Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2015, Article ID 408630, 16 pages
http://dx.doi.org/10.1155/2015/408630

Research Article

Explicit Wave Solutions and Qualitative Analysis of the (1 + 2)-Dimensional Nonlinear Schrödinger Equation with Dual-Power Law Nonlinearity

Qing Meng,[1] Bin He,[2] and Zhenyang Li[2]

[1]*Department of Physics, Honghe University, Mengzi, Yunnan 661100, China*
[2]*College of Mathematics, Honghe University, Mengzi, Yunnan 661100, China*

Correspondence should be addressed to Bin He; hebinmtc@163.com

Received 5 July 2015; Revised 30 August 2015; Accepted 31 August 2015

Academic Editor: Giorgio Kaniadakis

The (1 + 2)-dimensional nonlinear Schrödinger equation with dual-power law nonlinearity is studied using the factorization technique, bifurcation theory of dynamical system, and phase portraits analysis. From a dynamic point of view, the existence of smooth solitary wave, and kink and antikink waves is proved and all possible explicit parametric representations of these waves are presented.

1. Introduction

The nonlinear Schrödinger equation (NLSE) is studied in various areas of applied mathematics, theoretical physics, and engineering. In particular, it appears in the study of nonlinear optics, plasma physics, fluid dynamics, biochemistry, and many other areas. This equation is completely integrable by the inverse scattering transform [1]. There have been various forms of this equation that arise in the study of different areas. They have different forms of nonlinearity. Some previous works concentrated on the Kerr law nonlinearity media [2–5]. However, as one increases the intensity of the incident light power to produce shorter (femtosecond) pulses, non-Kerr nonlinearity effects become prominent, and the dynamics of pulses should be described by the nonlinear Schrödinger family of equations with higher order nonlinear terms [6]. Hence, it is very important that all higher order effects are considered in the propagation of femtosecond pulses [7].

The (1 + 2)-dimensional nonlinear Schrödinger equation with dual-power law nonlinearity is given in [8–13]:

$$iq_t + \frac{1}{2}\left(q_{xx} + q_{yy}\right) + \left(|q|^{2m} + k\,|q|^{4m}\right) q = 0, \qquad (1)$$

where $q(x, y, t)$ is a complex function that indicates the complex amplitude of the wave form, $i^2 = -1$, m is an arbitrary nonzero constant, and k is a constant which indicates the saturation of the nonlinear refractive index. Biswas [8] obtained the 1-soliton solution and calculated a few conserved quantities. With the aid of symbolic computation, Zhang and Si [9] obtained soliton solutions, combined soliton solutions, triangular periodic solutions, and rational function solutions. Some soliton solutions are obtained in [10] using the G/G'-expansion method. For $m = 1$, the dark 1-soliton solution is obtained by the ansatz method, the invariance, conservation laws, and double reductions in [11]. Bulut et al. [12] found some soliton solutions, rational and elliptic function solutions using the extended trial equation method. The bifurcation is considered in [13] and two implicit solutions are obtained (see (16) and (18) of [13]).

In this paper, using the factorization technique [14], bifurcation theory of dynamical system, and phase portraits analysis [15–17], we present some explicit expressions of the smooth solitary, kink, and antikink waves of (1).

Throughout this paper, we always suppose that $m \neq -1$ and $-1/2$, $k \neq 0$.

2. Preliminaries

Using transformation

$$q(x,y,t) = e^{i\eta}u(\xi),\quad \eta = x + c_1 y + c_2 t,\quad \xi = x + \omega_1 y + \omega_2 t, \tag{2}$$

where c_1, c_2, ω_1, and ω_2 are constants, (1) can be rewritten as

$$\left[\frac{1}{2}\left(1+\omega_1^2\right)u'' + ku^{4m+1} + u^{2m+1} - \frac{1}{2}\left(1+c_1^2+2c_2\right)u + i\left(1+c_1\omega_1+\omega_2\right)u'\right]e^{i\eta} = 0, \tag{3}$$

where "$\prime$" is the derivative with respect to ξ.

From (3), we have

$$\left(1+c_1\omega_1+\omega_2\right)u' = 0, \tag{4}$$

$$\frac{1}{2}\left(1+\omega_1^2\right)u'' + ku^{4m+1} + u^{2m+1} - \frac{1}{2}\left(1+c_1^2+2c_2\right)u = 0. \tag{5}$$

Obviously, if putting $\omega_2 = -(1+c_1\omega_1)$, then solution of (5) must be solution of (1).

Taking transformation

$$u = \phi^{1/2m}. \tag{6}$$

Equation (5) is converted into the following equation:

$$8km^2\phi^4 + 8m^2\phi^3 - 4\left(1+c_1^2+2c_2\right)m^2\phi^2 + 2\left(1+\omega_1^2\right)m\phi\phi'' + \left(1+\omega_1^2\right)(1-2m)\left(\phi'\right)^2 = 0. \tag{7}$$

Differentiating (7) once with respect to ξ, we have

$$2\left(1+\omega_1^2\right)m\phi\phi''' + 2\left(1+\omega_1^2\right)(1-m)\phi'\phi'' + \left(32km^2\phi^3 + 24m^2\phi^2 - 8\left(1+c_1^2+2c_2\right)m^2\phi\right)\phi' = 0. \tag{8}$$

Using Proposition 2 of [14], we know that (8) has the factorization

$$\left(f(\phi)\partial_\xi - \phi_1(\phi)\phi' - \phi_2(\phi)\right)\cdot\left(\partial_{\xi\xi} - \phi_3(\phi)\partial_\xi - \phi_4(\phi)\right)\phi = 0 \tag{9}$$

if and only if the following expressions are satisfied:

$$\begin{aligned} &g\left(\phi,\phi'\right) = -f(\phi)\phi_3(\phi) - \phi_1(\phi)\phi' - \phi_2(\phi),\\ &\phi_1(\phi)\phi_3(\phi) - f(\phi)\frac{d(\phi_3(\phi))}{d\phi} = 0,\\ &k(\phi) = \phi_2(\phi)\phi_4(\phi)\phi,\\ &h(\phi) = \phi_2(\phi)\phi_3(\phi) - f(\phi)\frac{d(\phi_4(\phi))}{d\phi}\phi - f(\phi)\phi_4(\phi) + \phi_1(\phi)\phi_4(\phi)\phi, \end{aligned} \tag{10}$$

where $f(\phi) = 2(1+\omega_1^2)m\phi$, $g(\phi,\phi') = 2(1+\omega_1^2)(1-m)\phi'$, and $h(\phi) = 32km^2\phi^3 + 24m^2\phi^2 - 8(1+c_1^2+2c_2)m^2\phi$, $k(\phi) = 0$.

Solving (10) and setting the integral constant being zero, we have

$$\begin{aligned} &\phi_1(\phi) = 2\left(1+\omega_1^2\right)(m-1),\\ &\phi_2(\phi) = \phi_3(\phi) = 0,\\ &\phi_4(\phi) = -\frac{4m^2\left(4k(m+1)\phi^2 + 3(2m+1)\phi - \left(1+c_1^2+2c_2\right)(m+1)(2m+1)\right)}{\left(1+\omega_1^2\right)(m+1)(2m+1)}. \end{aligned} \tag{11}$$

Thus, (8) has the following factorization:

$$\left(2\left(1+\omega_1^2\right)m\phi\partial_\xi - 2\left(1+\omega_1^2\right)(m-1)\phi'\right)\cdot\left(\partial_{\xi\xi} + \frac{4m^2\left(A\phi^2+B\phi+C\right)}{\left(1+\omega_1^2\right)(m+1)(2m+1)}\right)\phi = 0, \tag{12}$$

where $A = 4k(m+1)$, $B = 3(2m+1)$, and $C = -(1+c_1^2+2c_2)(m+1)(2m+1)$.

Some special solutions of (7) could be obtained by solving the following second-order differential equation:

$$\phi'' = \rho\left(\phi^2 + E\phi + F\right)\phi, \tag{13}$$

where $\rho = -16km^2/(1+\omega_1^2)(2m+1)$, $E = 3(2m+1)/4k(m+1)$, and $F = -(1+c_1^2+2c_2)(2m+1)/4k$.

Letting $y = d\phi/d\xi$, we get the following planar dynamical system:

$$\begin{aligned} \frac{d\phi}{d\xi} &= y, \\ \frac{dy}{d\xi} &= \rho\left(\phi^2 + E\phi + F\right)\phi \end{aligned} \tag{14}$$

with the first integral

$$H(\phi, y) = y^2 - \frac{1}{6}\rho\left(3\phi^2 + 4E\phi + 6F\right)\phi^2 = h. \tag{15}$$

For a fixed h, the level curve $H(\phi, y) = h$ defined by (15) determines a set of invariant curves of (14) which contain different branches. As h is varied, it defines different families of orbits of (14) with different dynamical behaviors.

The remainder of this paper is organized as follows. In Section 3, we consider bifurcation sets and phase portraits of (14). Some explicit smooth solitary, kink, and antikink wave solutions of (1) are presented in Section 4. A short conclusion will be provided in Section 5.

