Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2013, Article ID 801574, 7 pages
http://dx.doi.org/10.1155/2013/801574

Research Article

Study on Inertia as a Gravity Induced Property of Mass, in an Infinite Hubble Expanding Universe

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Received 2 July 2013; Accepted 24 September 2013

Academic Editor: Raj Bali

Mass is experienced to have two intrinsic properties: inertia (resistance to acceleration) and gravity (attraction to other masses). In this paper we evaluate the gravitational effect of all masses of the universe on an accelerated mass, starting from linearized general relativity. The gravitational interaction of all masses in a finite static universe model is shown to create a finite resistance to acceleration, which is inertia. Then, we propose a generalization of the linearized theory and evaluate the Hubble expanding universe. It is shown that the gravitational impact of an infinite expanding universe creates finite inertia, according to $\vec{F} = m\vec{a}$. The Friedmann critical mass density is found to be valid. The Mach principle is made explicit. The value and sign of the gravitational constant G are found to be of no consequence on an astronomical scale.

1. Introduction

The origin of inertia has been the subject of many debates in physics (see [1] for a review). Starting with Newton's "absolute space" as reference for acceleration, many physicists were quick to doubt anything that can act (cause an inertial force) but cannot be acted upon. However, Newton's famous thought experiment on the rotating water bucket [1, 2], see Figure 1, clearly showed that rotation, relative to "absolute space," results in locally observable facts.

Mach replaced "absolute space" with the "distant stars" as a source for inertia, without indicating an interaction mechanism [1–4]. The instantaneous "action at a distance" did not fit well into special relativity, which is based on a finite speed of interaction. The equivalence principle [2–4] postulated the equality of inertial and gravitational mass. In 1953, Sciama [5] proposed an explanation for inertia in a steady-state universe model, based on an analogy to electromagnetic field theory. However, the steady-state universe model is not in accordance with the observation of the Hubble expansion of the universe.

In this paper, we will pursue the origin of inertia further, assuming gravity as the interaction mechanism. First, we evaluate the origins of inertia in a steady-state universe model and we will reproduce the results by Sciama in a more explicit form. Then, we will study the Hubble expanding universe. We will start from the linearized general relativity equations in the weak field, low velocity approximation and later we will propose a more generalized approach.

2. Framework from General Relativity

The tensor equations describing general relativity can be simplified to linear differential equations resembling the Maxwell equations [1, 6], when only low mass densities and low velocities are considered. The Maxwell equations describe (in special relativity) the symmetrical field of a particle in 3 dimensions, assuming a finite field propagation velocity. These linear differential equations can be reformulated as the (retarded) Liénard-Wiechert fields [3, 7]:

$$\vec{F}g = -\text{Gmobs}\left(\vec{E}g + \vec{v} * \vec{B}g\right),$$
$$\vec{E}g = \left[\frac{\text{mpart}}{s^3}\right] \times \left(\left(1 - \left[\frac{v}{c}\right]^2\right)\left(\vec{r} - \frac{r\vec{v}}{c}\right) + \frac{\vec{r} * [(\vec{r} - r\vec{v}/c) * \vec{a}]}{c^2}\right),$$
$$\vec{B}g = \frac{\vec{r} * \vec{E}g}{(rc)}. \quad (1)$$

