Research Article

Structures and Low Dimensional Classifications of Hom-Poisson Superalgebras

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Hom-Poisson superalgebras can be considered as a deformation of Poisson superalgebras. We prove that Hom-Poisson superalgebras are closed under tensor products. Moreover, we show that Hom-Poisson superalgebras can be described using only the twisting map and one binary operation. Finally, all algebra endomorphisms on 2-dimensional complex Poisson superalgebras are computed, and their associated Hom-Poisson superalgebras are described explicitly.

1. Introduction

Poisson algebras are used in many fields in mathematics and physics. In mathematics, Poisson algebras play a fundamental role in Poisson geometry [1], quantum groups [2], and deformation of commutative associative algebras. In physics, Poisson algebras are a major part of deformation quantization, Hamiltonian mechanics [3], and topological field theories [4]. Poisson-like structures are also used in the study of vertex operator algebras [5]. Poisson superalgebras can be seen as a direct generalization of Poisson algebras. Remm show that a Poisson superalgebra can be described using only one binary operation in [6, 7].

Recently, a twisted generalization of noncommutative Poisson algebras, called Hom-noncommutative Poisson algebras, are studied in [8]. In a noncommutative Hom-Poisson algebra, there exists a twisted map, a Hom-Lie bracket and a Hom-associative product. The associativity, the Jacobi identity, and the Leibniz identity are considered as their Hom-type analogues in a noncommutative Hom-Poisson algebra. The purpose of this paper is to introduce and study a twisted generalization of Poisson superalgebras, called Hom-Poisson superalgebras.

This paper is organized as follows. In Section 2, we recall the construction of the Hom-Lie superalgebras. In Section 3, we give the definition of Hom-Poisson superalgebras. We show that starting with a Poisson superalgebra and an even Poisson superalgebra endomorphism, a Hom-Poisson superalgebra can be constructed. Moreover, we prove that Hom-Poisson superalgebras are closed under tensor products. In Section 4, we show that a Hom-Poisson superalgebra can be described using only the twisting map and one binary operation. Section 5 is devoted to classifying all the algebra endomorphisms α on all the 2-dimensional complex Poisson superalgebras and 2-dimensional Hom-Poisson superalgebras.

Throughout the paper $\mathbb{C}$ is the field of complex numbers. All algebras and vector spaces are considered over $\mathbb{C}$.

2. Hom-Lie and Hom-Associative Superalgebras

In this section, we first recall the notion of Hom-Lie superalgebras and then give some construction of the Hom-Lie superalgebras.

Definition 1 (see [9]). A Hom-associative superalgebra is a triple $(A, \circ, \alpha)$ consisting of a $\mathbb{Z}_2$ -graded vector space A , an even bilinear map $\circ : A \times A \rightarrow A$, and an even homomorphism of algebras $\alpha : A \rightarrow A$ satisfying

$$\alpha(x) \circ (y \circ z) = (x \circ y) \circ \alpha(z) \quad (1)$$

for all homogeneous elements $x, y, z \in A$.

Definition 2 (see [9]). A Hom-Lie superalgebra is a triple $(A, [\cdot, \cdot], \alpha)$ consisting of a $\mathbb{Z}_2$ -graded vector space A , an even bilinear map $[\cdot, \cdot] : A \times A \rightarrow A$ and an even homomorphism of algebras $\alpha : A \rightarrow A$ satisfying

$$[x, y] = -(-1)^{|x||y|} [y, x], \quad (2)$$

$$\begin{aligned} &(-1)^{|x||z|} [\alpha(x), [y, z]] + (-1)^{|z||y|} [\alpha(z), [x, y]] \\ &+ (-1)^{|x||y|} [\alpha(y), [z, x]] = 0 \end{aligned} \quad (3)$$

for all homogeneous elements $x, y, z \in A$.

Let $(V, [\cdot, \cdot], \alpha)$ and $(V', [\cdot, \cdot]', \alpha')$ be two Hom-Lie superalgebras. An even homomorphism $f : V \rightarrow V'$ is said to be a morphism of Hom-Lie superalgebras if

$$[f(x), f(y)]' = f([x, y]), \quad \forall x, y \in V, \quad (4)$$

$$f \circ \alpha = \alpha' \circ f. \quad (5)$$

Morphisms of Hom-associative superalgebras are defined similarly.

The following theorem provides a method to construct a Hom-Lie superalgebra by a Lie superalgebra and an even homomorphism of Lie superalgebras.

Proposition 3 (see [9]). Let $(V, [\cdot, \cdot])$ be a Lie superalgebra and let $\alpha : V \rightarrow V$ be an even endomorphism of Lie superalgebras. Then $(V, [\cdot, \cdot]_\alpha, \alpha)$ is a Hom-Lie superalgebra, where $[x, y]_\alpha = \alpha([x, y])$.

Moreover, suppose that $(V', [\cdot, \cdot]')$ is another Lie superalgebra and $\alpha' : V' \rightarrow V'$ is an even endomorphism of Lie superalgebras. If $f : V \rightarrow V'$ is a morphism of Lie superalgebras satisfying $f \circ \alpha = \alpha' \circ f$, then

$$f : (V, [\cdot, \cdot]_\alpha, \alpha) \longrightarrow (V', [\cdot, \cdot]'_{\alpha'}, \alpha') \quad (6)$$

is a morphism of Hom-Lie superalgebras.

Example 4 (see [9]). From the orthosymplectic Lie superalgebra $\text{osp}(1, 2) = V_0 \oplus V_1$, where V_0 is spanned by

$$\begin{aligned} H &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\ X &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ Y &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (7)$$

and V_1 is spanned by

$$\begin{aligned} F &= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ X &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (8)$$

The defining nonzero relations are

$$\begin{aligned} [H, X] &= 2X, \\ [H, Y] &= -2Y, \\ [X, Y] &= H, \\ [Y, G] &= F, \\ [X, F] &= G, \\ [H, F] &= -F, \\ [H, G] &= G, \\ [G, F] &= H, \\ [G, G] &= -2X, \\ [F, F] &= 2Y. \end{aligned} \quad (9)$$

Let $\lambda \in \mathbb{R}^*$ define a linear map $\alpha_\lambda : \text{osp}(1, 2) \rightarrow \text{osp}(1, 2)$ by

$$\begin{aligned} \alpha_\lambda(X) &= \lambda^2 X, \\ \alpha_\lambda(Y) &= \frac{1}{\lambda^2} (Y), \\ \alpha_\lambda(H) &= H, \\ \alpha_\lambda(F) &= \frac{1}{\lambda} (F), \\ \alpha_\lambda(G) &= \lambda G; \end{aligned} \quad (10)$$

then α_λ is an even Lie superalgebra automorphism; by Proposition 3, we obtain a family of Hom-Lie superalgebras $(\text{osp}(1, 2), [\cdot, \cdot]_\lambda, \alpha_\lambda)$. These Hom-Lie superalgebras are not Lie superalgebras for $\lambda \neq 1$.

