas model. In Section 3 we give brief derivation of the shear viscosity coefficient for the multicomponent hadronic matter using the relativistic Boltzmann equation in relaxation time approximation. In Section 4 we present the results and discuss their implications in the context of relativistic heavy-ion collision experiments. Finally we summarize and conclude in Section 5.

2. The Hadron Resonance Gas Model

Thermodynamical properties of the hadron resonance gas model can be deduced from the grand canonical partition function defined as

$$\mathcal{Z}(V, T, \mu_B) = \int dm \left[\rho_b(m) \ln Z_b(m, V, T, \mu_B) + \rho_f(m) \ln Z_f(m, V, T, \mu_B) \right], \quad (1)$$

where μ_B is the baryon chemical potential and ρ_b and ρ_f are the mass spectrum of the bosons and fermions, respectively. We assume that the hadron mass spectrum is a combination of discrete (HG) and continuous (HS) Hagedorn states given by

$$\rho(m) = \rho_{HG}(m) + \rho_{HS}(m), \quad (2)$$

where

$$\rho_{HG} = \sum_a^{\Lambda} g_a \delta(m - m_a) \theta(\Lambda - m). \quad (3)$$

This discrete mass spectrum consists of all the experimentally known hadrons with cut-off Λ . One can set different cut-off values for baryons and mesons. The Hagedorn density of states is assumed to be

$$\rho_{HS} = A e^{m/T_H}, \quad (4)$$

where A is constant and T_H is the Hagedorn temperature. One can physically interpret T_H as QCD phase transition temperature. It is important to note that one cannot define baryon chemical potential for the Hagedorn states. Hagedorn states depend on the trend in the increase of the number of states as the hadron mass increases. Since this is not a thermodynamic statement, concepts such as the baryon density or chemical potential would not apply to Hagedorn states. Further the Hagedorn states are defined by the density of states defined by (4) satisfying the bootstrap condition. Since the quark content of these states is unknown it is not legitimate to define baryon number and hence baryon chemical potential to these states. Thus for all practical purposes we set $\mu_B = 0$ for Hagedorn states in our calculation. It is to be noted that the heavy Hagedorn states have finite width and large width of Hagedorn states is of great importance to describe the lattice QCD thermodynamics and

to explain the chemical equilibrium of hadronic matter born from these states [80, 81]. But we will work in narrow width approximation for Hagedorn states.

Repulsive interaction can be accounted for in the ideal HRG model via excluded volume correction ($V - \nu N$) to the partition function. The pressure of the HRG model with the discrete mass spectrum and excluded volume correction (which we shall call EHRG) turns out to be [32]

$$P^{EV}(T, \mu_B) = \sum_a P_a^{id}(T, \tilde{\mu}_B), \quad (5)$$

where $\tilde{\mu}_B = \mu_B - \nu P^{EV}(T, \mu_B)$, P^{id} is the ideal gas pressure, and $\nu = 4(4/3)\pi r_h^3$ is the excluded volume parameter of the hadron with hard-core radius r_h . Note that we assume a uniform hard-core radius to all the hadrons. The excluded volume models can be extended to multicomponent gas of a different hard-core radius [82]. These models can further be extended to take into account the Lorentz contraction of hadrons due to their relativistic motion [83]. But we will neglect these effects for the sake of simplicity. In the Boltzmann approximation the contribution to the ideal gas pressure due to a^{th} hadronic species is

$$P_a^{id}(T, \mu_B) = \frac{g_a}{2\pi^2} T^2 m_a^2 K_2\left(\frac{m_a}{T}\right), \quad (6)$$

where g_a is the degeneracy factor and K_n is the modified Bessel function of the second kind. In the Boltzmann approximation, (5) is simplified to

$$P^{EV}(T, \mu_B) = \sum_a P_a^{id}(T, \mu_B) \exp\left(-\frac{\nu P^{EV}(T, \mu_B)}{T}\right). \quad (7)$$

In the case of the continuum Hagedorn spectrum the sum in (7) is replaced by the integration and the discrete mass spectrum given by the delta function is replaced by the Hagedorn mass spectrum given by (4). The other thermodynamical quantities, viz., energy density ($\epsilon(T, \mu_B)$), entropy density ($s(T, \mu_B)$), number density ($n(T, \mu_B)$), and speed of sound ($C_s^2(T, \mu_B)$), can be obtained from the pressure by taking appropriate derivatives as per thermodynamical identities.