3. Bifurcation Sets and Phase Portraits of (14)

We will use the following notations:

$$\begin{aligned} \Delta &= E^2 - 4F, \\ \phi_{1,2} &= \frac{1}{2}\left(-E \pm \sqrt{\Delta}\right). \end{aligned} \tag{16}$$

Obviously, (14) has only one equilibrium point at $(0,0)$ in ϕ-axis when $\Delta < 0$. Equation (14) has two equilibrium points at $(-(1/2)E, 0)$ and $(0,0)$ in ϕ-axis when $\Delta = 0$. Equation (14) has three equilibrium points at $(\phi_{1,2}, 0)$ and $(0,0)$ in ϕ-axis when $\Delta > 0$. From (15), the following conclusions hold:

$$\begin{aligned} h_1 &= H(\phi_1, 0) \\ &= \frac{1}{48}\rho\left(E - \sqrt{\Delta}\right)^2\left(E\left(E - \sqrt{\Delta}\right) - 6F\right), \\ h_2 &= H(\phi_2, 0) \\ &= \frac{1}{48}\rho\left(E + \sqrt{\Delta}\right)^2\left(E\left(E + \sqrt{\Delta}\right) - 6F\right). \end{aligned} \tag{17}$$

Let $M(\phi_e, 0)$ be the coefficient matrix of the linearized system of (14) at equilibrium point $(\phi_e, 0)$. We have

$$J(\phi_e, 0) = \det(M(\phi_e, 0)) = -\rho\left(3\phi_e^2 + 2E\phi_e + F\right). \tag{18}$$

For an equilibrium point $(\phi_e, 0)$ of (14), we know that $(\phi_e, 0)$ is a saddle point if $J(\phi_e, 0) < 0$, a center point if $J(\phi_e, 0) > 0$, and a cusp if $J(\phi_e, 0) = 0$ and the Poincaré index of $(\phi_e, 0)$ is zero.

Using the properties of equilibrium points and the bifurcation method, we can obtain four bifurcation curves of (14) as follows:

$$\begin{aligned} C_1 &: E = 0, \\ C_2 &: F = 0, \\ C_3 &: F = \frac{2}{9}E^2, \\ C_4 &: F = \frac{1}{4}E^2. \end{aligned} \tag{19}$$

The bifurcation curves C_1, C_2, C_3, and C_4 divide the (E, F)-parameter plane into 14 subregions. The bifurcation sets and phase portraits of (14) are drawn in Figures 1-2.

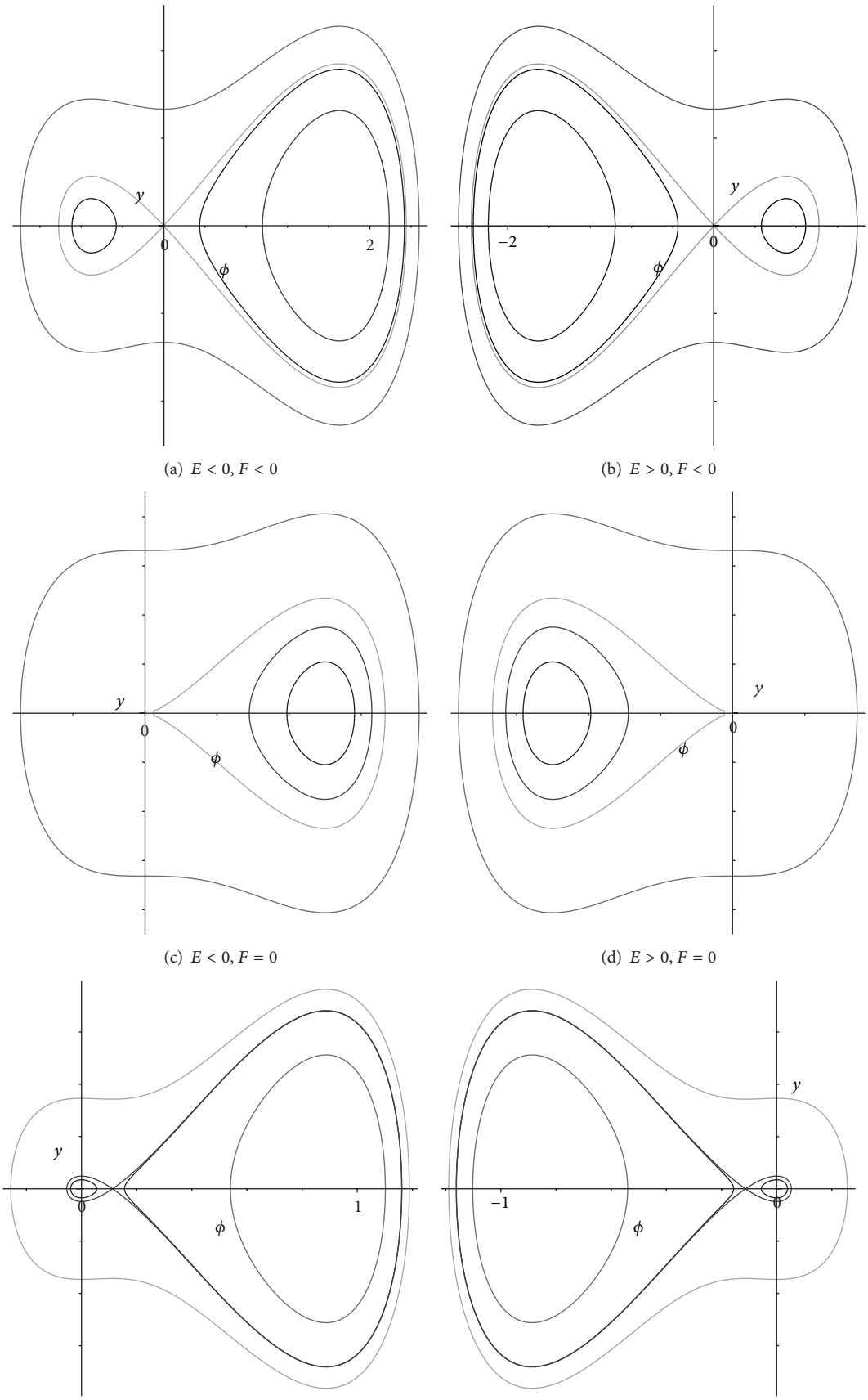
4. Explicit Smooth Solitary, Kink and Anti-Kink Wave Solutions of (1)

In this section we present all possible explicit smooth solitary, kink, and antikink wave solutions of (1) in the following propositions.

Proposition 1. *When $\rho < 0$, $E \neq 0$, and $F < 0$, (1) has two smooth solitary wave solutions as follows:*

$$q(x, y, t) = \frac{\left[\left(2\phi_m\phi_M\right)^{1/2m}\right]e^{i(x+c_1y+c_2t)}}{\left[\left(\phi_m + \phi_M\right) + \left(\phi_M - \phi_m\right)\cosh\left(\omega\left(x + \omega_1 y - \left(1 + c_1\omega_1\right)t\right)\right)\right]^{1/2m}}, \tag{20}$$

$$q(x, y, t) = \frac{\left[\left(2\phi_m\phi_M\right)^{1/2m}\right]e^{i(x+c_1y+c_2t)}}{\left[\left(\phi_m + \phi_M\right) - \left(\phi_M - \phi_m\right)\cosh\left(\omega\left(x + \omega_1 y - \left(1 + c_1\omega_1\right)t\right)\right)\right]^{1/2m}}, \tag{21}$$



(a) $E < 0, F < 0$

(b) $E > 0, F < 0$

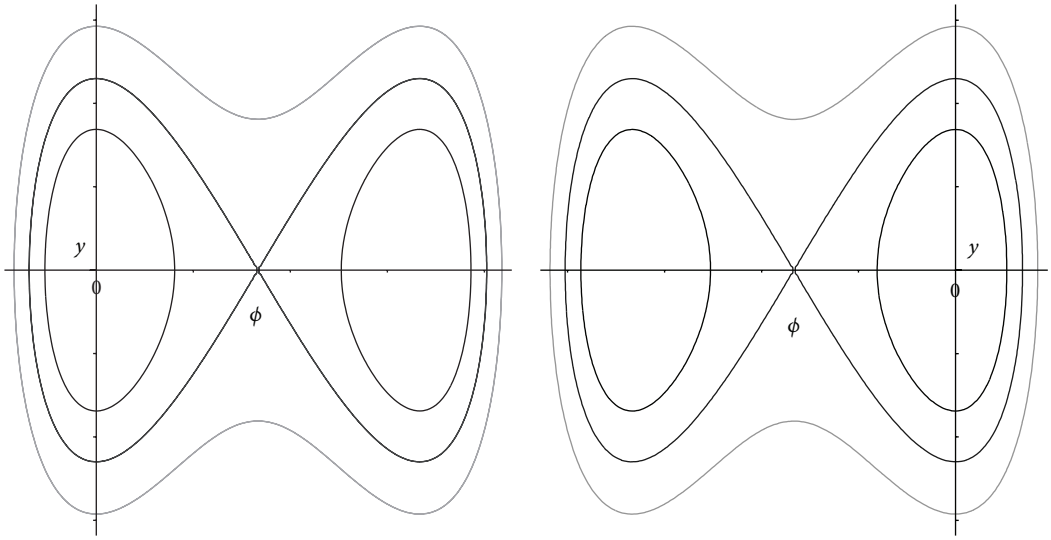
(c) $E < 0, F = 0$

(d) $E > 0, F = 0$

(e) $E < 0, 0 < F < (2/9)E^2$

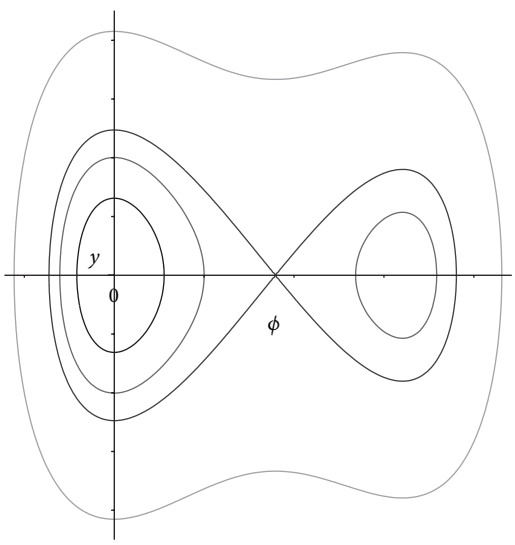
(f) $E > 0, 0 < F < (2/9)E^2$

Figure 1: Continued.

