Research Article

Local Fractional Laplace Variational Iteration Method for Fractal Vehicular Traffic Flow

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We discuss the line partial differential equations arising in fractal vehicular traffic flow. The nondifferentiable approximate solutions are obtained by using the local fractional Laplace variational iteration method, which is the coupling method of local fractional variational iteration method and Laplace transform. The obtained results show the efficiency and accuracy of implements of the present method.

1. Introduction

Fractional differential equations with arbitrary orders were applied to model the real-world problems for science and engineering. Many researchers present their applications with solid mechanics, heat transfer, fluid mechanics, transport process, water motion, and quantum mechanics. For example, Tarasov studied the wave equation for fractal solid string based on the fractional calculus [1]. Momani and Odibat presented the linear, nonlinear, and fractional partial differential equations arising in fluid mechanics [2, 3]. Povstenko suggested the fractional heat conduction equation [4], and Vázquez gave the second law of thermodynamics with fractional derivative [5]. Lutz considered the fractional transport equation for Levy stable processes [6], and Kadem et al. discussed its solution using the spectral method [7]. Laskin presented the fractional Schrodinger equation [8], and Muslih et al. suggested its solution [9]. There are many methods for finding the solutions to the fractional differentiable equations [10]. For example, Jafari and Seifi used the homotopy analysis method to deal with linear and nonlinear fractional diffusion-wave equations [11]. Jafari et al. applied the fractional subequation method to solve the Cahn-Hilliard and Klein-Gordon equations [12]. Hristov suggested the heat-balance integral method for fractional heat diffusion

and transient flow [13, 14]. Bhrawy and Alghamdi proposed the shifted Jacobi-Gauss-Lobatto collocation method to find the solution for nonlinear fractional Langevin equation with two variables [15] and the shifted Legendre spectral method for solving the fractional-order multipoint boundary value problems [16]. Bhrawy and Baleanu considered the spectral Legendre-Gauss-Lobatto collocation method to solve the space-fractional advection diffusion equations [17]. Atangana and Baleanu presented the two difference methods to solve the fractional parabolic equations [18].

Recently, the local fractional calculus is proposed and developed to describe the fractal problems in various fields, such as physics [19–21], applied mathematics [22, 23], signal processing [24–27], fluid mechanics [28], quantum mechanics [29], fractal forest gap [30], vehicular traffic flow [31], and silk cocoon hierarchy [32]. The linear differential equation arising in fractal vehicular traffic flow was suggested in [31]. The local fractional Laplace variational iteration method was suggested in [23] and developed in [33]. In this paper, we use the local fractional Laplace variational iteration method to solve the linear differential equation arising in fractal vehicular traffic flow. The structure of the paper is suggested as follows. In Section 2, the basic theory of local fractional calculus and local fractional Laplace transform are introduced. Section 3 gives the local fractional Laplace variational

iteration method. In Section 4, the nondifferentiable solutions for line partial differential equations arising in fractal vehicular traffic flow are presented. Finally, the conclusions are considered in Section 5.

2. The Lighthill-Whitham-Richards Model on a Finite Length Highway

In this section, we present the Lighthill-Whitham-Richards model on a finite length highway, the conceptions of local fractional derivative and integral, and the local fractional Laplace transform.

The Lighthill-Whitham-Richards model on a finite length highway reads as follows [31]:

$$\frac{\partial^\alpha \varphi(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi(x, t)}{\partial x^\alpha} = 0, \quad (1)$$

where the initial and boundary conditions are presented as follows:

$$\begin{aligned} \varphi(x, 0) &= \varphi_0(x), \\ \varphi(0, t) &= \omega(t), \end{aligned} \quad (2)$$

with

$$\begin{aligned} |\varphi_0(x) - \varphi_0(x_0)| &< \varepsilon^\alpha, \\ |\omega(t) - \omega(t_0)| &< \kappa^\alpha, \end{aligned} \quad (3)$$

for $|x - x_0| < \delta$, $|t - t_0| < \tau$ for $\varepsilon, \kappa, \delta, \tau > 0$, $0 < \alpha < 1$, and the local fractional partial derivative of $f(x)$ of order α is given as [19]

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \quad (4)$$

with

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha) [f(x) - f(x_0)]. \quad (5)$$

This is the line partial differential equation arising in fractal vehicular traffic flow.

