Research Article

LRS Bianchi Type II Massive String Cosmological Models with Magnetic Field in Lyra's Geometry

Raj Bali,¹ Mahesh Kumar Yadav,² and Lokesh Kumar Gupta¹

¹ Department of Mathematics, University of Rajasthan, Jaipur 302004, India

² Department of Mathematics, Dr. H.S. Gour Central University, Sagar 470003, India

Correspondence should be addressed to Raj Bali; balir5@yahoo.co.in

Received 10 May 2013; Accepted 17 September 2013

Academic Editor: Shri Ram

Copyright © 2013 Raj Bali et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Bianchi type II massive string cosmological models with magnetic field and time dependent gauge function (ϕ_t) in the frame work of Lyra's geometry are investigated. The magnetic field is in YZ -plane. To get the deterministic solution, we have assumed that the shear (σ) is proportional to the expansion (θ). This leads to $R = S^n$, where R and S are metric potentials and n is a constant. We find that the models start with a big bang at initial singularity and expansion decreases due to lapse of time. The anisotropy is maintained throughout but the model isotropizes when $n = 1$. The physical and geometrical aspects of the model in the presence and absence of magnetic field are also discussed.

1. Introduction

Bianchi type II space time successfully explains the initial stage of evolution of universe. Asseo and Sol [1] have given the importance of Bianchi type II space time for the study of universe. The string theory is useful to describe an event at the early stage of evolution of universe in a lucid way. Cosmic strings play a significant role in the structure formation and evolution of universe. The presence of string in the early universe has been explained by Kibble [2], Vilenkin [3], and Zel'dovich [4] using grand unified theories. These strings have stress energy and are classified as massive and geometric strings. The pioneer work in the formation of energy momentum tensor for classical massive strings is due to Letelier [5] who explained that the massive strings are formed by geometric strings (Stachel [6]) with particle attached along its extension. Letelier [5] first used this idea in obtaining some cosmological solutions for massive string for Bianchi type I and Kantowski-Sachs space-times. Many authors' namely, Banerjee et al. [7], Tikekar and Patel [8, 9], Wang [10], and Bali et al. [11–14], have investigated string cosmological models in different contexts.

Einstein introduced general theory of relativity to describe gravitation in terms of geometry and it helped

him to geometrize other physical fields. Motivated by the successful attempt of Einstein, Weyl [15] made one of the best attempts to generalize Riemannian geometry to unify gravitation and electromagnetism. Unfortunately Weyl's theory was not accepted due to nonintegrability of length. Lyra [16] proposed a modification in Riemannian geometry by introducing gauge function into the structureless manifold. This modification removed the main obstacle of the Weyl theory [15]. Sen [17] formulated a new scalar tensor theory of gravitation and constructed an analogue of Einstein field equations based on Lyra geometry. Halford [18] pointed out that the constant vector field (β) in Lyra geometry plays a similar role of cosmological constant (Λ) in general theory of relativity. The scalar tensor theory of gravitation in Lyra geometry predicts the same effects within the observational limits as in the Einstein theory. The main difference between the cosmological theories based on Lyra geometry and Riemannian geometry lies in the fact that the constant displacement vector (β) arises naturally from the concept of gauge in Lyra geometry whereas the cosmological constant (Λ) was introduced by Einstein in an ad hoc manner to find static solution of his field equations. Many authors, namely, Beesham [19], T. Singh and G. P. Singh [20], Chakraborty and Ghosh [21], Rahaman and Bera [22], Pradhan et al. [23, 24],

Bali and Chandnani [25, 26], and Ram et al. [27], have studied cosmological models in the frame work of Lyra's geometry.

The present day magnitude of magnetic field is very small as compared to estimated matter density. It might not have been negligible during early stage of evolution of universe. Asseo and Sol [1] speculated a primordial magnetic field of cosmological origin. Vilenkin [3] has pointed out that cosmic strings may act as gravitational lensing. Therefore, it is interesting to discuss whether it is possible to construct an analogue of cosmic string in the presence of magnetic field in the frame work of Lyra's geometry. Recently, Bali et al. [28] investigated Bianchi type I string dust magnetized cosmological model in the frame work of Lyra's geometry.