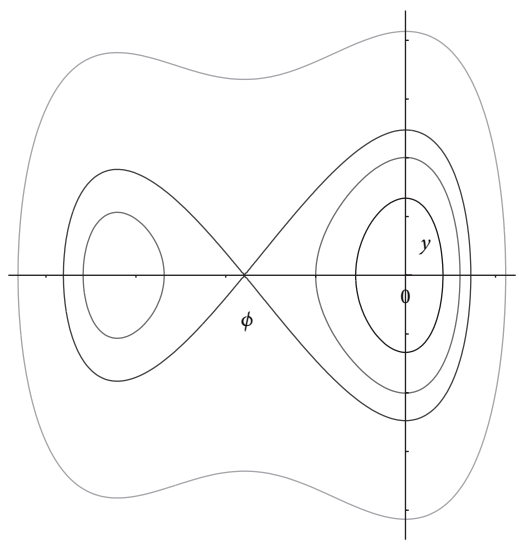


(g) $E < 0, F = (2/9)E^2$

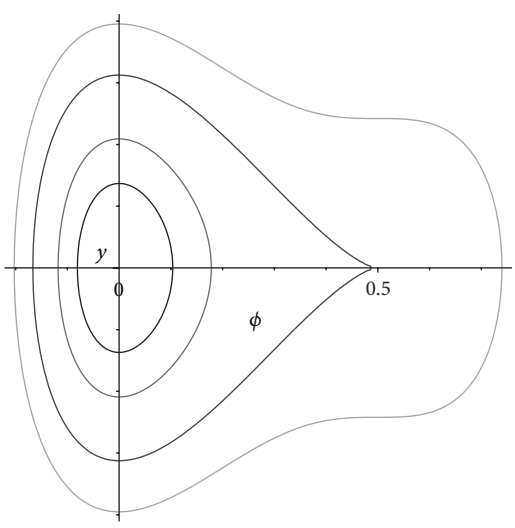
(h) $E > 0, F = (2/9)E^2$



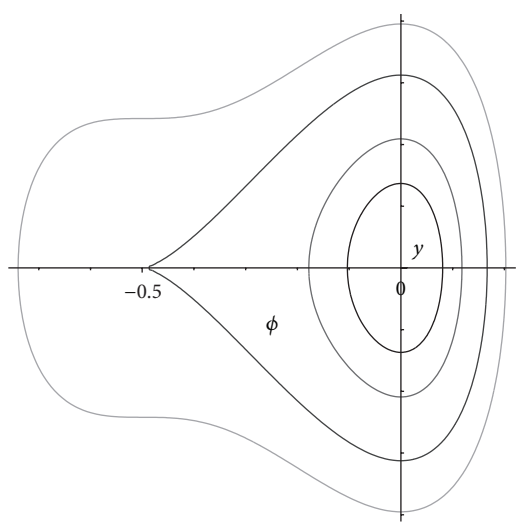
(i) $E < 0, (2/9)E^2 < F < (1/4)E^2$



(j) $E > 0, (2/9)E^2 < F < (1/4)E^2$

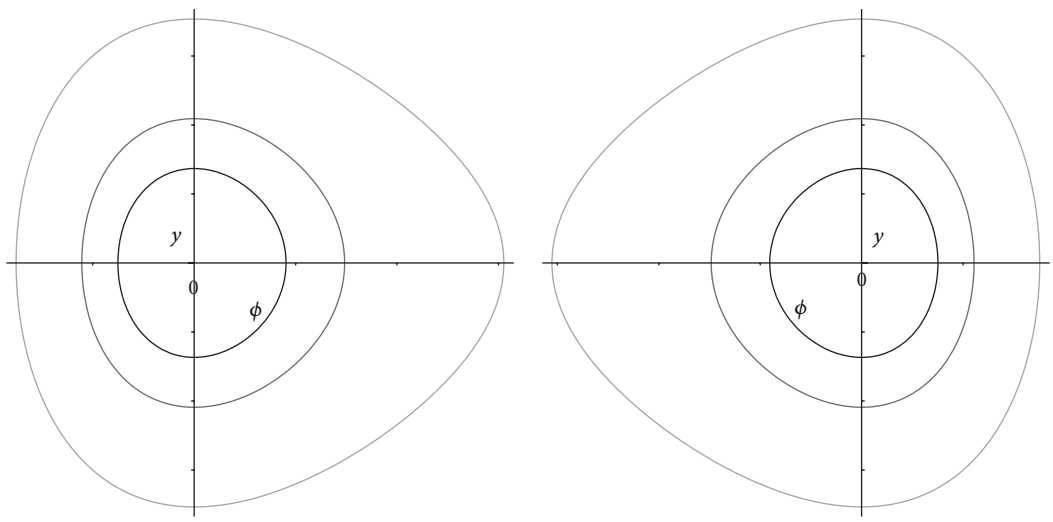


(k) $E < 0, F = (1/4)E^2$



(l) $E > 0, F = (1/4)E^2$

Figure 1: Continued.



(m) $E < 0, F > (1/4)E^2$ (n) $E > 0, F > (1/4)E^2$

Figure 1: Bifurcation sets and phase portraits of (14) when $\rho < 0$.

where $\phi_{M,m} = (1/3)(-2E \pm \sqrt{2(2E^2 - 9F)})$, $\omega = \sqrt{(1/2)\rho\phi_m\phi_M}$.

When $\rho < 0$, $E < 0$, $0 < F < (1/4)E^2$, *and* $F \neq (2/9)E^2$, *(1) has two smooth solitary wave solutions as follows:*

$$q(x,y,t) = \frac{\left[\left(\phi_2(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) - (\phi_2(\phi_m + \phi_M) - 2\phi_m\phi_M)\right)^{1/2m}\right] e^{i(x+c_1 y+c_2 t)}}{\left[(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) + (\phi_m + \phi_M - 2\phi_2)\right]^{1/2m}}, \tag{22}$$

$$q(x,y,t) = \frac{\left[\left(\phi_2(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) + (\phi_2(\phi_m + \phi_M) - 2\phi_m\phi_M)\right)^{1/2m}\right] e^{i(x+c_1 y+c_2 t)}}{\left[(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) - (\phi_m + \phi_M - 2\phi_2)\right]^{1/2m}}, \tag{23}$$

where $\phi_{M,m} = (1/6)(3\sqrt{\Delta} - E \pm 2\sqrt{E(E - 3\sqrt{\Delta})})$, $\omega = \sqrt{(1/2)\rho(\phi_M - \phi_2)(\phi_m - \phi_2)}$.

When $\rho < 0$, $E > 0$, $0 < F < (1/4)E^2$, *and* $F \neq (2/9)E^2$, *(1) has two smooth solitary wave solutions as follows:*

$$q(x,y,t) = \frac{\left[\left(\phi_1(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) - (\phi_1(\phi_m + \phi_M) - 2\phi_m\phi_M)\right)^{1/2m}\right] e^{i(x+c_1 y+c_2 t)}}{\left[(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) + (\phi_m + \phi_M - 2\phi_1)\right]^{1/2m}}, \tag{24}$$

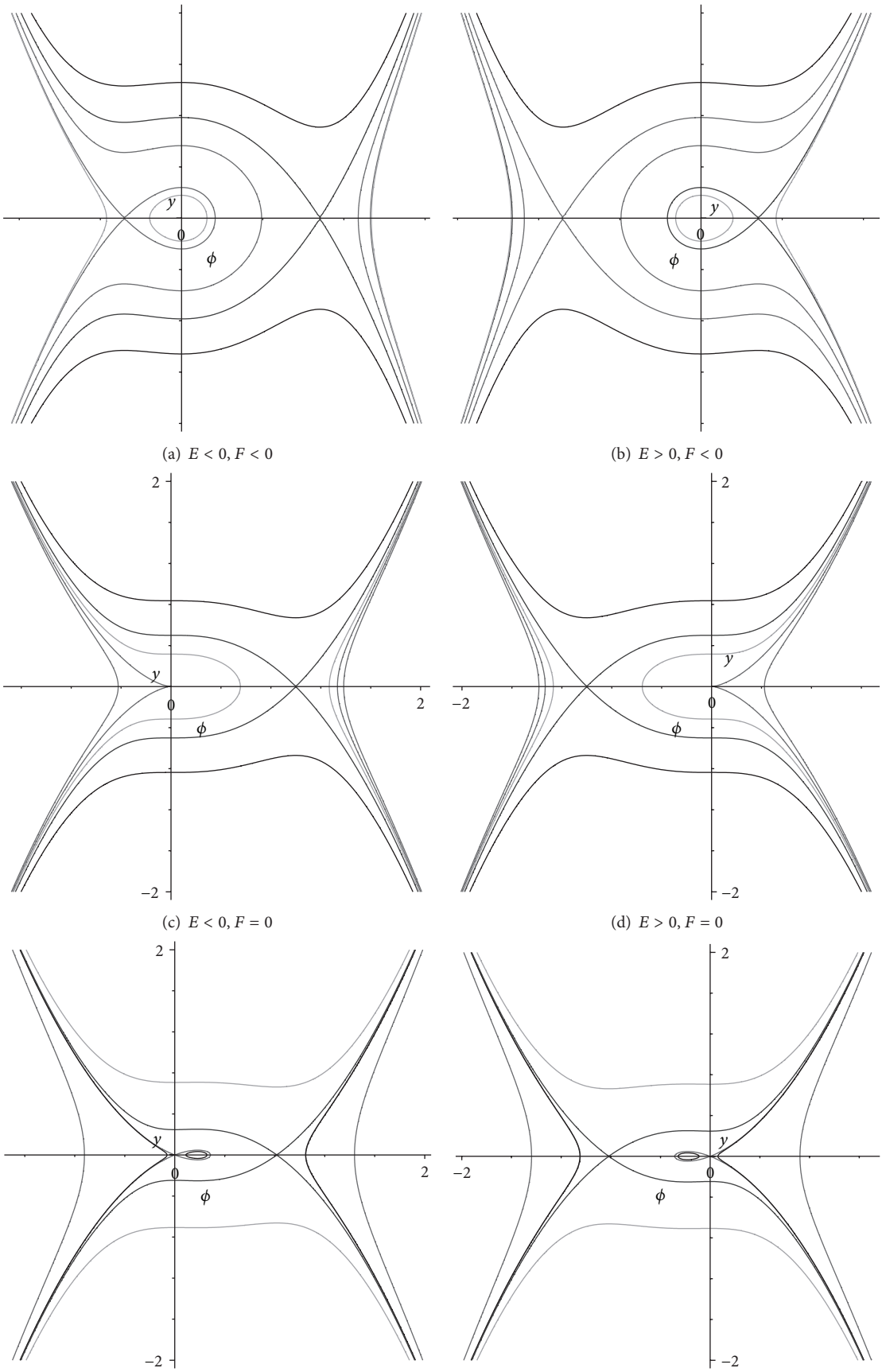
$$q(x,y,t) = \frac{\left[\left(\phi_1(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) + (\phi_1(\phi_m + \phi_M) - 2\phi_m\phi_M)\right)^{1/2m}\right] e^{i(x+c_1 y+c_2 t)}}{\left[(\phi_M - \phi_m)\cosh(\omega(x + \omega_1 y - (1 + c_1\omega_1)t)) - (\phi_m + \phi_M - 2\phi_1)\right]^{1/2m}}, \tag{25}$$

