Research Article

Approximate Solutions for Local Fractional Linear Transport Equations Arising in Fractal Porous Media

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We investigate the local fractional linear transport equations arising in fractal porous media by using the local fractional variational iteration method. Their approximate solutions within the nondifferentiable functions are obtained and their graphs are also shown.

1. Introduction

Transport equations [1–3] had successful applications in aeronomy [4], semiconductors [5], superconductor [6], turbulence [7], QCD [8], plasma [9], gas mixture [10], and biology [11]. The linear transport equation was written as follows [12]:

$$\frac{\partial u(x, t)}{\partial t} + a(x, t) \frac{\partial u(x, t)}{\partial x} = 0 \quad (1)$$

subject to the initial condition

$$u(x, 0) = f(x), \quad (2)$$

where $f(x)$ is a continuous function (differentiable function).

With the development of the transport theory in porous media [13], the disordered materials, which show fractal characteristics [14], had been investigated [15–17]. Recently, during anomalous transport [18] and evidence of its existence in point vortex flow [19], fractional theory of transport was developed by many researchers. Zaslavsky [20] and Tarasov [21] investigated the anomalous transport of fractional dynamics. Lutz presented the transport equations for Lévy stable processes with fractional derivative [22]. Uchaikin and

Sibatov considered the applications in disordered semiconductors [23]. Metzler and Klafter reported developments in the description of anomalous transport based upon fractional derivative [24]. Kadem et al. studied the solutions for the fractional transport equation by using the spectral method [25].

The above results via fractional calculus are set upon the differentiable functions. However, there are many nondifferentiable functions, which do not deal with the classical and fractional calculus. More recently, the local fractional calculus developed in [26–36] was the best candidate for scientists to deal with the nondifferentiable functions. In this paper, we consider the local fractional linear transport equations arising in fractal porous media in one-dimensional case [28]:

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + a(x, t) \frac{\partial^\alpha u(x, t)}{\partial x^\alpha} = 0, \quad (3)$$

subject to the initial condition

$$u(x, 0) = f(x), \quad (4)$$

where the velocity term $a(x, t)$ and the quantity $f(x)$ may be nondifferentiable functions. The purpose of the current

paper is to find the nondifferentiable solutions for the local fractional linear transport equations arising in fractal porous media in one-dimensional case by using the local fractional variational iteration method [31–36]. The plan of the paper is as follows. In Section 2, the conceptions of local fractional derivatives and local fractional integrals are given. In Section 3, the idea of local fractional variational iteration method is presented. In Section 4, the nondifferentiable behaviors for solutions of local fractional linear transport equations are studied. In Section 5, the conclusions are given.

2. On the Local Fractional Calculus

In this section, we introduce the definitions of local fractional derivatives and integrals which are used in the paper.

We set the function [26, 27]

$$f(x) \in C_\alpha(a, b), \quad (5)$$

where

$$|f(x) - f(x_0)| < \varepsilon^\alpha, \quad (6)$$

with $|x - x_0| < \delta$, for $\varepsilon > 0$, $0 < \alpha < 1$ and $\varepsilon \in R$.

Definition 1. Let $f(x) \in C_\alpha(a, b)$. We define the local fractional derivative of $f(x)$ of order α by [26–36]

$$\frac{d^\alpha f(x)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \quad (7)$$

where

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha) [f(x) - f(x_0)]. \quad (8)$$

Definition 2. Let $f(x) \in C_\alpha[a, b]$. We define the local fractional integral of $f(x)$ of order α in the interval $[a, b]$ by [26, 27, 29–36]

$$\begin{aligned} {}_a I_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t) (dt)^\alpha \\ &= \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)^\alpha, \end{aligned} \quad (9)$$

where the partitions of the interval $[a, b]$ are denoted as (t_j, t_{j+1}) , $j = 0, \dots, N-1$, $t_0 = a$ and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_2, \dots\}$.

