Intermittency Study of Charged Particles Generated in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV Using EPOS3

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1. Introduction

The strongly interacting dense state of matter, believed to represent QGP (quark-gluon plasma) after its creation in heavy-ion collision, rapidly cools into a spray of particles. This array of particles carry signals of QGP and its properties which can be directly and indirectly measured by detectors that are encircling the collision point. Of the myriad of analysis tools to understand the dynamics of this particle production [1] and phase changes in the matter while passing into the QGP phase from the hadronic phase and vice versa, an important one is the fluctuations study of the observables. Lattice QCD predicts large fluctuations being associated with the system undergoing phase transition. Multiplicity distributions characterize the system formed or any phase transition in the heavy-ion collisions. Studies of multiplicity fluctuations have prompted considerable advances in this area of research. Large particle density fluctuations in the JACEE event [2] and its explanation by normalized factorial moments triggered investigations of multiplicity fluctuation patterns in multihadronic events with decreasing domains of phase space [3]. The presence of power-law behaviour or scale invariance of normalized factorial moments with decreasing phase space interval or increasing bins is termed as intermittency [4, 5]. Observation of intermittency signals the presence of self-similar and fractal nature of the particle production. If fluctuations have a dynamical origin, the underlying probability density will be reflected as intermittency behaviour. The existence of dynamical fluctuations can thus be studied using normalized factorial moments (NFMs) [4] in one-, two-, or three-dimensional phase space.

The idea of intermittency has been obtained from the theory of turbulent flow. There, it signifies as a property of turbulent fluid: vortices of fluid with different size alternate in such a way that they form self-similar structures. These vortices do not necessarily fill in the entire volume, but instead create an intermittent pattern in the regions of laminar flow. This property is given by a power-law variation of the vortex-distribution moments on their size. So, the self-similar nature of vortices directly creates a relation between intermittency and fractality. Self-similar objects of nonintegral dimensions are called fractals [6]. A fractal dimension is a generalization of an ordinary topological dimensionality to nonintegers.
The proposal to look for intermittency also prompts a thorough study of phase-transition models. A very straightforward model that offers some hint on the nature of a second-order phase transition is the Ising model in two dimensions [7]. Intermittency in Ising model has been studied both analytically and numerically [8, 9], and the anomalous fractal dimension (\(d_q\)) is found to be 1/8, independent of the order of moment, \(q\). It has been conjectured on this account that intermittency may be monofractal in QCD second-order phase transition [10]. However, all types of interactions including heavy-ion collisions show multifractal behaviour [3, 11]. Also, Yang-Mills fields have been applied to QCD within asymptotic approximation where the fractal dimension is determined as a function of entropic index, and value obtained for entropic index is in good agreement with the experimental data [12]. For the first-order phase transition, all \(d_q\) are zero, and no intermittency was observed. Intermittency has also been studied in Ginzburg-Landau (GL) theory, which has been accustomed to explain the confinement of magnetic fields into fluxoids in a type-II superconductor. From the study of normalized factorial moments with decreasing phase space bins for the Ginzburg-Landau second-order formalism, the anomalous fractal dimension is observed not to be constant. It follows 

\[ d_q/d_2 = (q - 1)^{1/2} \],

where \(\nu\) is the scaling exponent [13]. \(d_q\) is observed to be a universal quantity valid for all systems describable by the GL theory for the second-order phase transition, and it is independent of the underlying dimensions or the parameters of the model. This is of particular importance for a QCD phase transition, since neither the transition temperature nor the other important parameters are known there. If a signature of quark-hadron phase transition depends on the details of the heavy-ion collisions, e.g., nuclear sizes, collision energy, and transverse energy, then even after the system has passed the thresholds for the creation of QGP, such a signature is likely to be sensitive to this theory.

In this work, intermittency analysis is performed for the charged particles generated in the midrapidity region of the central events \((b \leq 3.5 \text{ fm})\) from Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76 \text{ TeV}\) using EPOS3 (hydro) and EPOS3 (hydro+cascade).

