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Research Article

New Phase Transition Related to the Black Hole's Topological Charge

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The topological charge ϵ of AdS black hole is introduced by Tian et al. in their papers, where a complete thermodynamic first law is obtained. In this paper, we investigate a new phase transition related to the topological charge in Einstein-Maxwell theory. Firstly, we derive the explicit solutions corresponding to the divergence of specific heat C_ϵ and determine the phase transition critical point. Secondly, the $T-r$ curve and $T-S$ curve are investigated and they exhibit an interesting van der Waals system's behavior. Critical physical quantities are also obtained which are consistent with those derived from the specific heat analysis. Thirdly, a van der Waals system's swallow tail behavior is observed when $\epsilon > \epsilon_c$ in the $F-T$ graph. What is more, the analytic phase transition coexistence lines are obtained by using the Maxwell equal area law and free energy analysis, the results of which are consistent with each other.

1. Introduction

Black hole is a complicated object; there are Hawking radiation, entropy, phase transition, etc. Although black hole's microscopic mechanism is still not clear, its thermodynamic properties can be systematically studied as it is a thermodynamic system which is described by only few physical quantities, such as mass, charge, angular momentum, temperature, and entropy. Since the establishing of the four laws of black hole by Bardeen, Carter, and Hawking [1], black hole thermodynamics has become an exciting and extensively studied topic. Especially, in the anti-de Sitter space, there exists Hawking-Page phase transition [2] which is explained as the confinement/deconfinement phase transition of a gauge field [3] due to the AdS/CFT duality [4–6] and phase transition between small and large charged black holes which is reminiscent to the liquid-gas phase transition of van der Waals system [7]. This close relation between AdS black holes and van der Waals liquid-gas system has been further enhanced by the seminal work of Kubiznak and Mann in [8] where the cosmological constant is identified as thermodynamic pressure and the mass of the black hole is identified as the enthalpy [9]. The $P-V$ curves and the Gibbs free energy graphs in this extended phase space are shown to exhibit an interesting van der Waals systems behavior. For a review on this topic, one can refer to [10] and the references therein. However, in this paper, we will treat the cosmological constant as a constant, leaving the other case for future investigation [11].

Generally, the thermodynamic quantities are described on the horizon and they are related by the first law. However, they can be generalised on surface out of the horizon [12–14]. This has gotten new attention with the development of AdS/CFT, since the black hole thermodynamics on holographic screen has acquired a new and interesting interpretation as a duality of the correspondence field theory [15]. In [16, 17], a maximally symmetric black hole thermodynamics on holographic screen are studied in Einstein-Maxwell's gravity and Lovelock-Maxwell theory. The author found a topological charge naturally arisen in holography. Together with all other known charges (electric charge, mass, and entropy [18]), they satisfy an extended first law and the Gibbs-Duhem-like relation as a completeness. Based on the extended first law in Einstein-Maxwell's gravity, we will investigate the black hole's possible phase transition phenomenon related to the topological charge. Actually, we found that the $T-S$ curves and the free energy graphs also exhibit an interesting van der Waals systems behavior as the extended phase space case does. This result is unexpected, noting that the cosmological constant is not treated as thermodynamic pressure here.

This paper is organized as follows. In Section 2 we will briefly review how the extended first law is obtained in [17]. In Section 3, by analysing the specific heat, the phase transition of AdS black hole in 4-dimensional space-time is studied and the critical point is determined. Then the van der Waals like behavior of temperature is observed in both $T-r$ graph and $T-S$ graph in Section 4. In Section 5 we use the Maxwell equal area law and free energy to have obtained a consistent phase transition coexistence line. Conclusions are drawn in Section 6.