From (8) and (9), there are some properties in the following form [26, 27, 31–35]:

$$\begin{aligned} \frac{d^\alpha}{dx^\alpha} \frac{x^{n\alpha}}{\Gamma(1 + n\alpha)} &= \frac{x^{(n-1)\alpha}}{\Gamma(1 + (n-1)\alpha)}, \\ \frac{d^\alpha}{dx^\alpha} E_\alpha(x^\alpha) &= E_\alpha(x^\alpha), \end{aligned}$$

$$\frac{d^\alpha}{dx^\alpha} \sin_\alpha(x^\alpha) = \cos_\alpha(x^\alpha),$$

$$\frac{d^\alpha}{dx^\alpha} \sinh_\alpha(x^\alpha) = \cosh_\alpha(x^\alpha),$$

$${}_0 I_x^{(\alpha)} \frac{x^{n\alpha}}{\Gamma(1 + n\alpha)} = \frac{x^{(n+1)\alpha}}{\Gamma(1 + (n+1)\alpha)}, \quad (10)$$

where

$$\begin{aligned} E_\alpha(x^\alpha) &= \sum_{k=0}^{\infty} \frac{x^{\alpha k}}{\Gamma(1 + k\alpha)}, \\ \sin_\alpha(x^\alpha) &= \sum_{k=0}^{\infty} (-1)^k \frac{x^{(2k+1)\alpha}}{\Gamma(1 + (2k+1)\alpha)}, \\ \cos_\alpha(x^\alpha) &= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2\alpha k}}{\Gamma(1 + 2\alpha k)}, \\ \sinh_\alpha(x^\alpha) &= \sum_{k=0}^{\infty} \frac{x^{(2k+1)\alpha}}{\Gamma(1 + (2k+1)\alpha)}, \\ \cosh_\alpha(x^\alpha) &= \sum_{k=0}^{\infty} \frac{x^{2\alpha k}}{\Gamma(1 + 2\alpha k)}. \end{aligned} \quad (11)$$

3. The Local Fractional Variational Iteration Method

In this section, the local fractional variational iteration method first proposed in [31] is applied to deal with the local fractional linear differential equations of order α .

Let us consider the following local fractional operator equation:

$$L_\alpha u(s) + N_\alpha u(s) = 0, \quad (12)$$

where the linear local fractional differential operator is defined as $L_\alpha = d^\alpha/ds^\alpha$, a nonlinear local fractional operator N_α .

From (12), a correction local fractional functional can be structured as

$$u_{n+1}(t) = u_n(t) + {}_0 I_t^{(\alpha)} \{ \xi^\alpha [L_\alpha u_n(s) + N_\alpha u_n(s)] \}, \quad (13)$$

where ξ^α denotes a fractal Lagrange multiplier; that is, $\delta^\alpha \tilde{u}_n = 0$ [26].

Making use of (18), the new iteration formula reads as

$$\begin{aligned} \delta^\alpha u_{n+1}(t) &= \delta^\alpha u_n(t) + \delta^\alpha {}_0 I_t^{(\alpha)} \\ &\quad \times \{ \xi^\alpha [L_\alpha u_n(s) + N_\alpha \tilde{u}_n(s)] \}, \end{aligned} \quad (14)$$

which leads to

$$(\xi^\alpha(\tau))^{(\alpha)} = 0, \quad 1 + \xi^\alpha(\tau) \Big|_{\tau=t} = 0, \quad (15)$$

where $\delta^\alpha \tilde{u}_n$ denotes a restricted local fractional variation; that is, $\delta^\alpha \tilde{u}_n = 0$ [26].

Therefore, from (15) the fractal Lagrange multiplier can be identified as

$$\xi^\alpha(\tau) = -1. \quad (16)$$

Submitting (17) into (13), we have that

$$u_{n+1}(t) = u_n(t) + {}_0I_t^{(\alpha)} \left\{ -[L_\alpha u_n(s) + N_\alpha u_n(s)] \right\}. \quad (17)$$

Consequently, the nondifferentiable solution can be written as

$$u = \lim_{n \rightarrow \infty} u_n. \quad (18)$$

For more results, see [31–36].

4. Approximate Solutions

Example 3. Consider sample local fractional linear transport equations arising in fractal porous media in the form

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + \frac{\partial^\alpha u(x, t)}{\partial x^\alpha} = 0, \quad (19)$$

with the initial condition

$$u(x, 0) = \frac{x^\alpha}{\Gamma(1 + \alpha)}. \quad (20)$$

From (17) we derive the following iterative formula:

$$\begin{aligned} u_{n+1}(x, t) \\ = u_n(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_n(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_n(x, s)}{\partial x^\alpha} \right] \right\}, \end{aligned} \quad (21)$$

where the initial value condition is

$$u_0(x, t) = u(x, 0) = \frac{x^\alpha}{\Gamma(1 + \alpha)}. \quad (22)$$

From (21) the first approximate term reads as

$$\begin{aligned} u_1(x, t) &= u_0(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)}. \end{aligned} \quad (23)$$

In a like manner, the second approximate term is

$$\begin{aligned} u_2(x, t) &= u_1(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)}. \end{aligned} \quad (24)$$

FIGURE 1: The plot of solution of (19) when $\alpha = \ln 2 / \ln 3$.