In this paper, we have investigated LRS Bianchi type II massive string cosmological models with magnetic field in Lyra's geometry. We find that it is possible to construct an analogue of cosmic string solution in presence of magnetic field in the frame work of Lyra geometry. The physical and geometrical aspects of the model together with behavior of the model in the presence and absence of magnetic field are also discussed.

2. The Metric and Field Equations

We consider Locally Rotationally Symmetric (LRS) Bianchi type II metric as

$$ds^2 = \eta_{ab} \theta^a \theta^b, \quad (1)$$

where

$$\begin{aligned} \eta_{11} = \eta_{22} = \eta_{33} = 1, \quad \eta_{44} = (-1), \\ \theta^1 = R dx, \quad \theta^2 = S(dy - x dz), \\ \theta^3 = R dz, \quad \theta^4 = dt. \end{aligned} \quad (2)$$

Thus the metric (1) leads to

$$\begin{aligned} ds^2 = -dt^2 + R^2 dx^2 \\ + S^2(dy - x dz)^2 + R^2 dz^2, \end{aligned} \quad (3)$$

where R and S are functions of t alone.

Energy momentum tensor T_i^j for string dust in the presence of magnetic field is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j + E_i^j. \quad (4)$$

Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [17] is given by

$$\begin{aligned} R_i^j - \frac{1}{2} R g_i^j + \frac{3}{2} \phi_i \phi^j - \frac{3}{4} \phi_k \phi^k g_i^j = -T_i^j \\ (\text{in geometrized units, where } 8\pi G = 1, c = 1), \end{aligned} \quad (5)$$

where $v_i = (0, 0, 0, -1)$; $v^i v_i = -1$; $\phi_i = (0, 0, 0, \beta(t))$; $v_4 = -1$; $v^4 = 1 \cdot \rho$ is the matter density, λ the cloud string's tension density, v^j the fluid flow vector, β the gauge function, E_i^j the electromagnetic field tensor and x^a the space like 4-vectors representing the string's direction.

The electromagnetic field tensor E_i^j given by Lichnerowicz [29] is given as

$$E_i^j = \bar{\mu} \left[|h|^2 \left(v_i v^j + \frac{1}{2} g_i^j \right) - h_i h^j \right], \quad (6)$$

with $\bar{\mu}$ being the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \epsilon_{ijk\ell} F^{k\ell} v^j, \quad (7)$$

where $F^{k\ell}$ is the electromagnetic field tensor and $\epsilon_{ijk\ell}$ the Levi-Civita tensor density. We assume that the current is flowing along the x -axis, so magnetic field is in yz -plane. Thus $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$, and F_{23} is the only nonvanishing component of F_{ij} . This leads to $F_{12} = 0 = F_{13}$ by virtue of (7). We also find $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conductivity of the fluid (Maartens [30]). A cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic vector specifies a preferred spatial direction (Bronnikov et al. [31]). The Maxwell's equation

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \quad (8)$$

leads to

$$\begin{aligned} \frac{\partial F_{23}}{\partial t} = 0 \\ (\text{since } F_{23} \text{ is the only nonvanishing component} \\ \text{and } F_{ij} = -F_{ji}) \end{aligned} \quad (9)$$

which leads to

$$F_{23} = \text{constant} = H \text{ (say)}. \quad (10)$$

For $i = 1$, (7) leads to

$$h_1 = \frac{H}{\bar{\mu} S}. \quad (11)$$

Now the components of E_i^j corresponding to the line element (3) are as follows:

$$E_1^1 = -\frac{H^2}{2\bar{\mu} R^2 S^2} = -E_2^2 = -E_3^3 = E_4^4. \quad (12)$$

Now the modified Einstein's field equations (5) for the metric (3) lead to

$$\frac{S^2}{4R^4} + \frac{R_4 S_4}{RS} + \frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{3}{4} \beta^2 = \lambda + \frac{K}{R^2 S^2} \quad (13)$$

$$-\frac{3S^2}{4R^4} + \frac{R_4^2}{R^2} + \frac{2R_{44}}{R} + \frac{3}{4} \beta^2 = -\frac{K}{R^2 S^2} \quad (14)$$

$$-\frac{S^2}{4R^4} + \frac{2R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{3}{4} \beta^2 = \rho + \frac{K}{R^2 S^2}, \quad (15)$$

where $K = H^2/2\bar{\mu}$ and the direction of string is only along the x -axis so that $x_1 x^1 = 1$, $x_2 x^2 = 0 = x_3 x^3$.