2. Review of the Topologically Charged AdS Black Holes

A d-dimensional space-time AdS black hole solution with the extra topological charge in the Einstein-Maxwell theory was investigated in [16, 17]. The metric reads

$$ds^2 = \frac{dr^2}{f(r)} - f(r)\,dt^2 + r^2 d\Omega_{d-2}^{(k)2}, \tag{1}$$

where

$$\begin{aligned} f(r) &= k + \frac{r^2}{l^2} - \frac{m}{r^{d-3}} + \frac{q^2}{r^{2d-6}}, \\ d\Omega_{d-2}^{(k)2} &= \hat{g}_{ij}^{(k)}(x)\,dx^i dx^j, \\ A &= -\frac{\sqrt{d-2}q}{\sqrt{2(d-3)}r^{d-3}}dt. \end{aligned} \tag{2}$$

m, q, l are related to the ADM mass M, electric charge Q, and cosmological constant Λ by

$$\begin{aligned} M &= \frac{(d-2)\,\Omega_{d-2}^{(k)}}{16\pi}m, \\ Q &= \sqrt{2(d-2)(d-3)}\left(\frac{\Omega_{d-2}^{(k)}}{8\pi}\right)q, \\ \Lambda &= -\frac{(d-1)(d-2)}{2l^2}, \end{aligned} \tag{3}$$

and $\Omega_{d-2}^{(k)}$ is the volume of the "unit" sphere, plane, or hyperbola and k stands for the spatial curvature of the black hole. Under suitable compactifications for $k \le 0$, we assume that the volume of the unit space is a constant $\Omega_{d-2} = \Omega_{d-2}^{(k=1)}$ hereafter [16, 17].

Following [17], the first law can be obtained. Considering an equipotential surface $f(r) = c$ with fixed c, which can be rewritten as

$$f(r,k,m,q) - c = k + \frac{r^2}{l^2} - \frac{m}{r^{d-3}} + \frac{q^2}{r^{2d-6}} - c, \tag{4}$$

defining $K \equiv k - c$, we have

$$\begin{aligned} df(r,k,m,q) &= \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial K}dK + \frac{\partial f}{\partial m}dm + \frac{\partial f}{\partial q}dq \\ &= 0. \end{aligned} \tag{5}$$

Noting

$$\begin{aligned} \partial_r f &= 4\pi T, \\ \partial_K f &= 1, \\ \partial_m f &= -\frac{1}{r^{d-3}}, \\ \partial_q f &= \frac{2q}{r^{2d-6}}, \end{aligned} \tag{6}$$

we obtain

$$dm = \frac{4\pi T}{d-2}dr^{d-2} + r^{d-3}dK + \frac{2q}{r^{d-3}}dq. \tag{7}$$

Multiplying both sides with a constant factor $(d-2)\Omega_{d-2}/16\pi$, the above equation becomes

$$dM = TdS + \frac{(d-2)\,\Omega_{d-2}}{16\pi}r^{d-3}dK + \Phi dQ, \tag{8}$$

where $T = \partial_r f/4\pi$ is the Unruh-Verlinde temperature [12, 19], $S = (\Omega_{d-2}/4)r^{d-2}$ is the Wald-Padmanabhan entropy [18, 20], and $\Phi = \sqrt{(d-2)/2(d-3)}(q/r^{d-3})$ is the electric potential. If we introduce a new "charge" as in [16, 17]

$$\epsilon = \Omega_{d-2}K^{(d-2)/2}, \tag{9}$$

and denote its conjugate potential as $\omega = (1/8\pi)K^{(4-d)/2}r^{d-3}$, then the generalized first law is

$$dM = TdS + \omega d\epsilon + \Phi dQ. \tag{10}$$

This new charge ϵ is called the last (lost) charge of a black hole, and it together with all other known charges satisfies the Gibbs-Duhem-like relation as a completeness relation [17].

3. A New Phase Transition of AdS Black Hole

From the generalized first law, we see there is a topological charge ϵ. In this section, we will investigate the phase transition of AdS black hole in $d = 4$ dimensional space-time in canonical ensemble related to the topological charge rather than the electric charge. To do so, one can observe the behavior of the specific heat at constant topological charge [21].