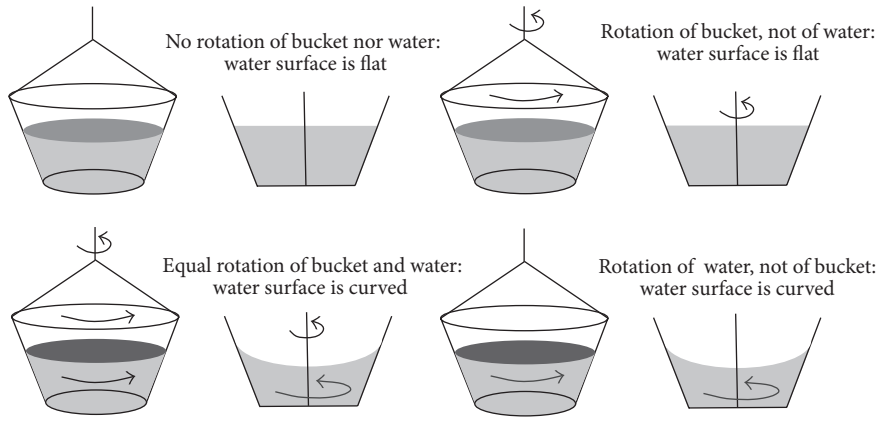

Figure 1: Newton's water bucket thought experiment, proving—to Newton—the existence of absolute space, as acting medium. The water shape only changes when the water is rotating relative to absolute space and is not influenced by the rotation state relative to the bucket.

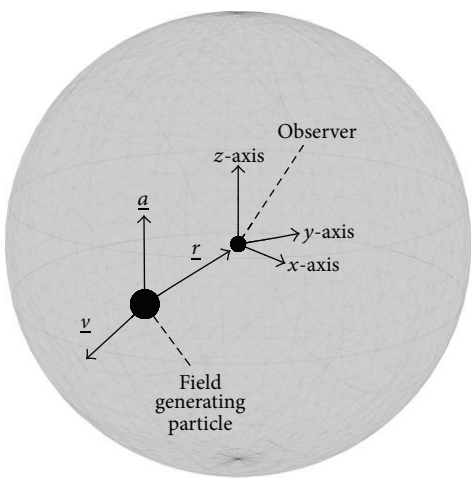

Figure 2: Coordinate definitions. Observer is in the center of the universe.

Table 1: Explanation of terms in (1).

$\vec{F}g$	Gravitational interaction force between particle and observer (=3D vector)
$\vec{E}g$, $\vec{B}g$	Field vectors to arrive at the interaction force vector
$\vec{r}$	Position of particle compared to observer, in direction particle to observer
$\vec{v}$, $\vec{a}$	Velocity $\vec{v} = -\, d\vec{r}/dt$ and acceleration $\vec{a} = d\vec{v}/dt$ vectors of particle compared to the observer
r, v	Vector norm of $\vec{r}$ and $\vec{v}$
c	Velocity of the expansion of gravity in vacuum
s	$(r - (\vec{r}, \vec{v}))/c$
G	Gravitational constant = 6.67×10^{-11} m^3 kg^{-1} s^{-2}
mobs	Gravitational mass of the observer
mpart	Gravitational mass of the field generating particle
$(,)^*$	The dot product and cross product

For details of the orientation of the coordinates in (1), see Figure 2. The formula abbreviations are given in Table 1. The static attractive Newton law of gravity reappears, under the conditions $\vec{v} = \vec{0}$ and $\vec{a} = \vec{0}$. Equation (1) is written in an addition form for mpart, allowing integration over all masses under consideration.

Inspired by Sciama [5], we will evaluate in this paper the consequences of (1). We start with a static universe calculation. We assume a constant (low) mass density ρ throughout the universe (cosmological principle). Under these boundary conditions, the usage of linearized general relativity is permitted, and we can compute the total interaction of the universe by integrating (1) over all masses of the entire universe. We have the following:

$$\vec{F}\text{tot} = \int^{\text{all mass}} d\vec{F}g. \tag{2}$$

3. Static Universe

In the static universe model, for a slow moving observer, the condition $v/c \ll 1$ is fulfilled. Under these conditions, (1) can be simplified further to the following:

$$\begin{aligned} \vec{F}g &= -\,\text{Gmobs}\left(\vec{E}g + \vec{v} * \vec{B}g\right) \\ &= -\,\text{Gmobs}\left(\vec{E}g + \left(\frac{\vec{v}}{c}\right) * \left[\left(\frac{\vec{r}}{r}\right) * \vec{E}g\right]\right) \\ &= -\,\text{Gmobs}\,\vec{E}g, \\ \vec{E}g &= \left(\frac{\text{mpart}}{r^3}\right)\left(\vec{r} + \frac{\vec{r} * [\vec{r} * \vec{a}]}{c^2}\right). \end{aligned} \tag{3}$$