The third approximate term can be written as

$$\begin{aligned} u_3(x, t) &= u_2(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)}. \end{aligned} \quad (25)$$

Continuing to calculate them in this manner, for $n > 1$, we have that

$$u_{n+1}(x, t) = u_n(x, t). \quad (26)$$

Hence, we obtain the nondifferentiable solution given by

$$u(x, t) = \frac{x^\alpha}{\Gamma(1 + \alpha)} - \frac{t^\alpha}{\Gamma(1 + \alpha)}, \quad (27)$$

and its graph is shown in Figure 1.

Example 4. The sample local fractional linear transport equations arising in fractal porous media take the form

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + \frac{\partial^\alpha u(x, t)}{\partial x^\alpha} = 0, \quad (28)$$

subject to the initial condition

$$u(x, 0) = E_\alpha(x^\alpha). \quad (29)$$

In view of (17) we structure the following iterative formula:

$$\begin{aligned} u_{n+1}(x, t) \\ = u_n(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_n(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_n(x, s)}{\partial x^\alpha} \right] \right\}, \end{aligned} \quad (30)$$

where the initial value condition is expressed by

$$u_0(x, t) = u(x, 0) = E_\alpha(x^\alpha). \quad (31)$$

In view of (31), we give the first approximation given by

$$\begin{aligned} u_1(x, t) &= u_0(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} \right]. \end{aligned} \quad (32)$$

Similarly, we obtain that

$$\begin{aligned} u_2(x, t) &= u_1(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} \right] \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \right], \\ u_3(x, t) &= u_2(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \right] \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} \right], \\ u_4(x, t) &= u_3(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) - E_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\} \\ &= E_\alpha(x^\alpha) \left[1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \right. \\ &\quad \left. - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{4\alpha}}{\Gamma(1+4\alpha)} \right], \\ &\vdots \end{aligned} \quad (33)$$

and so on.

FIGURE 2: The plot of solution of (28) when $\alpha = \ln 2 / \ln 3$.

Hence, the nondifferentiable solution is given by

$$\begin{aligned} u(x, t) &= \lim_{n \rightarrow \infty} u_n(x, t) \\ &= \lim_{n \rightarrow \infty} E_\alpha(x^\alpha) \left[\sum_{k=0}^{\infty} \frac{t^{2\alpha k}}{\Gamma(1+2\alpha k)} - \sum_{k=0}^{\infty} \frac{t^{(2k+1)\alpha}}{\Gamma[1+(2k+1)\alpha]} \right] \\ &= E_\alpha(x^\alpha) [\cosh_\alpha(t^\alpha) - \sinh_\alpha(t^\alpha)]. \end{aligned} \quad (34)$$

together with the graph shown in Figure 2.

Example 5. We now focus on the following local fractional linear transport equations arising in fractal porous media:

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + \frac{\partial^\alpha u(x, t)}{\partial x^\alpha} = 0, \quad (35)$$

subject to the initial condition

$$u(x, 0) = \cos_\alpha(x^\alpha). \quad (36)$$

In view of (17) the following iterative formula is given by

$$\begin{aligned} u_{n+1}(x, t) &= u_n(x, t) \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_n(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_n(x, s)}{\partial x^\alpha} \right] \right\}, \end{aligned} \quad (37)$$

where the initial value condition is expressed by

$$u_0(x, t) = u(x, 0) = \cos_\alpha(x^\alpha). \quad (38)$$

Using (37) gives the approximate terms

$$\begin{aligned}
 u_1(x, t) &= u_0(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \cos_\alpha(x^\alpha) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \cos_\alpha(x^\alpha) + \sin_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)}, \\
 u_2(x, t) &= u_1(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \cos_\alpha(x^\alpha) + \sin_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\
 &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \cos_\alpha(x^\alpha) + \sin_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\
 &\quad - \cos_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \\
 u_3(x, t) &= u_2(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \cos_\alpha(x^\alpha) + \sin_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\
 &\quad - \cos_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \\
 &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \sin_\alpha(x^\alpha) \sum_{k=0}^1 (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \\
 &\quad + \cos_\alpha(x^\alpha) \sum_{k=0}^1 (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)}, \\
 u_4(x, t) &= u_3(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\} \\
 &= \sin_\alpha(x^\alpha) \sum_{k=0}^1 (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \\
 &\quad + \cos_\alpha(x^\alpha) \sum_{k=0}^1 (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)} \\
 &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\}
 \end{aligned}$$

FIGURE 3: The plot of solution of (35) when $\alpha = \ln 2 / \ln 3$.

$$\begin{aligned}
 &= \sin_\alpha(x^\alpha) \sum_{k=0}^2 (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \\
 &\quad + \cos_\alpha(x^\alpha) \sum_{k=0}^2 (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)}, \\
 &\quad \vdots
 \end{aligned} \tag{39}$$

and so on.

