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Research Article

Thermal Ground State and Nonthermal Probes

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The Euclidean formulation of SU(2) Yang-Mills thermodynamics admits periodic, (anti)self-dual solutions to the fundamental, classical equation of motion which possess one unit of topological charge: (anti)calorons. A spatial coarse graining over the central region in a pair of such localised field configurations with trivial holonomy generates an inert adjoint scalar field ϕ, effectively describing the pure quantum part of the thermal ground state in the induced quantum field theory. Here we show for the limit of zero holonomy how (anti)calorons associate a temperature independent electric permittivity and magnetic permeability to the thermal ground state of $SU(2)_{CMB}$, the Yang-Mills theory conjectured to underlie the fundamental description of thermal photon gases.

1. Introduction

Quantum Mechanics is a highly efficient framework to describe the subatomic world [1–3], including coherence phenomena that extend to macroscopic length and time scales [4–10]. The key quantity to describe deviations from classical behavior is Planck's quantum of action $\hbar = h/2\pi = 6.58 \times 10^{16}$ eV s which determines the fundamental interaction between charged matter and the electromagnetic field and thus also the shape of blackbody spectra by relating frequency ω and wave vector $\mathbf{k}$ to particle-like energy $E = \hbar\omega$ and momentum $\mathbf{p} = \hbar\mathbf{k}$ and by appeal to Bose-Einstein statistics. In Quantum Mechanics, $\hbar$ sets the strength of multiplicative noncommutativity for a pair of canonically conjugate variables such as position and momentum, implying the respective uncertainty relations.

Despite being generally accepted as a universal constant of nature and in spite of the fact that we are able to efficiently compute quantum mechanical amplitudes and quantum statistical averages for a vast variety of processes in particle collisions, atoms, and molecules, extended condensed-matter systems, and astrophysical objects to match experiment and observation very well, one should remain curious concerning the principle mechanism that causes the emergence of a universal quantum of action. In [11, 12] it was argued that the irreconcilability of classical Euclidean and Minkowskian time evolution as expressed by a time-periodic SU(2) (anti)self-dual gauge field configuration—a (anti)caloron—whose action $\hbar$ is associated with one unit of winding about a central spacetime point gives rise to indeterminism in the process it mediates. That each unit of action assigned to (anti)calorons of radius $\rho = |\phi|^{-1}$, which dominate the emergence of the thermal ground state, equals $\hbar$ follows from the value of the coupling e in the induced, effective, thermal quantum field theory [13–17] of the deconfining phase in SU(2) Yang-Mills thermodynamics. The coupling e, in turn, obeys an evolution in temperature (flat almost everywhere) which represents the validity of Legendre transformations in the effective ensemble where the thermal ground state coexists with massive (adjoint Higgs mechanism) and massless (intact U(1)) thermal fluctuations. The thermal ground state thus is a spatially homogeneous ensemble of quantum fluctuations carried by (anti)caloron centers. At the same time, as we will see, this state provides electric and magnetic dipole densities supporting the propagation of certain electromagnetic waves in an SU(2) Yang-Mills of scale $\Lambda \sim 10^{-4}$ eV, $SU(2)_{CMB}$ [18].

In the present work, we establish this link between quantised action, represented by ϕ, and classical wave propagation enabled by the vacuum parameters ϵ_0 and μ_0 in terms of the central and peripheral structure of a trivial-holonomy (anti)caloron, respectively. That is, by allowing a fictitious temperature T to represent the energy density of an electromagnetic wave (nonthermal, external probe) via the thermal ground state through which it propagates we ask what this implies for ϵ_0 and μ_0. As a result, both ϵ_0 and μ_0 neither depend on T nor, as we will argue, on a singled-out inertial frame provided that a certain condition on intensity and frequency of the wave is satisfied. This is a first step in reviving the concept of the luminiferous aether, albeit now with the goal of constructing a Poincaré invariant object.

This paper is organised as follows. In the next section we shorty discuss key features of the effective theory for the deconfining phase of SU(2) Yang-Mills thermodynamics. Section 3 contains a reminder to principles in interpreting a Euclidean field configuration in terms of Minkowskian observables. In a next step, general facts are reviewed on Euclidean, periodic, (anti)self-dual field configurations of charge modulus unity concerning the central locus of action, their holonomy, and their behaviour under semiclassical deformation. Finally, we review the anatomy of a zero-holonomy Harrington-Shepard (HS) caloron in detail, pointing out its staticity for spatial distances from the center that exceed the inverse of temperature, and discuss which static charge configuration it resembles depending on two distinct spatial distance regimes. In Section 4 we briefly review the postulate that an SU(2) Yang-Mills theory of scale $\Lambda \sim 10^{-4}$ eV, SU(2)$_{\rm CMB}$, describes thermal photon gases [18]. Subsequently, the large-distance regime in an HS (anti)caloron is considered in order to deduce an expression for ϵ_0 based on knowledge about the electric dipole moment provided by a (anti)caloron of radius radius $\rho = |\phi|^{-1}$, the size of the spatial coarse-graining volume $V_{\rm cg}$, and the fact that the energy density of the probe must match that of the thermal ground state. As a result, ϵ_0 and μ_0 turn out to be T-independent, the former representing an electric charge, large on the scale of the electron charge, of the fictitious constituent monopoles giving rise to the associated dipole density. Zooming in to smaller spatial distances to the center, the HS (anti)caloron exhibits isolated (anti)self-dual monopoles. For them to turn into dipoles shaking by the probe fields is required. We then show that the definitions of ϵ_0 and μ_0, which were successfully applied to the large-distance regime, become meaningless. Finally, our results are discussed. Section 5 summarises the paper and discusses how a universality of ϵ_0 and μ_0 for the entire, experimentally investigated electromagnetic spectrum could emerge.