The Unruh-Verlinde temperature is

$$\begin{aligned} T &= \frac{f'(r)}{4\pi} = \frac{1}{4\pi r}\left(K - \frac{q^2}{r^2} + \frac{3r^2}{l^2}\right) \\ &= \frac{S\epsilon - 4\pi^2 Q^2 + 12S^2/l^2}{8\pi\Omega_2^{1/2}S^{3/2}} \end{aligned} \tag{11}$$

Setting $l = 1, Q = 1, \Omega_2 = 4\pi$ hereafter, the corresponding specific heat with topological charge ϵ fixed can be calculated as

$$\begin{aligned} C_\epsilon &= T\left(\frac{\partial S}{\partial T}\right)_\epsilon = \frac{24S^3 + 2\epsilon S^2 - 8\pi^2 S}{12S^2 - \epsilon S + 12\pi^2} \\ &= \frac{2\pi r^2\left(12\pi r^4 + \epsilon r^2 - 4\pi\right)}{12\pi r^4 - \epsilon r^2 + 12\pi} \end{aligned} \tag{12}$$

From the denominator, we can conclude the following.

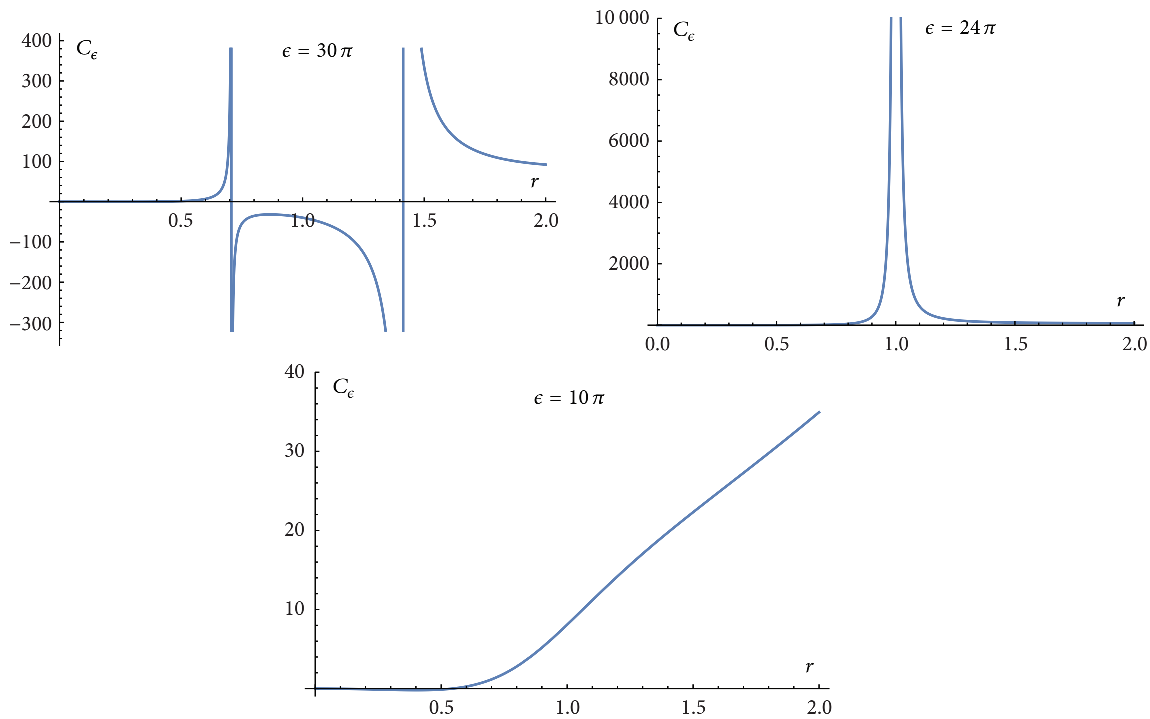

Figure 1: The specific heat C_ϵ versus r for $\epsilon = 30\pi > \epsilon_c$ which has two divergent points, $\epsilon = 24\pi = \epsilon_c$ which has only one divergent point, and $\epsilon = 10\pi < \epsilon_c$ which has no divergent point.

(1) When $\epsilon > 24\pi$, C_ϵ has two diverge points at

$$S_\pm = \frac{\epsilon \pm \sqrt{\epsilon^2 - (24\pi)^2}}{24}, \tag{13}$$

which corresponds to

$$r_\pm = \sqrt{\frac{\epsilon \pm \sqrt{\epsilon^2 - (24\pi)^2}}{24\pi}}. \tag{14}$$

(2) When $\epsilon = \epsilon_c = 24\pi$, C_ϵ has only one diverge point at

$$S = S_c = \pi, \tag{15}$$

which corresponds to

$$r = r_c = 1. \tag{16}$$

The temperature is $T_c = 2/\pi$.