Therefore, the nondifferentiable solution of (35) can be written as

$$\begin{aligned}
 u(x, t) &= \lim_{n \rightarrow \infty} u_n(x, t) \\
 &= \lim_{n \rightarrow \infty} \left[\sin_\alpha(x^\alpha) \sum_{k=0}^{\infty} (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \right. \\
 &\quad \left. + \cos_\alpha(x^\alpha) \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)} \right] \\
 &= \lim_{n \rightarrow \infty} [\sin_\alpha(x^\alpha) \sin_\alpha(t^\alpha) + \cos_\alpha(x^\alpha) \cos_\alpha(t^\alpha)]
 \end{aligned} \tag{40}$$

and its graph is given in Figure 3.

Example 6. We discuss the following local fractional linear transport equations arising in fractal porous media:

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + \frac{\partial^\alpha u(x, t)}{\partial x^\alpha} = 0, \tag{41}$$

subject to the initial condition

$$u(x, 0) = \sin_\alpha(x^\alpha). \tag{42}$$

From (17) the following iterative formula can be written as

$$\begin{aligned}
 u_{n+1}(x, t) &= u_n(x, t) \\
 &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_n(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_n(x, s)}{\partial x^\alpha} \right] \right\},
 \end{aligned} \tag{43}$$

where the initial value condition is given by

$$u_0(x, t) = u(x, 0) = \sin_\alpha(x^\alpha). \quad (44)$$

Making use of (43), we obtain the approximate terms

$$\begin{aligned} u_1(x, t) &= u_0(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_0(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_0(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)}, \\ u_2(x, t) &= u_1(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_1(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_1(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad - \sin_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \\ u_3(x, t) &= u_2(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad - \sin_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_2(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_2(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad - \sin_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \cos_\alpha(x^\alpha) \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \\ u_4(x, t) &= u_3(x, t) + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\} \\ &= \sin_\alpha(x^\alpha) - \cos_\alpha(x^\alpha) \frac{t^\alpha}{\Gamma(1+\alpha)} \\ &\quad - \sin_\alpha(x^\alpha) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \cos_\alpha(x^\alpha) \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} \\ &\quad + {}_0I_t^{(\alpha)} \left\{ - \left[\frac{\partial^\alpha u_3(x, s)}{\partial s^\alpha} + \frac{\partial^\alpha u_3(x, s)}{\partial x^\alpha} \right] \right\} \end{aligned}$$

FIGURE 4: The plot of solution of (41) when $\alpha = \ln 2 / \ln 3$.

$$\begin{aligned} &= \sin_\alpha(x^\alpha) \sum_{k=0}^2 (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)} \\ &\quad - \cos_\alpha(x^\alpha) \sum_{k=0}^1 (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}, \\ &\quad \vdots \end{aligned} \quad (45)$$

and so on.

Therefore, the nondifferentiable solution of (41) reads as

$$\begin{aligned} u(x, t) &= \lim_{n \rightarrow \infty} u_n(x, t) \\ &= \lim_{n \rightarrow \infty} \left[\sin_\alpha(x^\alpha) \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k\alpha}}{\Gamma(1+2k\alpha)} \right. \\ &\quad \left. - \cos_\alpha(x^\alpha) \sum_{k=0}^{\infty} (-1)^k \frac{t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)} \right] \\ &= \sin_\alpha(x^\alpha) \cos_\alpha(t^\alpha) - \cos_\alpha(x^\alpha) \sin_\alpha(t^\alpha) \end{aligned} \quad (46)$$

and its graph is given in Figure 4.

5. Conclusions

Local fractional calculus theory is a tool for modeling the nondifferentiable problems for science and engineering. In this work we studied the local fractional linear transport equations arising in fractal porous media by using the local fractional variational iteration method. The solutions with nondifferentiable functions were also obtained and some examples were also discussed. These results show the reliabilities and efficiencies of the proposed local fractional variational iteration method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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