3. Hom-Poisson Superalgebras

Definition 5. A Hom-Poisson superalgebra is a tuple $(A, \cdot, [\cdot, \cdot], \alpha)$ consisting of a $\mathbb{Z}_2$ -graded vector space V , two even bilinear maps $\cdot, [\cdot, \cdot] : V \times V \rightarrow V$, and an even homomorphism of algebras $\alpha : V \rightarrow V$, where $\alpha(x \cdot y) = \alpha(x) \cdot \alpha(y)$, $\alpha([x, y]) = [\alpha(x), \alpha(y)]$, satisfying the following axioms.

- (1) $(A, \cdot, \alpha)$ is a supercommutative Hom-associative superalgebra.
- (2) $(A, [\cdot, \cdot], \alpha)$ is a Hom-Lie superalgebra.
- (3) The Hom-Leibniz superidentity $[x \cdot y, \alpha(z)] = \alpha(x) \cdot [y, z] + (-1)^{|y||z|} [x, z] \cdot \alpha(y)$ holds for all homogeneous elements $x, y, z \in A$.

Definition 6. Let $(A, \cdot, [\cdot, \cdot], \alpha)$ and $(A', \cdot', [\cdot, \cdot]', \alpha')$ be two Hom-Poisson superalgebras. An even homomorphism $f : A \rightarrow A'$ is said to be a morphism of Hom-Poisson superalgebras if

$$\begin{aligned} f(x) \cdot' f(y) &= f(x \cdot y), \quad \forall x, y \in A, \\ [f(x), f(y)]' &= f([x, y]), \quad \forall x, y \in A, \\ f \cdot \alpha &= \alpha' \cdot f. \end{aligned} \quad (11)$$

Proposition 7. Let $(A, \cdot, \alpha)$ be a Hom-associative superalgebra and $[\cdot, \cdot] : A \times A \rightarrow A$ be a binary operation on A defined by

$$[x, y] = x \cdot y - (-1)^{|x||y|} y \cdot x, \quad \forall x, y \in A, \quad (12)$$

then $(A, \cdot, [\cdot, \cdot], \alpha)$ is a Hom-Poisson superalgebra with the same twisting map α .

Proof. It is straightforward. $\square$

From Proposition 7, there is the following construction of Hom-Poisson superalgebras by Poisson superalgebras and homomorphisms.

Theorem 8. Let $(A, \cdot, [\cdot, \cdot])$ be a Poisson superalgebra and let $\alpha : A \rightarrow A$ be an even endomorphism of Poisson superalgebras. Then $(A, \cdot_\alpha, [\cdot, \cdot]_\alpha, \alpha)$ is a Hom-Poisson superalgebra, where $x \cdot_\alpha y = \alpha(x \cdot y)$ and $[x, y]_\alpha = \alpha([x, y])$.

Moreover, suppose that $(A', \cdot', [\cdot, \cdot]', \alpha')$ is another Poisson superalgebra and $\alpha' : A' \rightarrow A'$ is an even endomorphism of Poisson superalgebras. If $f : A \rightarrow A'$ is a morphism of Poisson superalgebras satisfying $f \cdot \alpha = \alpha' \cdot f$, then

$$f : (A, \cdot_\alpha, [\cdot, \cdot]_\alpha, \alpha) \longrightarrow (A', \cdot_{\alpha'}, [\cdot, \cdot]_{\alpha'}, \alpha') \quad (13)$$

is a morphism of Hom-Poisson superalgebras.

Proof. By Proposition 3, $(A, [\cdot, \cdot]_\alpha, \alpha)$ is a Hom-Lie superalgebra.

We will show that $(A, \cdot_\alpha, \alpha)$ satisfies the axioms (1) and (3) of Definition 5. In fact

$$\begin{aligned} \alpha(x) \cdot_\alpha (y \cdot_\alpha z) &= \alpha(x) \cdot_\alpha \alpha(y \cdot z) = \alpha(\alpha(x) \cdot (y \cdot z)) \\ &= \alpha((x \cdot y) \cdot \alpha(z)) = \alpha(x \cdot y) \cdot_\alpha \alpha(z) \\ &= (x \cdot_\alpha y) \cdot_\alpha \alpha(z), \\ [x \cdot_\alpha y, \alpha(z)]_\alpha &= \alpha([x \cdot_\alpha y, \alpha(z)]) \\ &= \alpha([\alpha(x \cdot y), \alpha(z)]) \\ &= \alpha^2([x \cdot y, z]) \\ &= \alpha^2((-1)^{|y||z|} [x, z] \cdot y + x \cdot [y, z]) \quad (14) \\ &= (-1)^{|y||z|} \alpha(\alpha([x, z]) \cdot \alpha(y)) \\ &\quad + \alpha(\alpha(x) \cdot \alpha([y, z])) \\ &= (-1)^{|y||z|} \alpha([x, z]) \cdot_\alpha \alpha(y) \\ &\quad + \alpha(x) \cdot_\alpha \alpha([y, z]) \\ &= (-1)^{|y||z|} [x, z]_\alpha \cdot_\alpha \alpha(y) + \alpha(x) \\ &\quad \cdot_\alpha [y, z]_\alpha. \end{aligned}$$

Hence $(A, \cdot_\alpha, [\cdot, \cdot]_\alpha, \alpha)$ is a Hom-Poisson superalgebra. Then second assertion follows from

$$\begin{aligned} f([x, y]_\alpha) &= f(\alpha([x, y])) = f \cdot \alpha([x, y]) \\ &= \alpha' \cdot f([x, y]) = \alpha'([f(x), f(y)]) \\ &= [f(x), f(y)]_{\alpha'}; \\ f(x \cdot_\alpha y) &= f(\alpha(x \cdot y)) = f \cdot \alpha(x \cdot y) \\ &= \alpha' \cdot f(x \cdot y) = \alpha'(f(x) \cdot f(y)) \\ &= f(x) \cdot_{\alpha'} f(y). \end{aligned} \quad (15)$$

$\square$

This theorem provides a method to construct a Hom-Poisson superalgebra by a Poisson superalgebra and an even homomorphism of Poisson superalgebras.

Example 9. Let $A = A_{\bar{0}} \oplus A_{\bar{1}}$ be a 2-dimensional $\mathbb{Z}_2$ -graded vector space, where $A_{\bar{0}}$ is generated by e_1 and $A_{\bar{1}}$ is generated by e_2 and nonzero products are given by

$$\begin{aligned} e_1 \cdot e_1 &= e_1, \\ e_2 \cdot e_2 &= e_1, \\ e_1 \cdot e_2 &= e_2 \cdot e_1 = e_2, \\ [e_2, e_2] &= 2e_1; \end{aligned} \quad (16)$$

then $(A, \cdot, [\cdot, \cdot])$ is a Poisson superalgebra. For any $a \in \mathbb{C}$, we consider the homomorphism $\alpha : A \rightarrow A$ defined by $\alpha(e_1) = ae_1, \alpha(e_2) = ae_2$. By Theorem 8, for any $a \in \mathbb{C}$,

there is the corresponding Hom-Poisson superalgebra $A_a = (A, \cdot_\alpha, [\cdot, \cdot]_\alpha, \alpha)$ with the nonzero products

$$\begin{aligned} e_1 \cdot_\alpha e_1 &= ae_1, \\ e_2 \cdot_\alpha e_2 &= ae_1, \\ e_1 \cdot_\alpha e_\alpha &= ae_2, \\ [e_2, e_2]_\alpha &= 2ae_1. \end{aligned} \quad (17)$$

It is not a Poisson superalgebra when $a \neq 0, 1$.