The energy conservation equation $T_{ij}^j = 0$ leads to

$$\rho_4 + \rho \left(\frac{2R_4}{R} + \frac{S_4}{S} \right) - \lambda \frac{R_4}{R} = 0 \quad (16)$$

and conservation of left hand side of (5) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j \right)_{,j} + \frac{3}{2} (\phi_i \phi^j)_{,j} - \frac{3}{4} (\phi_k \phi^k g_i^j)_{,j} = 0 \quad (17)$$

which again leads to

$$\begin{aligned} & \frac{3}{2} \phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^\ell \Gamma_{\ell j}^j \right] + \frac{3}{2} \phi^j \left[\frac{\partial \phi_i}{\partial x^j} - \phi_\ell \Gamma_{ij}^\ell \right] \\ & - \frac{3}{4} g_i^j \phi^k \left[\frac{\partial \phi_k}{\partial x^j} + \phi^\ell \Gamma_{\ell j}^j \right] \\ & - \frac{3}{4} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} - \phi_\ell \Gamma_{ij}^\ell \right] = 0. \end{aligned} \quad (18)$$

Equation (18) is automatically satisfied for $i = 1, 2, 3$.

For $i = j = 4$, (18) leads to

$$\begin{aligned} & \frac{3}{2} \beta \left[\frac{\partial}{\partial t} (g^{44} \phi_4) + \phi^4 \Gamma_{44}^4 \right] \\ & + \frac{3}{2} g^{44} \phi_4 \left[\frac{\partial \phi_4}{\partial t} - \phi_4 \Gamma_{44}^4 \right] \\ & - \frac{3}{4} g_4^4 \phi_4 \left[\frac{\partial \phi_4}{\partial t} + \phi^4 \Gamma_{44}^4 \right] \\ & - \frac{3}{2} g_4^4 g^{44} \phi_4 \left[\frac{\partial \phi_4}{\partial t} - \phi^4 \Gamma_{44}^4 \right] = 0 \end{aligned} \quad (19)$$

which again leads to

$$\frac{3}{2} \beta \beta_4 + \frac{3}{2} \beta^2 \left(\frac{2R_4}{R} + \frac{S_4}{S} \right) = 0 \quad (20)$$

where

$$\phi_i = (0, 0, 0, \beta(t)). \quad (21)$$

3. Solution of Field Equations

For the complete determination of the model of the universe, we assume that the shear tensor (σ) is proportional to the expansion (θ) which leads to

$$R = S^n. \quad (22)$$

From (20), we have

$$\beta = \frac{\alpha}{R^2 S}, \quad (23)$$

with α being constant of integration.

Using (22) and (23) in (14), we have

$$\begin{aligned} & 2S_{44} + (3n-2) \frac{S_4^2}{S} \\ & = \frac{3}{4n} S^{-4n+3} - \frac{3\alpha^2}{4n} S^{-4n-1} - \frac{K}{n} S^{-2n-1}. \end{aligned} \quad (24)$$

Now we assume that

$$S_4 = f(S). \quad (25)$$

Thus

$$S_{44} = f f', \quad (26)$$

where

$$f' = \frac{df}{dS}. \quad (27)$$

Therefore, (24) leads to

$$\begin{aligned} & \frac{df^2}{dS} + \frac{3n-2}{S} f^2 \\ & = \frac{3}{4n} S^{-4n+3} - \frac{3\alpha^2}{4n} S^{-4n-1} - \frac{K}{n} S^{-2n-1} \end{aligned} \quad (28)$$

which again leads to

$$\begin{aligned} f^2 & = \frac{3}{4n(2-n)} S^{4-4n} \\ & + \frac{3\alpha^2}{4n(n+2)} S^{-4n} - \frac{K}{n(n-2)} S^{-2n}. \end{aligned} \quad (29)$$

Equation (29) leads to

$$f = \left(\frac{dS}{dt} \right) = \frac{\sqrt{L\tau^4 - N\tau^{2n} + M}}{\tau^{2n}}, \quad (30)$$

where $L = 3/(4n(2-n))$, $M = 3\alpha^2/(4n(n+2))$, $N = K/(n(n-2))$, $S = \tau$, a new coordinate is used, and $\alpha \neq 0$.