The local fractional derivative of $f(x)$ of order α is expressed as [19, 20]

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \quad (6)$$

where

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha) [f(x) - f(x_0)]. \quad (7)$$

The local fractional integral of $f(x)$ of order α in the interval $[a, b]$ is defined through [19, 21, 22]

$$\begin{aligned} {}_a I_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t) (dt)^\alpha \\ &= \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{j=N-1} f(t_j) (\Delta t_j)^\alpha, \end{aligned} \quad (8)$$

where the partitions of the interval $[a, b]$ are (t_j, t_{j+1}) , with $\Delta t_j = t_{j+1} - t_j$, $t_0 = a$, $t_N = b$, and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \dots\}$, $j = 0, \dots, N - 1$.

The local fractional Laplace transform is given as [23, 26, 33–35]

$$\begin{aligned} \tilde{L}_\alpha \{f(x)\} &= f_s^{\tilde{L}, \alpha}(s) \\ &= \frac{1}{\Gamma(1 + \alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x) (dx)^\alpha, \end{aligned} \quad (9)$$

$$0 < \alpha \leq 1,$$

where $f(x)$ is local fractional continuous and $s^\alpha = \beta^\alpha + i^\alpha \infty^\alpha$.

The inverse formula of local fractional Laplace transform is suggested as [23, 26, 33–35]

$$\begin{aligned} f(x) &= \tilde{L}_\alpha^{-1} \{f_s^{\tilde{L}, \alpha}(s)\} \\ &= \frac{1}{(2\pi)^\alpha} \int_{\beta - i\infty}^{\beta + i\infty} E_\alpha(s^\alpha x^\alpha) f_s^{\tilde{L}, \alpha}(s) (ds)^\alpha, \end{aligned} \quad (10)$$

where $s^\alpha = \beta^\alpha + i^\alpha \infty^\alpha$ and $\text{Re}(s^\alpha) = \beta^\alpha$.

The local fractional convolution of two functions is presented as [23, 33]

$$f_1(x) * f_2(x) = \frac{1}{\Gamma(1 + \alpha)} \int_{-\infty}^\infty f_1(t) f_2(x - t) (dt)^\alpha. \quad (11)$$

The properties for local fractional Laplace transform are listed as follows [23, 26, 33–35]:

$$\tilde{L}_\alpha \{af(x) + bg(x)\} = a\tilde{L}_\alpha \{f(x)\} + b\tilde{L}_\alpha \{g(x)\},$$

$$\tilde{L}_\alpha \{f^{(n\alpha)}(x)\} = s^{n\alpha} \tilde{L}_\alpha \{f(x)\} - \sum_{k=1}^n s^{(k-1)\alpha} f^{(n-k)\alpha}(0),$$

$$F_\alpha \{f_1(x) * f_2(x)\} = f_{\omega,1}^{F,\alpha}(\omega) f_{\omega,2}^{F,\alpha}(\omega), \quad (12)$$

$$\tilde{L}_\alpha \{\sin_\alpha(c^\alpha x^\alpha)\} = \frac{c^\alpha}{s^{2\alpha} + c^{2\alpha}},$$

$$\tilde{L}_\alpha \{\cos_\alpha(c^\alpha x^\alpha)\} = \frac{s^\alpha}{s^{2\alpha} + c^{2\alpha}},$$

$$\tilde{L}_\alpha \{x^{k\alpha}\} = \frac{\Gamma(1 + k\alpha)}{s^{(k+1)\alpha}}.$$

3. Local Fractional Laplace Variational Iteration Method

In this section, we introduce the local fractional Laplace variational iteration method. Let us consider the following local fractional differential operator:

$$L_\alpha u - R_\alpha u = 0, \quad (13)$$

where the linear local fractional differential operator is $L_\alpha = (d^\alpha/ds^\alpha)$ and $u(x)$ is a nondifferential function.