Example 10. Let $A = A_{\bar{0}} \oplus A_{\bar{1}}$ be a 3-dimensional $\mathbb{Z}_2$ -graded vector space, where $A_{\bar{0}}$ is generated by e_1, e_2 and $A_{\bar{1}}$ is generated by e_3 and the nonzero products are given by

$$\begin{aligned} e_1 \cdot e_2 &= e_1, \\ e_2 \cdot e_2 &= e_2, \\ e_3 \cdot e_2 &= e_3, \\ [e_1, e_2] &= ae_1; \end{aligned} \quad (18)$$

then $(A, \cdot, [\cdot, \cdot])$ is a Poisson superalgebra. For any $a \in \mathbb{C}$, we consider the homomorphism $\alpha : A \rightarrow A$ defined by

$$\begin{aligned} \alpha(e_1) &= ae_1, \\ \alpha(e_2) &= e_1 + e_2. \end{aligned} \quad (19)$$

By Theorem 8, for any $a \in \mathbb{C}$, there is the corresponding Hom-Poisson superalgebra $A_\alpha = (A, \cdot_\alpha, [\cdot, \cdot]_\alpha, \alpha)$ with the nonzero products

$$\begin{aligned} e_1 \cdot_\alpha e_2 &= ae_1, \\ e_2 \cdot_\alpha e_2 &= e_1 + e_2, \\ [e_1, e_2]_\alpha &= ae_1. \end{aligned} \quad (20)$$

It is not a Poisson superalgebra when $a \neq 0, 1$.

We know Hom-Poisson algebras are closed under tensor products ([8] Theorem 2.9). Here we aim to consider it in detail in the superalgebra case.

Theorem 11. Let $(A_i, \cdot_i, [\cdot, \cdot]_i, \alpha_i)$ be Hom-Poisson superalgebras for $i = 1, 2$, and let $A = A_1 \otimes A_2$. Define the operations $\alpha : A \rightarrow A$ and $\cdot, [\cdot, \cdot] : A^{\otimes 2} \rightarrow A$ by

$$\begin{aligned} \alpha &= \alpha_1 \otimes \alpha_2, \\ (x_1 \otimes x_2) \cdot (y_1 \otimes y_2) &= (-1)^{|x_2||y_1|} (x_1 \cdot_1 y_1) \\ &\quad \otimes (x_2 \cdot_2 y_2), \\ [x_1 \otimes x_2, y_1 \otimes y_2] &= (-1)^{|x_2||y_1|} [x_1, y_1]_1 \otimes (x_2 \cdot_2 y_2) \\ &\quad + (-1)^{|x_2||y_1|} (x_1 \cdot_1 y_1) \\ &\quad \otimes [x_2, y_2]_2, \end{aligned} \quad (21)$$

for $x_i, y_i \in A_i$. Then $(A, \cdot, [\cdot, \cdot], \alpha)$ is a Hom-Poisson superalgebra.

Proof. The $(A, \cdot, \alpha)$ is a supercommutative Hom-associative superalgebra following from the supercommutativity and Hom-associativity of both $\cdot_i$. Also, the supercommutativity of the $\cdot_i$ and the antisymmetry of the $[\cdot, \cdot]_i$ imply the antisymmetry of $[\cdot, \cdot]$. It remains to prove the Hom-Jacobi superidentity and the Hom-Leibniz superidentity in A .

To simplify the typography, we abbreviate $\cdot_1, \cdot_2$, and $\cdot$ using juxtaposition and drop the subscripts in $[\cdot, \cdot]_i$ and α_i . Pick $x = x_1 \otimes x_2, y = y_1 \otimes y_2, z = z_1 \otimes z_2 \in A$. Then

$$\begin{aligned} &(-1)^{|x||z|} [[x, y], \alpha(z)] \\ &= (-1)^{s_1} [[x_1, y_1] \otimes (x_2 y_2), \alpha_1(z_1) \otimes \alpha_2(z_2)] \\ &\quad + (-1)^{s_1} [(x_1 y_1) \otimes [x_2, y_2], \alpha_1(z_1) \otimes \alpha_2(z_2)] \\ &= (-1)^{s_2} [[x_1, y_1], \alpha_1(z_1)] \otimes (x_2 y_2) \alpha_2(z_2) \\ &\quad + (-1)^{s_2} [x_1, y_1] \alpha_1(z_1) \otimes [x_2 y_2, \alpha_2(z_2)] \\ &\quad + (-1)^{s_2} [x_1 y_1, \alpha_1(z_1)] \otimes [x_2, y_2] \alpha_2(z_2) \\ &\quad + (-1)^{s_2} (x_1 y_1) \alpha_1(z_1) \otimes [[x_2, y_2], \alpha_2(z_2)], \end{aligned} \quad (22)$$

where $s_1 = |x_1||z_1| + |x_1||z_2| + |x_2||z_1| + |x_2||z_2| + |x_2||y_1|$, $s_2 = |x_1||z_1| + |x_1||z_2| + |x_2||z_2| + |x_2||y_1| + |y_2||z_1|$. Consider

$$\begin{aligned} &(-1)^{|x||y|} [[y, z], \alpha(x)] \\ &= (-1)^{s_3} [[y_1, z_1], \alpha_1(x_1)] \otimes (y_2 z_2) \alpha_2(x_2) \\ &\quad + (-1)^{s_3} [y_1, z_1] \alpha_1(x_1) \otimes [y_2 z_2, \alpha_2(x_2)] \\ &\quad + (-1)^{s_3} [y_1 z_1, \alpha_1(x_1)] \otimes [y_2, z_2] \alpha_2(x_2) \\ &\quad + (-1)^{s_3} (y_1 z_1) \alpha_1(x_1) \otimes [[y_2, z_2], \alpha_2(x_2)], \end{aligned} \quad (23)$$

where $s_3 = |y_1||x_1| + |y_1||x_2| + |y_2||x_2| + |y_2||z_1| + |z_2||x_1|$. Consider

$$\begin{aligned} &(-1)^{|y||z|} [[z, x], \alpha(y)] \\ &= (-1)^{s_4} [[z_1, x_1], \alpha_1(y_1)] \otimes (z_2 x_2) \alpha_2(y_2) \\ &\quad + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \otimes [z_2 x_2, \alpha_2(y_2)] \\ &\quad + (-1)^{s_4} [z_1 x_1, \alpha_1(y_1)] \otimes [z_2, x_2] \alpha_2(y_2) \\ &\quad + (-1)^{s_4} (z_1 x_1) \alpha_1(y_1) \otimes [[z_2, x_2], \alpha_2(y_2)], \end{aligned} \quad (24)$$

where $s_4 = |z_1||y_1| + |z_1||y_2| + |z_2||y_2| + |z_2||x_1| + |x_2||y_1|$. Using the Hom-Jacobi superidentity in A_1 and the supercommutativity and Hom-associativity in A_2 , we have

$$\begin{aligned} &(-1)^{s_2} [[x_1, y_1], \alpha_1(z_1)] \otimes (x_2 y_2) \alpha_2(z_2) \\ &\quad + (-1)^{s_3} [[y_1, z_1], \alpha_1(x_1)] \otimes (y_2 z_2) \alpha_2(x_2) \\ &\quad + (-1)^{s_4} [[z_1, x_1], \alpha_1(y_1)] \otimes (z_2 x_2) \alpha_2(y_2) \\ &= 0. \end{aligned} \quad (25)$$