By (22), we have

$$R = S^n \quad (31)$$

which leads to

$$R = \tau^n, \quad (32)$$

where $S = \tau$.

Using (30) and (32), the metric (3) leads to

$$\begin{aligned} ds^2 & = - \left(\frac{dt}{dS} \right)^2 dS^2 + R^2 dx^2 \\ & + S^2 (dy - x dz)^2 + R^2 dz^2 \end{aligned} \quad (33)$$

which again leads to

$$\begin{aligned} ds^2 & = - \frac{d\tau^2}{L\tau^{4-4n} - N\tau^{-2n} + M\tau^{-4n}} \\ & + \tau^{2n} (dx^2 + dz^2) + \tau^2 (dy - x dz)^2, \end{aligned} \quad (34)$$

where the cosmic time t is defined as

$$t = \int \frac{d\tau}{L\tau^{4-4n} - N\tau^{-2n} + M\tau^{-4n}}. \quad (35)$$

4. Some Physical and Geometrical Features

Using (22), (23), (30), and (32) in (15), we have

$$\rho = A\tau^{2-4n} + B\tau^{-2n-2}, \quad (36)$$

where $A = (n+1)/(2-n)$ and $B = 2nk/(2-n)$.

Similarly from (15), the string tension density λ is given as

$$\begin{aligned} \lambda &= a\tau^{2-4n} + b\tau^{-2n-2} + d\tau^{-4n-2} \\ \rho_p = \rho - \lambda &= (A-a)\tau^{(2-4n)} \\ &\quad + (B-b)\tau^{(-2n-2)} - d\tau^{(-4n-2)}, \end{aligned} \quad (37)$$

where

$$\begin{aligned} a &= \frac{(n+1)(3-2n)}{2n(2-n)}, \quad b = \frac{K(3-n)}{(n-2)}, \\ d &= \frac{3\alpha^2(n+4)}{(n+2)}. \end{aligned} \quad (38)$$

Equation (23) gives

$$\beta = \frac{\alpha}{\tau^{2n+1}}. \quad (39)$$

The expansion (θ) is given as

$$\theta = \frac{2R_4}{R} + \frac{S_4}{S} \quad (40)$$

which leads to

$$\theta = \frac{(2n+1)}{\tau^{2n+1}} \sqrt{L\tau^4 - N\tau^{2n} + M}. \quad (41)$$

Shear (σ) is given by

$$\sigma = \frac{1}{\sqrt{3}} \left(\frac{R_4}{R} - \frac{S_4}{S} \right) \quad (42)$$

which leads to

$$\sigma = \frac{(n-1)}{\sqrt{3}\tau^{2n+1}} \sqrt{L\tau^4 - N\tau^{2n} + M}. \quad (43)$$

The deceleration parameter q is given by

$$q = -\frac{R_{44}/R}{R_4^2/R^2} \quad (44)$$

which leads to

$$\begin{aligned} q &= 2 + \frac{1}{n} \left[\frac{NS^{2n}(n+2) + 2M}{LS^4 - NS^{2n} + M} \right] \\ &= 2 + \frac{1}{n} \left[\frac{N\tau^{2n}(n+2) + 2M}{L\tau^4 - N\tau^{2n} + M} \right]. \end{aligned} \quad (45)$$

5. Model in Absence of Magnetic Field

To discuss the model in the absence of the magnetic field, we put $K = 0$ in (29) and have

$$f^2 = LS^{4-4n} + MS^{-4n}, \quad (46)$$

where

$$L = \frac{3}{4n(2-n)}, \quad M = \frac{3\alpha^2}{4n(n+2)}. \quad (47)$$

Equation (45) leads to

$$\frac{ds}{dt} = S_4 = \frac{\sqrt{L\tau^4 + M}}{\tau^{2n}}, \quad (48)$$

where $S = \tau$, and a new coordinate is used.