Note that the term involving $\vec{B}g$ has dropped out of the equations due to $v/c \ll 1$. A further simplification of (3) can be obtained when using the triple product expansion: $\vec{r} * [\vec{r} * \vec{a}] = \vec{r}\,(\vec{r}, \vec{a}) - \vec{a}\,(\vec{r}, \vec{r}) = \vec{r}(\vec{r}, \vec{a}) - r^2\vec{a}$.

We now start our evaluations of a symmetrical static universe: a sphere with the slow moving observer in its

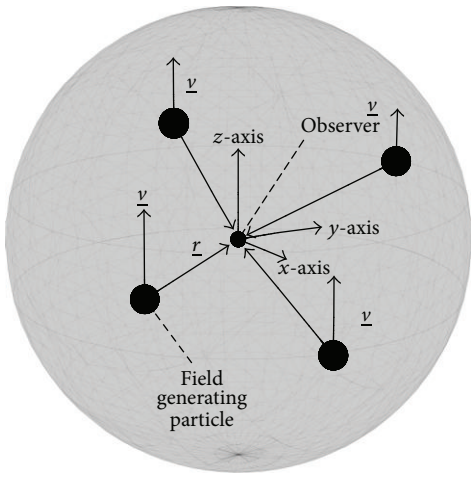

Figure 3: When the observer moves, the universe appears to *instantaneously* countermove.

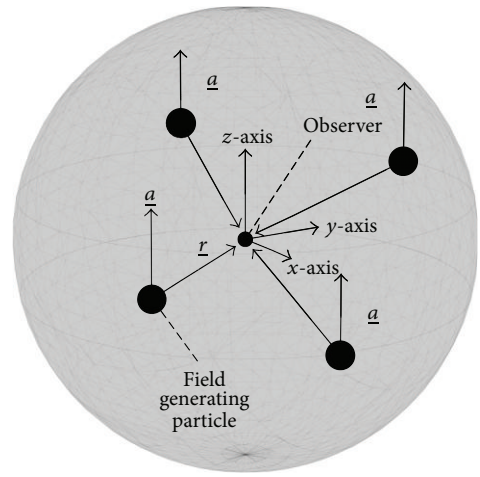

Figure 4: When the observer accelerates, the universe appears to *instantaneously* counteraccelerate.

center. When the observer is moving in the static universe, it appears as if all particles of the universe are *instantaneously* countermoving; see Figure 3. Note that this is not a violation of special relativity, as the observer moves in the preexisting retarded Liénard-Wiechert field.

Now, we take a full integration of (2) using (3) and $\vec{a} = \vec{0}$, over a sphere in spherical coordinates: $(x, y, z) = (r\cos(\theta)\sin(\phi), r\sin(\theta)\sin(\phi), r\cos(\phi))$, and replace mpart by the integration quantity as follows:

$$d\text{mpart}(r) = \rho \sin(\phi)\, r^2 d\theta\, d\phi\, dr. \tag{4}$$

This results in a zero net force, which means that a moving observer experiences no interaction of the entire static universe (for $v/c \ll 1$). This means that any object in slow uniform motion remains in uniform motion, which is Newton's first law.