2. Sketch of Deconfining SU(2) Yang-Mills Thermodynamics

For deconfining SU(2) Yang-Mills thermodynamics, a spatial coarse graining over the (anti)self-dual, that is, the non-propagating [19], topological sector with charge modulus $|Q| = 1$ can be performed; see [18] and references therein, to yield an inert adjoint scalar field ϕ. Its modulus $|\phi|$ sets the maximal possible resolution in the effective theory whose ground state energy density essentially is given as $\mathrm{tr}(\Lambda^6/\phi^2) = 4\pi\Lambda^3 T$ (Λ, a constant of integration of dimension mass) and whose propagating sector is, in a totally fixed, physical gauge (unitary-Coulomb) characterised by a massless mode (γ, unbroken U(1) subgroup of SU(2)) and two thermal quasi-particle modes of equal mass $m = 2e|\phi|$ ($V^{\pm}$, mass induced by adjoint Higgs mechanism) which propagate thermally, that is, on-shell only. Interactions within this propagating sector are mediated by isolated (anti)calorons whose action is argued to be $\hbar$ [11, 12]. Judged in terms of inclusive quantities such as radiative corrections to the one-loop pressure or the energy density of blackbody radiation, these interactions are feeble, and their expansion into 1-PI irreducible bubble diagrams is conjectured to terminate at a finite number of loops [18]. However, spectrally seen, the effects of $V^{\pm}$ interacting with γ lead to severe consequences at low frequencies and temperatures comparable to the critical temperature T_c where screened (anti)monopoles, released by (anti)caloron dissociation upon large-holonomy deformations [20], rapidly become massless and thus start to condense.

3. Caloron Structure

3.1. Euclidean Field Theory and Interpretable Quantities. Nontrivial solutions to an elliptic differential equation, such as the Euclidean Yang-Mills equation $D_\mu F_{\mu\nu} = 0$, no longer are solutions of the corresponding hyperbolic equation upon analytic continuation $x_4 \equiv \tau \to ix_0$ (Wick rotation). To endow meaning to quantities computed on classical field configurations on a 4D Euclidean spacetime in SU(2) Yang-Mills thermodynamics in terms of observables in a Minkowskian spacetime we thus must insist that these quantities are not affected by the Wick rotation. That is, to assign a real-world interpretation to a Euclidean quantity it needs to be (i) either stationary (not depend on τ) or (ii) associated with an instant in Euclidean spacetime because, by exploiting time translational invariance of the Yang-Mills action, this instant can be picked as $(\tau = 0, \mathbf{x})$ in Euclidean spacetime.

3.2. Review of General Facts. If not stated otherwise we work in supernatural units, $\hbar = c = k_B = 1$, where $\hbar$ is the reduced quantum of action, c is the speed of light in vacuum, and k_B is Boltzmann's constant. A trivial-holonomy caloron of topological charge unity on the cylinder $S_1 \times \mathbf{R}^3$, where S_1 is the circle of circumference $\beta \equiv 1/T$ (T temperature) describing the compactified Euclidean time dimension ($0 \le \tau \le \beta$), is constructed by an appropriate superposition of charge-one singular-gauge instanton prepotentials [21, 22] with the temporal coordinate of their instanton centers equidistantly stacked along the infinitely extended Euclidean time dimension [23] to enforce temporal periodicity, $A_\mu(\tau = 0, \mathbf{x}) = A_\mu(\tau = \beta, \mathbf{x})$. For gauge group SU(2) this Harrington-Shepard (HS) caloron is given as (antihermitian generators t_a $(a = 1, 2, 3)$ with $\mathrm{tr}\, t_a t_b = -(1/2)\delta_{ab}$):

$$A_\mu = \bar{\eta}^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r), \tag{1}$$

where $r \equiv |\mathbf{x}|$, $\bar{\eta}^a_{\mu\nu}$ denotes the antiself-dual 't Hooft symbol, $\bar{\eta}^a_{\mu\nu} = \epsilon^a_{\mu\nu} - \delta_{a\mu}\delta_{\nu 4} + \delta_{a\nu}\delta_{\mu 4}$, and

$$\Pi(\tau, r) = 1 + \frac{\pi\rho^2}{\beta r} \frac{\sinh(2\pi r/\beta)}{\cosh(2\pi r/\beta) - \cos(2\pi\tau/\beta)}. \tag{2}$$

Here ρ is the scale parameter of the singular-gauge instanton to seed the "mirror sum" within $S^1 \times \mathbf{R}^3$, leading to (2). The associated antiself-dual field configuration is obtained in replacing $\bar{\eta}^a_{\mu\nu}$ by $\eta^a_{\mu\nu}$ (self-dual 't Hooft symbol) in (1).