By (22), we have

$$R = \tau^n. \quad (49)$$

Using (48) and (49) in metric (3), we get

$$\begin{aligned} ds^2 &= -\left(\frac{dt}{dS}\right)^2 dS^2 + R^2 dx^2 \\ &\quad + S^2(dy - xdz)^2 + R^2 dz^2 \end{aligned} \quad (50)$$

which again leads to

$$\begin{aligned} ds^2 &= -\frac{d\tau^2}{L\tau^{4-4n} + M\tau^{-4n}} + \tau^{2n} dx^2 \\ &\quad + \tau^2(dy - xdz)^2 + \tau^{2n} dz^2. \end{aligned} \quad (51)$$

In this case, the energy density (ρ), the string tension density (λ), gauge function (β), the expansion (θ), shear (σ), and deceleration parameter (q) are given by

$$\begin{aligned} \rho &= \frac{n+1}{2-n} \tau^{-4n-2} \\ \lambda &= \frac{(2n-3)(n+1)}{2n(n-2)} \tau^{-4n+2} \\ \rho_p &= \rho - \lambda \\ \beta &= \frac{\alpha}{R^2 S} \\ &= \frac{\alpha}{\tau^{2n+1}} \\ \theta &= \frac{2R_4}{R} + \frac{S_4}{S} \\ &= \frac{(2n+1)}{\tau^{2n+1}} \sqrt{L\tau^4 + M} \\ \sigma &= \frac{1}{\sqrt{3}} \left(\frac{R_4}{R} - \frac{S_4}{S} \right) \\ &= \frac{(n-1)}{\sqrt{3}\tau^{2n+1}} \sqrt{L\tau^4 + M} \\ q &= -\frac{R_{44}/R}{R_4^2/R^2} \\ &= n^2 + n \left(\frac{M - L\tau^4}{M + L\tau^4} \right). \end{aligned} \quad (52)$$

6. Discussion

Model (34) in the presence of magnetic field starts with a big bang at $\tau = 0$ and the expansion in the model decreases as τ increases. The spatial volume increases as τ increases. Thus inflationary scenario exists in the model. The model has point-type singularity at $\tau = 0$ where $n > 0$. Since $\sigma/\theta \neq 0$, hence anisotropy is maintained throughout. However, if $n = 1$, then the model isotropizes. The displacement vector β is initially large but decreases due to lapse of time where $2n + 1 > 0$; however, β increases continuously when $2n + 1 < 0$. The matter density $\rho > 0$ when $0 < n < 2$.

Model (51) starts with a big bang at $\tau = 0$ when $n = -1/2$ and the expansion in the model decreases as time increases. The displacement vector (β) is initially large but decreases due to lapse of time. The model (51) has point-type singularity at $\tau = 0$, where $n > 0$. Since $\sigma/\theta \neq 0$, hence anisotropy is maintained throughout. However, if $n = 1$, then the model isotropizes.

Thus it is possible to construct globally regular Bianchi type II solutions with displacement vector (β) using geometric condition shear which is proportional to expansion.