According to the local fractional Laplace variational iteration method [23, 33], the local fractional functional formula is presented as follows:

$$u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)} \left\{ \frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) - R_\alpha u_n] \right\}. \quad (14)$$

Applying the local fractional Laplace transform gives the following:

$$\begin{aligned} \tilde{L}_\alpha \{u_{n+1}(x)\} &= \tilde{L}_\alpha \{u_n(x)\} \\ &+ \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} \tilde{L}_\alpha \{L_\alpha u_n(x) - R_\alpha u_n(x)\}. \end{aligned} \quad (15)$$

Taking the local fractional variation of (15), we obtain

$$\begin{aligned} \delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} &= \delta^\alpha \{ \tilde{L}_\alpha \{u_n(x)\} \} \\ &+ \delta^\alpha \left\{ \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} \tilde{L}_\alpha \{L_\alpha u_n(x) - R_\alpha u_n(x)\} \right\}. \end{aligned} \quad (16)$$

From (16), we have

$$\begin{aligned} \delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} &= \delta^\alpha \{ \tilde{L}_\alpha \{u_n(x)\} \} \\ &+ \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} \{ \delta^\alpha \{ \tilde{L}_\alpha \{L_\alpha u_n(x)\} \} \} = 0 \end{aligned} \quad (17)$$

such that

$$1 + \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} s^\alpha = 0, \quad (18)$$

where

$$\begin{aligned} \delta^\alpha \{ \tilde{L}_\alpha \{L_\alpha u_n(x)\} \} &= \delta^\alpha \{ s^\alpha \tilde{L}_\alpha \{u_n(x)\} - u_n(0) \} \\ &= s^\alpha \delta^\alpha \tilde{L}_\alpha \{u_n(x)\}. \end{aligned} \quad (19)$$

Therefore, we get

$$\tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} = -\frac{1}{s^\alpha}. \quad (20)$$

Using formula (20), we arrive at new iteration algorithm as follows:

$$\begin{aligned} \tilde{L}_\alpha \{u_{n+1}(x)\} &= \tilde{L}_\alpha \{u_n(x)\} \\ &- \frac{1}{s^\alpha} \tilde{L}_\alpha \{ (L_\alpha u_n(x) - R_\alpha u_n(x)) \}, \end{aligned} \quad (21)$$

where the initial value reads as

$$\tilde{L}_\alpha \{u_0(x)\} = u(0). \quad (22)$$

Thus, the local fractional series solution of (13) is

$$\tilde{L}_\alpha \{u\} = \lim_{n \rightarrow \infty} \tilde{L}_\alpha \{u_n\} \quad (23)$$

which leads to

$$u = \lim_{n \rightarrow \infty} \tilde{L}_\alpha^{-1} \{ \tilde{L}_\alpha u_n \}. \quad (24)$$

4. The Nondifferentiable Solutions for Line Partial Differential Equations Arising in Fractal Vehicular Traffic Flow

In this section, we present the boundary value problems for line partial differential equations arising in fractal vehicular traffic flow.