Configuration (1) is singular at $\tau = r = 0$. This point is the locus of the configuration's topological charge $Q = 1$ in the sense that the integral of the Chern-Simons current $K_\mu = (1/16\pi^2)\epsilon_{\mu\alpha\beta\gamma}(A^a_\alpha \partial_\beta A^a_\gamma + (1/3)\epsilon^{abc} A^a_\alpha A^b_\beta A^c_\gamma)$ over a three-sphere S^3_δ of radius δ, which is centered there, yields unity independently of $\delta \geq 0$. Self-duality implies that the action of the HS caloron is given as

$$S_C = \frac{8\pi^2}{g^2} \int_{S^3_\delta} d\Sigma_\mu K_\mu = \frac{8\pi^2}{g^2}, \tag{3}$$

where g is the coupling constant in Euclidean (classical) theory. Equation (3) holds in the limit $\delta \to 0$, meaning that S_C can be attributed to the singularity of the HS solution at $\tau = r = 0$ and thus has a Minkowskian interpretation; see Section 3.1. Based on [13–16] and on the fact that the thermal ground state emerges from $|Q| = 1$ caloron/anticalorons, whose scale parameter ρ essentially coincides with the inverse of maximal resolution, $|\phi|^{-1}$, in the effective theory for deconfining SU(2) Yang-Mills thermodynamics, it was argued in [11] (see also [12]) that S_C (as well as the action of an HS anticaloron S_A with $\rho \sim |\phi|^{-1}$) equals $\hbar$ if the effective theory is to be interpreted as a local quantum field theory.

The HS caloron is the trivial-holonomy limit of the self-dual Lee-Lu-Kraan-van-Baal (LLKvB) configuration with $Q = 1$ and total magnetic charge zero [24–26] which is constructed via the Nahm transformation of self-dual fields on the Euclidean four torus [27–29]. For nontrivial holonomy ($A_4(r \to \infty) = iut^3$ with $0 < u < 2\pi/\beta$) the LLKvB solution exhibits a pair of a magnetic monopole (m) and its antimonopole (a) with respect to the Abelian subgroup $U(1) \subset SU(2)$ left unbroken by $A_4(r \to \infty) \neq 0$. Their masses are $m_m = 4\pi u$ and $m_a = 4\pi(2\pi/\beta - u)$ such that, in the trivial-holonomy limits $u \to 0, 2\pi/\beta$, one of these magnetic constituents becomes massless and thus completely spatially delocalised. For nontrivial holonomy, where both monopole and antimonopole are of finite mass, localised, and separated by a spatial distance

$$s = \pi \frac{\rho^2}{\beta}, \tag{4}$$

they can be considered static by an exact cancellation of attraction, mediated by their U(1) magnetic fields, and repulsion due to the field A_4. As was shown in [20] by investigating the effective action of a LLKvB caloron (integrating out Gaussian fluctuations), this balance is distorted, leading to monopole-antimonopole attraction for

$$\begin{gathered} 0 < u \leq \frac{\pi}{\beta}\left(1 - \frac{1}{\sqrt{3}}\right) \\ \text{or } \frac{\pi}{\beta}\left(1 + \frac{1}{\sqrt{3}}\right) < u \leq \frac{2\pi}{\beta}, \end{gathered} \tag{5}$$

and to repulsion in the complementary range of (large) holonomy. Because there is no localised counterpart to a monopole or antimonopole in the trivial-holonomy limit, HS calorons must be considered stable under Gaussian fluctuations, in contrast to the case of nontrivial holonomy which is unstable. The latter statement is also mirrored by the fact that a nontrivial, static holonomy leads to zero quantum weight in the infinite-volume limit (which is realistic at high temperatures [18] where the radius of the spatial coarse-graining volume for a single caloron diverges as $|\phi|^{-1} = \sqrt{2\pi T/\Lambda^3}$, where Λ being the Yang-Mills scale). As a consequence, nontrivial holonomy can only occur transiently in configurations which do not saturate (anti)self-duality bounds to the Yang-Mills action. Again, this is equivalent to stating the instability of the LLKvB solution. It can be shown [18] that the small-holonomy case of monopole-antimonopole attraction by far dominates the situation of monopole-antimonopole repulsion when a caloron dissociates into its constituents.

The spatial coarse graining over (anti)self-dual calorons of charge modulus $|Q| = 1$, which do not propagate (due to (anti)self-duality their energy-momentum tensor vanishes identically [19]), yielding a highly accurate a priori estimate of the deconfining thermal ground state in terms of an inert, adjoint scalar field ϕ and a pure-gauge configuration a^{gs}_μ, is performed over isolated and stable HS solutions [18]. The coarse-grained field a^{gs}_μ represents a posteriori the effects of small holonomy changes due to (anti)caloron overlap and interaction.