References

- [1] E. Asseo and H. Sol, "Extragalactic magnetic fields," *Physics Reports*, vol. 148, no. 6, pp. 307–436, 1987.
- [2] T. W. B. Kibble, "Topology of cosmic domains and strings," *Journal of Physics A*, vol. 9, no. 8, pp. 1387–1398, 1976.
- [3] A. Vilenkin, "Cosmic strings," *Physical Review D*, vol. 24, no. 8, pp. 2082–2089, 1981.
- [4] Y. B. Zel'dovich, "A hypothesis, unifying the structure and the entropy of the universe," *Monthly Notices of the Royal Astronomical Society*, vol. 160, pp. 1–3, 1972.
- [5] P. S. Letelier, "String cosmologies," *Physical Review D*, vol. 28, no. 10, pp. 2414–2419, 1983.
- [6] J. Stachel, "Thickening the string. I. The string perfect dust," *Physical Review D*, vol. 21, no. 8, pp. 2171–2181, 1980.
- [7] A. Banerjee, A. K. Sanyal, and S. Chakraborty, "String cosmology in Bianchi I space-time," *Pramana*, vol. 34, no. 1, pp. 1–11, 1990.
- [8] R. Tikekar and L. K. Patel, "Some exact solutions of string cosmology in Bianchi III space-time," *General Relativity and Gravitation*, vol. 24, no. 4, pp. 397–404, 1992.
- [9] R. Tikekar and L. K. Patel, "Some exact solutions in Bianchi VI string cosmology," *Pramana*, vol. 42, no. 6, pp. 483–489, 1994.
- [10] X. Wang, "Bianchi type-III string cosmological model with bulk viscosity and magnetic field," *Chinese Physics Letters*, vol. 23, no. 7, pp. 1702–1704, 2006.
- [11] R. Bali and R. D. Upadhyaya, "L.R.S. Bianchi type I string dust magnetized cosmological models," *Astrophysics and Space Science*, vol. 283, no. 1, pp. 97–108, 2003.
- [12] R. Bali and A. Anjali, "Bianchi type I magnetized string cosmological model in general relativity," *Astrophysics and Space Science*, vol. 302, no. 1–4, pp. 201–205, 2006.
- [13] R. Bali and D. K. Singh, "Bianchi type-V bulk viscous fluid string dust cosmological model in general relativity," *Astrophysics and Space Science*, vol. 300, no. 4, pp. 387–394, 2005.
- [14] R. Bali, U. K. Pareek, and A. Pradhan, "Bianchi type-I massive string magnetized barotropic perfect fluid cosmological model in general relativity," *Chinese Physics Letters*, vol. 24, no. 8, pp. 2455–2458, 2007.
- [15] H. Weyl, *Gravitation and Electricität*, pp. 465–475, Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, Berlin, Germany, 1918.
- [16] G. Lyra, "Über eine Modifikation der Riemannschen Geometrie," *Mathematische Zeitschrift*, vol. 54, pp. 52–64, 1951.
- [17] D. K. Sen, "A static cosmological model," *Zeitschrift für Physik*, vol. 149, pp. 311–323, 1957.
- [18] W. D. Halford, "Scalar-tensor theory of gravitation in a Lyra manifold," *Journal of Mathematical Physics*, vol. 13, no. 11, pp. 1699–1703, 1972.
- [19] A. Beesham, "Vacuum friedmann cosmology based on Lyra's manifold," *Astrophysics and Space Science*, vol. 127, no. 1, pp. 189–191, 1986.
- [20] T. Singh and G. P. Singh, "Bianchi type-I cosmological models in Lyra's geometry," *Journal of Mathematical Physics*, vol. 32, no. 9, pp. 2456–2458, 1992.
- [21] S. Chakraborty and A. Ghosh, "Generalized scalar tensor theory in four and higher dimension," *International Journal of Modern Physics D*, vol. 9, no. 5, pp. 543–549, 2000.
- [22] F. Rahaman and J. K. Bera, "Higher dimensional cosmological model in Lyra geometry," *International Journal of Modern Physics D*, vol. 10, no. 5, pp. 729–733, 2001.
- [23] A. Pradhan and A. K. Vishwakarma, "A new class of LRS Bianchi type-I cosmological models in Lyra geometry," *Journal of Geometry and Physics*, vol. 49, no. 3–4, pp. 332–342, 2004.
- [24] A. Pradhan, V. K. Yadav, and I. Chakraborty, "Bulk viscous FRW cosmology in Lyra geometry," *International Journal of Modern Physics D*, vol. 10, no. 3, pp. 339–349, 2001.
- [25] R. Bali and N. K. Chandnani, "Bianchi type-I cosmological model for perfect fluid distribution in Lyra geometry," *Journal of Mathematical Physics*, vol. 49, no. 3, Article ID 032502, 8 pages, 2008.
- [26] R. Bali and N. K. Chandnani, "Bianchi type-III bulk viscous dust filled universe in Lyra geometry," *Astrophysics and Space Science*, vol. 318, no. 3–4, pp. 225–229, 2009.
- [27] S. Ram, M. Zeyauddin, and C. P. Singh, "Bianchi type V cosmological models with perfect fluid and heat conduction in Lyra's geometry," *International Journal of Modern Physics A*, vol. 23, no. 31, pp. 4991–5005, 2008.
- [28] R. Bali, L. K. Gupta, and N. K. Chandnani, "Bianchi type I string dust magnetized cosmological models in Lyra geometry," *Communications in Theoretical Physics*, vol. 54, pp. 197–202, 2010.
- [29] A. Lichnerowicz, *Relativistic Hydrodynamics and Magneto Hydrodynamics*, Benjamin, Elmsford, NY, USA, 1967.
- [30] R. Maartens, "Cosmological magnetic fields," *Pramana*, vol. 55, no. 4, pp. 575–583, 2000.
- [31] K. A. Bronnikov, E. N. Chudayeva, and G. N. Shikin, "Magneto-dilatonic Bianchi-I cosmology: isotropization and singularity problems," *Classical and Quantum Gravity*, vol. 21, no. 14, pp. 3389–3403, 2004.