where $\phi_{M,m} = -(1/6)(3\sqrt{\Delta} + E \mp 2\sqrt{E(E + 3\sqrt{\Delta})})$, $\omega = \sqrt{(1/2)\rho(\phi_M - \phi_1)(\phi_m - \phi_1)}$.

When $\rho < 0$, $E \neq 0$, *and* $F = (2/9)E^2$, *(1) has two smooth solitary wave solutions as follows:*

$$q(x,y,t) = \left[\left(\frac{1}{3}E\left(-1 \pm \sqrt{2}\right)\right.\right.$$
$$\left.\left.\cdot \operatorname{sech}(\omega(x + \omega_1 y - (1 + c_1\omega_1)t))\right)^{1/2m}\right]$$
$$\cdot e^{i(x+c_1 y+c_2 t)}, \tag{26}$$

where $\omega = -(E/3)\sqrt{-\rho}$.



(a) $E < 0, F < 0$

(b) $E > 0, F < 0$

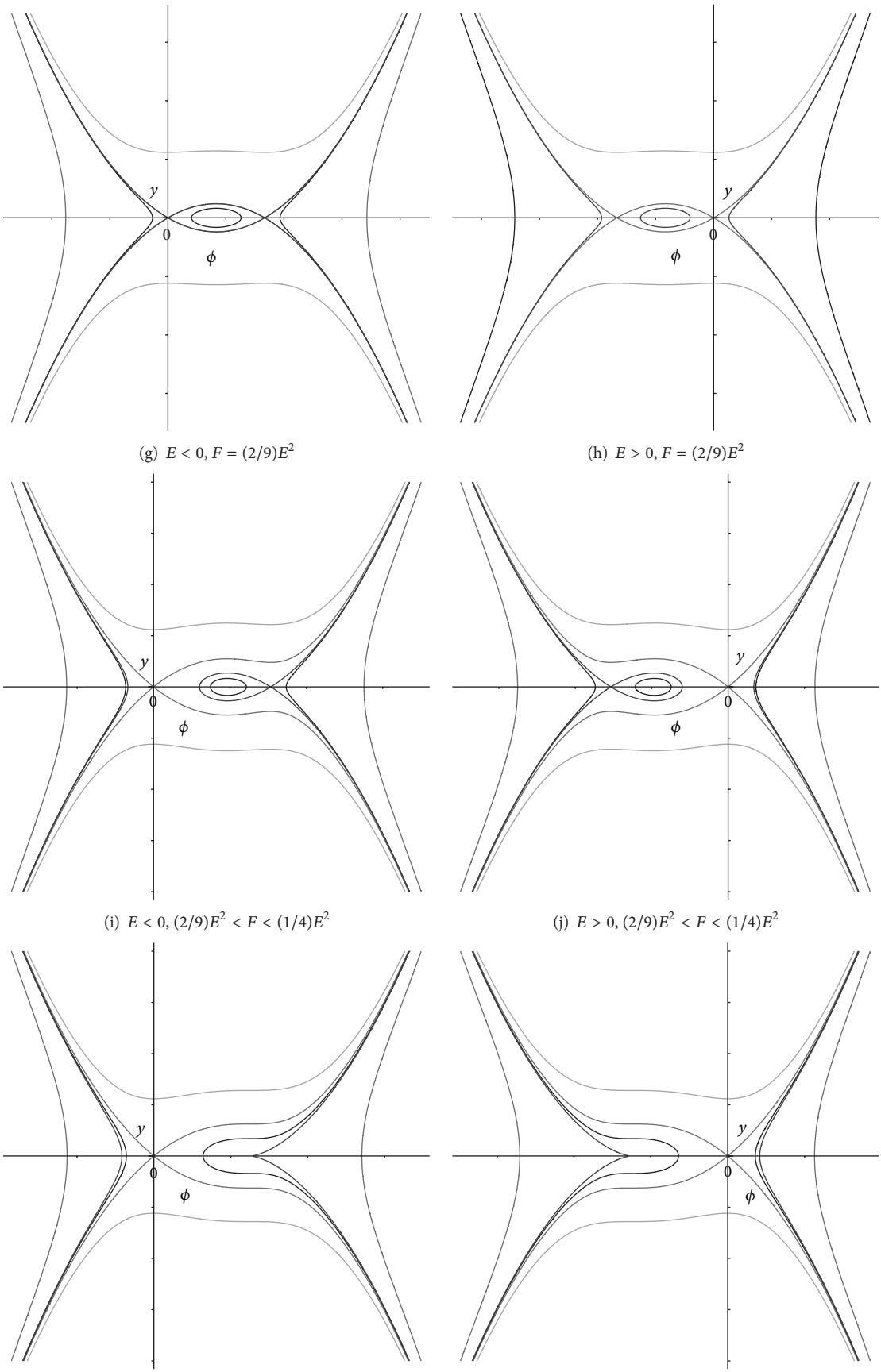
(c) $E < 0, F = 0$

(d) $E > 0, F = 0$

(e) $E < 0, 0 < F < (2/9)E^2$

(f) $E > 0, 0 < F < (2/9)E^2$

Figure 2: Continued.



(g) $E < 0, F = (2/9)E^2$

(h) $E > 0, F = (2/9)E^2$

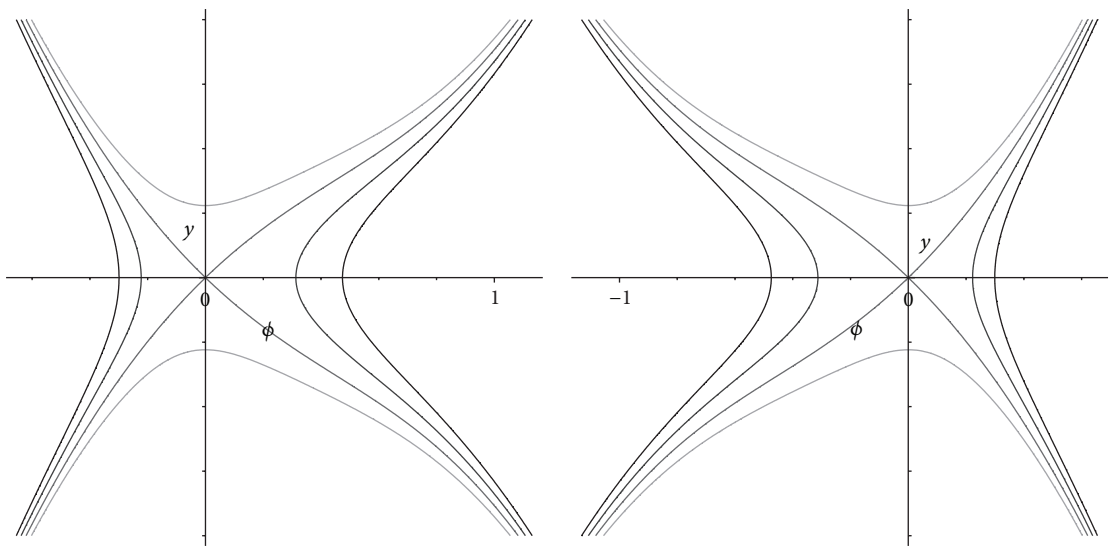
(i) $E < 0, (2/9)E^2 < F < (1/4)E^2$

(j) $E > 0, (2/9)E^2 < F < (1/4)E^2$

(k) $E < 0, F = (1/4)E^2$

(l) $E > 0, F = (1/4)E^2$

Figure 2: Continued.



(m) $E < 0, F > (1/4)E^2$ (n) $E > 0, F > (1/4)E^2$

Figure 2: Bifurcation sets and phase portraits of (14) when $\rho > 0$.