3. Transport Coefficients: A Relaxation Time Approximation

In the relativistic kinetic theory the evolution of distribution function $f_p(\mathbf{x}, \mathbf{t})$ is determined by the Boltzmann equation [84]

$$p^\mu \partial_\mu f_p = \mathcal{C}[f_p], \quad (8)$$

where $P^\mu = (E_p, \mathbf{P})$ and \mathcal{C} is called the collision term. Solving this equation, which is in general an ‘‘integrodifferential’’ equation, for f_p is rather a very difficult task, if not impossible. So it is customary to resort to certain approximations so that solving (8) becomes feasible. We assume that the system is only slightly away from the equilibrium; i.e.,

$$f_p = f_p^0 + \delta f_p \quad (9)$$

with $f_p^0 \gg \delta f_p$. f_p^0 is an equilibrium distribution function. If we further assume that the collisions bring the system towards equilibrium with the time scale $\sim \tau$, then the collision term can be approximated as

$$\mathcal{C}[f_p] \simeq -\frac{P^\mu u_\mu}{\tau(E_p)} \delta f_p. \quad (10)$$

In this so-called relaxation time approximation, Boltzmann equation (8) becomes

$$p^\mu \partial_\mu f_p^0 = -\frac{P^\mu u_\mu}{\tau} \delta f_p. \quad (11)$$

The equilibrium distribution function f_p^0 in the Boltzmann approximation is given by

$$f_p^0 = \exp\left\{-\frac{(E_p - \mathbf{p} \cdot \mathbf{u} - \mu_B)}{T}\right\}, \quad (12)$$

where \mathbf{u} is the fluid velocity.

In the theory of fluid dynamics shear (η) and bulk (ζ) viscosities enter as coefficients of the space-space component of the energy-momentum tensor away from equilibrium as

$$T^{\mu\nu} = T_0^{\mu\nu} + T_{dissi}^{\mu\nu}, \quad (13)$$

where $T_0^{\mu\nu}$ is the ideal part of the stress tensor.

In the local Lorentz frame the dissipative part of the stress energy tensor can be written as

$$T_{dissi}^{ij} = -\eta \left(\frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i} \right) - \left(\zeta - \frac{2}{3}\eta \right) \frac{\partial u^i}{\partial x^j} \delta^{ij}. \quad (14)$$

In the kinetic theory $T^{\mu\nu}$ is defined as

$$T^{\mu\nu} = \int d\Gamma \frac{P^\mu P^\nu}{E_p} f_p = \int d\Gamma \frac{P^\mu P^\nu}{E_p} (f_p^0 + \delta f_p), \quad (15)$$

where $d\Gamma = g d^3 p / (2\pi)^3$, and g is the degeneracy. The space-space component of the above equation is

$$T^{ij} = T_0^{ij} + T_{dissi}^{ij}, \quad (16)$$

where

$$T_{dissi}^{ij} = \int d\Gamma p^i p^j \delta f_p. \quad (17)$$

In the local rest frame, (11) is simplified to

$$\delta f_p = -\tau(E_p) \left(\frac{\partial f_p^0}{\partial t} + v_p^j \frac{\partial f_p^0}{\partial x^j} \right). \quad (18)$$

Assuming steady flow of the form $u^i = (u_x(y), 0, 0)$ and space-time independent temperature, (14) is simplified to

$$T_{dissi}^{xy} = -\frac{\eta \partial u_x}{\partial y}. \quad (19)$$

From (17) and (18) we get (using (12) with $\mu_B = 0$)

$$T_{dissi}^{xy} = \left\{ -\frac{1}{T} \int d\Gamma \tau(E_p) \left(\frac{p_x p_y}{E_p} \right)^2 f_p^0 \right\} \frac{\partial u_x}{\partial y}. \quad (20)$$

Comparing the coefficients of gradients in (19) and (20) we get the coefficient of shear viscosity

$$\eta = \frac{1}{15T} \int d\Gamma \tau(E_p) \frac{p^4}{E_p^2} f_p^0. \quad (21)$$

Thus for multicomponent hadron gas at finite chemical potential the shear viscosity coefficient is [68]

$$\eta = \frac{1}{15T} \sum_a \int d\Gamma_a \frac{p^4}{E_a^2} \tau_a(E_a) f_p^0. \quad (22)$$

The relaxation time is in general energy dependent. But for simplicity we use averaged relaxation time ($\bar{\tau}$) which is rather a good approximation as energy dependent relaxation time [85]. Thus the averaged partial relaxation time is defined by

$$\bar{\tau}_a^{-1} = \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle, \quad (23)$$

where $n_b = \int d\Gamma f_b^0$ is the number density of b^{th} hadronic species and $\langle \sigma_{ab} v_{ab} \rangle$ is the thermal average of the cross section given by [86]

$$\begin{aligned} \langle \sigma_{ab} v_{ab} \rangle &= \frac{\sigma}{8T m_a^2 m_b^2 K_2(m_a/T) K_2(m_b/T)} \\ &\cdot \int_{(m_a+m_b)^2}^{\infty} dS \frac{[S - (m_a - m_b)^2]}{\sqrt{S}} \\ &\cdot [S - (m_a + m_b)^2] K_1\left(\frac{\sqrt{S}}{T}\right), \end{aligned} \quad (24)$$

where S is a center of mass energy and K_n is the modified Bessel function of the second kind. Note that we have assumed the cross section σ to be constant for all the species in the system.