Likewise, using the supercommutativity and Hom-associativity in A_1 and the Hom-Jacobi superidentity in A_2 , we obtain

$$\begin{aligned}
 & (-1)^{s_2} (x_1 y_1) \alpha_1(z_1) \otimes [[x_2, y_2], \alpha_2(z_2)] \\
 & + (-1)^{s_3} (y_1 z_1) \alpha_1(x_1) \otimes [[y_2, z_2], \alpha_2(x_2)] \\
 & + (-1)^{s_4} (z_1 x_1) \alpha_1(y_1) \otimes [[z_2, x_2], \alpha_2(y_2)] \\
 & = 0.
 \end{aligned} \tag{26}$$

Using the Hom-Leibniz superidentity in A_i , then we have

$$\begin{aligned}
 & (-1)^{s_2} [x_1, y_1] \alpha_1(z_1) \otimes [x_2 y_2, \alpha_2(z_2)] + (-1)^{s_2} \\
 & \cdot [x_1 y_1, \alpha_1(z_1)] \otimes [x_2, y_2] \alpha_2(z_2) + (-1)^{s_3} [y_1, z_1] \\
 & \cdot \alpha_1(x_1) \otimes [y_2 z_2, \alpha_2(x_2)] + (-1)^{s_3} [y_1 z_1, \alpha_1(x_1)] \\
 & \otimes [y_2, z_2] \alpha_2(x_2) + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes [z_2 x_2, \alpha_2(y_2)] + (-1)^{s_4} [z_1 x_1, \alpha_1(y_1)] \\
 & \otimes [z_2, x_2] \alpha_2(y_2) = (-1)^{s_2 + |y_2||z_2|} [x_1, y_1] \alpha_1(z_1) \\
 & \otimes [x_2, (z_2)] \alpha_2(y_2) + (-1)^{s_2} [x_1, y_1] \alpha_1(z_1) \\
 & \otimes \alpha_2(x_2) [y_2, z_2] + (-1)^{s_3 + |z_2||x_2|} [y_1, z_1] \alpha_1(x_1) \\
 & \otimes [y_2, x_2] \alpha_2(z_2) + (-1)^{s_3} [y_1, z_1] \alpha_1(x_1) \\
 & \otimes \alpha_2(y_2) [z_2, x_2] + (-1)^{s_3 + |z_1||x_1|} [y_1, x_1] \alpha_1(z_1) \\
 & \otimes [y_2, z_2] \alpha_2(x_2) + (-1)^{s_3} \alpha_1(y_1) [z_1, x_1] \\
 & \otimes [y_2, z_2] \alpha_2(x_2) + (-1)^{s_4 + |y_2||z_2|} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes [z_2 x_2, \alpha_2(y_2)] + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes [z_2, y_2] \alpha_2(x_2) + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes \alpha_2(z_2) [x_2, y_2] + (-1)^{s_4 + |x_1||y_1|} [z_1, y_1] \alpha_1(x_1) \\
 & \otimes [z_2, x_2] \alpha_2(y_2) + (-1)^{s_4} \alpha_1(z_1) [x_1, y_1] \\
 & \otimes [z_2, x_2] \alpha_2(y_2) = \{(-1)^{s_2 + |y_2||z_2|} [x_1, y_1] \alpha_1(z_1) \\
 & \otimes [x_2, z_2] \alpha_2(y_2) + (-1)^{s_4} \alpha_1(z_1) [x_1, y_1] \\
 & \otimes [z_2, x_2] \alpha_2(y_2)\} + \{(-1)^{s_2} [x_1, y_1] \alpha_1(z_1) \\
 & \otimes \alpha_2(x_2) [y_2, z_2] + (-1)^{s_3 + |z_1||x_1|} [y_1, x_1] \alpha_1(z_1) \\
 & \otimes [y_2, z_2] \alpha_2(x_2)\} + \{(-1)^{s_3} [y_1, z_1] \alpha_1(x_1) \\
 & \otimes \alpha_2(y_2) [z_2, x_2] + (-1)^{s_3 + |z_1||x_1|} [y_1, x_1] \alpha_1(z_1) \\
 & \otimes [y_2, z_2] \alpha_2(x_2)\} + \{(-1)^{s_3} [y_1, z_1] \alpha_1(x_1) \\
 & \otimes \alpha_2(y_2) [z_2, x_2] + (-1)^{s_4 + |x_1||y_1|} [z_1, y_1] \alpha_1(x_1) \\
 & \otimes [z_2, x_2] \alpha_2(y_2)\} + \{(-1)^{s_3} \alpha_1(y_1) [z_1, x_1]
 \end{aligned}$$

$$\begin{aligned}
 & \otimes [y_2, z_2] \alpha_2(x_2) + (-1)^{s_4 + |x_2||y_2|} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes [z_2, y_2] \alpha_2(x_2)\} + \{(-1)^{s_2 + |y_1||z_1|} [x_1, z_1] \alpha_1(y_1) \\
 & \otimes [x_2, y_2] \alpha_2(z_2) + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \\
 & \otimes \alpha_2(z_2) [x_2, y_2]\} = 0 + 0 + 0 + 0 + 0 + 0 = 0.
 \end{aligned} \tag{27}$$

This shows that $(A, [\cdot, \cdot], \alpha)$ satisfies the Hom-Jacobi superidentity:

$$\begin{aligned}
 & (-1)^{|x||z|} [[x, y], \alpha(z)] + (-1)^{|x||y|} [[y, z], \alpha(x)] \\
 & + (-1)^{|y||z|} [[z, x], \alpha(y)] = 0.
 \end{aligned} \tag{28}$$