Proof of Proposition 1. From Figures 1(a) and 1(b), when $\rho < 0$, $E \neq 0$, and $F < 0$, there are two homoclinic orbits connecting with the saddle point $(0, 0)$ and passing through the points $(\phi_m, 0)$ and $(\phi_M, 0)$, respectively, where $\phi_{M,m} = (1/3)(-2E \pm \sqrt{2(2E^2 - 9F)})$. Their expressions are, respectively,

$$y = \pm\sqrt{-\frac{1}{2}\rho}\phi\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_m \le \phi < 0, \tag{27}$$

$$y = \pm\sqrt{-\frac{1}{2}\rho}\phi\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad 0 < \phi \le \phi_M. \tag{28}$$

Substituting (27) and (28) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{\phi_m}^{\phi} \frac{ds}{s\sqrt{(\phi_M - s)(s - \phi_m)}} = -\sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{29}$$

$$\int_{\phi}^{\phi_M} \frac{ds}{s\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{30}$$

Completing the integrals in (29) and (30), we obtain two smooth solitary wave solutions of (7) as follows:

$$\phi(\xi) = \frac{2\phi_m\phi_M}{(\phi_m + \phi_M) \pm (\phi_M - \phi_m)\cosh(\omega\xi)}, \tag{31}$$

where $\omega = \sqrt{(1/2)\rho\phi_m\phi_M}$. The profiles of (31) are shown in Figures 3(a) and 3(b). From (2), (6), (31), and $\omega_2 = -(1+c_1\omega_1)$, we obtain two smooth solitary wave solutions of (1) with $\rho < 0$, $E \neq 0$, and $F < 0$ given in (20) and (21).

From Figures 1(e) and 1(i), when $\rho < 0$, $E < 0$, $0 < F < (1/4)E^2$, and $F \neq (2/9)E^2$, there are two homoclinic orbits connecting with the saddle point $(\phi_2, 0)$ and passing through the points $(\phi_m, 0)$ and $(\phi_M, 0)$, respectively, where $\phi_{M,m} = (1/6)(3\sqrt{\Delta} - E \pm 2\sqrt{E(E - 3\sqrt{\Delta})})$. Their expressions are, respectively,

$$y = \pm\sqrt{-\frac{1}{2}\rho}(\phi_2 - \phi)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_m \le \phi < \phi_2, \tag{32}$$

$$y = \pm\sqrt{-\frac{1}{2}\rho}(\phi - \phi_2)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_2 < \phi \le \phi_M. \tag{33}$$

Substituting (32) and (33) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{\phi_m}^{\phi} \frac{ds}{(\phi_2 - s)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{34}$$

$$\int_{\phi}^{\phi_M} \frac{ds}{(s - \phi_2)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{35}$$

Completing the integrals in (34) and (35), we obtain two smooth solitary wave solutions of (7) as follows:

$$\phi(\xi) = \frac{\phi_2(\phi_M - \phi_m)\cosh(\omega\xi) - (\phi_2(\phi_m + \phi_M) - 2\phi_m\phi_M)}{(\phi_M - \phi_m)\cosh(\omega\xi) + (\phi_m + \phi_M - 2\phi_2)}, \tag{36}$$

$$\phi(\xi) = \frac{\phi_2(\phi_M - \phi_m)\cosh(\omega\xi) + (\phi_2(\phi_m + \phi_M) - 2\phi_m\phi_M)}{(\phi_M - \phi_m)\cosh(\omega\xi) - (\phi_m + \phi_M - 2\phi_2)}, \tag{37}$$

where $\omega = \sqrt{-(1/2)\rho(\phi_M - \phi_2)(\phi_2 - \phi_m)}$. The profiles of (36) and (37) are shown in Figures 3(c) and 3(d), respectively.

From (2), (6), (36), (37), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain two smooth solitary wave solutions of (1) with $\rho < 0$, $E < 0$, $0 < F < (1/4)E^2$, and $F \neq (2/9)E^2$ given in (22) and (23).

From Figures 1(f) and 1(j), when $\rho < 0$, $E > 0$, $0 < F < (1/4)E^2$, and $F \neq (2/9)E^2$, there are two homoclinic orbits connecting with the saddle point $(\phi_1, 0)$ and passing through the points $(\phi_m, 0)$ and $(\phi_M, 0)$, respectively, where $\phi_{M,m} = -(1/6)(3\sqrt{\Delta} + E \mp 2\sqrt{E(E + 3\sqrt{\Delta})})$. Their expressions are, respectively,

$$y = \pm\sqrt{-\frac{1}{2}\rho}\,(\phi_1 - \phi)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_m \le \phi < \phi_1, \tag{38}$$

$$y = \pm\sqrt{-\frac{1}{2}\rho}\,(\phi - \phi_1)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_1 < \phi \le \phi_M. \tag{39}$$

Substituting (38) and (39) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{\phi_m}^{\phi} \frac{ds}{(\phi_1 - s)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{40}$$

$$\int_{\phi}^{\phi_M} \frac{ds}{(s - \phi_1)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{41}$$

Completing the integrals in (40) and (41), we obtain two smooth solitary wave solutions of (7) as follows:

$$\phi(\xi) = \frac{\phi_1(\phi_M - \phi_m)\cosh(\omega\xi) - (\phi_1(\phi_m + \phi_M) - 2\phi_m\phi_M)}{(\phi_M - \phi_m)\cosh(\omega\xi) + (\phi_m + \phi_M - 2\phi_1)}, \tag{42}$$

$$\phi(\xi) = \frac{\phi_1(\phi_M - \phi_m)\cosh(\omega\xi) + (\phi_1(\phi_m + \phi_M) - 2\phi_m\phi_M)}{(\phi_M - \phi_m)\cosh(\omega\xi) - (\phi_m + \phi_M - 2\phi_1)}, \tag{43}$$

where $\omega = \sqrt{-(1/2)\rho(\phi_M - \phi_1)(\phi_1 - \phi_m)}$. The profiles of (42) and (43) are shown in Figures 3(e) and 3(f), respectively. From (2), (6), (42), (43), and $\omega_2 = -(1+c_1\omega_1)$, we obtain two smooth solitary wave solutions of (1) with $\rho < 0$, $E > 0$, $0 < F < (1/4)E^2$, and $F \neq (2/9)E^2$ given in (24) and (25).

From Figures 1(g) and 1(h), when $\rho < 0$, $E \neq 0$, and $F = (2/9)E^2$, there are two homoclinic orbits connecting with the saddle point $(-(1/3)E, 0)$ and passing through the points $(\phi_m, 0)$ and $(\phi_M, 0)$, respectively, where $\phi_{M,m} = -(1/3)(1 \pm \sqrt{2})E$ when $\rho < 0$, $E < 0$, $F = (2/9)E^2$, and $\phi_{M,m} = -(1/3)(1 \mp \sqrt{2})E$ when $\rho < 0$, $E > 0$, and $F = (2/9)E^2$. Their expressions are, respectively,

$$y = \pm\sqrt{-\frac{1}{2}\rho}\left(-\frac{1}{3}E - \phi\right)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad \phi_m \le \phi < -\frac{1}{3}E, \tag{44}$$

$$y = \pm\sqrt{-\frac{1}{2}\rho}\left(\phi + \frac{1}{3}E\right)\sqrt{(\phi_M - \phi)(\phi - \phi_m)}, \quad -\frac{1}{3}E < \phi \le \phi_M. \tag{45}$$

Substituting (44) and (45) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{\phi_m}^{\phi} \frac{ds}{(-(1/3)E - s)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{46}$$

$$\int_{\phi}^{\phi_M} \frac{ds}{(s + (1/3)E)\sqrt{(\phi_M - s)(s - \phi_m)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{47}$$

Completing the integrals in (46) and (47), we obtain two smooth solitary wave solutions of (7) as follows:

$$\phi(\xi) = \frac{1}{3}E\left(-1 \pm \sqrt{2}\right)\operatorname{sech}(\omega\xi), \tag{48}$$

where $\omega = -(E/3)\sqrt{-\rho}$. The profiles of (48) are shown in Figures 3(g) and 3(h). From (2), (6), (48), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain two smooth solitary wave solutions of (1) with $\rho < 0$, $E \neq 0$, and $F = (2/9)E^2$ given in (26).

The proof of the Proposition 1 is completed. □

Proposition 2. *When $\rho < 0$, $E \neq 0$, and $F = 0$, (1) has one smooth solitary wave solution as follows:*

$$q(x, y, t) = \frac{\left[(-4E)^{1/2m}\right]e^{i(x + c_1 y + c_2 t)}}{\left[3\left(1 + \left(\omega\left(x + \omega_1 y - (1 + c_1\omega_1)t\right)\right)^2\right)\right]^{1/2m}}, \tag{49}$$

where $\omega = (2E/3)\sqrt{-(1/2)\rho}$.

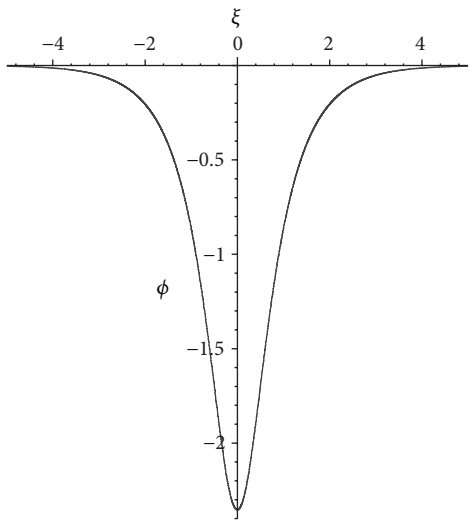
When $\rho < 0$, $E \neq 0$, and $F = (1/4)E^2$, (1) has one smooth solitary wave solution as follows:

$$q(x, y, t) = \frac{\left[\left(E\left(1 - 3\left(\omega\left(x + \omega_1 y - (1 + c_1\omega_1)t\right)\right)^2\right)\right)^{1/2m}\right]e^{i(x + c_1 y + c_2 t)}}{\left[6\left(1 + \left(\omega\left(x + \omega_1 y - (1 + c_1\omega_1)t\right)\right)^2\right)\right]^{1/2m}}, \tag{50}$$

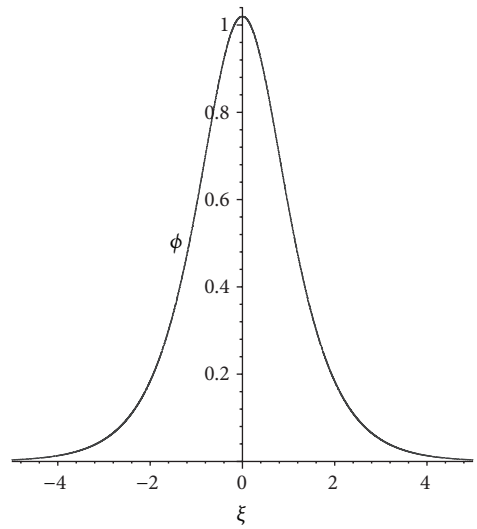
where $\omega = (E/3)\sqrt{-(1/2)\rho}$.