The massive Hagedorn states cannot be described rigorously using the Boltzmann equation. Thus it is very difficult to compute the contribution of these massive and highly unstable hadrons to the shear viscosity of low temperature QCD matter. But since these states contribute significantly to the thermodynamics of hadronic matter close to QCD transition temperature their contribution cannot be ignored. In fact they decay so rapidly that it is legitimate to assume that their presence will affect the relaxation time of the system. In [87] the authors studied the effect of Hagedorn states on the shear viscosity of QCD matter near T_c . They assumed that the relaxation time τ is inversely related to the decay width through the relation $\tau = 1/\Gamma$. The decay width of Hagedorn states can be obtained from the linear fit to the decay widths of all the resonances in the particle data book. This prescription corresponds to the decay cross section and neglects the

collisional cross section for the momentum transport that might contribute the shear viscosity as per the Boltzmann equation. Such approximation gives only a rough estimate of the shear viscosity. Since we are also interested in the rough estimate of shear viscosity we assume that the Hagedorn states contribute to the shear viscosity through collisions with other hadrons. We further assume that these states are hard sphere particles of radius r_h . Thus the thermodynamics of the Hagedorn states can be estimated using the excluded volume HRG model. It can be shown that in the excluded volume approximation the shear viscosity of the Hagedorn gas is [88]

$$\begin{aligned} \eta_{HS} &= \frac{5}{64r_h^2} \sqrt{\frac{T}{\pi}} \frac{T}{2\pi^2 n(T)} \\ &\cdot \int_0^{\infty} dm \rho_{HS}(m) m^{5/2} K_{5/2}\left(\frac{m}{T}\right), \end{aligned} \quad (25)$$

where $n(T)$ is the number density of the Hagedorn gas. Viscosity coefficients of the pure Hagedorn fluid in the relaxation time approximation of the Boltzmann equation have also been studied in [77]. We again mention here that the heavy Hagedorn states have finite width but we are working in the narrow width approximation of these states.

4. Results and Discussion

We include all the hadrons and resonances up to 2.225 GeV listed in [89]. More specifically we choose cut-offs $\Lambda_M = 2.011$ GeV for mesons and $\Lambda_B = 2.225$ GeV for the baryons. Note here that in HRGM usually one has to include all hadrons and resonances up to 2.5 – 2.6 GeV in order to get a good description of the experimental data. But the mass spectrum of hadrons with masses between 1 GeV and 2.2 GeV is approximately Hagedorn-like. We set uniform hard-core radius $r_h = 0.4$ fm to all the hadrons. Note here that the excluded volume corrections to the ideal HRG model are consistent only if we choose a uniform hard-core radius to all the hadrons. To specify the Hagedorn mass spectrum we choose $A = 0.4$ GeV⁻¹. Finally we set the Hagedorn temperatures $T_H = 198$ MeV for $\mu_B = 0$ and $T_H = 188$ MeV for $\mu_B = 300$ MeV. This specific choice is made to get the best fit with the LQCD data at zero as well as finite chemical potential.

Figure 1 shows scaled pressure (P/T^4) and the scaled interaction measure ($I = (\epsilon - 3P)/T^4$) at zero chemical potential. The black dots (with error bars) correspond to lattice QCD data taken from [42]. The blue curve corresponds to the ideal HRG model with only the discrete mass spectrum while the green dashed curve corresponds to the ideal HRG with both the discrete and continuous Hagedorn mass spectrum. The brown curve corresponds to the excluded volume HRG model with only experimentally known hadrons included. The solid magenta curve corresponds to excluded volume corrections to the ideal HRG model in which both experimentally known hadrons and continuous Hagedorn states are included. We shall call HRG with excluded volume corrections and Hagedorn spectrum MEHRG (modified excluded volume HRG) for brevity. It can