Now, we compute the interaction of an accelerated universe with the mass of the observer in the center [5]; see Figure 4. We execute the integration of (2) and (3) with $v/c \ll 1$ and $\vec{a} \neq \vec{0}$. Without loss of generality, $\vec{a}$ is taken in the z-direction. Now we have

$$\begin{aligned}
\vec{F}g &= -G\text{mobs} \int_0^{r\,\max} \int_0^{\pi} \int_0^{2\pi} \rho r^2 \sin(\phi)\, \vec{E}g\, d\theta\, d\phi\, dr \\
&= \text{mobs}\vec{a}\left(\frac{2\pi\rho G}{c^2}\right) \int_0^{r\,\max} \int_0^{\pi} r\sin^3(\phi)\, d\phi\, dr \\
&= \text{mobs}\vec{a}\left(\frac{8\pi\rho G}{3c^2}\right) \int_0^{r\,\max} r\, dr \\
&= \text{mobs}\vec{a}\left(\frac{4\pi\rho G}{3c^2}\right) r\text{max}^2.
\end{aligned} \tag{5}$$

The resulting force of the accelerated universe is proportional to mobs and $\vec{a}$, in the direction of the acceleration, which means that the mass of the observer attempts to join the acceleration of the universe. It resists the acceleration initiated by an external force, which is called inertia!

The upper integration limit r max can be derived from a side step to the Hubble expanding universe [2, 3]. At r max, it is assumed that the masses of the universe move away at the maximum allowable velocity c and, consequentially, have negligible interaction. Starting from the linear Hubble expansion formula $v = Hr$, r max can be found to equal c/H (which we label as rHubble). Equation (5) reduces to the following:

$$\vec{F}g = \text{mobs}\vec{a}\left(\frac{4\pi\rho G}{3H^2}\right). \tag{6}$$

Identifying (6) as Newton's 2nd law $\vec{F} = m\vec{a}$, $(4\pi\rho G/3H^2)$ must be close to unity. From WMAP data [8] (Ωtot $= 8\pi\rho G/3H^2 = 1.02 \pm 0.02$), this value is found to be 0.51 ± 0.01.

So far, we assumed a finite spherical universe with the observer in its center. However, when evaluating the gravitational impact of the universe, assuming the observer is not in its center (see Figure 5), it becomes clear that the experienced inertial reaction must be asymmetric. This asymmetry has not been observed in practice in Eötvös type experiments [2, 3] and would be counterintuitive: its absence would imply (in the static universe model) that the earth has a privileged position in the center of the universe.

A solution to this dilemma can be found by considering the Hubble expansion of the universe. However, this requires a further generalization of the meaning of (1).

4. Interaction Pur Sang

The Liénard-Wiechert fields, described by (1), have been introduced as limiting cases of the general relativity tensor equations. Here, we will reintroduce the Liénard-Wiechert field equations, taking a first principles approach. We define interaction of a particle as the ability to influence the distance between itself and other particles as function of time, starting from a static situation. The simplest of interactions is the one

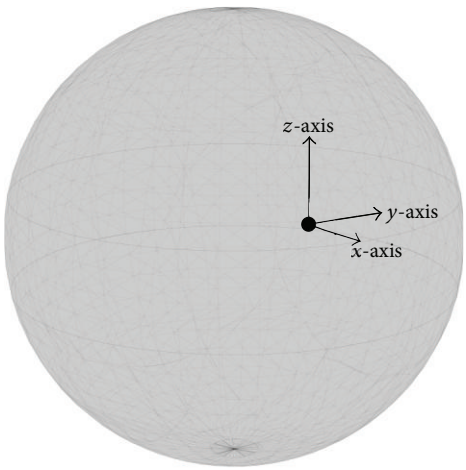

Figure 5: When the observer is not in the center of the static universe, asymmetrical inertia results.