When $\rho > 0$, $E < 0$, and $F < 0$ (or $\rho > 0$, $E > 0$, and $(2/9)E^2 < F < (1/4)E^2$), (1) has one smooth solitary wave solution the same as in (22).

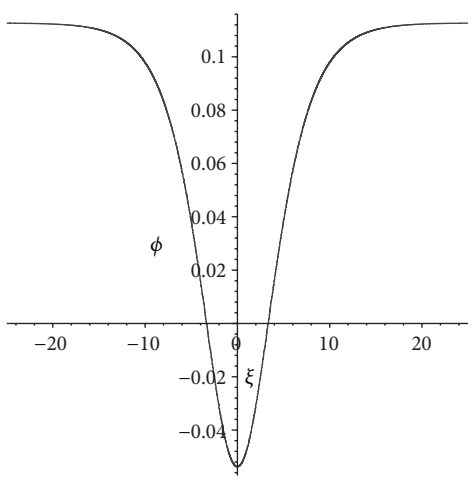
When $\rho > 0$, $E > 0$, and $F < 0$ (or $\rho > 0$, $E < 0$, and $(2/9)E^2 < F < (1/4)E^2$), (1) has one smooth solitary wave solution the same as in (25).



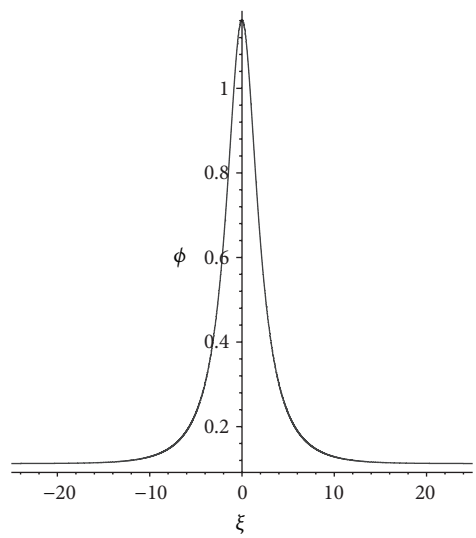
(a) $\rho = -1.5$, $E = 1.0$, and $F = -1.2$



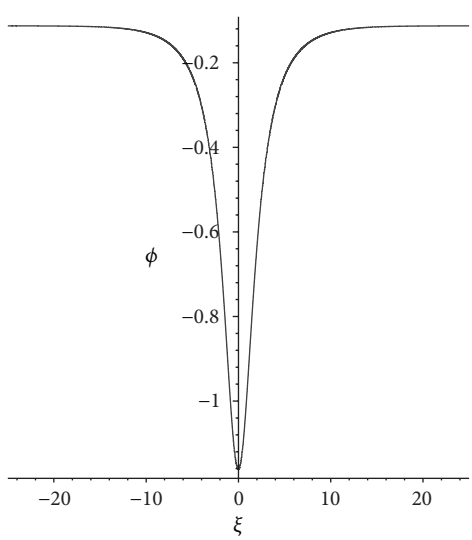
(b) $\rho = -1.5$, $E = 1.0$, and $F = -1.2$



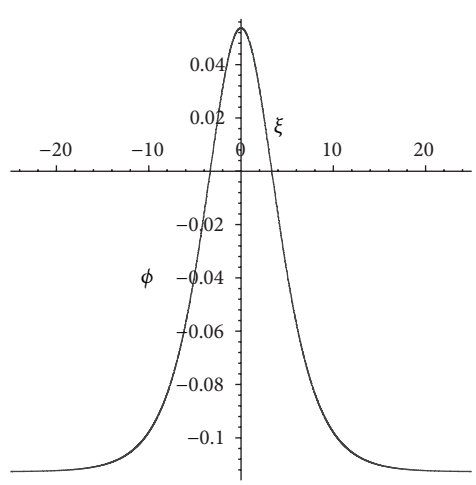
(c) $\rho = -1.5$, $E = -1.0$, and $F = 0.1$



(d) $\rho = -1.5$, $E = -1.0$, and $F = 0.1$

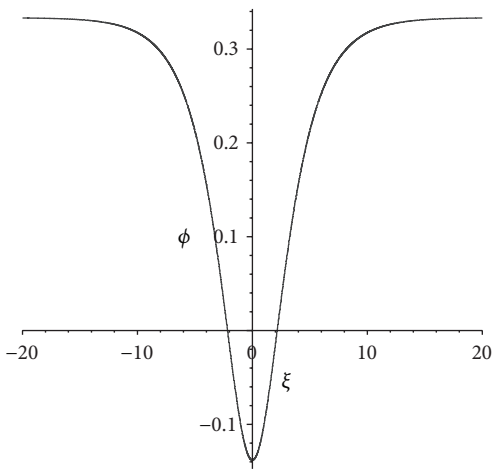


(e) $\rho = -1.5$, $E = 1.0$, and $F = 0.1$

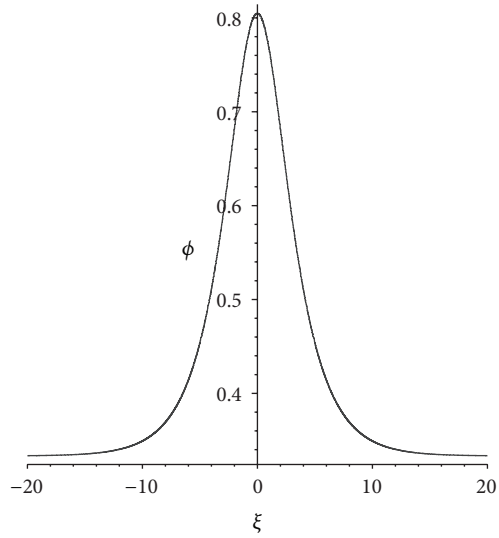


(f) $\rho = -1.5$, $E = 1.0$, and $F = 0.1$

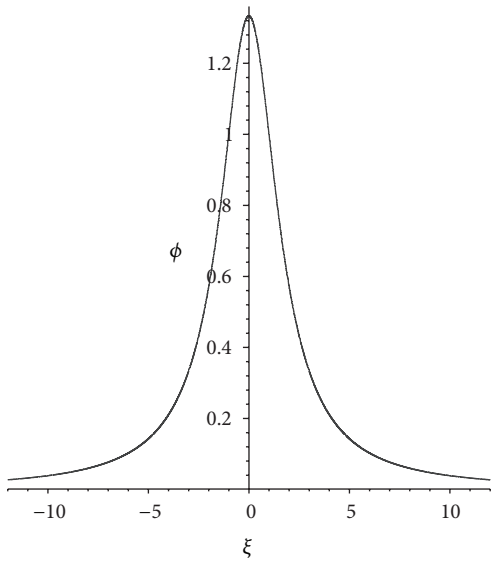
Figure 3: Continued.



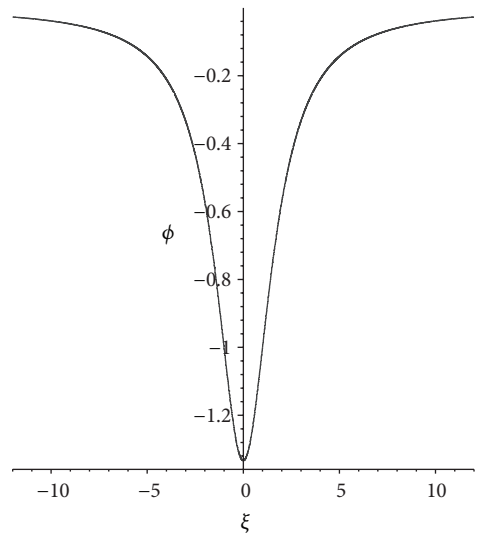
(g) $\rho = -1.5$, $E = -1.0$, and $F = 2/9$



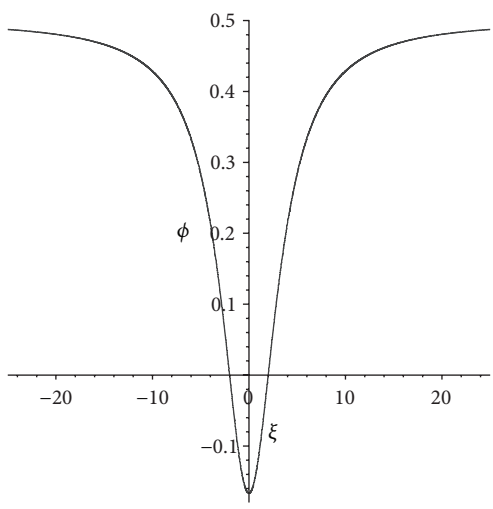
(h) $\rho = -1.5$, $E = -1.0$, and $F = 2/9$



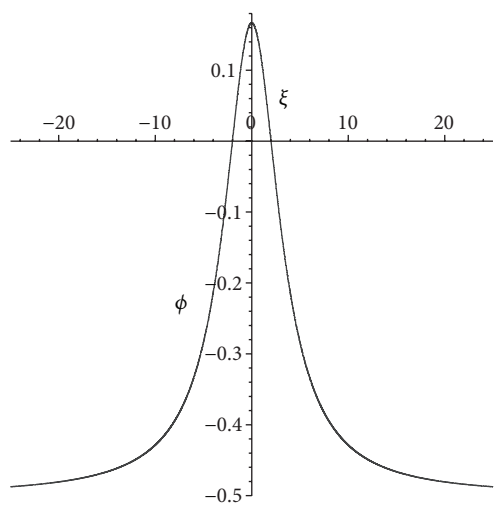
(i) $\rho = -1.5$, $E = -1.0$, and $F = 0$



(j) $\rho = -1.5$, $E = 1.0$, and $F = 0$



(k) $\rho = -1.5$, $E = -1.0$, and $F = 1/4$



(l) $\rho = -1.5$, $E = 1.0$, $F = 1/4$

Figure 3: Smooth solitary waves of (7).