3.3. Anatomy of a Relevant Harrington-Shepard Caloron. Let us now review [30] how the field strength of an HS caloron depends on the distance from its center at $\tau = r = 0$. For $|x| \ll \beta$ ($|x| \equiv \sqrt{x^2} \equiv \sqrt{x_\mu x_\mu}$, $x_4 \equiv \tau$) one has

$$\Pi(x) = \left(1 + \frac{\pi}{3}\frac{s}{\beta}\right) + \frac{\rho^2}{x^2} + O\left(\frac{x^2}{\beta^2}\right), \tag{6}$$

where s is defined in (4). From (6) and (1) one obtains with $|x| \ll \beta$ the following expression for $F_{\mu\nu} = (1/2)\epsilon_{\mu\nu\kappa\lambda}F_{\kappa\lambda} \equiv \widetilde{F}_{\mu\nu}$:

$$F^a_{\mu\nu} = -4\rho'^2 \frac{\bar{\eta}^a_{\alpha\beta}}{(x^2 + \rho'^2)^2} I_{\alpha\mu} I_{\beta\nu} + O\left(\frac{x^2}{\beta^4}\right), \tag{7}$$

where $I_{\alpha\mu} \equiv \delta_{\alpha\mu} - 2(x_\alpha x_\mu / x^2)$. At small four-dimensional distances from the caloron center the field strength thus behaves like the one of a singular-gauge instanton with a renormalised scale parameter $\rho'^2 = \rho^2/(1 + (\pi/3)(s/\beta))$. Therefore, the field strength of the HS solution exhibits a dependence on τ and as such has no Minkowskian interpretation; see Section 3.1.

For a Minkowskian spacetime one can infer, however, that the action of the configuration is attributable to winding of the caloron around the group manifold S_3 as induced by a spacetime point, the instanton center. This is because, in the sense of (3), an instant has no analytic continuation or Wick rotation. (The 4D action or topological-charge *density* of the caloron is regular at $\tau = r = 0$, does depend on

Euclidean spacetime in the vicinity of this point, and thus has no Minkowskian interpretation.)

For $r \gg \beta$ the self-dual electric and magnetic fields E_i^a and B_i^a are static and can be written as

$$E_i^a = B_i^a \sim -\frac{\left(\hat{x}^a \hat{x}_i / r^2\right) - (1/rs)\left(\delta_i^a - 3\hat{x}^a \hat{x}_i\right)}{(1 + r/s)^2}, \tag{8}$$

where $\hat{x}_i \equiv x_i / r$ and $\hat{x}^a \equiv x^a / r$. For $\beta \ll r \ll s$ (8) simplifies as

$$E_i^a = B_i^a \sim -\frac{\hat{x}^a \hat{x}_i}{r^2} \tag{9}$$

and thus describes a static non-Abelian monopole of unit electric and magnetic charges (dyon). For $r \gg s \gg \beta$ (8) reduces to

$$E_i^a = B_i^a \sim s\frac{\delta_i^a - 3\hat{x}^a \hat{x}_i}{r^3}. \tag{10}$$

This is the field strength of a static, self-dual non-Abelian dipole field, its dipole moment p_i^a given as

$$p_i^a = s\delta_i^a. \tag{11}$$

Interestingly, the same distance s, which sets the separation between the charge centers of an Abelian magnetic monopole and its antimonopole in a nontrivial-holonomy caloron, prescribes here for the case of trivial holonomy how small r needs to be in order to reduce the non-Abelian dipole of (10) to the non-Abelian monopole constituent; see (9). For the HS anticaloron one simply replaces $E_i^a = B_i^a$ by $E_i^a = -B_i^a$ in (8), (9), and (10).

Finally, let us remark that the condition $s \gg \beta$, which is required for (9) and (10) to be valid, is always satisfied for the caloron scale $\rho \sim |\phi|^{-1}$ which is relevant for the building of the thermal ground state in the deconfining phase of SU(2) Yang-Mills thermodynamics [18]. Namely, one has

$$\frac{s}{\beta} = \pi\left(\frac{\rho}{\beta}\right)^2 = \pi\left(\frac{\lambda^{3/2}}{2\pi}\right)^2 = \frac{\lambda^3}{4\pi} \geq 212.3, \tag{12}$$

where $\lambda \equiv 2\pi T/\Lambda \geq \lambda_c = 13.87$.