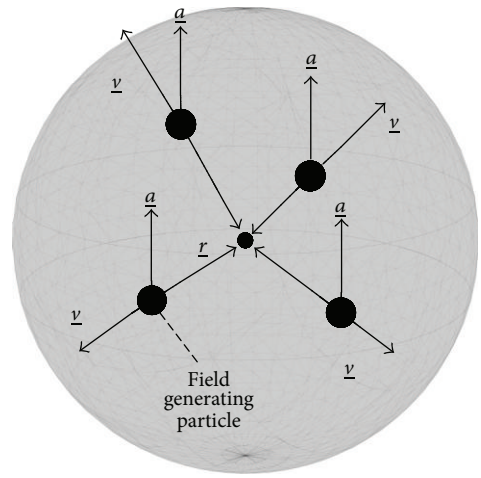

Figure 6: Hubble expansion of the universe. Particles move away from the observer in the center, in radial direction.

that arises from the pure existence of a particle. This is also known as gravity!

When evaluating this (gravitational) interaction, it is postulated that

(1) interactions spread evenly over space and time;

(2) the interaction speed is finite and independent of the state of motion of the interacting particles.

The first postulate implies that static interactions have spherical symmetry and thus only act in the direction of the vector connecting the interacting particles. When the static field of a single particle is studied, it is found experimentally that this interaction reduces with distance according to $1/r^2$. This implies, using the Gauss flux law, that the interaction spreads in 3 dimensions. From a 1-dimensional experiment, we conclude that we live in a 3-dimensional space!

The second postulate is in fact a copy of the special relativity postulate [2, 3].

The combination of the two postulates, incorporating the static gravity law of $1/r^2$, results in the special relativity description of fields [3, 7]. This can be expressed by the Maxwell equations, as well as through the Liénard-Wiechert fields of (1) [3, 7]. Rather than taking these field equations as a limiting case of general relativity, they are taken here as starting point and considered valid for all velocities and mass densities.

5. Expanding Universe

Prior to the study of the gravitational impact of the expanding universe on an accelerated mass, we need to establish its mass-velocity distribution for the integration according to (2). It was observed by Hubble that objects move away from each other in radial direction [2, 3] (see Figure 6). The velocity increases linearly as function of distance, for small values of the resulting velocity compared to c. Our interest is in the interaction between particles at large distances, where the condition $v \ll c$ does not hold. This requires a relativistic evaluation of the resulting velocity, based on [3, 7]

$$v\text{new}\,(v1, v2) = \frac{(v1 + v2)}{\left(1 + (v1 \cdot v2)/c^2\right)}. \tag{7}$$

In order to compute the relativistic Hubble law, we perform a thought experiment with many observers in a row, separated by equal distances (ΔrHubble, with $0 < \Delta \ll 1$). Starting from the origin, we assume each observer (labeled i) will move with a Hubble expansion velocity of $v = H\Delta r\text{Hubble} = \Delta c$, with respect to observer $i - 1$. To compute the velocity of observer i, compared to the origin, we utilize (7) in a recursive way and find it to numerically equal $c\tanh(i\Delta)$. Thus, the velocity formula for Hubble expansion, as observed by a (static) observer in the center, is as follows:

$$\vec{v}\text{Hubble}\,(r) = c\left(\frac{\vec{r}}{r}\right)\tanh\left(\frac{r}{r\text{Hubble}}\right). \tag{8}$$

The behavior of this function for $v \ll c$ corresponds to the original linear Hubble expansion law; see Figure 7.

A further consideration must be given to (4). The mass density integration was previously evaluated statically, whereas now we need to take the Hubble expansion into consideration. The cosmological principle implies that the mass density (as found by a comoving observer, at distance r) is the same everywhere: ρ. The nonmoving observer in the center notes a relativistic length contraction [2, 3, 6] in the radial direction, in line with (8). Therefore, (4) transforms to the following:

$$\begin{aligned} d\text{mpart}\,(r) &= \rho\sqrt{1 - \left(\frac{v(r)}{c}\right)^2}\,\sin(\phi)\,r^2 d\theta\,d\phi\,dr \\ &= \rho\sqrt{1 - \tanh^2\left(\frac{r}{r\text{Hubble}}\right)}\,\sin(\phi)\,r^2 d\theta\,d\phi\,dr. \end{aligned} \tag{9}$$

