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Research Article

A Regime Switching Model of Schooling Choice as a Job Search Process

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We propose a regime switching model of schooling choice as a job search process. We adopt a two-state Markov process and the derived coupled Bellman equations are solved by seeking the root of an auxiliary algebraic equation. Some numerical examples are also considered.

1. Introduction

In the course of schooling as a job search process, the role of uncertainty that gives rise to an option was studied by Fan [1]. Fan [1] used an analytic formulation, in which the wage offer process $w(t)$ was modeled as the following arithmetic Brownian motion:

$$dw(t) = g dt + \sigma dB_t, \tag{1}$$

where g is a positive constant and B_t is a standard Brownian motion. If Q is the schooling preference net of education costs, an individual's objective is to maximize the expected value V with the schooling duration τ:

$$V := \mathbb{E}\left[\int_0^\tau e^{-rt} Q\, dt + \int_\tau^\infty e^{-rt} w(\tau)\, dt\right], \tag{2}$$

where r is the constant discount rate and Q was also assumed to be constant in Fan [1]. However, this abstraction may not be able to fully capture reality. Specifically, in an economic recession, the desire to obtain education may rise; for example, graduate program enrollment increases in an economic recession (Fan [1]), but the cost of education, for example, tuition fees, may decline (Binder [2]). In addition, wage offer volatility may have a higher value in an economic recession (Gronau [3]) than in an economic expansion. Therefore, we extend the model by allowing wage offer dynamics and schooling preference (net of education costs) to be modulated by a two-state Markov process. Since an arithmetic Brownian motion allows for a negative wage offer, we utilize a geometric Brownian motion as the wage offer process in this paper.

Our methodology to derive the closed-form solutions is in line with that used in the real option framework with regime switching, such as Dixit and Pindyck [4], Guo et al. [5], and Bensoussan et al. [6].

2. The Model

We assume that a risk-neutral individual receives a wage offer $w(t)$, which follows a geometric Brownian motion with coefficients modulated by a two-state Markov process as follows:

$$dw(t) = \mu_{\Theta_t} w(t)\, dt + \sigma_{\Theta_t} w(t)\, dB_t, \quad w(0) = w > 0, \tag{3}$$

where $(B_t)_{t\geq 0}$ is a standard Brownian motion under a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $(\Theta_t)_{t\geq 0}$ is a two-state Markov process defined on $(\Omega, \mathcal{F}, \mathbb{P})$ and can take value 1 or 2. We also assume that $(\Theta_t)_{t\geq 0}$ is independent of $(B_t)_{t\geq 0}$ and has the following generator:

$$\Lambda = \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{pmatrix}, \quad \lambda_1 > 0,\ \lambda_2 > 0. \tag{4}$$

For each regime $i, (i = 1, 2)$, μ_i is a constant drift and σ_i is a constant volatility. We allow that schooling preference (net of education costs), denoted by $Q_t = Q_{\Theta_t}$, is also regime switching.

We assume the following as in Fan [1] (see also verifications therein). An individual will stop schooling at time τ to accept a wage offer, which will be his constant wage throughout the remainder of the individual's infinite lifetime. Further, this stopping process is irreversible. Then an individual's objective is to choose optimally the schooling duration τ_i that maximizes the expected value $V_i(w)$, which consists of the present value of lifetime earnings and schooling preference (net of education costs) for each regime $i, (i = 1, 2)$,

$$\begin{aligned} V_i(w) &:= \max_{\tau_i} \mathbb{E}\left[\int_0^{\tau_i} e^{-rt} Q_{\Theta_t} dt \right. \\ &\quad \left. + \int_{\tau_i}^{\infty} e^{-rt} w(\tau_i)\, dt \mid \Theta_0 = i, w(0) = w\right] \\ &= \max_{\tau_i} \mathbb{E}\left[\int_0^{\tau_i} e^{-rt} Q_{\Theta_t} dt + e^{-r\tau_i} \frac{w(\tau_i)}{r} \mid \Theta_0 \right. \\ &\quad \left. = i, w(0) = w\right], \end{aligned} \tag{5}$$

where r is the discount rate and τ_i can be considered as an optimal stopping time of education. We denote $w(\tau_1) := \overline{w}_1$, $w(\tau_2) := \overline{w}_2$ and we only consider the case where

$$\overline{w}_1 < \overline{w}_2. \tag{6}$$

$\overline{w}_i$ is called the reservation wage in labor economics.

Remark 1. For each regime $i, (i = 1, 2)$, we define the functions $f_i(\cdot)$ and $G(\cdot)$:

$$\begin{aligned} f_i(m) &:= -\frac{1}{2}\sigma_i^2 m(m-1) - \mu_i m + (r + \lambda_i), \\ G(m) &:= f_1(m) f_2(m) - \lambda_1\lambda_2. \end{aligned} \tag{7}$$

Let $n_1 < 0$ and $n_2 > 0$ be two real roots of the quadratic equation $f_2(n) = 0$. We also consider the quartic equation $G(m) = 0$, which has four real roots $m_1 < m_2 < 0 < m_3 < m_4$.