(3) When $\epsilon < 24\pi$, $C_\epsilon > 0$.

Figure 1 shows the behavior of specific heat for the cases $\epsilon > \epsilon_c$, $\epsilon = \epsilon_c$, $\epsilon < \epsilon_c$. For $\epsilon > \epsilon_c$, there are two divergent points on the specific heat curve; they divide the region into three parts: the large radius region, the medium radius region, and the small radius region. With positive specific heat, both the large radius region and the small radius region are thermodynamically stable. While with negative specific heat, the medium radius region is unstable. So there is a phase transition taking place between small black hole and large black hole. For $\epsilon = \epsilon_c$, the curve of specific heat has only one divergent point and is always positive which denote that ϵ_c is the phase transition critical point. While, for $\epsilon < \epsilon_c$, the curve of specific heat has no divergent point and is always positive, which denote that the black holes are stable and no phase transition will take place.

4. Van der Waals Like Behavior of Temperature

It was shown in [8] that when the cosmological constant is identified as thermodynamic pressure [9], $P-v$ graph exhibits van der Waals like behavior. Since this pioneering work, this universal property is discovered in various black holes [22–45]. Here, we find that, for different topological charges, temperature of AdS black holes also possess the interesting van der Waals like property.

In $T - r$ curve, the possible critical point can be obtained by

$$\left(\frac{\partial T}{\partial r}\right)_{\epsilon=\epsilon_c, r=r_c} = 0,$$
$$\left(\frac{\partial^2 T}{\partial r^2}\right)_{\epsilon=\epsilon_c, r=r_c} = 0. \tag{17}$$

Solving the above equations, one can obtain

$$\epsilon_c = 24\pi,$$
$$r_c = 1, \tag{18}$$

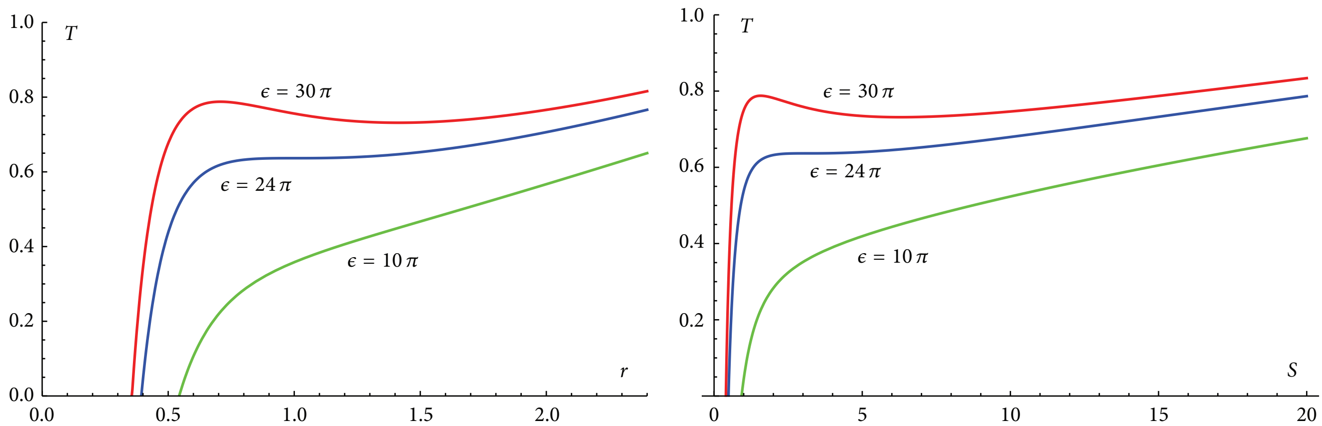

FIGURE 2: T versus r (left graph) and T versus S (right graph) for different topological charge ϵ. There is an oscillating behavior when $\epsilon > \epsilon_c = 24\pi$ for both $T(r)$ and $T(S)$ which is reminiscent of the van der Waals phase transition behavior.

which are exactly the same critical point we obtained by analysing the divergent behavior of specific heat.