When $\rho > 0$, $E < 0$, and $0 < F < (2/9)E^2$, (1) has one smooth solitary wave solution the same as in (20).

When $\rho > 0$, $E > 0$, and $0 < F < (2/9)E^2$, (1) has one smooth solitary wave solution the same as in (21).

Proof of Proposition 2. From Figures 1(c) and 1(d), when $\rho < 0$, $E \neq 0$, and $F = 0$, there is a homoclinic orbit connecting with the cusp $(0, 0)$ and passing through the point $(-(4/3)E, 0)$. When $\rho < 0$, $E < 0$, and $F = 0$, its expression is

$$y = \pm\sqrt{-\frac{1}{2}\rho}\phi\sqrt{\phi\left(-\frac{4}{3}E - \phi\right)}, \quad 0 < \phi \leq -\frac{4}{3}E. \tag{51}$$

When $\rho < 0$, $E > 0$, and $F = 0$, its expression is

$$y = \pm\sqrt{-\frac{1}{2}\rho}\phi\sqrt{-\phi\left(\phi + \frac{4}{3}E\right)}, \quad -\frac{4}{3}E \leq \phi < 0. \tag{52}$$

Substituting (51) and (52) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{\phi}^{-4E/3} \frac{ds}{s\sqrt{s(-(4/3)E - s)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{53}$$

$$\int_{-4E/3}^{\phi} \frac{ds}{s\sqrt{-s(s + (4/3)E)}} = -\sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{54}$$

Completing the integral in (53), we obtain a smooth solitary wave solution of (7) as follows:

$$\phi(\xi) = -\frac{4E}{3\left(1 + (\omega\xi)^2\right)}, \tag{55}$$

where $\omega = (2E/3)\sqrt{-(1/2)\rho}$. Completing the integral in (54), we obtain a smooth solitary wave solution of (7) the same as in (55). The profiles of (55) are shown in Figures 3(i) and 3(j). From (2), (6), (55), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho < 0$, $E \neq 0$, and $F = 0$ given in (49).

From Figures 1(k) and 1(l), when $\rho < 0$, $E \neq 0$, and $F = (1/4)E^2$, there is a homoclinic orbit connecting with the cusp $(-(1/2)E, 0)$ and passing through the point $((1/6)E, 0)$. When $\rho < 0$, $E < 0$, and $F = (1/4)E^2$, its expression is

$$y = \pm\sqrt{-\frac{1}{2}\rho}\left(-\frac{1}{2}E - \phi\right)\sqrt{\left(-\frac{1}{2}E - \phi\right)\left(\phi - \frac{1}{6}E\right)}, \quad \frac{1}{6}E \leq \phi < -\frac{1}{2}E. \tag{56}$$

When $\rho < 0$, $E > 0$, and $F = (1/4)E^2$, its expression is

$$y = \pm\sqrt{-\frac{1}{2}\rho}\left(\phi + \frac{1}{2}E\right)\sqrt{\left(\phi + \frac{1}{2}E\right)\left(\frac{1}{6}E - \phi\right)}, \quad -\frac{1}{2}E < \phi \leq \frac{1}{6}E. \tag{57}$$

Substituting (56) and (57) into $d\phi/d\xi = y$, respectively, and integrating them along the homoclinic orbits, we have

$$\int_{E/6}^{\phi} \frac{ds}{(-(1/2)E - s)\sqrt{(-(1/2)E - s)(s - (1/6)E)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|, \tag{58}$$

$$\int_{\phi}^{E/6} \frac{ds}{(s + (1/2)E)\sqrt{(s + (1/2)E)((1/6)E - s)}} = \sqrt{-\frac{1}{2}\rho}\,|\xi|. \tag{59}$$

Completing the integral in (58), we obtain a smooth solitary wave solution of (7) as follows:

$$\phi(\xi) = \frac{E\left(1 - 3(\omega\xi)^2\right)}{6\left(1 + (\omega\xi)^2\right)}, \tag{60}$$

where $\omega = (E/3)\sqrt{-(1/2)\rho}$. Completing the integral in (59), we obtain a smooth solitary wave solution of (7) the same as in (60). The profiles of (60) are shown in Figures 3(k) and 3(l). From (2), (6), (60), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho < 0$, $E \neq 0$, and $F = (1/4)E^2$ given in (50).

From Figures 2(a) and 2(j), when $\rho > 0$, $E < 0$, and $F < 0$ (or $\rho > 0$, $E > 0$, and $(2/9)E^2 < F < (1/4)E^2$), there is a homoclinic orbit connecting with the saddle point $(\phi_2, 0)$ and passing through the point $(\phi_m, 0)$, and its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}(\phi - \phi_2)\sqrt{(\phi_M - \phi)(\phi_m - \phi)}, \quad \phi_2 < \phi \leq \phi_m, \tag{61}$$

where $\phi_{M,m} = (1/6)(3\sqrt{\Delta} - E \pm 2\sqrt{E(E - 3\sqrt{\Delta})})$. Substituting (61) into $d\phi/d\xi = y$ and integrating it along the homoclinic orbit, we have

$$\int_{\phi}^{\phi_m} \frac{ds}{(s - \phi_2)\sqrt{(\phi_M - s)(\phi_m - s)}} = \sqrt{\frac{1}{2}\rho}\,|\xi|. \tag{62}$$

Completing the integral in (62), and using (2), (6), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho > 0$, $E < 0$, and $F < 0$ (or $\rho > 0$, $E > 0$, and $(2/9)E^2 < F < (1/4)E^2$) given in (22).

From Figures 2(b) and 2(i), when $\rho > 0$, $E > 0$, and $F < 0$ (or $\rho > 0$, $E < 0$, and $(2/9)E^2 < F < (1/4)E^2$), there is a homoclinic orbit connecting with the saddle point $(\phi_1, 0)$ and passing through the point $(\phi_M, 0)$, and its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}(\phi_1 - \phi)\sqrt{(\phi - \phi_M)(\phi - \phi_m)}, \quad \phi_M \leq \phi < \phi_1, \tag{63}$$

where $\phi_{M,m} = -(1/6)(3\sqrt{\Delta} + E \mp 2\sqrt{E(E+3\sqrt{\Delta})})$. Substituting (63) into $d\phi/d\xi = y$ and integrating it along the homoclinic orbit, we have

$$\int_{\phi_M}^{\phi} \frac{ds}{(\phi_1 - s)\sqrt{(s-\phi_M)(s-\phi_m)}} = \sqrt{\frac{1}{2}\rho}\,|\xi|. \tag{64}$$

Completing the integral in (64), and using (2), (6), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho > 0$, $E > 0$, and $F < 0$ (or $\rho > 0$, $E < 0$, and $(2/9)E^2 < F < (1/4)E^2$) given in (25).

From Figure 2(e), when $\rho > 0$, $E < 0$, and $0 < F < (2/9)E^2$, there is a homoclinic orbit connecting with the saddle point $(0,0)$ and passing through the point $(\phi_m, 0)$, and its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}\phi\sqrt{(\phi_M - \phi)(\phi_m - \phi)}, \quad 0 < \phi \le \phi_m, \tag{65}$$

where $\phi_{M,m} = (1/3)(-2E \pm \sqrt{2(2E^2 - 9F)})$. Substituting (65) into $d\phi/d\xi = y$ and integrating it along the homoclinic orbit, we have

$$\int_{\phi}^{\phi_m} \frac{ds}{s\sqrt{(\phi_M - s)(\phi_m - s)}} = \sqrt{\frac{1}{2}\rho}\,|\xi|. \tag{66}$$

Completing the integral in (66), and using (2), (6), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho > 0$, $E < 0$, and $0 < F < (2/9)E^2$ given in (20).

From Figure 2(f), when $\rho > 0$, $E > 0$, and $0 < F < (2/9)E^2$, there is a homoclinic orbit connecting with the saddle point $(0,0)$ and passing through the point $(\phi_M, 0)$, and its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}\phi\sqrt{(\phi - \phi_M)(\phi - \phi_m)}, \quad \phi_M \le \phi < 0, \tag{67}$$

where $\phi_{M,m} = (1/3)(-2E \pm \sqrt{2(2E^2 - 9F)})$. Substituting (67) into $d\phi/d\xi = y$ and integrating it along the homoclinic orbit, we have

$$\int_{\phi_M}^{\phi} \frac{ds}{s\sqrt{(s-\phi_M)(s-\phi_m)}} = -\sqrt{\frac{1}{2}\rho}\,|\xi|. \tag{68}$$

Completing the integral in (68), and using (2), (6), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain a smooth solitary wave solution of (1) with $\rho > 0$, $E > 0$, and $0 < F < (2/9)E^2$ given in (21).

The proof of the Proposition 2 is completed. □

Proposition 3. *When $\rho > 0$, $E \neq 0$, and $F = (2/9)E^2$, (1) has the kink and antikink wave solutions as follows:*

$$q(x,y,t) = \frac{\left[(-2E)^{1/2m}\right] e^{i(x+c_1y+c_2t)}}{\left[3\left(1 + e^{\pm\omega(x+\omega_1 y-(1+c_1\omega_1)t)}\right)\right]^{1/2m}}, \tag{69}$$

where $\omega = (2E/3)\sqrt{(1/2)\rho}$.