The plan of the paper is as follows: the EPOS3 model [14] is introduced in Section 2. The methodology of analysis is given in Section 3. In Section 4, observations and results are given followed by a summary in Section 5.

2. A Brief Introduction to EPOS3

EPOS3 [14–16] is a hybrid Monte-Carlo event generator with a 3+1D hydrodynamical expanding system. This model is based on flux tube initial conditions which are generated in the Gribov-Regge multiple scattering framework. The formalism is referred to as "Parton based Gribov Regge Theory", which is detailed in [17]. An individual scattering gives rise to a parton ladder and is called a Pomeron. Each parton ladder eventually shows up as flux tubes (or strings) and is identified by a PQCD hard process, plus initial, and final state linear parton emission. Satura-

![Figure 1: Charged particle pseudorapidity density distributions of EPOS3 (hydro) and EPOS3 (hydro+cascade) compared with that of the ATLAS data [20], for Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76 \text{ TeV}\).](image-url)
centralities and are compared with that of ATLAS data [20] for the same system and energy. Where for the polar angle \( \theta \) of the particle, measured with respect to the beam axis, the pseudorapidity \( \eta \) is defined as \( \eta = -\ln \tan (\theta/2) \). In this work, analysis is performed for the charged particles generated in full azimuthal space with \( |\eta| \leq 0.8 \) in the most central events. It is observed (Figure 1) in that the midrapidity region of our interest \( (|\eta| \leq 0.8) \), charged particle pseudorapidity density of the EPOS3 generated central (0-10\%) events, slightly overestimates the ATLAS data within errors.

Intermittency studies at low energies had limitation of statistics because a lesser number of particles were available per bin for the order of the moment \( q \geq 2 \). In the present collider experiments, with the availability of high multiplicity events per pseudorapidity unit both in pp and AA collisions the studies of local multiplicity fluctuations, dependent on the bin contents can be taken up, to get a clear and complete picture of the multiparticle production. Predictions for intermittency analysis of data at present collider energies are still not available. Present work is carried to study scaling behaviours of the charged particles multiplicity fluctuations and hence the intermittency in the EPOS3 model, which is based on the hydrodynamic particle production mechanism.

3. Methodology

Observation of spike events first noticed in the cosmic ray interaction [2] and later in the laboratory [3, 11] lead to great spurt of interest in the studies of intermittency in particle production in high-energy collisions. In [4, 5], groundbreaking work was done theoretically formulating the features of intermittency in the field of particle physics.

Intermittency is defined as the scale-invariance of NFM, \( F_q \), with respect to changes in the size of phase space cells (bins) [4]. For one-dimensional phase space of rapidity \( Y \), with cell \( \delta y \) (say), it is defined as

\[
F_q(\delta y) \propto (\delta y)^{-\phi_q}(\delta y \to 0),
\]

where \( F_q \)'s are the NFM [4], of order \( q \), where \( q \) is a positive integer and takes values \( \geq 2 \) and \( \phi_q > 0 \) is called the “intermittency index” or “intermittency slope”. In terms of the number of bins \( M \) in the phase space, where \( M \propto 1/\delta y \); Equation (1) can be written as

\[
F_q(M) \propto M^{\phi_q}.
\]

In [21, 22], it is proposed that NFM using event NFM be investigated at LHC energies where the charged particle density is very high. The event NFM, \( F_q^e \), is defined as

\[
F_q^e(M) = \frac{f_q^e(M)}{[f_q^e(M)]^{1/2}},
\]

with \( f_q^e(M) = \langle n_m(n_m-1) \cdots (n_m-q+1) \rangle \), where \( \langle \cdots \rangle \) is the averaging over all bins in an \( e^{th} \) event, called horizontal averaging, and \( n_m \) is bin multiplicity of the \( m^{th} \) bin. NFM \( F_q \) for a sample of events, \( N_{\text{evt}} \), is then

\[
F_q(M) = \frac{1}{N_{\text{evt}}} \sum_{\text{evt} = 1}^{N_{\text{evt}}} F_q^e(M).
\]