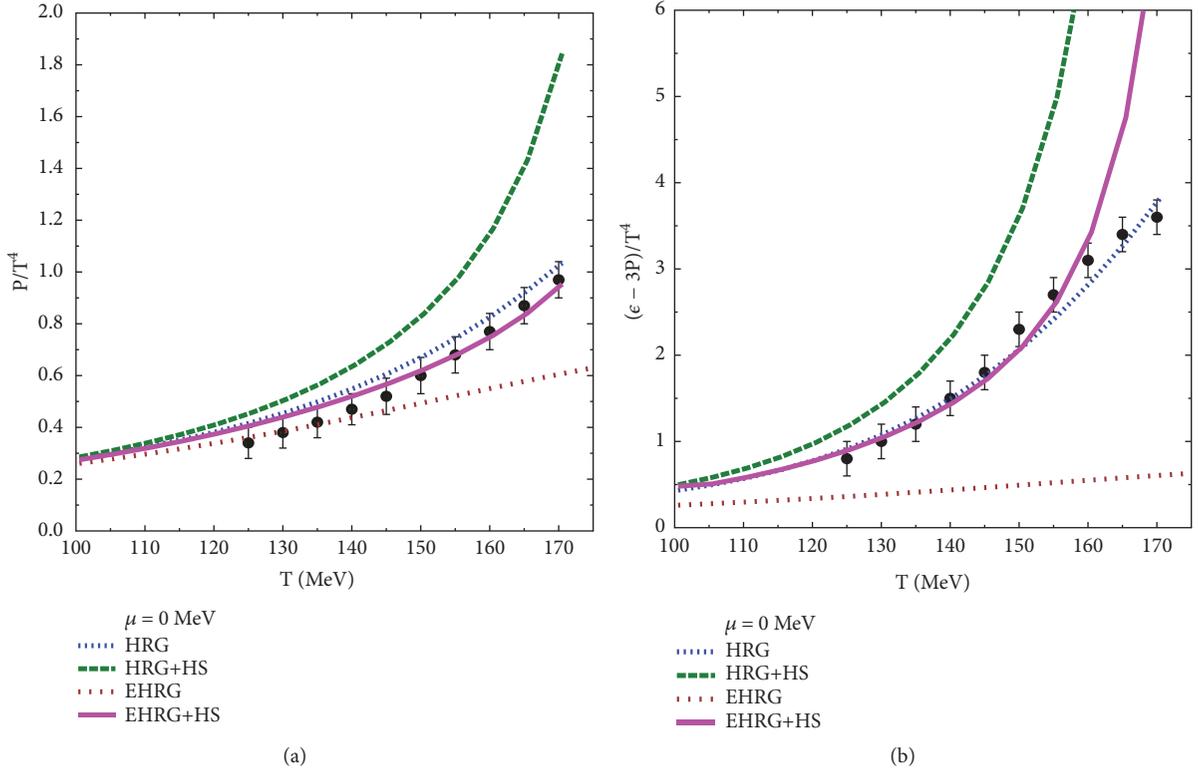


FIGURE 1: Thermodynamical functions, pressure (a) and trace anomaly (b), at zero chemical potential. The hard-core radius of all the hadrons has been set to the value $r_h = 0.4$ fm. The Hagedorn mass spectrum is given by (4). The LQCD data has been taken from [42].

be noted that the HRG model alone cannot reproduce lattice data for the pressure as well as the interaction measure. The HRG estimates for the pressure are merely within the error bars. In case of interaction measure, while the LQCD predict the rapid rise till $T = 160$ MeV, HRG estimates do not show such rapid rise. Similar points can be noted at finite chemical potential as shown in Figure 2. Further, the inclusion of the Hagedorn mass spectrum or the excluded volume corrections to the ideal HRG model does not improve the results. But if excluded volume corrections are made to the ideal HRG model with the discrete as well as continuous Hagedorn mass spectrum then the resulting model (MEHRG) reproduces the LQCD results up to temperature $T = 160$ MeV. This observation has already been made at zero chemical potential in [41] where the authors confronted the LQCD data at $\mu_B = 0$ with the HRG model with the Hagedorn mass spectrum and excluded volume correction. We observe that a similar conclusion can be drawn at finite chemical potential as well. While the ideal HRG does not satisfactorily reproduce all the features of LQCD data, MEHRG reproduces the lattice data all the way up to $T = 160$ MeV. It is to be noted in passing that at finite chemical potential QCD phase transition occurs at lower temperature than that at zero chemical potential.

Figure 3 shows P/T^4 estimated in MEHRG with the Hagedorn mass spectrum of the form $\rho_{HS} = C/(m^2 + m_0^2)e^{m/T_H}$. The values of the parameters are the same as in [41]. We note that our conclusion, i.e., the ideal HRG needing to be improved with excluded volume corrections as well as

Hagedorn density of states, does not depend on the choice of the Hagedorn mass spectrum.