Finally, we check the Hom-Leibniz superidentity in A . Using the Hom-associativity and the Hom-Leibniz superidentity in the A_i , we have

$$\begin{aligned}
 & [xy, \alpha(z)] = [(x_1 \otimes y_1)(y_1 \otimes y_2), \alpha_1(z_1) \otimes \alpha_2(z_2)] \\
 & = (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1|} [x_1 y_1, \alpha_1(z_1)] \otimes (x_2 y_2) \\
 & \cdot \alpha_2(z_2) + (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1|} (x_1 y_1) \alpha_1(z_1) \\
 & \otimes [x_2 y_2, \alpha_2(z_2)] = (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1| + |y_1||z_1|} \\
 & \cdot [x_1, z_1] \alpha_1(y_1) \otimes (x_2 y_2) \alpha_2(z_2) \\
 & + (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1|} \alpha_1(x_1) [y_1, z_1] \otimes (x_2 y_2) \\
 & \cdot \alpha_2(z_2) + (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1| + |y_2||z_2|} (x_1 y_1) \\
 & \cdot \alpha_1(z_1) \otimes [x_2, z_2] \alpha_2(y_2) \\
 & + (-1)^{|x_2||y_1| + |x_2||z_1| + |y_2||z_1|} (x_1 y_1) \alpha_1(z_1) \otimes \alpha_2(x_2) \\
 & \cdot [y_2, z_2], \\
 & (-1)^{|y||z|} [x, z] \alpha(y) + \alpha(x) [y, z] \\
 & = (-1)^{|y_1||z_1| + |y_1||z_2| + |y_2||z_1| + |y_2||z_2| + |x_2||z_1|} \\
 & \cdot ([x_1, z_1] \otimes (x_2 z_2)) (\alpha_1(y_1) \otimes \alpha_2(y_2)) \\
 & + (-1)^{|y_1||z_1| + |y_1||z_2| + |y_2||z_1| + |y_2||z_2| + |x_2||z_1|} \\
 & \cdot ((x_1 z_1) \otimes [x_2, z_2]) (\alpha_1(y_1) \otimes \alpha_2(y_2)) \\
 & + (-1)^{|y_2||z_1|} \alpha_1(x_1 \otimes \alpha_2(x_2)) ([y_1, z_1] \otimes (y_2 z_2)) \\
 & + (-1)^{|y_2||z_1|} \alpha_1(x_1 \otimes \alpha_2(x_2)) ((y_1 z_1) \otimes [y_2, z_2]) \\
 & = (-1)^{|y_1||z_1| + |y_2||z_1| + |x_2||z_1| + |x_2||y_1|} [x_1, z_1] \alpha_1(y_1) \\
 & \otimes (x_2 z_2) \alpha_2(y_2) + (-1)^{|y_2||z_1| + |y_2||z_2| + |x_2||z_1| + |x_2||y_1|} \\
 & \cdot (x_1 z_1) \alpha_1(y_1) \otimes [x_2, z_2] \alpha_2(y_2) \\
 & + (-1)^{|y_2||z_1| + |x_2||y_1| + |x_2||z_1|} \alpha_1(x_1) [y_1, z_1] \otimes \alpha_2(x_2)
 \end{aligned}$$

$$\begin{aligned} & \cdot (y_2 z_2) + (-1)^{|y_2||z_1|+|x_2||y_1|+|x_2||z_1|} \alpha_1(x_1)(y_1 z_1) \\ & \otimes \alpha_2(x_2)[y_2, z_2]. \end{aligned} \quad (29)$$

Therefore, we have

$$[xy, \alpha(z)] = (-1)^{|y||z|} [x, z] \alpha(y) + \alpha(x) [y, z]. \quad (30)$$

□

Setting $\alpha_i = Id_{A_i}$ in Theorem 11, we obtain the result about Poisson superalgebras.

Corollary 12. Let $(A_i, \cdot, [\cdot, \cdot], \alpha_i)$ be Poisson superalgebras for $i = 1, 2$, and let $A = A_1 \otimes A_2$. Define the operations $\cdot, [\cdot, \cdot] : A^{\otimes 2} \rightarrow A$ by

$$\begin{aligned} (x_1 \otimes x_2) \cdot (y_1 \otimes y_2) &= (-1)^{|x_2||y_1|} (x_1 \cdot y_1) \\ &\quad \otimes (x_2 \cdot y_2), \\ [x_1 \otimes x_2, y_1 \otimes y_2] &= (-1)^{|x_2||y_1|} [x_1, y_1]_1 \otimes (x_2 \cdot y_2) \\ &\quad + (-1)^{|x_2||y_1|} (x_1 \cdot y_1) \\ &\quad \otimes [x_2, y_2]_2, \end{aligned} \quad (31)$$

for $x_i, y_i \in A_i$. Then $(A, \cdot, [\cdot, \cdot])$ is a Poisson superalgebra.

4. Admissible Hom-Poisson Superalgebras

A Poisson algebra has two binary operations, the Lie bracket and the commutative associative product. It is shown in [10] that Poisson algebras can be described using only one binary operation via the polarization-depolarization process. Moreover, the result of Poisson algebras is extended to Hom-Poisson algebras in [8]. In other words, the paper shows that a Hom-Poisson algebra can be described using only the twisting map and one binary operation. The purpose of this section is to extend this alternative description of Poisson algebras to Hom-Poisson superalgebras.

Definition 13. An admissible Hom-Poisson superalgebra A is a Hom-superalgebra satisfying

$$\begin{aligned} & 3A(x, y, z) + (-1)^{|x||y|} (yx) \alpha(z) \\ & - (-1)^{|y||z|} (xz) \alpha(y) \\ & - (-1)^{|x||y|+|x||z|} (yz) \alpha(x) \\ & + (-1)^{|x||z|+|y||z|} (zx) \alpha(y) = 0, \end{aligned} \quad (32)$$

where $A(x, y, z) = (xy)\alpha(z) - \alpha(x)(yz)$, for any homogeneous elements $x, y, z \in A$, the identity (32) is called the Hom-Remm identity.

Remark 14. In particular, taking $\alpha = Id_A$, we find the notion of admissible Poisson superalgebra presented in [8].

Theorem 15. Let $(A, \cdot, [\cdot, \cdot], \alpha)$ be a double Hom-superalgebra. Then $(A, \cdot, [\cdot, \cdot], \alpha)$ is a Hom-Poisson superalgebra if and only if there exists on A a nonassociative product xy such that $(A, \cdot, \alpha)$ is an admissible Hom-Poisson superalgebra.