In $T-S$ curve, the possible critical point can be obtained by

$$\left(\frac{\partial T}{\partial S}\right)_{\epsilon=\epsilon_c,S=S_c} = 0,$$
$$\left(\frac{\partial^2 T}{\partial S^2}\right)_{\epsilon=\epsilon_c,S=S_c} = 0. \tag{19}$$

Solving the above equations, one can obtain

$$\epsilon_c = 24\pi,$$
$$S_c = \pi, \tag{20}$$

which are also exactly the same critical point we obtained by analysing the divergent behavior of specific heat. As the entropy S is proportional to r^2, the $T-r$ behavior will be similar to the $T-S$ behavior. However r and S are different physical quantities, so we show the $T-r$ graph and $T-S$ graph in the same figure to demonstrate their minor differences.

Figure 2 shows the temperature behavior as a function of r or S for different values of topological charge ϵ. When $\epsilon > \epsilon_c$, the curve can be divided into three branches. The slopes of the large radius branch and the small radius branch are both positive, while the slope of the medium radius branch is negative. When $\epsilon < \epsilon_c$, the temperature increases monotonically as r or S increases. This phenomenon is analogous to that of the van der Waals liquid-gas system.

From above, one can find that, by analysing the specific heat curves, the $T-r$ curves, and the $T-S$ curves, the critical points are obtained and they are consistent with each other. In the above section, we have shown that both the large radius branch and the small radius branch are stable with positive specific heat, while the medium radius branch is unstable with negative specific heat. As argued in [25], one can use the Maxwell equal area law to remove the unstable branch in $T-S$ curve with a bar vertical to the temperature axis $T = T^*$ and obtain the phase transition point (T^*, ϵ). In the next section, we will use the Maxwell equal area law and analyse the free energy to determine the phase transition coexistence line.

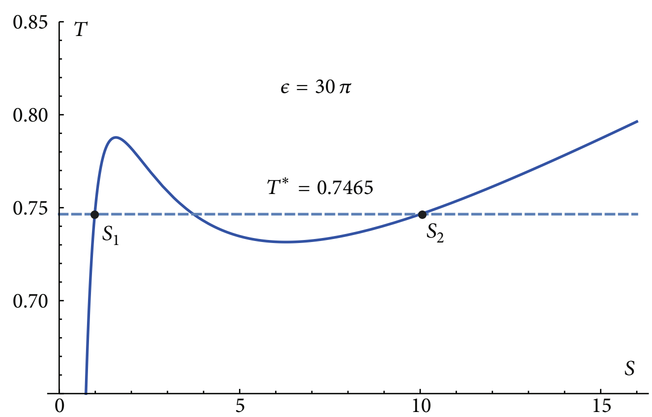

FIGURE 3: T versus S at $\epsilon = 30\pi > \epsilon_c$. The dashed line $T = 0.7465$ equally separates the oscillating part. According to the Maxwell's equal area law, the phase transition point is $(T = 0.7465, \epsilon = 30\pi)$.

5. Maxwell Equal Area Law, Free Energy, and Phase Diagram

In Figure 3, for fixed topological charge $\epsilon > \epsilon_c$, temperature $T(S,\epsilon)$ curve shows an oscillating behavior which denotes a phase transition. The oscillating part needs to be replaced by an isobar (denote as T^*) such that the areas above and below it are equal to each other. This treatment is called Maxwell's equal area law. In what follows, we will analytically determine this isobar T^* for fixed ϵ [25, 26].

Maxwell's equal area law is manifested as

$$T^*\left(S_2 - S_1\right) = \int_{S_1}^{S_2} T\left(S,\epsilon\right) dS = \frac{1}{8\pi^{3/2}}\left(4S_2^{3/2} + \epsilon S_2^{1/2} + 4\pi^2 S_2^{-1/2} - 4S_1^{3/2} - \epsilon S_1^{1/2} + 4\pi^2 S_1^{-1/2}\right). \tag{21}$$