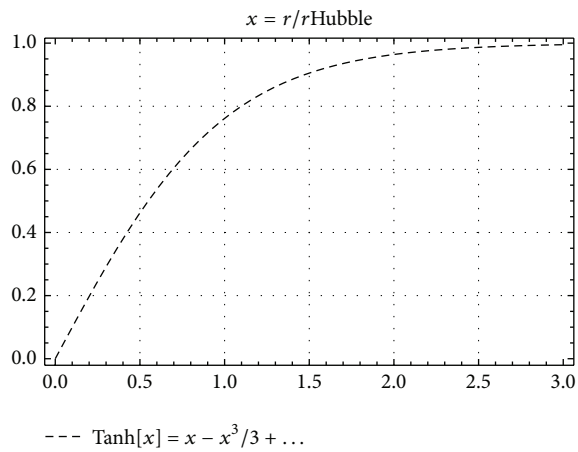

Figure 7: Relativistic Hubble expansion: tanh function.

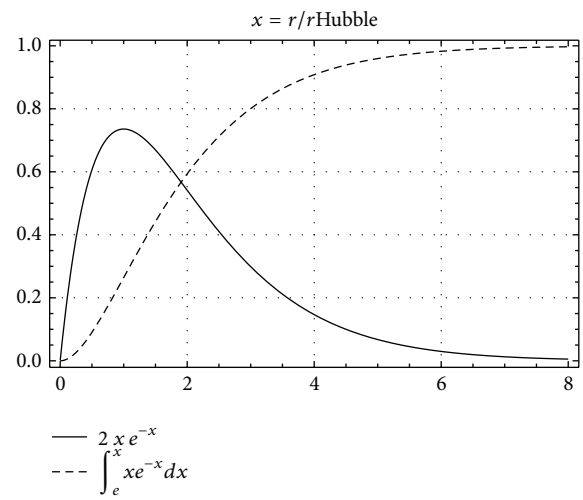

Figure 8: Relative contribution to inertia versus distance: most inertia is generated at distances larger than rHubble.

Now, we can compute the total force of the Hubble expanding universe (accelerated in z-direction) on the mass of the observer in the center, who is initially comoving with his direct surroundings. Defining the help integration variable $x = r/r\text{Hubble}$, (2) transforms into the following:

$$\vec{F}g\,(x\,\max) = \text{mobs}\vec{a}\left(\frac{8\pi\rho G}{3H^2}\right)\int_0^{x\,\max} x\; e^{-x}dx. \tag{10}$$

Taking $x\,\max \;\rightarrow\; \infty$ (implying an infinite universe, filled with constant mass density), (10) is evaluated as follows:

$$\vec{F}g = \text{mobs}\vec{a}\;\left(\frac{8\pi\rho G}{3H^2}\right). \tag{11}$$

Again, identifying the gravitational interaction on the accelerated observer as inertia, the following equality must hold:

$$\frac{8\pi\rho G}{3H^2} = 1. \tag{12}$$

Utilizing the WMAP values [8] mentioned before, the equality of (12) is found by measurement as well. This proves that gravity by the entire expanding universe is the root cause of inertia, which is evidence for the intuitive Mach principle [1–4].

Equation (12) is also known in general relativity, for it defines the critical mass density ρ in the Friedmann metric [2, 3, 6]. With this equation proven, it is clear that the model of choice for the universe at large is an expanding model of infinite size and homogeneous mass density without intrinsic space curvature. This can be understood as well from the cosmological principle for an infinite universe: as each cube of space is symmetrically surrounded by other equal cubes of space, any space curvature generated by the central cube is counteracted by the other cubes. No cube has priority to bend space, so space is not curved on a cosmological scale.

A further analysis of the convergence of (10) shows that the contribution of masses at distances larger than rHubble is dominant, as shown in Figure 8. This means that the universe is infinitely filled with expanding mass see Figure 9.