Proof. Assume that $(A, \cdot, [\cdot, \cdot], \alpha)$ is a Hom-Poisson superalgebra. Consider the multiplication

$$xy = x \cdot y + [x, y]. \quad (33)$$

We deduce that

$$x \cdot y = \frac{1}{2} (xy + (-1)^{|x||y|} yx). \quad (34)$$

Thus the associativity condition can be denoted by

$$\begin{aligned} v_1(x, y, z) &= A(x, y, z) \\ &\quad - (-1)^{|x||y|+|x||z|+|y||z|} A(z, y, x) \\ &\quad + (-1)^{|x||y|} (yx) \alpha(z) \\ &\quad - (-1)^{|y||z|} \alpha(x)(zy) \\ &\quad - (-1)^{|x||y|+|x||z|} (yz) \alpha(x) \\ &\quad + (-1)^{|x||z|+|y||z|} \alpha(z)(xy) = 0, \end{aligned} \quad (35)$$

where $A(x, y, z) = (xy)\alpha(z) - \alpha(x)(yz)$. Likewise, the Hom-Poisson bracket can be denoted by

$$[x, y] = \frac{1}{2} (xy - (-1)^{|x||y|} yx) \quad (36)$$

and the Hom-super Jacobi condition

$$\begin{aligned} v_2(x, y, z) &= (-1)^{|x||z|} A(x, y, z) \\ &\quad - (-1)^{|x||y|+|x||z|} A(y, x, z) \\ &\quad - (-1)^{|x||y|+|y||z|} A(z, y, x) \\ &\quad - (-1)^{|x||z|+|y||z|} A(x, z, y) \\ &\quad + (-1)^{|x||y|} A(y, z, x) \\ &\quad + (-1)^{|y||z|} A(z, x, y) = 0. \end{aligned} \quad (37)$$

The Hom-Leibniz superidentity can be denoted by

$$\begin{aligned} v_3(x, y, z) &= A(x, y, z) - (-1)^{|x||y|} A(y, x, z) \\ &\quad + (-1)^{|x||y|+|x||z|+|y||z|} A(z, y, x) \\ &\quad + (-1)^{|y||z|} A(x, z, y) \\ &\quad + (-1)^{|x||y|+|x||z|} A(y, z, x) \\ &\quad - (-1)^{|x||z|+|y||z|} A(z, x, y) = 0. \end{aligned} \quad (38)$$

Let us consider the vector

$$\begin{aligned} v(x, y, z) = & \frac{1}{3} \left\{ (-1)^{|x||y|} (yx) \alpha(z) \right. \\ & - (-1)^{|y||z|} (xz) \alpha(y) - (-1)^{|x||y|+|x||z|} (yz) \alpha(x) \\ & + (-1)^{|x||z|+|y||z|} (zx) \alpha(y) \left. \right\} + (xy) \alpha(z) - \alpha(x) \\ & \cdot (yz). \end{aligned} \quad (39)$$

Then

$$\begin{aligned} v(x, y, z) = & \frac{1}{6} \left\{ 2v_1(x, y, z) + (-1)^{|x||z|} v_2(x, y, z) \right. \\ & + v_3(x, y, z) + 2(-1)^{|x||z|+|y||z|} v_3(z, x, y) \left. \right\}. \end{aligned} \quad (40)$$

We deduce that the product xy satisfies

$$v(x, y, z) = 0 \quad (41)$$

for any homogeneous elements $x, y, z \in A$.

Conversely, assume that the produce of the nonassociative product A satisfies $v(x, y, z) = 0$ for any homogeneous elements $x, y, z \in A$. Let $v_1(x, y, z)$, $v_2(x, y, z)$, $v_3(x, y, z)$ be the elements of A defined in the first part, respectively, in relation to the Hom-associativity, Hom-super Jacobi, and Hom-super Leibniz relations. We have

$$\begin{aligned} v_1(x, y, z) = & v(x, y, z) \\ & - (-1)^{|x||y|+|x||z|+|y||z|} v(z, y, x) \\ & + (-1)^{|y||z|} v(x, z, y) \\ & - (-1)^{|x||z|+|y||z|} v(z, x, y), \\ v_2(x, y, z) = & (-1)^{|x||z|} v(x, y, z) \\ & - (-1)^{|x||y|+|x||z|} v(y, x, z) \\ & - (-1)^{|x||y|+|y||z|} v(z, y, x) \\ & - (-1)^{|x||z|+|y||z|} v(x, z, y) \\ & + (-1)^{|x||y|} v(y, z, x) \\ & + (-1)^{|y||z|} v(z, x, y), \\ v_3(x, y, z) = & v(x, y, z) - (-1)^{|x||y|} v(y, x, z) \\ & + (-1)^{|x||y|+|x||z|+|y||z|} v(z, y, x) \\ & + (-1)^{|y||z|} v(x, z, y) \\ & + (-1)^{|x||y|+|x||z|} v(y, z, x) \\ & - (-1)^{|x||z|+|y||z|} v(z, x, y). \end{aligned} \quad (42)$$

□

Taking $\alpha = Id_A$ in Theorem 15, we obtain the following result, which is Theorem 1 in [7].

Corollary 16. *Let $(A, \cdot, [\cdot, \cdot])$ be a double superalgebra. Then $(A, \cdot, [\cdot, \cdot])$ is a Poisson superalgebra if and only if there exists on A a nonassociative product xy satisfying*

$$\begin{aligned} 3A(x, y, z) + (-1)^{|x||y|} (yx) z - (-1)^{|y||z|} (xz) y \\ - (-1)^{|x||y|+|x||z|} (yz) x + (-1)^{|x||z|+|y||z|} (zx) y \\ = 0, \end{aligned} \quad (43)$$

where $A(x, y, z) = (xy)z - x(yz)$, for any homogeneous elements $x, y, z \in A$.

Definition 17. A Hom-nonassociative superalgebra $(A, \cdot, \alpha)$ is called Hom-superflexive if the multiplication xy satisfies

$$A(x, y, z) + (-1)^{|x||y|+|x||z|+|y||z|} A(z, y, x) = 0 \quad (44)$$

for any homogeneous elements $x, y, z \in A$, where $A_\alpha(x, y, z) = (xy)\alpha(z) - \alpha(x)(yz)$ is called a Hom-associator of the multiplication.

Proposition 18. *Let $(A, \cdot, [\cdot, \cdot], \alpha)$ be a Hom-Poisson superalgebra. Then the Hom-Remm product defining the Hom-Poisson superalgebra structure is Hom-superflexive.*

Proof. Let

$$\begin{aligned} B(x, y, z) \\ = 3 \left\{ A(x, y, z) + (-1)^{|x||y|+|x||z|+|y||z|} A(z, y, x) \right\}. \end{aligned} \quad (45)$$

We have

$$\begin{aligned} B(x, y, z) = & -(-1)^{|x||y|} (yx) \alpha(z) + (-1)^{|y||z|} (xz) \\ & \cdot \alpha(y) + (-1)^{|x||y|+|x||z|} (yz) \alpha(x) - (-1)^{|x||z|+|y||z|} \\ & \cdot (zx) \alpha(y) + (-1)^{|x||y|+|x||z|+|y||z|} \\ & \cdot \left\{ -(-1)^{|z||y|} (yz) \alpha(x) \right. \\ & + (-1)^{|y||x|} (zx) \alpha(y) + (-1)^{|z||y|+|z||x|} (yx) \alpha(y) \\ & - (-1)^{|z||x|+|y||z|} (xz) \alpha(y) \left. \right\} = \left\{ -(-1)^{|x||y|} \right. \\ & + (-1)^{|x||y|} \left. \right\} (yx) \alpha(z) + \left\{ (-1)^{|y||z|} - (-1)^{|y||z|} \right\} \\ & \cdot (xz) \alpha(y) + \left\{ (-1)^{|x||y|+|x||z|} - (-1)^{|x||y|+|x||z|} \right\} \\ & \cdot (yz) \alpha(x) + \left\{ -(-1)^{|x||z|+|y||z|} + (-1)^{|x||z|+|y||z|} \right\} \\ & \cdot (zx) \alpha(y) = 0. \end{aligned} \quad (46)$$

□

Taking $\alpha = Id_A$ in Proposition 18, we obtain the following result, which is Proposition 3 in [7].