Example 1. The initial and boundary conditions for line partial differential equations arising in fractal vehicular traffic flow read as follows:

$$\begin{aligned} \varphi(x, 0) &= E_\alpha(x^\alpha), \\ \varphi(0, t) &= 0. \end{aligned} \quad (25)$$

In view of (21), we have

$$\begin{aligned} \tilde{L}_\alpha \{ \varphi_{n+1}(x, t) \} &= \tilde{L}_\alpha \{ \varphi_n(x, t) \} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_n(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_n(x, t)}{\partial x^\alpha} \right\} \\ &= \varphi_n(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_n(x, s) - \varphi_n(x, 0) + \mu \frac{\partial^\alpha \varphi_n(x, s)}{\partial x^\alpha} \right\} \\ &= \frac{1}{s^\alpha} \varphi_n(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_n(s, t)}{\partial x^\alpha}, \end{aligned} \quad (26)$$

where the initial value is

$$\tilde{L}_\alpha \{ \varphi_0(x, s) \} = \varphi_0(x, s) = \tilde{L}_\alpha \{ E_\alpha(x^\alpha) \} = \frac{E_\alpha(x^\alpha)}{s^\alpha}. \quad (27)$$

Making use of (26) and (27), we have the first approximation

$$\begin{aligned} \tilde{L}_\alpha \{ \varphi_1(x, t) \} &= \tilde{L}_\alpha \{ \varphi_0(x, t) \} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_0(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_0(x, t)}{\partial x^\alpha} \right\} \\ &= \varphi_0(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_0(x, s) - \varphi_0(x, 0) + \mu \frac{\partial^\alpha \varphi_0(x, s)}{\partial x^\alpha} \right\} \\ &= \frac{1}{s^\alpha} \varphi_0(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_0(s, t)}{\partial x^\alpha} \\ &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}}. \end{aligned} \quad (28)$$

In view of (26) and (28), we arrive at the second approximation

$$\begin{aligned}
 & \tilde{L}_\alpha \{\varphi_2(x, t)\} \\
 &= \tilde{L}_\alpha \{\varphi_1(x, t)\} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_1(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_1(x, t)}{\partial x^\alpha} \right\} \\
 &= \varphi_1(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_1(x, s) - \varphi_1(x, 0) + \mu \frac{\partial^\alpha \varphi_1(x, s)}{\partial x^\alpha} \right\} \\
 &= \frac{1}{s^\alpha} \varphi_1(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_1(s, t)}{\partial x^\alpha} \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu}{s^\alpha} \left(\frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} \right) \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}}.
 \end{aligned} \tag{29}$$

From (26) and (29), the third approximation is

$$\begin{aligned}
 & \tilde{L}_\alpha \{\varphi_3(x, t)\} \\
 &= \tilde{L}_\alpha \{\varphi_2(x, t)\} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_2(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_2(x, t)}{\partial x^\alpha} \right\} \\
 &= \varphi_2(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_2(x, s) - \varphi_2(x, 0) + \mu \frac{\partial^\alpha \varphi_2(x, s)}{\partial x^\alpha} \right\} \\
 &= \frac{1}{s^\alpha} \varphi_2(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_2(s, t)}{\partial x^\alpha} \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu}{s^\alpha} \left(\frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} \right) \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} - \frac{\mu^3 E_\alpha(x^\alpha)}{s^{4\alpha}}.
 \end{aligned} \tag{30}$$

Applying (26) and (29) gives the fourth approximation

$$\begin{aligned}
 & \tilde{L}_\alpha \{\varphi_4(x, t)\} \\
 &= \tilde{L}_\alpha \{\varphi_3(x, t)\} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_3(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_3(x, t)}{\partial x^\alpha} \right\} \\
 &= \varphi_3(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_3(x, s) - \varphi_3(x, 0) + \mu \frac{\partial^\alpha \varphi_3(x, s)}{\partial x^\alpha} \right\} \\
 &= \frac{1}{s^\alpha} \varphi_3(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_3(s, t)}{\partial x^\alpha} \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu}{s^\alpha} \left(\frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} \right. \\
 &\quad \left. + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} - \frac{\mu^3 E_\alpha(x^\alpha)}{s^{4\alpha}} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} \\
 &\quad - \frac{\mu^3 E_\alpha(x^\alpha)}{s^{4\alpha}} + \frac{\mu^4 E_\alpha(x^\alpha)}{s^{4\alpha}}.
 \end{aligned} \tag{31}$$