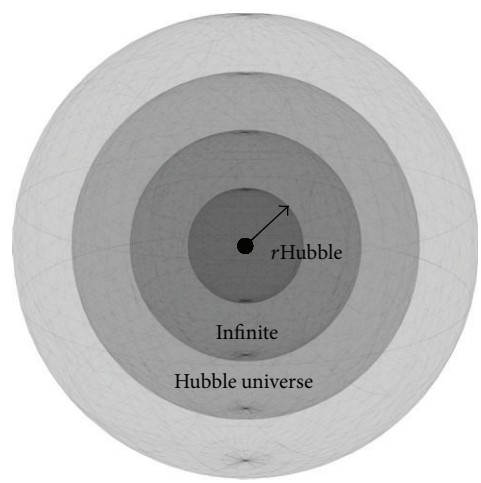

Figure 9: Infinitely mass filled expanding Hubble universe.

Local deviations from the average mass density ρ have no effect on the overall gravitational force on an accelerated observer. Also moving the observer to another position in the expanding universe does not change the analysis (assuming the observer comoves, as per Hubble law, with the local masses). The asymmetry issue of the finite, static universe does not hold in an infinite, expanding universe.

6. Redshift

It was shown that the model of choice for the universe at large is an expanding model of homogeneous mass density and infinite size, without intrinsic space curvature. The infinite homogeneity implies that no net gravitational force exists when moving from one place to another. Therefore, observed redshifts from faraway galaxies are not due to gravitational effects but originate purely from velocity Doppler effects,

due to Hubble expansion. For a pure motion away from the observer, the redshift z is given by [2, 3]

$$z = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} - 1. \tag{13}$$

With the relativistic Hubble velocity (8), the redshift formula is simplified to the following:

$$z(r) = e^{r/r\text{Hubble}} - 1. \tag{14}$$

7. Evolution of the Universe

We have shown the expanding universe to be infinitely large. This means that it will always be infinitely large but also that it has always been infinitely large. In this section, we evaluate the time evolution of a small part of the universe.

So far, we have assumed a constant value for the Hubble expansion factor H. However, when studying a small expanding sphere of $r \ll r\text{Hubble}$ with fixed mass M, the Friedmann (critical mass density) equality of (12) implies a change in mass density ρ, even for constant H.

We now study the time evolution of the radius $r(t)$ in which the mass M is contained, with assumed homogeneous mass density ρ. We will assume that G is a universal constant (indicating that the gravitational force between two objects is a constant) and we assume that the Friedmann equality of (12) will be valid over time, leading to constancy of the $\vec{F} = m\vec{a}$ formula. We rewrite H as follows:

$$H(t) = \frac{(dr/dt)}{r}, \tag{15}$$

and we identify the mass M of the sphere under study as constant, in flat space-time ($M = 4/3\pi\rho r^3$). With this, we rearrange (12) to arrive at the following:

$$\left(\frac{dr}{dt}\right) - \sqrt{\frac{2MG}{r}} = 0, \tag{16}$$

which equals the Einstein-de Sitter equation for the flat universe [2, 3], which can be solved as follows:

$$r(t) = r0\left(1 + \sqrt{6\pi G\rho 0}t\right)^{2/3}, \tag{17}$$

where $r0$ and $\rho 0$ indicate the initial size and initial mass density of the small part of the infinite universe. Equation (17) can be further simplified using (12) and introducing $H0$ as initial Hubble expansion coefficient to arrive at the following:

$$r(t) = r0\left(1 + \frac{3}{2}H0t\right)^{2/3}. \tag{18}$$

For small time scales compared to the initial Hubble time scale ($t \ll 1/H0$), the linear Hubble expansion law is regained, by series expansion.