Corollary 19. *Let $(A, \cdot, [\cdot, \cdot])$ be a Poisson superalgebra. Then the Remm product defining Poisson superalgebra structure is superflexive.*

Remark 20. The deformation cohomology of Hom-Poisson superalgebras can be computed with the Hom-Remm identity, which is similar to the method in [11]. This content is not primary in the paper, we do not have a detailed discussion here.

5. A Classification of 2-Dimensional Hom-Poisson Superalgebras

In this section, we only consider that $A_{\bar{1}}$ is nontrivial. A_2^i denotes one of the 2-dimensional admissible Poisson superalgebra types. α_2^i denotes one of the homomorphism types corresponding to A_2^i . $\bar{A}_2^i$ denotes one of the 2-dimensional admissible Hom-Poisson superalgebra types corresponding to A_2^i . In the following, the products equal to zero are omitted.

Lemma 21. *Let $(A, ())$ be an admissible Poisson superalgebra and let $\alpha : A \rightarrow A$ be an even Poisson superalgebra endomorphism. Then $(A, (), \alpha)$ is an admissible Hom-Poisson superalgebra, where $(xy)_\alpha = \alpha(xy)$.*

Proof. It is straightforward by Definition 13. $\square$

Lemma 22 (see [7]). *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with a basis $\{e_0, e_1\}$, where $e_0 \in A_{\bar{0}}, e_1 \in A_{\bar{1}}$. Then A is one of the following types:*

$$\begin{aligned} A_2^1 : e_0 e_0 &= ae_0, e_0 e_1 = ae_1, e_1 e_0 = ae_1, e_1 e_1 = de_0, \\ d &\neq 0. \\ A_2^2 : e_0 e_0 &= ae_0. \\ A_2^3 : e_0 e_0 &= ae_0, e_0 e_1 = ae_1, e_1 e_0 = ae_1, a \neq 0. \\ A_2^4 : e_0 e_1 &= be_1, e_1 e_0 = -be_1, b \neq 0. \end{aligned}$$

Proof. Let

$$\begin{aligned} e_0 e_0 &= ae_0, \\ e_0 e_1 &= be_1, \\ e_1 e_0 &= ce_1, \\ e_1 e_1 &= de_0. \end{aligned} \quad (47)$$

By Corollary 16, we have

$$\begin{aligned} 3(a-b)b + ab - 2bc + c^2 &= 0, \\ d(b-a) &= 0, \\ 3(a-c)c + ab - 2bc + c^2 &= 0, \\ d(3c-b-2a) &= 0, \\ d(b-c) &= 0. \end{aligned} \quad (48)$$

Now we consider the cases as follows.

Case 1. If $d \neq 0$, then $b = a = c$; hence we have A_2^1 .

Case 2. If $d = 0$, then $(b-c)(a-b-c) = 0$.

Subcase 2.1. If $b = c$, then $b(a-b) = 0$.

If $b = 0$, then we have A_2^2 .

If $b \neq 0$, then $a = b = c \neq 0$; hence we have A_2^3 .

Subcase 2.2. If $b \neq c$, then $b = -c \neq 0, a = 0$; hence we have A_2^4 . $\square$

Remark 23. Lemma 5.2 in [6] has been obtained; however, there is one minor inaccuracy on the product operation in that proof.

Lemma 24. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of type A_2^1 is as follows:*

$$\begin{aligned} \alpha_{2(1)}^1 : \begin{cases} \alpha(e_0) = k^2 e_0 \\ \alpha(e_1) = ke_1; \end{cases} \\ \alpha_{2(2)}^1 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases} \\ \alpha_{2(3)}^1 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = \pm e_1. \end{cases} \end{aligned} \quad (49)$$

Proof. Let

$$\begin{aligned} \alpha(e_0) &= a_{10} e_0, \\ \alpha(e_1) &= a_{11} e_1. \end{aligned} \quad (50)$$

From α is an even homomorphism, we obtain

$$\begin{aligned} \alpha(e_0 e_0) &= \alpha(e_0) \alpha(e_0), \\ \alpha(e_0 e_1) &= \alpha(e_0) \alpha(e_1), \\ \alpha(e_1 e_0) &= \alpha(e_1) \alpha(e_0), \\ \alpha(e_1 e_1) &= \alpha(e_1) \alpha(e_1). \end{aligned} \quad (51)$$

By Lemma 22 and (50) and (51), we obtain

$$\begin{aligned} aa_{10}(a_{10} - 1) &= 0, \\ aa_{11}(a_{10} - 1) &= 0, \\ a_{10} &= a_{11}^2. \end{aligned} \quad (52)$$

Case 1. If $a = 0$, then we have $\alpha_{2(1)}^1$.

Case 2. If $a \neq 0$, then we consider two cases as follows.

If $a_{10} = 0$, then $a_{11} = 0$; hence we have $\alpha_{2(2)}^1$.

If $a_{10} \neq 0$, then $a_{10} = 1, a_{11} = \pm 1$; hence we have $\alpha_{2(3)}^1$. $\square$

Corollary 25. *Let $(\bar{A}, (), \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\bar{A}$ with respect to α_2^1 is as follows:*

$$\begin{aligned} \bar{A}_{2(1)}^1 : (e_1 e_1)_\alpha &= dk^2 e_0, d \neq 0. \\ \bar{A}_{2(2)}^1 : (AA)_\alpha &= 0. \end{aligned}$$

$$\begin{aligned} \bar{A}_{2(3)}^1 : (e_0 e_0)_\alpha &= ae_0, (e_0 e_1)_\alpha = \pm ae_1, (e_1 e_0)_\alpha = \pm ae_1, \\ (e_1 e_1)_\alpha &= de_0, a \neq 0, d \neq 0. \end{aligned}$$

Proof. Apply Lemmas 21, 22, and 24. $\square$

Lemma 26. Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of type A_2^2 is as follows:

$$\begin{aligned} \alpha_{2(1)}^2 : & \begin{cases} \alpha(e_0) = k_0 e_0 \\ \alpha(e_1) = ke_1; \end{cases} \\ \alpha_{2(2)}^2 : & \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = ke_1; \end{cases} \\ \alpha_{2(3)}^2 : & \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1. \end{cases} \end{aligned} \quad (53)$$

Proof. By Lemma 22 and (20) and (21), we obtain $aa_{10}(a_{10} - 1) = 0$.

Case 1. Suppose that $a = 0$, we have $\alpha_{2(1)}^2$.

Case 2. Suppose that $a \neq 0$, we consider two cases as follows.

If $a_{10} = 0$, then we have $\alpha_{2(2)}^2$.