\( F_q(M) \) enjoys the property of filtering out statistical fluctuations (or noise) [4, 23]. The scaling of the NFM, \( F_q^e \) with number of bins \( M \) as in Equation (2) is referred here as \( M \)-\textit{scaling}. Observation of this scaling would indicate the self-similarity in the spatial distribution of the particles. It has been observed that the Ginzburg-Landau formalism [13] for second-order phase transition, \( F_q^e \) follows power-law as

\[
F_q \propto \tau^{-\beta_q},
\]

such that \( \beta_q = (q-1)\nu \) with \( \nu = 1.304 \). Equation (5) is referred here as \( F \)-\textit{scaling}. Its validity is independent of the scaling behaviour in Equation (2).

There exist more complicated self-similar objects which include fractal patterns with different noninteger dimensions, \textit{multifractals} [3, 11, 24, 25]. Multifractals are characterized by generalized (or \( R_q \)-\textit{enyi}) dimensions \( D_q \), which are decreasing functions of \( q \). The thought of \( R_\text{enyi} \) dimensions \( D_q \) generalizes the idea of fractal dimension \( D_0 = D_p \), information dimension \( D_1 \), and correlation dimension \( D_2 \). Consequently, the \( R_\text{enyi} \) dimension is often known as the generalized dimension. The anomalous fractal dimension \( d_q \) is related to the generalized dimension \( D_q \) by the relation

\[
d_q = D - D_q,
\]

where \( D \) is the topological dimension that represents the number of dimensions. A relation between the exponents of factorial moments, intermittency index \( \phi_q \), and generalized moments can be devised at low values of \( q \) as

\[
\phi_q + \tau(q) = (q-1)D_q,
\]

where the exponents are related to \( R_\text{enyi} \) dimensions and codimension as

\[
\tau(q) = (q-1)\phi_q,
\]

It is needed to stress that the slope \( \tau_q \) has no dynamical feature of \( \phi_q \) and needs to be corrected for the statistical contribution to be removed [26]. Increasing \( d_q \) with \( q \) is a signal of the multifractal system.

Here, intermittency and notion of fractality for charged particle multiplicity distribution is studied in the two-dimensional phase space \((\eta, \phi)\) of the events generated using EPOS3 for the Pb-Pb collision system at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \).
collisions at \( \sqrt{s} \), with kaons, and protons) generated in the kinematical region have been analyzed. In this work, charged particles (pions, \( \pi \)) for the SM AMPT model. The been studied.

there are a total of \( n_{\text{m}} \) \( \leq \Delta p_T \leq 0.6 \text{ GeV/c} \) and \( 0.6 \leq p_T \leq 0.8 \text{ GeV/c} \).

**Figure 2:** Log-Log \( F_q \) dependence on number of bins (\( M^2 \)) for EPOS3-hydro events for the \( p_T \) bins \( 0.4 \leq p_T \leq 0.6 \text{ GeV/c} \) and \( 0.6 \leq p_T \leq 0.8 \text{ GeV/c} \).

**Figure 3:** Log-Log \( F_q \) dependence on number of bins (\( M^2 \)) for EPOS3-hydro+cascade events for the \( p_T \) bins \( 0.4 \leq p_T \leq 0.6 \text{ GeV/c} \) and \( 0.6 \leq p_T \leq 0.8 \text{ GeV/c} \).