In view of (26) and (31), we give the fifth approximation

$$\begin{aligned}
 & \tilde{L}_\alpha \{\varphi_5(x, t)\} \\
 &= \tilde{L}_\alpha \{\varphi_4(x, t)\} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_4(x, t)}{\partial t^\alpha} + \mu \frac{\partial^\alpha \varphi_4(x, t)}{\partial x^\alpha} \right\} \\
 &= \varphi_4(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_4(x, s) - \varphi_4(x, 0) + \mu \frac{\partial^\alpha \varphi_4(x, s)}{\partial x^\alpha} \right\} \\
 &= \frac{1}{s^\alpha} \varphi_4(x, 0) - \frac{\mu}{s^\alpha} \frac{\partial^\alpha \varphi_4(s, t)}{\partial x^\alpha} \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu}{s^\alpha} \left(\frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} \right. \\
 &\quad \left. - \frac{\mu^3 E_\alpha(x^\alpha)}{s^{4\alpha}} + \frac{\mu^4 E_\alpha(x^\alpha)}{s^{4\alpha}} \right) \\
 &= \frac{E_\alpha(x^\alpha)}{s^\alpha} - \frac{\mu E_\alpha(x^\alpha)}{s^{2\alpha}} + \frac{\mu^2 E_\alpha(x^\alpha)}{s^{3\alpha}} - \frac{\mu^3 E_\alpha(x^\alpha)}{s^{4\alpha}} \\
 &\quad + \frac{\mu^4 E_\alpha(x^\alpha)}{s^{4\alpha}} - \frac{\mu^5 E_\alpha(x^\alpha)}{s^{5\alpha}}.
 \end{aligned} \tag{32}$$

Therefore, we obtain

$$\begin{aligned}
 & \tilde{L}_\alpha \{\varphi_n(x, t)\} \\
 &= \lim_{n \rightarrow \infty} \tilde{L}_\alpha \left\{ \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{(1+2i)} E_\alpha(x^\alpha)}{s^{(1+2i)\alpha}} - \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{2i} E_\alpha(x^\alpha)}{s^{2i\alpha}} \right\},
 \end{aligned} \tag{33}$$

which reduces to

$$\begin{aligned}
 & \varphi(x, t) \\
 &= \lim_{n \rightarrow \infty} \left\{ \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{(1+2i)} E_\alpha(x^\alpha)}{s^{(1+2i)\alpha}} - \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{2i} E_\alpha(x^\alpha)}{s^{2i\alpha}} \right\} \\
 &= E_\alpha(x^\alpha) \lim_{n \rightarrow \infty} \left\{ \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{(1+2i)}}{s^{(1+2i)\alpha}} - \frac{1}{\mu} \sum_{i=1}^n \frac{\mu^{2i}}{s^{2i\alpha}} \right\} \\
 &= E_\alpha(x^\alpha) \left(\sum_{i=1}^n \frac{\mu^{2i} t^{2i\alpha}}{\Gamma(1+2i\alpha)} - \sum_{i=1}^n \frac{\mu^{2i-1} t^{(2i-1)\alpha}}{\Gamma(1+(2i-1)\alpha)} \right) \\
 &= E_\alpha(x^\alpha) [\cos_\alpha(\mu t^\alpha) - \sin_\alpha(\mu t^\alpha)],
 \end{aligned} \tag{34}$$

and its graph is shown in Figure 1 with the parameters $\alpha = \ln 2 / \ln 3$ and $\mu = 2$.

FIGURE 1: The graph of (1) with the parameters $\alpha = \ln 2 / \ln 3$ and $\mu = 2$.