4. Thermal Ground State as Induced by a Probe

The postulate that thermal photon propagation should be described by an SU(2) rather than a U(1) gauge principle was put forward a decade ago and has undergone various levels of investigation ever since; see [18, 31]. As a result, the associated Yang-Mills scale $\Lambda \sim 1.0638 \times 10^{-4}$ eV is fixed by low-frequency observation of the Cosmic Microwave Background (CMB) [32] to correspond to the critical temperature for the deconfining-preconfining phase transition being the CMB's present baseline temperature $T_0 = 2.725$ K [18]. This prompted the name $SU(2)_{CMB}$. In the following we would like to investigate in what sense the vacuum parameters of classical electrodynamics, namely, the electric permittivity ϵ_0 and the magnetic permeability μ_0, can be reduced to the physics of the static, non-Abelian, and (anti)self-dual monopole and dipole configurations represented by HS (anti)calorons in the regimes $\beta \ll r \ll s$ and $r \gg s \gg \beta$, respectively; see Section 3.3. To do this, the concept of a thermal ground state together with information on how it is obtained [18] as well as the results of Section 3.3 [30] are invoked.

4.1. Preexisting Dipole Densities. Let us discuss the case $r \gg s$. In order not to affect spatial homogeneity on scales comparable to or smaller than s the electromagnetic field, which propagates through the deconfining thermal ground state in the absence of any explicit electric charges, is considered a plane wave of wavelength l much larger than s. Such a field effectively sees a density of self-dual dipoles; see (10). Because they are given by $p_i^a = s\delta_i^a$ their dipole moments align along the direction of the exciting electric or magnetic field both in space and in the SU(2) algebra su(2). Note that at this stage the definition of what is to be viewed as an Abelian direction in su(2) is a global gauge convention such that *all* spatial directions of the dipole moment p_i^a are a priori thinkable. That is, dynamical Abelian projection of the non-Abelian situation of (10) is owed to the Abelian and dipole aligning nature of the exciting, massless field [18]. Up to global gauge transformations, this field exists because of the adjoint Higgs mechanism invoked by the inert field ϕ.

Per spatial coarse-graining volume V_{cg} of radius $|\phi|^{-1} = \rho = \sqrt{\Lambda^3/2\pi T}$ with

$$V_{cg} = \frac{4}{3}\pi|\phi|^{-3}, \tag{13}$$

the center of a self-dual HS caloron and the center of an antiself-dual HS anticaloron [18] reside. Note the large hierarchy between s (the minimal spatial distance to the center of a (anti)caloron, which allows us to identify the static, (anti)self-dual dipole) and the radius of the sphere $|\phi|^{-1}$ defining V_{cg},

$$\frac{s}{|\phi|^{-1}} = \frac{1}{2}\lambda^{3/2} \geq 25.83\left(\frac{\lambda}{\lambda_c}\right)^{3/2}. \tag{14}$$

If the exciting field is electric then it sees *twice* the electric dipole p_i^a (cancellation of magnetic dipole between caloron and anticaloron), and if it is magnetic it sees *twice* the magnetic dipole p_i^a (cancellation of electric dipole between caloron and anticaloron, $\mathbf{E} = -\mathbf{B} \Leftrightarrow -\mathbf{E} = \mathbf{B}$). To be definite, let us discuss the electric case in detail, characterised by an exciting Abelian field $\mathbf{E}_e$. The modulus of the according dipole density $\mathbf{D}_e \parallel \mathbf{E}_e$ is given as

$$|\mathbf{D}_e| = \frac{2s}{V_{cg}} = \frac{3}{4\pi}\Lambda^2\lambda_c^{1/2}\left(\frac{\lambda}{\lambda_c}\right)^{1/2}. \tag{15}$$

In classical electromagnetism the relation between the fields $\mathbf{E}_e$ and $\mathbf{D}_e$ is

$$\mathbf{D}_e = \epsilon_0 \mathbf{E}_e, \tag{16}$$

where

$$\epsilon_0 = 5.52703 \times 10^7 \frac{\mathrm{Q}}{\mathrm{Vm}} \quad (17)$$

is the electric permittivity of the vacuum, and $Q = 1.602 \times 10^{-19}$ A s denotes the electron charge (unit of elementary charge), now both in SI units.

According to electromagnetism the energy density ρ_{EM} carried by an external electromagnetic wave with $|\mathbf{E}_e| = |\mathbf{B}_e|$ is

$$\rho_{\mathrm{EM}} = \frac{1}{2}\left(\epsilon_0 \mathbf{E}_e^2 + \frac{1}{\mu_0}\mathbf{B}_e^2\right) = \frac{1}{2}\left(\epsilon_0 + \frac{1}{\mu_0}\right)\mathbf{E}_e^2. \quad (18)$$

In natural units we have $\epsilon_0\mu_0 = 1/c^2 = 1$, and therefore (to assume $\epsilon_0\mu_0 = 1$ just represents a short cut, it would have come out automatically if we had treated the magnetic case explicitly) $\mu_0 = 1/\epsilon_0$. Thus

$$\rho_{\mathrm{EM}} = \epsilon_0 \mathbf{E}_e^2. \quad (19)$$

The $\mathbf{E}_e$-field dependence of ρ_{EM} is converted into a fictitious temperature dependence by demanding that the temperature of the thermal ground state of $\mathrm{SU(2)_{CMB}}$ adjusts itself so as to accommodate ρ_{EM},

$$\rho_{\mathrm{EM}} = 4\pi\Lambda^3 T \Longleftrightarrow$$
$$\left|\mathbf{E}_e\right| = \Lambda^2 \sqrt{2\frac{\lambda_c}{\epsilon_0}}\left(\frac{\lambda}{\lambda_c}\right)^{1/2}. \quad (20)$$