### 4. Analysis and Observations

A two-dimensional intermittency analysis in (\( \eta, \phi \)) phase space in different \( p_T \) \( \left( p_T = \sqrt{p_x^2 + p_y^2} \right) \), where \( p_x \) and \( p_y \) are the momentum components in the transverse momentum plane) bins of varying widths \( 0.2 \leq p_T \leq 0.4 \text{ GeV/c} \), \( 0.4 \leq p_T \leq 0.6 \text{ GeV/c} \), \( 0.6 \leq p_T \leq 0.8 \text{ GeV/c} \), \( 0.8 \leq p_T \leq 1.0 \text{ GeV/c} \), \( 0.2 \leq p_T \leq 0.6 \text{ GeV/c} \), \( 0.2 \leq p_T \leq 0.8 \text{ GeV/c} \), and \( 0.2 \leq p_T \leq 1.0 \text{ GeV/c} \) are performed for two event samples for Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{TeV} \) generated using two modes of EPOS3. Central events with impact parameter \( b \leq 3.5 \text{ fm} \) have been analyzed. In this work, charged particles (pions, kaons, and protons) generated in the kinematical region with \( |\eta| \leq 0.8 \), full \( \phi \) coverage, and \( p_T \leq 1.0 \text{ GeV/c} \) have been studied.

The methodology adopted for analysis is the same as in [27] for the SM AMPT model. The (\( \eta, \phi \)) phase space in a \( p_T \) bin, for an event, is divided into a \( M \times M \) matrix such that there are a total of \( M^2 \) bins. \( M \) is taken from 2 to 32 in an interval of 2. Number of charged particles in a bin, \( n_{m,n} \), is the bin multiplicity in the \( m^{th} \) bin. Event factorial moment, \( F_q^c(M) \) (Equation (3)), is determined for \( n_{m,n} \geq q \), where \( q = 2, 3, 4, \text{ and } 5 \) is the order of the moment. \( F_q^c(M) \) is obtained for all the events in the event sample. This gives the event factorial moment distribution and hence the \( F_q^c(M) \) (Equation (4)). \( F_q^c(M) \) are thus studied for their dependence on \( M \) and the second-order normalized factorial moments (\( F_2^c(M) \)).

From the study of dependence of \( F_q^c \) on \( M \) (M-scaling) for the various \( p_T \) bins, it is observed that for the small \( p_T \) bins with width \( \Delta p_T = 0.2 \text{ GeV/c} \), \( \Delta p_T = 0.4 \text{ GeV/c} \), \( 0.4 \leq p_T \leq 0.6 \text{ GeV/c} \), \( 0.6 \leq p_T \leq 0.8 \text{ GeV/c} \), and \( 0.8 \leq p_T \leq 1.0 \text{ GeV/c} \) M-scaling is absent in the case of both hydro and hydro+cascade events. For two bins, \( \ln F_q^c \) vs \( \ln M^2 \) graphs for \( q = 2, 3, 4, 5 \) are given in Figure 2 (EPOS3 hydro) and Figure 3 (EPOS3 hydro+cascade). For the wider \( p_T \) bins with \( \Delta p_T \geq 0.6 \text{ GeV/c} \) that is for \( 0.2 \leq p_T \leq 0.8 \text{ GeV/c} \) and \( 0.2 \leq p_T \leq 1.0 \text{ GeV/c} \), scaling of \( F_q^c \) with \( M \) is observed in the lower \( M \) region followed by saturation effects at higher \( M \) region as observed in \( \ln F_q^c \) vs \( \ln M^2 \) graph in Figure 4 for EPOS3 (hydro). For the same \( p_T \) bins that is \( 0.2 \leq p_T \leq 0.8 \text{ GeV/c} \) and \( 0.2 \leq p_T \leq 1.0 \text{ GeV/c} \). Figure 5 shows the same graphs from EPOS3 (hydro+cascade) events. M-scaling is observed to be present in the low \( M \) region with saturation and
overlapping effects at higher $M$. Absence of power-law or $M$-scaling in narrow $p_T$ bins clearly indicates the absence of local density fluctuations and hence, the intermittency signal. The presence of weak intermittency in the wider $p_T$ bins is probably due to number effect as average bin content increases in the given phase space. The error bars are the statistical uncertainties, calculated using the error propagation formula as suggested in [28].