Example 2. The initial and boundary conditions for line partial differential equations arising in fractal vehicular traffic flow with the parameter $\mu = 1$ are

$$\begin{aligned}\varphi(x, 0) &= \sinh_{\alpha}(x^{\alpha}), \\ \varphi(0, t) &= 0.\end{aligned}\quad (35)$$

From (21), we obtain

$$\begin{aligned}\tilde{L}_{\alpha}\{\varphi_{n+1}(x, t)\} &= \tilde{L}_{\alpha}\{\varphi_n(x, t)\} - \frac{1}{s^{\alpha}}\tilde{L}_{\alpha}\left\{\frac{\partial^{\alpha}\varphi_n(x, t)}{\partial t^{\alpha}} + \frac{\partial^{\alpha}\varphi_n(x, t)}{\partial x^{\alpha}}\right\} \\ &= \varphi_n(x, s) - \frac{1}{s^{\alpha}}\left\{s^{\alpha}\varphi_n(x, s) - \varphi_n(x, 0) + \frac{\partial^{\alpha}\varphi_n(x, s)}{\partial x^{\alpha}}\right\} \\ &= \frac{1}{s^{\alpha}}\varphi_n(x, 0) - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}\varphi_n(s, t)}{\partial x^{\alpha}},\end{aligned}\quad (36)$$

where the initial value is

$$\tilde{L}_{\alpha}\{\varphi_0(x, t)\} = \varphi_0(x, s) = \tilde{L}_{\alpha}\{\sinh_{\alpha}(x^{\alpha})\} = \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}}.\quad (37)$$

In view of (36) and (37), we get the first approximation

$$\begin{aligned}\tilde{L}_{\alpha}\{\varphi_1(x, t)\} &= \tilde{L}_{\alpha}\{\varphi_0(x, t)\} - \frac{1}{s^{\alpha}}\tilde{L}_{\alpha}\left\{\frac{\partial^{\alpha}\varphi_0(x, t)}{\partial t^{\alpha}} + \frac{\partial^{\alpha}\varphi_0(x, t)}{\partial x^{\alpha}}\right\} \\ &= \varphi_0(x, s) - \frac{1}{s^{\alpha}}\left\{s^{\alpha}\varphi_0(x, s) - \varphi_0(x, 0) + \frac{\partial^{\alpha}\varphi_0(x, s)}{\partial x^{\alpha}}\right\} \\ &= \frac{1}{s^{\alpha}}\varphi_0(x, 0) - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}\varphi_0(s, t)}{\partial x^{\alpha}} \\ &= \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{\cosh_{\alpha}(x^{\alpha})}{s^{\alpha}}.\end{aligned}\quad (38)$$

From (36) and (38), we obtain the second approximation as follows:

$$\begin{aligned}\tilde{L}_{\alpha}\{\varphi_2(x, t)\} &= \tilde{L}_{\alpha}\{\varphi_1(x, t)\} - \frac{1}{s^{\alpha}}\tilde{L}_{\alpha}\left\{\frac{\partial^{\alpha}\varphi_1(x, t)}{\partial t^{\alpha}} + \frac{\partial^{\alpha}\varphi_1(x, t)}{\partial x^{\alpha}}\right\} \\ &= \varphi_1(x, s) - \frac{1}{s^{\alpha}}\left\{s^{\alpha}\varphi_1(x, s) - \varphi_1(x, 0) + \frac{\partial^{\alpha}\varphi_1(x, s)}{\partial x^{\alpha}}\right\} \\ &= \frac{1}{s^{\alpha}}\varphi_1(x, 0) - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}\varphi_1(s, t)}{\partial x^{\alpha}} \\ &= \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{1}{s^{\alpha}}\left(\frac{\cosh_{\alpha}(x^{\alpha})}{s^{\alpha}} + \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}}\right) \\ &= \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{\cosh_{\alpha}(x^{\alpha})}{s^{2\alpha}} - \frac{\sinh_{\alpha}(x^{\alpha})}{s^{2\alpha}}.\end{aligned}\quad (39)$$