If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha_{2(3)}^2$. $\square$

Corollary 27. Let $(\bar{A}, (), \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\bar{A}$ with respect to α_2^2 is as follows:

$$\bar{A}_{2(1)}^2 : (AA)_\alpha = 0.$$

$$\bar{A}_{2(2)}^2 : (AA)_\alpha = 0.$$

$$\bar{A}_{2(3)}^2 : (e_0 e_0)_\alpha = ae_0, a \neq 0.$$

Proof. Apply Lemmas 21, 22, and 26. $\square$

Lemma 28. Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of type A_2^3 is as follows:

$$\begin{aligned} \alpha_{2(1)}^3 : & \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases} \\ \alpha_{2(2)}^3 : & \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1; \end{cases} \\ \alpha_{2(3)}^3 : & \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases} \end{aligned}$$

$$\alpha_{2(4)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(5)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0.$$

(54)

Proof. By Lemma 22 and (20) and (21), we obtain

$$\begin{aligned} a_{10}(a_{10} - 1) &= 0, \\ a_{11}(a_{10} - 1) &= 0. \end{aligned} \quad (55)$$

Case 1. If $a_{10} = 0$, then $a_{11} = 0$; hence we have $\alpha_{2(1)}^3$.

Case 2. If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha_{2(2)}^3$.

Case 3. If $a_{11} = 0$, then we consider two cases as follows.

If $a_{10} = 0$, we have $\alpha_{2(3)}^3$.

If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha_{2(4)}^3$.

Case 4. If $a_{11} \neq 0$, then $a_{10} = 1$; hence we have $\alpha_{2(5)}^3$. $\square$

Corollary 29. Let $(\bar{A}, (), \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\bar{A}$ with respect to α_2^3 is as follows:

$$\bar{A}_{2(1)}^3 : (AA)_\alpha = 0.$$

$$\bar{A}_{2(2)}^3 : (e_0 e_0)_\alpha = ae_0, (e_0 e_1)_\alpha = kae_1, (e_1 e_0)_\alpha = kae_1, a \neq 0.$$

$$\bar{A}_{2(3)}^3 : (AA)_\alpha = 0.$$

$$\bar{A}_{2(4)}^3 : (e_0 e_0)_\alpha = ae_0, a \neq 0.$$

$$\bar{A}_{2(5)}^3 : (e_0 e_0)_\alpha = ae_0, (e_0 e_1)_\alpha = kae_1, (e_1 e_0)_\alpha = kae_1, a \neq 0, k \neq 0.$$

Proof. Apply Lemmas 21, 22, and 28. $\square$

Lemma 30. Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of type A_2^4 is as follows:

$$\alpha_{2(1)}^4 : \begin{cases} \alpha(e_0) = ke_0 \\ \alpha(e_1) = 0; \end{cases} \quad (56)$$

$$\alpha_{2(2)}^4 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0.$$

Proof. By Lemma 22 and (20) and (21), we obtain $a_{11}(a_{10} - 1) = 0$.

Case 1. If $a_{10} = 0$, then we have $\alpha_{2(1)}^4$.

Case 2. If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha_{2(2)}^4$. $\square$

Corollary 31. Let $(\tilde{A}, (\cdot)_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\tilde{A}$ with respect to α_2^4 is as follows:

$$\tilde{A}_{2(1)}^4 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(2)}^4 : (e_0 e_1)_\alpha = k b e_1, (e_1 e_0)_\alpha = -k b e_1, b \neq 0, k \neq 0.$$

Proof. Apply Lemmas 21, 22, and 30. $\square$

Theorem 32. Let $(A, (\cdot))$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of A is as follows:

$$\alpha_{2(1)}^1 : \begin{cases} \alpha(e_0) = k^2 e_0 \\ \alpha(e_1) = k e_1; \end{cases}$$

$$\alpha_{2(2)}^1 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(3)}^1 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = \pm e_1; \end{cases}$$

$$\alpha_{2(1)}^2 : \begin{cases} \alpha(e_0) = k_0 e_0 \\ \alpha(e_1) = k e_1; \end{cases}$$

$$\alpha_{2(2)}^2 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = k e_1; \end{cases}$$

$$\alpha_{2(3)}^2 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = k e_1; \end{cases}$$

$$\alpha_{2(1)}^3 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(2)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = k e_1; \end{cases}$$

$$\alpha_{2(3)}^3 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(4)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(5)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = k e_1, \end{cases} \quad k \neq 0;$$

$$\alpha_{2(1)}^4 : \begin{cases} \alpha(e_0) = k e_0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\sigma_{2(2)}^4 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = k e_1, \end{cases} \quad k \neq 0.$$

(57)

Proof. Apply Lemmas 24–30. $\square$

Corollary 33. Let $(\tilde{A}, (\cdot)_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\tilde{A}$ with respect to α_2^i is as follows:

$$\tilde{A}_{2(1)}^1 : (e_1 e_1)_\alpha = d k^2 e_0, d \neq 0.$$

$$\tilde{A}_{2(2)}^1 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(3)}^1 : (e_0 e_0)_\alpha = a e_0, (e_0 e_1)_\alpha = \pm a e_1, (e_1 e_0)_\alpha = \pm a e_1, (e_1 e_1)_\alpha = d e_0, a \neq 0, d \neq 0.$$

$$\tilde{A}_{2(1)}^2 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(2)}^2 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(3)}^2 : (e_0 e_0)_\alpha = a e_0, a \neq 0.$$

$$\tilde{A}_{2(1)}^3 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(2)}^3 : (e_0 e_0)_\alpha = a e_0, (e_0 e_1)_\alpha = k a e_1, (e_1 e_0)_\alpha = k a e_1, a \neq 0.$$

$$\tilde{A}_{2(3)}^3 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(4)}^3 : (e_0 e_0)_\alpha = a e_0, a \neq 0.$$

$$\tilde{A}_{2(5)}^3 : (e_0 e_0)_\alpha = a e_0, (e_0 e_1)_\alpha = k a e_1, (e_1 e_0)_\alpha = k a e_1, a \neq 0, k \neq 0.$$

$$\tilde{A}_{2(1)}^4 : (AA)_\alpha = 0.$$

$$\tilde{A}_{2(2)}^4 : (e_0 e_1)_\sigma = k b e_1, (e_1 e_0)_\alpha = -k b e_1, b \neq 0, k \neq 0.$$

Proof. Apply Corollaries 25–31. $\square$

Remark 34. (1) Some nonisomorphic 2-dimensional admissible Poisson superalgebras have isomorphic admissible Hom-Poisson deformations. For example, the admissible Poisson superalgebras A_2^2 (with $k = 0$ in its algebra homomorphism $\alpha_{2(3)}^2$) and A_2^3 can be deformed into isomorphic admissible Hom-Poisson superalgebras $\tilde{A}_{2(3)}^2$ and $\tilde{A}_{2(4)}^3$. There are several other such pairs in the cases above.

(2) We give a classification of 2-dimensional admissible Hom-Poisson superalgebras in Corollary 33. Using Theorem 15, a classification of 2-dimensional Hom-Poisson superalgebras will be obviously obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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