At points (S_1, T^*), (S_2, T^*), we have two equations:

$$\begin{aligned} T^* &= T(S_1, \epsilon) \\ &= \frac{1}{16\pi^{3/2}}\left(12S_1^{1/2} + \epsilon S_1^{-1/2} - 4\pi^2 S_1^{-3/2}\right), \\ T^* &= T(S_2, \epsilon) \\ &= \frac{1}{16\pi^{3/2}}\left(12S_2^{1/2} + \epsilon S_2^{-1/2} - 4\pi^2 S_2^{-3/2}\right). \end{aligned} \tag{22}$$

The above three equations can be solved as

$$\begin{aligned} S_1 &= \frac{\epsilon - 16\pi - \sqrt{(\epsilon - 16\pi)^2 - 64\pi^2}}{8}, \\ S_2 &= \frac{\epsilon - 16\pi + \sqrt{(\epsilon - 16\pi)^2 - 64\pi^2}}{8}, \\ T^* &= \frac{\epsilon^2 - \epsilon\sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2} - 28\pi\epsilon + 12\pi\sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2} + 160\pi^2}{\sqrt{2}\pi^{3/2}\left(\epsilon - 16\pi - \sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2}\right)^{3/2}}. \end{aligned} \tag{23}$$

The last equation $T^*(\epsilon)$ is the phase transition curve we are looking for.

To double-check the phase transition curve obtained by Maxwell's equal area law, we will probe the behavior of free energy, which is derived as

$$F = M - TS = \frac{12\pi^2 + \epsilon S - 4S^2}{16\pi^{3/2}\sqrt{S}}. \tag{24}$$

Since temperature is also a function of S and ϵ, we can plot F versus T in Figure 4. When $\epsilon > 24\pi = \epsilon_c$, $F - T$ curve shows a swallow tail behavior which is reminiscent of $G - T$ curve for the van der Waals system. In this sense, the free energy here should be regarded as Gibbs free energy, and the inner energy M should be regarded as enthalpy. Anyway, the cross point is determined by the equations below.

$$\begin{aligned} T^* &= T(S_1, \epsilon) = T(S_2, \epsilon), \\ F^* &= F(S_1, \epsilon) = F(S_2, \epsilon). \end{aligned} \tag{25}$$

The right side equations can be rewritten as

$$\begin{aligned} &\frac{1}{16\pi^{3/2}}\left(12S_1^{1/2} + \epsilon S_1^{-1/2} - 4\pi^2 S_1^{-3/2}\right) \\ &\quad = \frac{1}{16\pi^{3/2}}\left(12S_2^{1/2} + \epsilon S_2^{-1/2} - 4\pi^2 S_2^{-3/2}\right), \\ &\frac{1}{16\pi^{3/2}}\left(12\pi^2 S_1^{-1/2} + \epsilon S_1^{1/2} - 4S_1^{3/2}\right) \\ &\quad = \frac{1}{16\pi^{3/2}}\left(12\pi^2 S_2^{-1/2} + \epsilon S_2^{1/2} - 4S_2^{3/2}\right). \end{aligned} \tag{26}$$

These equations can be solved as

$$\begin{aligned} S_1 &= \frac{\epsilon - 16\pi - \sqrt{(\epsilon - 16\pi)^2 - 64\pi^2}}{8}, \\ S_2 &= \frac{\epsilon - 16\pi + \sqrt{(\epsilon - 16\pi)^2 - 64\pi^2}}{8}, \\ F^* &= \frac{\epsilon - \sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2} - 8\pi}{2\sqrt{2\pi}\sqrt{\epsilon - 16\pi - \sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2}}}, \\ T^* &= \frac{\epsilon^2 - \epsilon\sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2} - 28\pi\epsilon + 12\pi\sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2} + 160\pi^2}{\sqrt{2}\pi^{3/2}\left(\epsilon - 16\pi - \sqrt{\epsilon^2 - 32\pi\epsilon + 192\pi^2}\right)^{3/2}}. \end{aligned} \tag{27}$$