Figure 4 shows comparison of the ratio η/s estimated within our model with that of various other methods [43–46]. The red dashed curve corresponds to the Chapman-Enskog method with constant cross sections [43]. The dashed green curve corresponds to the relativistic Boltzmann equation in relaxation time approximation. The thermodynamical quantities in this model have been estimated using the scaled hadron masses and coupling (SHMC) model [44]. The brown dashed curve corresponds to estimations made using the relativistic Boltzmann equation in RTA. The thermodynamical quantities are estimated within the EHRG model [45]. The dot-dashed orchid curve corresponds to η/s of meson gas estimated using the chiral perturbation theory [46]. While the ratio η/s in our model is relatively large at low temperature as compared to other models it rapidly falls and approaches closer to the Kovtun-Son-Starinets (KSS) bound [67], $\eta/s = 1/4\pi$ at high temperature. This rapid fall may be attributed to the rapidly rising entropy density due to Hagedorn states which are absent in the other models.

Figure 5(a) shows the ratio of shear viscosity to entropy density at two different chemical potentials. The ratio η/s decreases with increase in temperature and approaches the KSS bound at high temperature. The rapid fall in η/s estimated within MEHRG can again be attributed to the rapid rise in the entropy density due to exponentially rising Hagedorn states. At finite μ_B this ratio approaches the

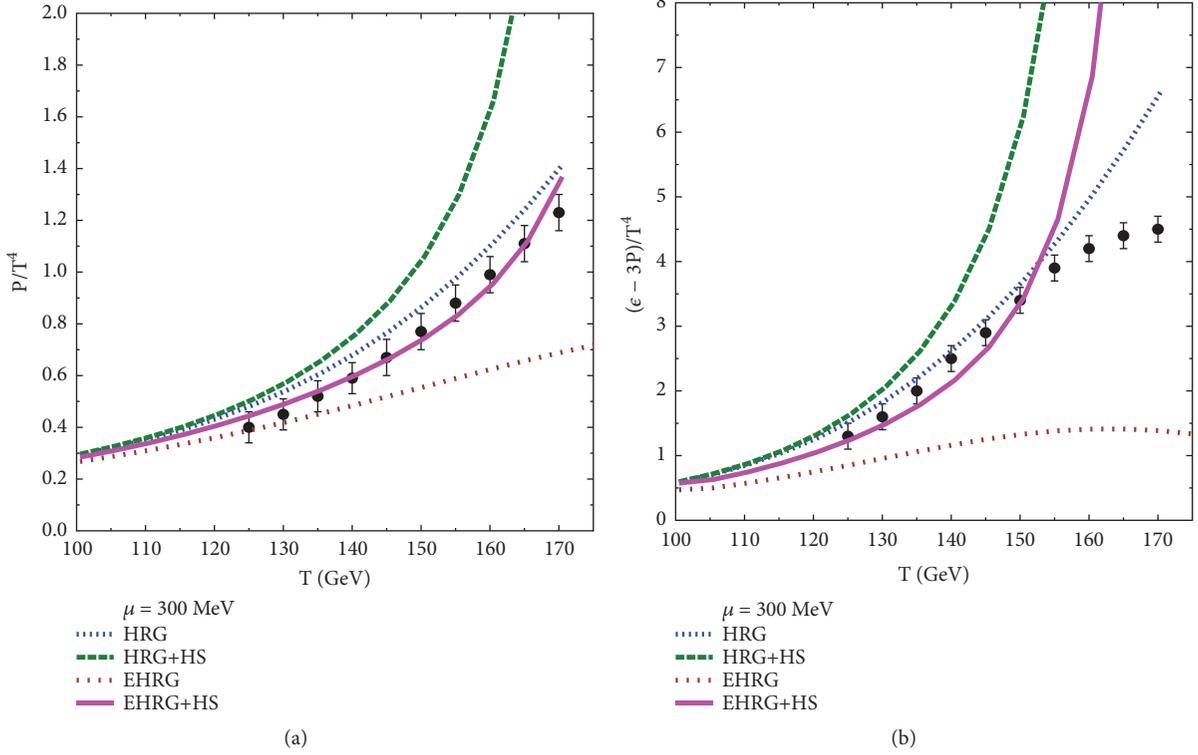


FIGURE 2: Thermodynamical functions, pressure (a) and trace anomaly (b), at finite chemical potential. The hard-core radius of all the hadrons has been set to the value $r_h = 0.4$ fm. The LQCD data has been taken from [42].