8. Planetary Motion: Mach

Another computation is the slow motion ($v/c \ll 1$) of the observer around a central mass (comparable to a planetary orbit). Assuming movement in only (x, y) plane ($z = 0$), the addition of one large mass (given as mass), close to the observer, to the evaluation leading to the Friedmann equality of (12) results in the differential equations for the x and y coordinates (dropping mobs) as follows:

$$\begin{aligned} \left(\frac{8\pi\rho G}{3H^2}\right)ax + \frac{G\text{mass}\cos(\phi)}{rm^2} &= 0, \\ \left(\frac{8\pi\rho G}{3H^2}\right)ay + \frac{G\text{mass}\sin(\phi)}{rm^2} &= 0, \end{aligned} \tag{19}$$

with $rm(t)$ being distance from observer to the central mass and $\vec{a} = (ax, ay, 0)$. The equations in (19) allow a circular orbit (as well as an elliptical orbit), as $x(t) = rm\cos(\omega t)$, $y(t) = rm\sin(\omega t)$, resulting in the Kepler law as follows:

$$\omega^2 rm^3 = \frac{3\text{mass}H^2}{(8\pi\rho)}. \tag{20}$$

Equation (20) is an explicit form of Mach's principle: planetary movements are determined by the mass distribution of the universe, with gravity as interaction.

The realization that inertia is due to the gravitational impact of all masses in the infinite expanding universe ends the debate on "absolute space" (as introduced by Newton, following his famous rotating water bucket thought experiment [1, 2]—see Figure 1—criticized by Mach). "Absolute space" in itself does not exist. Everything needs to be evaluated relative to everything else, but our (only available) universe provides an absolute inertial reference frame, through the gravitational interaction with all masses of the infinite expanding universe.

9. Gravitational Impact of G

Previously, we have identified inertia as a gravitational effect. When only gravitational effects are present, (1) learns that these effects are proportional to G. Therefore, G drops out of the equation. The actual value (or sign!) of G is not an orbit influencing factor! This implies that it is inconsequential for astronomical phenomena, such as star or planet formation, whether we call gravity attractive or repulsive. The net effect of 2 gravitating particles (initially at rest, as part of the expanding universe) remains the same: the reduction of their distance over time. Purely, the fact that particles exist all over the universe is sufficient.

10. Conclusions

In this paper, it is demonstrated that inertia is the result of the gravitational interaction of all masses of the infinitely sized, expanding universe. Therefore, the so-called inertial and gravitational masses are equal. It was also shown that the Friedmann critical mass density holds true, which indicates a flat space-time topology for the universe. The Mach principle was thus made explicit. The value and sign of the gravitational constant G are of no consequence on an astronomical scale.

11. Suggestions for Further Research

As further verification (or falsification) for this theory, the following cases can be studied further:

(1) variation of the amount of inertia for a fast moving particle ($0 < 1 - v/c \ll 1$);

(2) the perihelion shift of Mercury;

(3) the net interaction of a (nonaccelerated) fast moving particle with the expanding universe.

References

[1] J. Giné, "On the Origin of the Inertial Force and Gravitation," *International Journal of Theoretical Physics*, vol. 50, no. 2, pp. 607–617, 2011.

[2] W. Rindler, *Relativity*, 2nd edition, 2006.

[3] R. K. Pathria, *The Theory of Relativity*, 2nd edition, 1974.

[4] D. F. Lawden, *Introduction to Tensor Calculus, Relativity and Cosmology*, 3rd edition, 1982.

[5] D. W. Sciama, "On the origin of inertia," *Monthly Notices of the Royal Astronomical Society*, vol. 113, p. 34, 1953.

[6] M. P. Hobson, G. Elstathiou, and A. N. Lasenby, *General Rrelativity*, Cambridge University Press, Cambridge, UK, 2006.

[7] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, vol. 2, Butterworth-Heinemann, Oxford, UK, 4th edition, 1975.

[8] D. N. Spergel, L. Verde, H. V. Peiris et al., "First-year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) observations: determination of cosmological parameters," *The Astrophysical Journal Supplement*, vol. 148, no. 1, p. 175, 2003.