Using (36) and (39), we have the third approximation

$$\begin{aligned}\tilde{L}_{\alpha}\{\varphi_3(x, t)\} &= \tilde{L}_{\alpha}\{\varphi_2(x, t)\} - \frac{1}{s^{\alpha}}\tilde{L}_{\alpha}\left\{\frac{\partial^{\alpha}\varphi_2(x, t)}{\partial t^{\alpha}} + \frac{\partial^{\alpha}\varphi_2(x, t)}{\partial x^{\alpha}}\right\} \\ &= \varphi_2(x, s) - \frac{1}{s^{\alpha}}\left\{s^{\alpha}\varphi_2(x, s) - \varphi_2(x, 0) + \frac{\partial^{\alpha}\varphi_2(x, s)}{\partial x^{\alpha}}\right\} \\ &= \frac{1}{s^{\alpha}}\varphi_2(x, 0) - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}\varphi_2(s, t)}{\partial x^{\alpha}} \\ &= \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{\cosh_{\alpha}(x^{\alpha})}{s^{2\alpha}} - \frac{\sinh_{\alpha}(x^{\alpha})}{s^{3\alpha}}\right) \\ &= \frac{\sinh_{\alpha}(x^{\alpha})}{s^{\alpha}} - \frac{\cosh_{\alpha}(x^{\alpha})}{s^{2\alpha}} - \frac{\sinh_{\alpha}(x^{\alpha})}{s^{3\alpha}} + \frac{\cosh_{\alpha}(x^{\alpha})}{s^{4\alpha}}.\end{aligned}\quad (40)$$

Making use of (36) and (40), we have the fourth approximation

$$\begin{aligned}\tilde{L}_{\alpha}\{\varphi_4(x, t)\} &= \tilde{L}_{\alpha}\{\varphi_3(x, t)\} - \frac{1}{s^{\alpha}}\tilde{L}_{\alpha}\left\{\frac{\partial^{\alpha}\varphi_3(x, t)}{\partial t^{\alpha}} + \frac{\partial^{\alpha}\varphi_3(x, t)}{\partial x^{\alpha}}\right\} \\ &= \varphi_3(x, s) - \frac{1}{s^{\alpha}}\left\{s^{\alpha}\varphi_3(x, s) - \varphi_3(x, 0) + \frac{\partial^{\alpha}\varphi_3(x, s)}{\partial x^{\alpha}}\right\} \\ &= \frac{1}{s^{\alpha}}\varphi_3(x, 0) - \frac{1}{s^{\alpha}}\frac{\partial^{\alpha}\varphi_3(s, t)}{\partial x^{\alpha}}\end{aligned}$$

$$\begin{aligned}
&= \frac{\sinh_\alpha(x^\alpha)}{s^\alpha} - \frac{1}{s^\alpha} \frac{\partial^\alpha}{\partial x^\alpha} \left(\frac{\sinh_\alpha(x^\alpha)}{s^\alpha} - \frac{\cosh_\alpha(x^\alpha)}{s^{2\alpha}} \right. \\
&\quad \left. - \frac{\sinh_\alpha(x^\alpha)}{s^{3\alpha}} + \frac{\cosh_\alpha(x^\alpha)}{s^{4\alpha}} \right) \\
&= \frac{\sinh_\alpha(x^\alpha)}{s^\alpha} - \frac{\cosh_\alpha(x^\alpha)}{s^{2\alpha}} - \frac{\sinh_\alpha(x^\alpha)}{s^{3\alpha}} \\
&\quad + \frac{\cosh_\alpha(x^\alpha)}{s^{4\alpha}} + \frac{\sinh_\alpha(x^\alpha)}{s^{5\alpha}}.
\end{aligned} \tag{41}$$