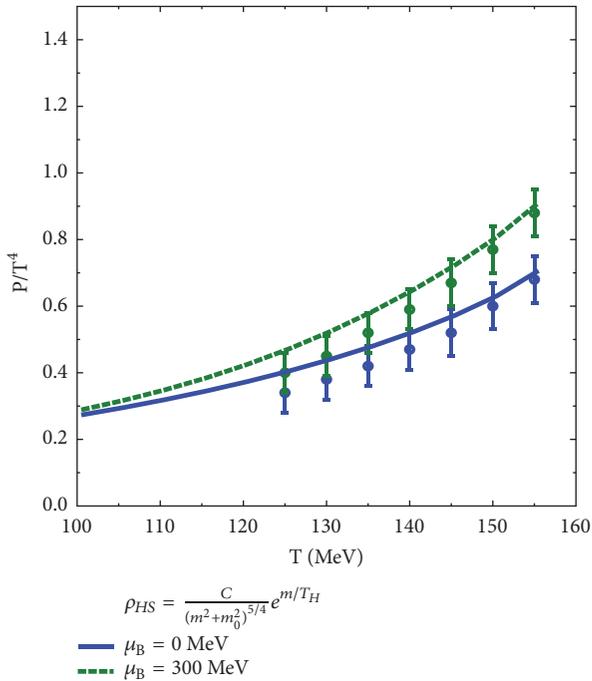


FIGURE 3: P/T^4 at zero as well as finite baryon chemical potential for the Hagedorn spectrum used in [41]. The values of the parameters are also the same as in [41].

KSS bound more closely as compared to the zero chemical potential case.

It has been argued in [90] that at finite chemical potential the correct fluidity measure is not η/s but the quantity $\eta T/(\epsilon + P)$. At zero chemical potential the basic thermodynamical identity implies that two fluidity measures η/s and $\eta T/(\epsilon + P)$ are the same. However, at finite chemical potential they may differ. Figure 5(b) shows the fluidity measure $\eta T/(\epsilon + P)$ at zero as well as at finite chemical potential. It can be noted that $\eta T/(\epsilon + P)$ shows behavior similar to that of η/s .

One can make estimations of η/s in the context of heavy-ion collision experiments by finding the beam energy (\sqrt{S}) dependence of the temperature and chemical potential. This is extracted from a statistical thermal model description of the particle yield at various \sqrt{S} [47, 91–96]. The freeze-out curve $T(\mu_B)$ is parametrized by [47]

$$T(\sqrt{S_{NN}}) = c_+ \left(T_{10} + T_{20} \sqrt{S_{NN}} \right) + c_- \left(T_0^{\text{lim}} + \frac{T_{30}}{\sqrt{S_{NN}}} \right) \quad (26)$$

$$\mu(\sqrt{S_{NN}}) = \frac{a_0}{1 + b_0 \sqrt{S_{NN}}}, \quad (27)$$

where $T_{10} = 34.4$ MeV, $T_{20} = 30.9$ MeV/GeV, $T_{30} = 176.8$ GeV MeV, $T_0^{\text{lim}} = 161.5$ MeV, $a_0 = 1481.6$ MeV, and $b_0 = 0.365$ GeV⁻¹. The functions c_+ and c_- smoothly connect the different behaviors of $\sqrt{S_{NN}}$. Figure 6 shows fluidity measures, η/s and $\eta T/(\epsilon + P)$, along the chemical freeze-out line. It can be noted that the fluidity measures