Equation (20) generalises the thermal situation of ground-state energy density of Section 3.2, where ground-state thermalisation is induced by a thermal ensemble of excitations, to the case where the thermal ensemble is missing but the probe field induces a fictitious temperature and energy density to the ground state. Combining (15), (16), and (20) and introducing the ratio ξ between the non-Abelian monopole charge Q' in the dipole and the (Abelian) electron charge (in natural units, the actual charge of the monopole constituents within the (anti)self-dual dipole is $1/g$, where g is the undetermined fundamental gauge coupling. This is absorbed into ξ) Q, we obtain

$$\epsilon_0\left[\mathrm{Q}\,(\mathrm{V\,m})^{-1}\right] = \frac{3}{\sqrt{32}\pi}\left(\frac{\Lambda\left[\mathrm{m}^{-1}\right]}{\Lambda\,[\mathrm{eV}]}\right)^{1/2}\xi\sqrt{\epsilon_0\left[\mathrm{Q}\,(\mathrm{V\,m})^{-1}\right]} \Longleftrightarrow \quad (21)$$
$$\epsilon_0\left[\mathrm{Q}\,(\mathrm{V\,m})^{-1}\right] = \frac{9}{32\pi^2}\frac{\Lambda\left[\mathrm{m}^{-1}\right]}{\Lambda\,[\mathrm{eV}]}\xi^2.$$

Notice that ϵ_0 does not exhibit any temperature dependence and thus no dependence on the field strength $\mathbf{E}_e$. It is a universal constant. In particular, ϵ_0 does *not* relate to the state of fictitious ground-state thermalisation which would associate with the rest frame of a local heat bath.

To produce the measured value for ϵ_0 as in (17) the ratio ξ in (21) is required to be

$$\xi \equiv \frac{Q'}{Q} = 19.56. \quad (22)$$

Thus, compared to the electron charge, the charge unit associated with a (anti)self-dual non-Abelian dipole, residing in the thermal ground state, is gigantic.

Discussing μ_0, we could have proceeded in complete analogy to the case of ϵ_0. (It would be μ_0^{-1} defining the ratio between the modulus of the magnetic dipole density and the magnetic flux density $|\mathbf{B}|$.) Here, however, the comparison between non-Abelian magnetic charge and an elementary, magnetic, and Abelian charge is not facilitated since the latter does not exist in electrodynamics.

Finally, let us see what the condition that the wavelength l of the electromagnetic disturbance considered in this section is much larger than s implies when invoking $\mathrm{SU(2)_{CMB}}$. One has

$$l \gg \frac{\lambda_c^2}{2\Lambda}\left(\frac{\lambda}{\lambda_c}\right)^2 = 1.1254\,\mathrm{m}\left(\frac{T}{2.725\,\mathrm{K}}\right)^2. \quad (23)$$

Setting $T = T_c = 2.725$ K in (23), we obtain a lower bound on the wavelength of $l_{\min} = 1.1254$ m.

4.2. Explicitly Induced Dipole Densities. Let us now discuss the case $\beta \ll |\phi|^{-1} \ll r \ll s$. To rely on the presence of the inert adjoint scalar field ϕ of the effective theory, r needs to be larger than the spatial coarse-graining scale $|\phi|^{-1} = (1/2\pi)\lambda_c^{3/2}(\lambda/\lambda_c)^{3/2}\beta \geq 8.22\beta$. Within the according regime $|\phi|^{-1} \leq r \ll s$ of spatial distances from the caloron center at ($\tau = 0$, $\mathbf{x} = 0$) an electromagnetic wave of wavelength l sees the self-dual field of a static, non-Abelian monopole of electric and magnetic charge as in (9) which is centered at $\mathbf{x} = 0$. A self-dual Abelian field strength $E_i = B_i$ of this monopole is obtained [33] as

$$E_i = B_i = \frac{\phi^a}{|\phi|}E_i^a = \frac{\phi^a}{|\phi|}B_i^a \quad (24)$$

with the field ϕ gauged from unitary gauge $\phi^a = 2|\phi|\delta^{a3}$ into "hedgehog" gauge $\phi^a = 2|\phi|\hat{x}^a$. The according gauge transformation is give in terms of the group element $\Omega \equiv \cos(1/2)\psi - i\hat{k}\cdot\sigma\sin(1/2)\psi$, where σ_i, ($i = 1, 2, 3$), are the Pauli matrices, $\hat{k} \equiv (\hat{e}_3 \times \hat{x})/\sin\theta$, $\hat{e}_3$ is the third vector of an orthonormal basis of space, $\theta \equiv \angle(\hat{e}_3, \hat{x})$, and $\psi = \theta$ for $0 \leq \theta \leq \pi - \epsilon$, which smoothly drops to zero at $\theta = \pi$, and the limit $\epsilon \rightarrow 0$ is understood [33]. For the monopole field E_i to be normalized to charge $-2Q'$ one (the factor two in front of the monopole charge Q' is due to a contribution to the monopole field strength of the anticaloron identical to that of the caloron) thus has

$$E_i = B_i = -\frac{2Q'}{4\pi\epsilon_0}\frac{\hat{x}_i}{r^2} = -\frac{2Q'\mu_0}{4\pi}\frac{\hat{x}_i}{r^2}. \quad (25)$$