$F_q$ is observed to show a linear dependence on $F_2$ even in the absence of $M$-scaling [13]. In Figures 6 and 7, $\ln F_q$ vs
is 1 phase transition [30]. Since the lattice QCD predicts continuous crossover type of (hydro+cascade), which is different from the value of 1.304 as is obtained from the GL formalism for the second-order ν heavy-ion experiments. The hydro and hydro+cascade mode do not have fractal behaviour. This observation indicates that EPOS3 in dimensions are observed to be much smaller than one ϕ intermittency index (ϕq) and thus the fractal dimensions Dq are determined and are plotted against q in Figure 8. The Dq grows in a way such that that the fractal (Rényi) dimensions Dq are close to one. However in the data, the fractal dimensions are observed to be much smaller than one [3, 11, 31]. This observation indicates that EPOS3 in hydro and hydro+cascade mode do not have fractal behaviour. The Dq decreases faster with increasing order of the moment q and has similar behaviour for both the bins for the two modes of the EPOS3 modes. However, D2 < D3 contradicts the data [31, 32].

\[ \frac{\partial p_T}{\partial T} \geq 0.6 \text{ GeV/c}, \text{ the same bins in which for M-scaling is observed for low M values. } F_q \text{ is observed to follow power-law in } F_2, \text{ whereas in the smaller } p_T \text{ bins, } F-\text{scaling is also absent.} \]

Scaling index, ν, is determined from the slope for ln βq against ln (q − 1). The scaling index, (v) obtained for the two cases, is enlisted in Table 1. The NA22 data on particle production in hadronic collisions gives ν = 1.45 ± 0.04, heavy-ion experiments ν = 1.55 ± 0.12 [13], and ν = 1.459 ± 0.021 [29]. However, the average value of ν obtained here is 1.795 ± 0.156 EPOS3 (hydro) and 1.824 ± 0.295 EPOS3 (hydro+cascade), which is different from the value of 1.304 as is obtained from the GL formalism for the second-order phase transition. The values obtained here are significant, since the lattice QCD predicts continuous crossover type of phase transition [30].

For the two pT bins in which M-scaling is observed for the low M-region, the d_q s have been calculated from the intermittency index (ϕq) and thus the fractal dimensions Dq are determined and are plotted against q in Figure 8. The d_q grows in a way such that that the fractal (Rényi) dimensions Dq are close to one. However in the data, the fractal dimensions are observed to be much smaller than one [3, 11, 31]. This observation indicates that EPOS3 in hydro and hydro+cascade mode do not have fractal behaviour. The Dq decreases faster with increasing order of the moment q and has similar behaviour for both the bins for the two modes of the EPOS3 modes. However, D2 < D3 contradicts the data [31, 32].

<table>
<thead>
<tr>
<th>Event sample</th>
<th>p_T bins (GeV/c)</th>
<th>Value of v</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>0.2-0.8</td>
<td>1.84 ± 0.19</td>
</tr>
<tr>
<td></td>
<td>0.2-1.0</td>
<td>1.75 ± 0.12</td>
</tr>
<tr>
<td>Hydro+cascade</td>
<td>0.2-0.8</td>
<td>1.85 ± 0.33</td>
</tr>
<tr>
<td></td>
<td>0.2-1.0</td>
<td>1.80 ± 0.26</td>
</tr>
</tbody>
</table>

\[ \ln F_2 \text{ plots are given for the bins with } \Delta p_T \geq 0.6 \text{ GeV/c, the same bins in which for M-scaling is observed for low M values. } F_q \text{ is observed to follow power-law in } F_2, \text{ whereas in the smaller } p_T \text{ bins, } F-\text{scaling is also absent.} \]

\[ \ln \frac{F_q}{\ln F_q} \text{ for EPOS3-hydro+cascade events for the } p_T \text{ bins } 0.2 \leq p_T \leq 1.0 \text{ GeV/c.} \]