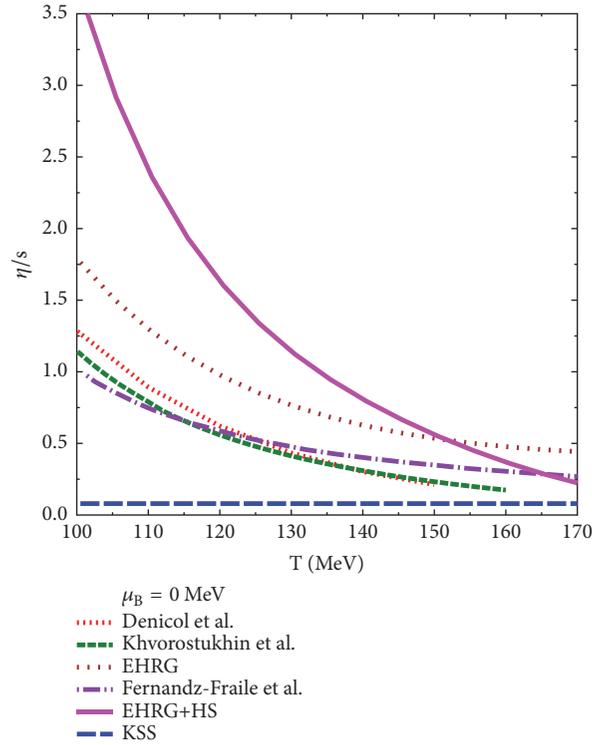


FIGURE 4: Comparison of the ratio η/s estimated within our model, i.e., MEHRG (solid magenta curve), with various other methods. The red dashed curve corresponds to the Chapman-Enskog method [43] with constant cross sections. The dashed green curve corresponds to estimations made within the SHMC model of the hadronic matter [44]. The brown dashed curve corresponds to estimations made within the EHRG model [45]. The dot-dashed orchid curve corresponds to η/s of meson gas estimated using the chiral perturbation theory [46].

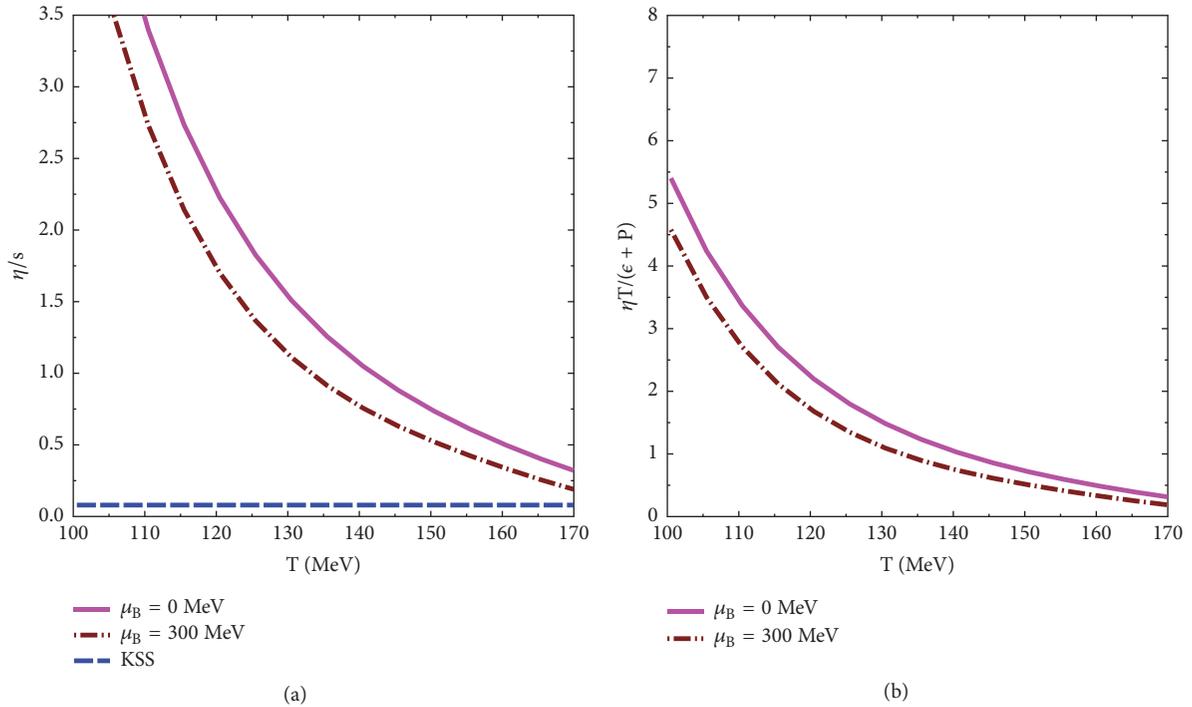


FIGURE 5: (a) shows η/s estimated within the MEHRG model at $\mu_B = 0$ and 300 MeV. The blue dashed curve corresponds to KSS bound, $\eta/s = 1/4\pi$. (b) shows the fluidity measure $\eta T/(\epsilon + P)$ estimated within MEHRG at $\mu_B = 0$ and 300 MeV.