The electric or magnetic poles of (25) should independently react by harmonic and linear acceleration to the presence of an external electric or magnetic field $\mathbf{E}_e$ or $\mathbf{B}_e$, respectively, forming a monochromatic electromagnetic wave of frequency $\omega = 2\pi/l$. At $\mathbf{x} = 0$ one has

$$\mathbf{E}_e = \mathbf{E}_0 \sin(\omega t) \tag{26}$$

and readily derives (as in Thomson scattering) that the induced dipole moment $\mathbf{p}$, say, for the electric case, is given as

$$\mathbf{p} = -\frac{\mathbf{E}_e \left(2Q'\right)^2}{m\omega^2}. \tag{27}$$

Interestingly, by virtue of (25) the squared charge of the pole, $(2Q')^2$, cancels out in $\mathbf{p}$ because its mass m carries an identical factor (only the electric (magnetic) monopole is linearly and harmonically accelerated by the external electric (magnetic) field $\mathbf{E}_e$ ($\mathbf{B}_e$) and hence m carries electric (magnetic) field energy only):

$$\begin{aligned} m &= \frac{1}{2}\epsilon_0 4\pi \int_{|\phi|^{-1}}^{\infty} dr r^2 E_i E_i = \frac{1}{8\pi\epsilon_0}\left(2Q'\right)^2 \int_{|\phi|^{-1}}^{\infty} \frac{dr}{r^2} \\ &= \frac{1}{8\pi\epsilon_0}\left(2Q'\right)^2 |\phi| \Longrightarrow \\ \mathbf{p} &= -\frac{8\pi\epsilon_0 \mathbf{E}_e}{|\phi|\,\omega^2}. \end{aligned} \tag{28}$$

Again, the volume V_{cg}, which underlies the dipole moment $\mathbf{p}$ by containing a caloron and an anticaloron center, is given by (13), and we have

$$\left|\mathbf{D}_e\right| = \frac{|\mathbf{p}|}{V_{\text{cg}}} = 6\epsilon_0 \frac{\left|\mathbf{E}_e\right| |\phi|^2}{\omega^2}, \tag{29}$$

and therefore

$$\epsilon_0 \equiv \frac{\left|\mathbf{D}_e\right|}{\left|\mathbf{E}_e\right|} = 6\epsilon_0 \frac{|\phi|^2}{\omega^2}. \tag{30}$$

In (30) also the vacuum permittivity ϵ_0 cancels out, and we are left with the condition

$$\begin{aligned} &\omega = \sqrt{6}\,|\phi| \Longleftrightarrow \\ &l = \sqrt{\frac{2}{3}}\pi\,|\phi|^{-1} = \sqrt{\frac{2}{3}}\pi\Lambda^{-1}\lambda_c^{1/2}\left(\frac{\lambda}{\lambda_c}\right)^{1/2}, \end{aligned} \tag{31}$$

where temperature T (or λ), again, is set by the local field strengths of the electromagnetic probe according to (18) and (20). Let us see whether the second of (31) is consistent with $|\phi|^{-1} \le r = l \ll s$. The former inequality is self-evident, and the latter follows from

$$\frac{s}{l} = \sqrt{\frac{3}{8}}\frac{\lambda_c^{3/2}}{\pi}\left(\frac{\lambda}{\lambda_c}\right)^{3/2} = 10.069\left(\frac{\lambda}{\lambda_c}\right)^{3/2}. \tag{32}$$

By setting $\lambda = \lambda_c$ we obtain from (31) a minimal wavelength

$$l_{\min} = \sqrt{\frac{2}{3}}\pi\Lambda^{-1}\lambda_c^{1/2} = 0.112\,\text{m}. \tag{33}$$

This wavelength is about a factor of ten smaller than the lowest possible value as expressed by (23).

4.3. Discussion. In Sections 4.1 and 4.2 an analysis was performed to clarify to what extent the thermal ground state of $\text{SU}(2)_{\text{CMB}}$ can be regarded as the luminiferous aether, supporting the propagation of an external electromagnetic wave (probe) of field strengths $|\mathbf{E}_e| = |\mathbf{B}_e|$ and wavelength l which, by itself, is not thermal.

Section 4.1 has focussed on wavelengths that are large compared to the distance $s = \pi|\phi|^{-2}/\beta$, very large compared to the resolution limit $|\phi|^{-1}$ of the effective theory for deconfining $\text{SU}(2)_{\text{CMB}}$ and even more so on the scale of inverse temperature β (see (12)) when (anti)calorons of $\text{SU}(2)_{\text{CMB}}$ manifest themselves as static (anti)self-dual dipoles whose dipole moment is set by a fictitious temperature representing the intensity of the probe via (20). And indeed, in this case vacuum permittivity ϵ_0 and permeability μ_0 turn out to be universal constants; see (21). When confronted with their experimental values the charges of the "constituent" non-Abelian monopoles in a dipole follow in units of electron charge; see (22).