Proof of Proposition 3. From Figures 2(g) and 2(h), when $\rho > 0$, $E \neq 0$, and $F = (2/9)E^2$, there are two heteroclinic orbits connecting with the saddle points $(0,0)$ and $(-(2/3)E, 0)$. When $\rho > 0$, $E < 0$, and $F = (2/9)E^2$, its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}\phi\left(-\frac{2}{3}E - \phi\right), \quad 0 < \phi < -\frac{2}{3}E. \tag{70}$$

When $\rho > 0$, $E > 0$, and $F = (2/9)E^2$, its expression is

$$y = \pm\sqrt{\frac{1}{2}\rho}\phi\left(\phi + \frac{2}{3}E\right), \quad -\frac{2}{3}E < \phi < 0. \tag{71}$$

Substituting (70) and (71) into $d\phi/d\xi = y$, respectively, and integrating them along the heteroclinic orbits, we have

$$\int_{\phi}^{-E/3} \frac{ds}{s(-(2/3)E - s)} = \pm\sqrt{\frac{1}{2}\rho}\xi, \tag{72}$$

$$\int_{-E/3}^{\phi} \frac{ds}{s(s + (2/3)E)} = \pm\sqrt{\frac{1}{2}\rho}\xi. \tag{73}$$

Completing the integral in (72), we obtain the kink and antikink wave solutions of (7) as follows:

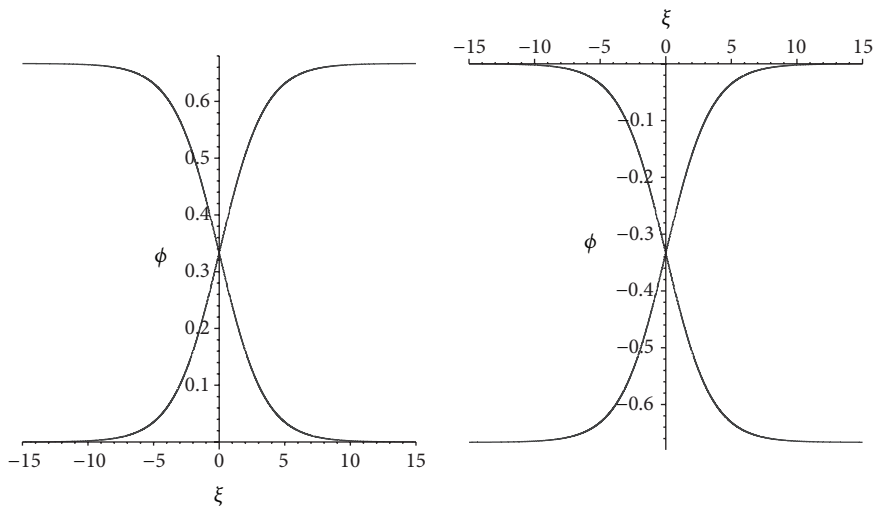
$$\phi(\xi) = -\frac{2E}{3\left(1 + e^{\pm\omega\xi}\right)}, \tag{74}$$

where $\omega = (2E/3)\sqrt{(1/2)\rho}$. Completing the integral in (73), we obtain the kink and antikink wave solutions of (7) the same as in (74). The profiles of (74) are shown in Figures 4(a) and 4(b). From (2), (6), (74), and $\omega_2 = -(1 + c_1\omega_1)$, we obtain the kink and antikink wave solutions of (1) with $\rho > 0$, $E \neq 0$, and $F = (2/9)E^2$ given in (69).

The proof of Proposition 3 is completed. □

Remark 4. The implicit solutions in [13] can be deduced if setting that concrete values of a set of the parameters. For instance, if one set $a_1 \to 1$, $a_2 \to c_1$, $a_3 \to c_2$, $a_4 \to 0$, $A_1 \to 1$, $A_2 \to \omega_1$, $A_3 \to \omega_2 = -(1+c_1\omega_1)$, $A_4 \to 0$, $m \to 2m$, and $v \to \phi$ in [13], (16) and (18) in [13], respectively, can be deduced as follows:

$$\frac{4m\left(4m^2\left(1 + 2c_2 + c_1^2\right) + 2m\left(1 + 2c_2 + c_1^2\right) - 2m\phi^2 + (2m+1)\Phi\sqrt{\left(1 + 2c_2 + c_1^2\right)\left(1 + \omega_1^2\right)}\right)}{(2m+1)\left(1 + \omega_1^2\right)\phi} = e^{\mp 4m\sqrt{1+2c_2+c_1^2}\xi/\sqrt{1+\omega_1^2}}, \tag{75}$$



(a) $\rho = 1.5$, $E = -1.0$, and $F = 2/9$ (b) $\rho = 1.5$, $E = 1.0$, and $F = 2/9$

Figure 4: Kink and antikink waves of (7).

$$\xi = \pm \int \left(\frac{4m^2\phi^2 \left(\left(1 + 2c_2 + c_1^2\right)\left(8m^2 + 6m + 1\right) - 2\left(1 + 4m\right)\phi^2 - 2k\left(2m + 1\right)\phi^4 \right) + 2h\left(8m^2 + 6m + 1\right)\phi^{2-1/m}}{\left(8m^2 + 6m + 1\right)\left(1 + \omega_1^2\right)} \right)^{-1/2} d\phi, \tag{76}$$

where

$$\xi = x + \omega_1 y - \left(1 + c_1\omega_1\right)t,$$
$$\Phi = 2\sqrt{\frac{m^2\left(\left(1 + 2c_2 + c_1^2\right)(2m + 1)(4m + 1) - 2k(2m + 1)\phi^4 - 2(4m + 1)\phi^2\right)}{(2m + 1)(4m + 1)\left(1 + \omega_1^2\right)}} \tag{77}$$

and h is given in (10) of [13]. Equations (75) and (76) maybe can be reduced to some of results in this paper under concrete values of some sets of the parameters. We omit the discussions here.

5. Conclusion

In this paper, we present some explicit smooth solitary wave solutions for (1) expressed in (20)–(26), (49), and (50), and also some explicit kink and antikink wave solutions shown in (69). We will continue to discuss the properties of (1), and more results will be obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant no. 11461022 and the Major Natural Science Foundation of Yunnan Province, China, under Grant no. 2014FA037.

References

[1] A. Hasegawa and Y. Kodama, *Solitons in Optical Communication*, Oxford University Press, New York, NY, USA, 1995.

[2] Z.-Y. Zhang, Y.-X. Li, Z.-H. Liu, and X.-J. Miao, "New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity via modified trigonometric function series method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 8, pp. 3097–3106, 2011.

[3] M. S. Abdel Latif, "Bright and dark soliton solutions for the perturbed nonlinear Schrödinger's equation with Kerr law and

non-Kerr law nonlinearity," *Applied Mathematics and Computation*, vol. 247, pp. 501–510, 2014.

[4] Z.-Y. Zhang, Z.-H. Liu, X.-J. Miao, and Y.-Z. Chen, "New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity," *Applied Mathematics and Computation*, vol. 216, no. 10, pp. 3064–3072, 2010.

[5] A. Biswas and D. Milovic, "Bright and dark solitons of the generalized nonlinear Schrödinger's equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1473–1484, 2010.

[6] W.-P. Hong, "Optical solitary wave solutions for the higher order nonlinear Schrödinger equation with cubic-quintic non-Kerr terms," *Optics Communications*, vol. 194, no. 1–3, pp. 217–223, 2001.

[7] Z. Li, L. Li, H. Tian, and G. Zhou, "New types of solitary wave solutions for the higher order nonlinear Schrödinger equation," *Physical Review Letters*, vol. 84, no. 18, pp. 4096–4099, 2000.

[8] A. Biswas, "1-Soliton solution of (1 + 2)-dimensional nonlinear Schrödinger's equation in dual-power law media," *Physics Letters A*, vol. 372, no. 38, pp. 5941–5943, 2008.

[9] L.-H. Zhang and J.-G. Si, "New soliton and periodic solutions of (1+2)-dimensional nonlinear Schrödinger equation with dual-power law nonlinearity," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 10, pp. 2747–2754, 2010.

[10] M. Mirzazadeha, M. Eslamib, D. Milovicc, and A. Biswasd, "Topological solitons of resonant nonlinear Schödinger's equation with dual-power law nonlinearity by G'/G-expansion technique," *Optik—International Journal for Light and Electron Optics*, vol. 125, no. 19, pp. 5480–5489, 2014.

[11] H. Triki, S. Lepule, A. Love, A. H. Kara, and A. Biswas, "Dark optical solitons and conservation laws for parabolic and dual-power law nonlinearities in $(2+1)$-dimensions," *Optik*, vol. 125, no. 12, pp. 2784–2792, 2014.

[12] H. Bulut, Y. Pandir, and S. T. Demiray, "Exact solutions of nonlinear Schrodinger's equation with dual power-law nonlinearity by extended trial equation method," http://arxiv.org/abs/1402.0745.

[13] H. Liu, F. Yan, and C. Xu, "The bifurcation and exact travelling wave solutions of (1+2)-dimensional nonlinear Schrödinger equation with dual-power law nonlinearity," *Nonlinear Dynamics*, vol. 67, no. 1, pp. 465–473, 2012.

[14] D.-S. Wang and H. B. Li, "Single and multi-solitary wave solutions to a class of nonlinear evolution equations," *Journal of Mathematical Analysis and Applications*, vol. 343, no. 1, pp. 273–298, 2008.

[15] J. B. Li and H. H. Dai, *On the Study of Singular Nonlinear Traveling Wave Equation: Dynamical System Approach*, Science Press, Beijing, China, 2007.

[16] B. He, "Bifurcations and exact bounded travelling wave solutions for a partial differential equation," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 364–371, 2010.

[17] Q. Meng and B. He, "Notes on 'solitary wave solutions of the generalized two-component Hunter-Saxton system'," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 103, pp. 33–38, 2014.