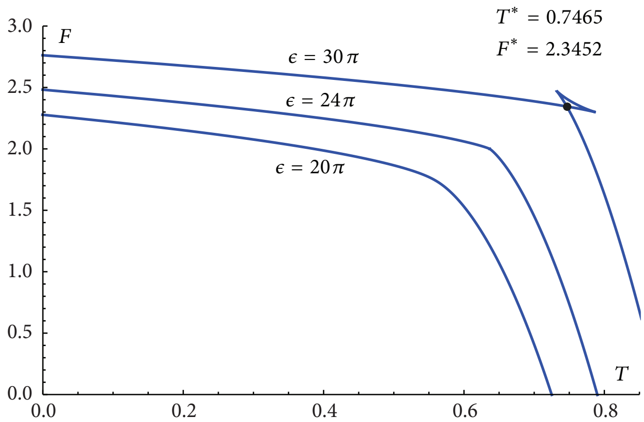

FIGURE 4: F versus T for different topological charge ϵ. When $\epsilon > 24\pi = \epsilon_c$, the curve shows a swallow tail behavior.

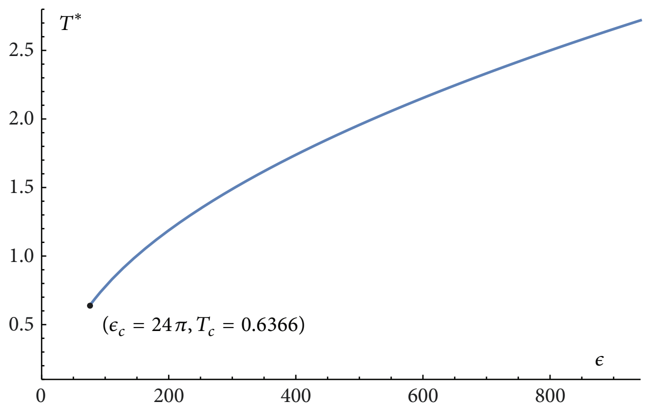

FIGURE 5: The phase transition coexistence line $T^* - \epsilon$ for fixed electric charge $Q = 1$ and AdS radius $l = 1$.

They are consistent with the results obtained by Maxwell's equal area law.

Finally, we can show the phase transition coexistence line in Figure 5 for fixed electric charge Q and AdS radius l. This kind of phase transition is special, as the critical point is at the small value of (T^*, ϵ) in phase diagram.

6. Conclusion

In this paper, the phase transition phenomenon of Reissner-Nordström AdS black holes relating to the topological charge ϵ in canonical ensemble in 4-dimensional space-time are studied. As we are interested in the effects of the topological charge, so the electric charge is fixed $Q = 1$. Firstly, the black hole's specific heat C_ϵ is calculated and the corresponding divergence solutions are derived. The two solutions merge into one denoting the critical point where $\epsilon_c = 24\pi$, $r_c = 1$. When $\epsilon > \epsilon_c$, the curve of specific heat has two divergent points and is divided into three regions. The specific heat is positive for both the large radius region and the small radius region which are thermodynamically stable, while it is negative for the medium radius region which is unstable. When $\epsilon < \epsilon_c$, the specific heat is always positive implying the black holes are stable and no phase transition will take place.

Secondly, the behavior of temperature in both the $T - r$ graph and $T - S$ graph is studied. They exhibit the interesting van de Waals gas-liquid system's behavior. The critical points correspond to the inflection points of $T - r$ curve and $T - S$ curve, and they are consistent with that derived from the specific heat analysis. When $\epsilon > \epsilon_c$, the curves can be divided into three regions. The slopes of the large radius regions and the small radius regions are positive while those of the medium radius region are negative. When $\epsilon < \epsilon_c$, the temperature increase monotonically.

Thirdly, a van der Waals system's swallow tail behavior is observed when $\epsilon > \epsilon_c$ in the $F - T$ graph. What is more, by using the Maxwell's equal area law and analysing the free energy, the analytic phase transition coexistence lines are obtained, and they are consistent with each other.

From the above detailed study, one can find that this van der Waals like system exhibits phase transition of special property. The phase transition take place at large topological charge $\epsilon > \epsilon_c$ and high temperature which can be clearly seen from the phase transition coexistence line in Figure 5. Whether this phase transition property is universal in other gravity theories (such as the Lovelock, Gauss-Bonnet theory) and different dimensional space-time is unknown.