Equations (23) and (20) indicate that an uncertainty-like relation between field $|\mathbf{E}_e|$ strength and wavelength l takes place as follows:

$$\left|\mathbf{E}_e\right|^4 l^{-1} \ll \frac{8\Lambda^9}{\epsilon_0^2}. \tag{34}$$

Therefore, the larger the probe intensity is, the longer its wavelength is required to be in order to be supported by thermal ground-state physics. Equation (34) is a condition with no reference to temperature and as such should be regarded valid independently of the constraint that, thermodynamically speaking, $\lambda \ge \lambda_c$.

Things are different for wavelengths that are large on the scale $|\phi|^{-1}$ but short on the scale $s = \pi|\phi|^{-2}/\beta$. This case is investigated in Section 4.2. Then a (anti)caloron can no longer be viewed as a static, (anti)self-dual dipole but rather is represented by a static, (anti)self-dual monopole. However, an attempt to consider dipole moments as induced dynamically by monopole shaking through the probe fields renders the definitions of vacuum parameters ϵ_0 and μ_0 meaningless; see (30). It does yield a fixation of the probe's wavelength l in terms of $|\phi|^{-1}$ though; see (31). While the former situation is not surprising because single magnetic charges violate the Bianchi identities for the electromagnetic field strength tensor $F_{\mu\nu}$ it is nontrivial that l turns out to self-consistently satisfy the constraint that $s \gg l > |\phi|^{-1}$. Note that the minimal wavelengths $l_{\min} = 1.1254\,\text{m}$ and $l_{\min} = \sqrt{2/3}\pi\Lambda^{-1}\lambda_c^{1/2} = 0.112\,\text{m}$ as obtained in Sections 4.1 and 4.2, respectively, are off by a factor of ten only.

5. Summary and Conclusions

We have addressed the question how the concept of a thermal ground state of $\text{SU}(2)_{\text{CMB}}$, which in a fully thermalised situation coexists with a spectrum of partially massive (adjoint Higgs mechanism) thermal excitations of the same temperature, can be employed to understand the propagation of a nonthermal probe (monochromatic electromagnetic wave) in vacuum, characterised by electric permittivity ϵ_0 and magnetic permeability μ_0. To do this, we have appealed to the fact that the thermal ground state emerges by a spatial coarse graining over (anti)self-dual fundamental Yang-Mills fields of topological charge modulus unity at finite temperature: Harrington-Shepard (anti)calorons of trivial holonomy. Note that this coarse graining does not require the consideration of thermal excitations. Therefore, it is suggestive that the concept of the thermal ground state can be extended to the description of a nonthermal situation with the parameter T acting as a period in compactified Euclidean spacetime $S_1 \times \mathbf{R}^3$ and no longer as a thermodynamical temperature.

Knowing how large the coarse-graining volume is, which contains one caloron and one anticaloron center, where the fundamental unit of action $\hbar$ is localised (Section 3.2), and by exploiting the structure of these field configurations spatially far away (Section 3.3) from their centers, we were able to deduce densities of electric and magnetic dipoles in Section 4.1. Dividing these dipole densities by the respective field strengths of the probe, self-consistently adjusted to the energy-density of the thermal ground state (small, transient (anti)caloron holonomies), yields definitions of ϵ_0 and μ_0. In the electric case a match with the experimental value predicts the charge of one of the monopoles, which constitutes the dipole, in terms of electron charge. The former charge turns out to be substantially larger than the latter.

As shown in Section 4.2 this way of reasoning, which is valid for large wavelengths ($l \gg s$) only, cannot be extended to smaller wavelengths l. Namely, in a region of spatial distances to the (anti)caloron center, where the configuration resembles (anti)self-dual, static monopoles, the definition of ϵ_0 and μ_0 in terms of dipole densities that are explicitly induced by the probes oscillating field strengths becomes meaningless. This is expected since the existence of resolved magnetic monopoles would violate the Bianchi identities for the field strength tensor $F_{\mu\nu}$ of electromagnetism.

We conclude that the thermal ground state of $SU(2)_{CMB}$ supports the propagation of a nonthermal probe purely in terms of Harrington-Shepard (anti)calorons (trivial holonomy) if an uncertainty-like relation between wavelength l and the square of the probe's intensity is obeyed; see (34).

To address the nonthermal propagation of shorter wavelength and/or higher intensities (see (34)), additional, mixing SU(2) gauge factors of hierarchically larger Yang-Mills scales have to be postulated; see discussion in [34]. At present, however, it is not clear how the effectiveness of the very successful Standard Model of particle physics in describing electroweak processes can be achieved in terms of such a more fundamental framework of pure Yang-Mills dynamics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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