\[ \phi \text{ dependence on } \ln F_q \text{ and has similar behaviour for both the bins in the } \phi \text{ region, } \phi \text{ scale dependence of fractal dimensions, } D_q \text{ for the two cases, is enlisted in Table 1. The NA22 data on particle production in hadronic collisions gives } \nu = 1.45 \pm 0.04, \text{ heavy-ion experiments } \nu = 1.55 \pm 0.12 [13], \text{ and } \nu = 1.459 \pm 0.021 [29]. \text{ However, the average value of } \nu \text{ obtained here is } 1.795 \pm 0.156 \text{ EPOS3 (hydro) and } 1.824 \pm 0.295 \text{ EPOS3 (hydro+cascade), which is different from the value of 1.304 as is obtained from the GL formalism for the second-order phase transition. The values obtained here are significant, since the lattice QCD predicts continuous crossover type of phase transition [30].} \]

\[ \nu \text{ grows in a way such that that the fractal (Rényi) dimensions } D_q \text{ are close to one. However in the data, the fractal dimensions are observed to be much smaller than one [3, 11, 31]. This observation indicates that EPOS3 in hydro and hydro+cascade mode do not have fractal behaviour. The } D_q \text{ decreases faster with increasing order of the moment } q \text{ and has similar behaviour for both the bins for the two modes of the EPOS3 modes. However, } D_2 < D_3 \text{ contradicts the data [31, 32].} \]

5. Summary

An event-by-event intermittency analysis is performed for the charged particle multiplicity distributions of the events generated using two different modes of EPOS3 hydrodynamical model. Central events with b ≤ 3.5 fm generated from Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) have been studied. The two-dimensional intermittency analysis is performed in (η, φ) phase space with |η| ≤ 0.8 and full azimuth space in the narrow transverse momentum \( p_T \) bins in the region with \( p_T ≤ 1.0 \text{ GeV/c} \) with the objective to study the scaling behaviour of the charged particle multiplicity fluctuations as are introduced by the hydro and hydro+cascade modes of the EPOS3 model. In narrow \( p_T \) bins in the (η, φ) space, M-scaling is found to be absent.
whereas weak M-scaling in two larger $p_T$ bins with $\Delta p_T \geq 0.6 \text{ GeV}/c$ viz, $0.2 \leq p_T \leq 0.8 \text{ GeV}/c$, and $0.2 \leq p_T \leq 1.0 \text{ GeV}/c$ is observed. Absence of power-law of $F_q$ with $M$ indicates the absence of intermittency and hence self-similar behaviour in the local multiplicity fluctuations in charged particle generation in the events and hence the EPOS3 model. For the narrow $p_T$ bins $\Delta p_T < 0.6 \text{ GeV}/c$, F-scaling which is independent of the observation of M-scaling is also absent. However, in the wider $p_T$ bins $0.2 \leq p_T \leq 0.8 \text{ GeV}/c$ and $0.2 \leq p_T \leq 1.0 \text{ GeV}/c$, $F_q$ shows power-law with $F_q$. This is in contrast to what is observed in [27], where M-scaling as well as F-scaling is observed in the small $p_T$ bins with $\Delta p_T \leq 0.2 \text{ GeV}/c$. The average value of $v$, the scaling exponent for these two bins from the two modes of EPOS3 is 1.809, a value different from 1.304, the value as obtained from Ginzburg-Landau theory for second-order phase transition. This suggests the absence of spatial fluctuations in the local charged particle generation that was not the case with the transport String Melting AMPT model [27]. In the larger phase space bins corresponding to $\Delta p_T \geq 0.6 \text{ GeV}/c$ in the low $p_T$-region, M-scaling observed for the low $M$ values is reflected in the value of generalized fractal dimension, $D_q$. $D_q$ shows an inverse dependence on $q$ for $q > 2$, thus the presence of multifractality in the larger phase space bins. This is in contrast to the observations at lower energies. Similar studies of experimental data from RHIC and LHC are yet not available. It would be interesting to see whether we get similar observations from the experiment or not.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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