From (36) and (41), we get the fifth approximation as follows:

$$\begin{aligned}
&\tilde{L}_\alpha \{ \varphi_5(x, t) \} \\
&= \tilde{L}_\alpha \{ \varphi_4(x, t) \} - \frac{1}{s^\alpha} \tilde{L}_\alpha \left\{ \frac{\partial^\alpha \varphi_4(x, t)}{\partial t^\alpha} + \frac{\partial^\alpha \varphi_4(x, t)}{\partial x^\alpha} \right\} \\
&= \varphi_4(x, s) - \frac{1}{s^\alpha} \left\{ s^\alpha \varphi_4(x, s) - \varphi_4(x, 0) + \frac{\partial^\alpha \varphi_4(x, s)}{\partial x^\alpha} \right\} \\
&= \frac{1}{s^\alpha} \varphi_4(x, 0) - \frac{1}{s^\alpha} \frac{\partial^\alpha \varphi_4(x, s)}{\partial x^\alpha} \\
&= \frac{\sinh_\alpha(x^\alpha)}{s^\alpha} \\
&\quad - \frac{1}{s^\alpha} \frac{\partial^\alpha}{\partial x^\alpha} \left(\frac{\sinh_\alpha(x^\alpha)}{s^\alpha} - \frac{\cosh_\alpha(x^\alpha)}{s^{2\alpha}} - \frac{\sinh_\alpha(x^\alpha)}{s^{3\alpha}} \right. \\
&\quad \left. + \frac{\cosh_\alpha(x^\alpha)}{s^{4\alpha}} + \frac{\sinh_\alpha(x^\alpha)}{s^{5\alpha}} \right) \\
&= \frac{\sinh_\alpha(x^\alpha)}{s^\alpha} - \frac{\cosh_\alpha(x^\alpha)}{s^{2\alpha}} - \frac{\sinh_\alpha(x^\alpha)}{s^{3\alpha}} + \frac{\cosh_\alpha(x^\alpha)}{s^{4\alpha}} \\
&\quad + \frac{\sinh_\alpha(x^\alpha)}{s^{5\alpha}} - \frac{\cosh_\alpha(x^\alpha)}{s^{6\alpha}}.
\end{aligned} \tag{42}$$

Thus, we obtain the local fractional series as follows:

$$\begin{aligned}
&\tilde{L}_\alpha \{ \varphi_n(x, t) \} \\
&= \lim_{n \rightarrow \infty} \tilde{L}_\alpha \left\{ \sum_{i=0}^n (-1)^i \frac{\sinh_\alpha(x^\alpha)}{s^{(2i+1)\alpha}} - \sum_{i=0}^n (-1)^i \frac{\cosh_\alpha(x^\alpha)}{s^{2i\alpha}} \right\},
\end{aligned} \tag{43}$$

which yields

$$\begin{aligned}
&\varphi(x, t) \\
&= \lim_{n \rightarrow \infty} \left\{ \sum_{i=0}^n (-1)^i \frac{\sinh_\alpha(x^\alpha) t^{2i\alpha}}{\Gamma(1 + 2i\alpha)} \right. \\
&\quad \left. - \sum_{i=0}^n (-1)^i \frac{\cosh_\alpha(x^\alpha) t^{(2i+1)\alpha}}{\Gamma(1 + (2i+1)\alpha)} \right\} \\
&= \sinh_\alpha(x^\alpha) \cos_\alpha(t^\alpha) - \cosh_\alpha(x^\alpha) \sin_\alpha(t^\alpha),
\end{aligned} \tag{44}$$

and its graph is shown in Figure 2.

FIGURE 2: The graph of (1) with the parameters $\alpha = \ln 2 / \ln 3$ and $\mu = 1$.

5. Conclusions

In this paper, the boundary value problems for line partial differential equations arising in fractal vehicular traffic flow were solved by using the local fractional Laplace variational iteration method, which is the coupling method of local fractional variational iteration method (a generalization of variational iteration method based upon the local fractional calculus) and Laplace transform (a generalization of Fourier transform based upon the local fractional calculus). The nondifferentiable approximate solutions were obtained and their graphs were also shown.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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