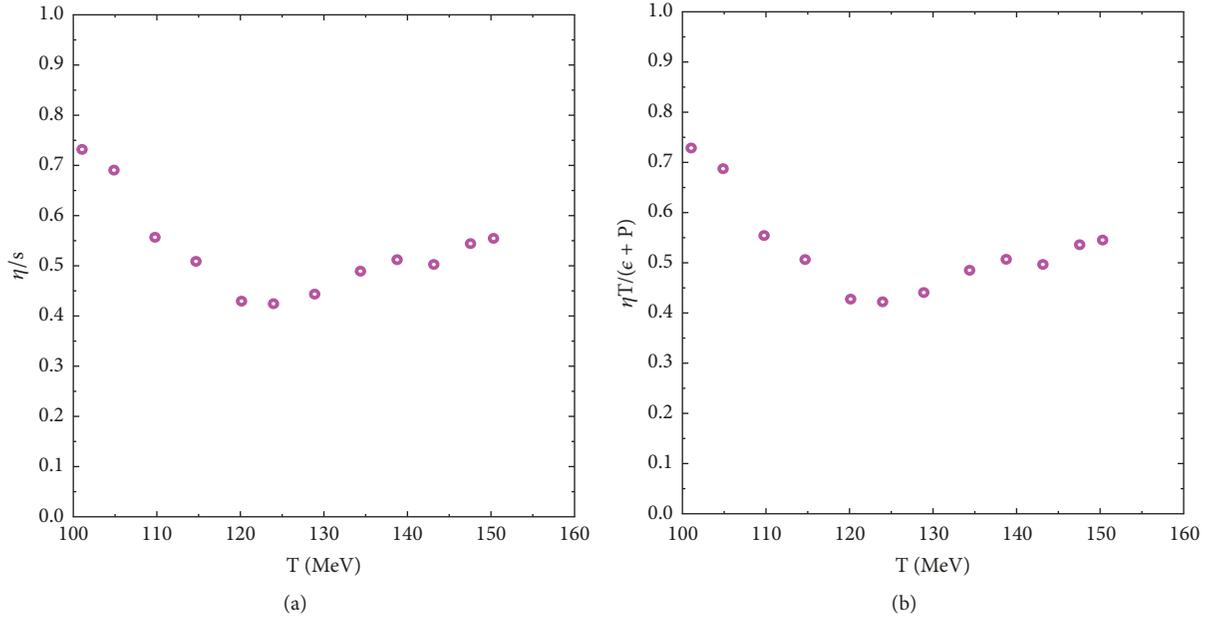


FIGURE 6: Fluidity measures along the chemical freeze-out line ((26) and (27)). The freeze-out curve parametrization is taken from [47].

decrease as the center of mass energy increases and attains the minimum, and then it does not vary much as \sqrt{s} increases. This indicates that the fluid behavior of hadronic matter does not change much for the wide range of higher values of collision energies. Thus the matter produced in heavy-ion collision experiments with the wide range of high collision energies can exhibit substantial elliptic flow.

5. Summary and Conclusion

In this work we confronted the HRG equation of state with the LQCD results at finite baryon chemical potential. We noted that the ideal HRG model along with its extended forms, viz., HRG with excluded volume corrections and HRG with discrete as well as continuous Hagedorn states, cannot explain the lattice data simultaneously at zero as well as finite chemical potential. We found that the LQCD data can be accurately reproduced up to $T = 160$ MeV at zero as well as finite baryon chemical potential if both the excluded volume corrections and Hagedorn states are simultaneously included in the ideal HRG model. We noted that while the excluded volume corrections suppress the thermodynamical quantities as compared to ideal HRG results, the Hagedorn states provide necessary rapid rise in the trace anomaly observed in LQCD simulation results. With these observations we conclude that the unobserved heavy Hagedorn states play an important role in the thermodynamics of hadronic matter especially near QCD transition temperature.

We also estimated the shear viscosity coefficient within the ambit of the HRG model augmented with excluded volume corrections and Hagedorn states. We found that the behaviors of the fluidity measures η/s and $\eta T/(\epsilon + P)$ are in agreement with the existing results. We noted that both fluidity measures fall rapidly at high temperature. This fall

may be attributed to the rapid rise in entropy density, s , and the quantity $(\epsilon + P)/T$ which are thermodynamically related to each other. We further noted that the fluidity measures do not change much at high collision energies. This indicates that the matter produced in heavy-ion collision experiments with the wide range of higher collision energies can exhibit substantial elliptic flow.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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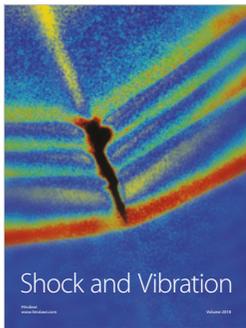
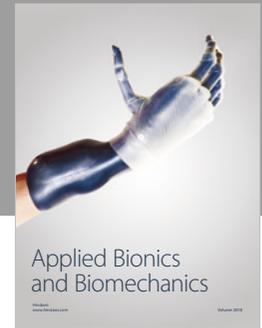
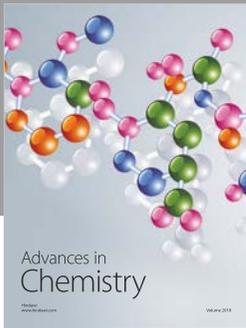
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