There are some other interesting topics that are worth investigating, such as the holographic duality in the field theory of this kind of phase transition; the cases of space-time dimension $d > 4$, as the topological charge's conjugate potential $\omega = (1/8\pi)K^{(4-d)/2}r^{d-3}$ decayed to $1/8\pi r$ which is irrelevant to K at $d = 4$ dimensional space-time, the black hole thermodynamics in the extended phase space where the cosmological constant is identified as thermodynamic pressure and the spatial curvature of black hole is treated as topological charge.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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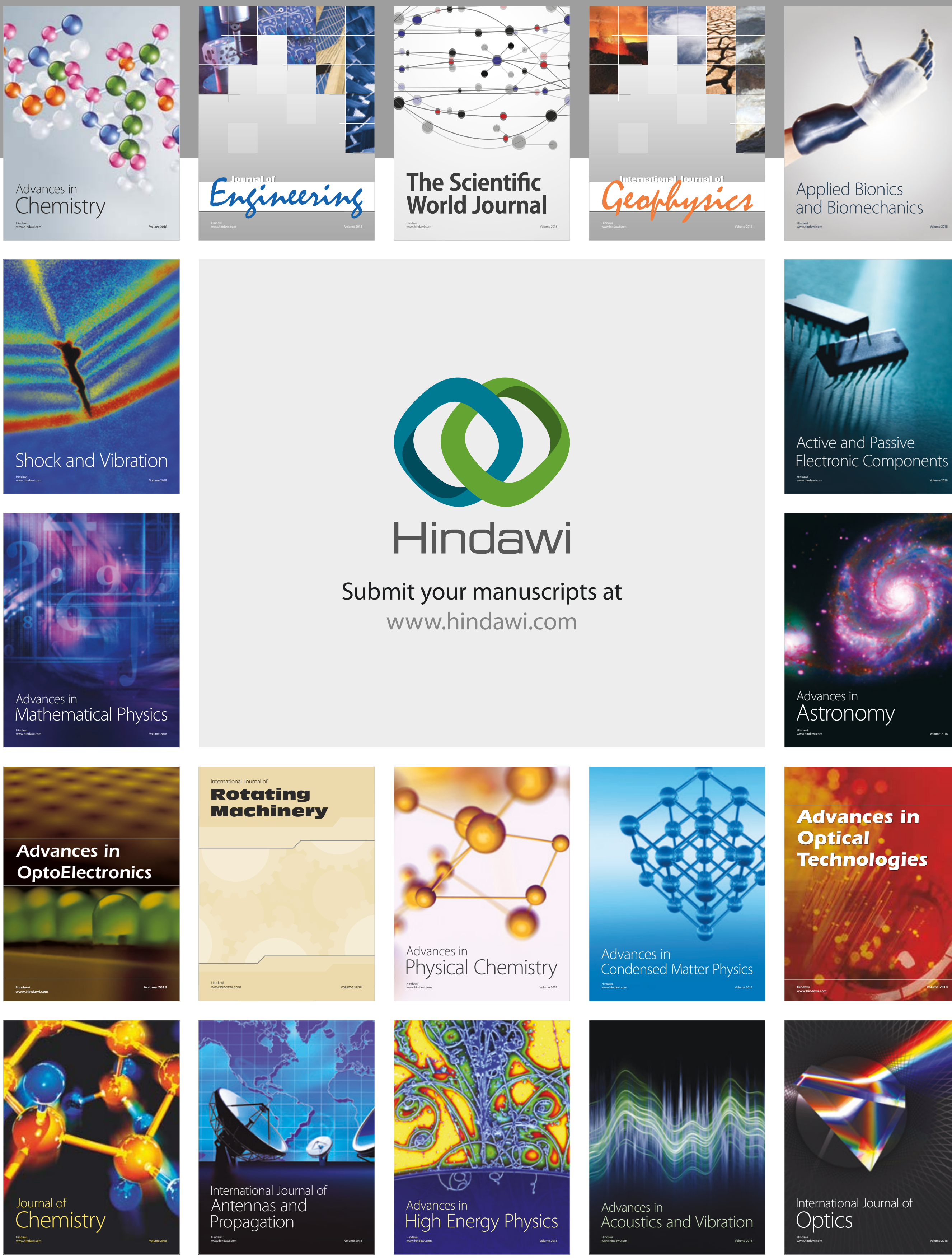
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