Theorem 2. *Under assumption (6), for each regime $i, (i = 1, 2)$, the value function $V_i(\cdot)$ is derived as follows:*

$$\begin{aligned} V_1(w) &= \begin{cases} A_3 w^{m_3} + A_4 w^{m_4} + \dfrac{(r+\lambda_2)Q_1 + \lambda_1 Q_2}{(r+\lambda_1)(r+\lambda_2) - \lambda_1\lambda_2}, & \text{for } 0 < w < \overline{w}_1, \\ \dfrac{w}{r}, & \text{for } w \geq \overline{w}_1, \end{cases} \\ V_2(w) &= \begin{cases} \dfrac{f_1(m_3)}{\lambda_1} A_3 w^{m_3} + \dfrac{f_1(m_4)}{\lambda_1} A_4 w^{m_4} + \dfrac{(r+\lambda_1)Q_2 + \lambda_2 Q_1}{(r+\lambda_1)(r+\lambda_2) - \lambda_1\lambda_2}, & \text{for } 0 < w < \overline{w}_1, \\ C_1 w^{n_1} + C_2 w^{n_2} + \dfrac{\lambda_2}{r(r-\mu_2+\lambda_2)} w + \dfrac{Q_2}{r+\lambda_2}, & \text{for } \overline{w}_1 \leq w < \overline{w}_2, \\ \dfrac{w}{r}, & \text{for } w \geq \overline{w}_2. \end{cases} \end{aligned} \tag{8}$$

The values of $A_3, A_4, C_1, C_2, \overline{w}_1$, and $\overline{w}_2$ are expanded in the proof.

Proof. From (5), for $0 < w < \overline{w}_1$, we obtain the following coupled Bellman equations:

$$\begin{aligned} \mathscr{L}_1 V_1(w) - Q_1 &= \lambda_1 V_2(w), \\ \mathscr{L}_2 V_2(w) - Q_2 &= \lambda_2 V_1(w), \end{aligned} \tag{9}$$

where the differential operator $\mathscr{L}_i$ is given by

$$\mathscr{L}_i \bullet = -\frac{1}{2}\sigma_i^2 w^2 \frac{\partial^2 \bullet}{\partial w^2} - \mu_i w \frac{\partial \bullet}{\partial w} + (r + \lambda_i)\bullet, \tag{10}$$

for each regime $i, (i = 1, 2)$. Thus the solutions to (9) are of the forms

$$\begin{aligned} V_1(w) &= A_3 w^{m_3} + A_4 w^{m_4} + a, \\ V_2(w) &= \frac{f_1(m_3)}{\lambda_1} A_3 w^{m_3} + \frac{f_1(m_4)}{\lambda_1} A_4 w^{m_4} + b, \end{aligned} \tag{11}$$

where

$$\begin{aligned} a &= \frac{(r+\lambda_2)Q_1 + \lambda_1 Q_2}{(r+\lambda_1)(r+\lambda_2) - \lambda_1\lambda_2}, \\ b &= \frac{(r+\lambda_1)Q_2 + \lambda_2 Q_1}{(r+\lambda_1)(r+\lambda_2) - \lambda_1\lambda_2}. \end{aligned} \tag{12}$$

For $\overline{w}_1 \leq w < \overline{w}_2$, we have the equation

$$\mathscr{L}_2 V_2(w) - Q_2 = \lambda_2 \frac{w}{r}, \tag{13}$$

since $V_1(w) = w/r$. Thus the solution to (13) is of the form

$$V_2(w) = C_1 w^{n_1} + C_2 w^{n_2} + \frac{\lambda_2}{r(r-\mu_2+\lambda_2)} w + \frac{Q_2}{r+\lambda_2}. \tag{14}$$

Now we determine $A_3, A_4, C_1, C_2, \overline{w}_1$, and $\overline{w}_2$ using the C^1-conditions of V_1 and V_2 at $w = \overline{w}_1$ and $w = \overline{w}_2$. The C^1-condition of $V_1(w)$ at $w = \overline{w}_1$ implies

$$A_3 = \frac{(1/r)(m_4-1)\overline{w}_1 - am_4}{(m_4-m_3)\overline{w}_1^{m_3}}, \qquad A_4 = \frac{(1/r)(1-m_3)\overline{w}_1 + am_3}{(m_4-m_3)\overline{w}_1^{m_4}}. \tag{15}$$

The C^1-condition of $V_2(w)$ at $w = \overline{w}_2$ yields

$$C_1 = \frac{((n_2-1)(r-\mu_2)/r(r-\mu_2+\lambda_2))\overline{w}_2 - n_2 Q_2/(r+\lambda_2)}{(n_2-n_1)\overline{w}_2^{n_1}}, \qquad C_2 = \frac{((1-n_1)(r-\mu_2)/r(r-\mu_2+\lambda_2))\overline{w}_2 + n_1 Q_2/(r+\lambda_2)}{(n_2-n_1)\overline{w}_2^{n_2}}. \tag{16}$$

From the C^1-condition of $V_2(w)$ at $w = \overline{w}_1$ we obtain the equations

$$\frac{f_1(m_3)}{\lambda_1} A_3 \overline{w}_1^{m_3} + \frac{f_1(m_4)}{\lambda_1} A_4 \overline{w}_1^{m_4} + b = C_1 \overline{w}_1^{n_1} + C_2 \overline{w}_1^{n_2} + \frac{\lambda_2}{r(r-\mu_2+\lambda_2)} \overline{w}_1 + \frac{Q_2}{r+\lambda_2}, \tag{17}$$

$$\frac{m_3 f_1(m_3)}{\lambda_1} A_3 \overline{w}_1^{m_3} + \frac{m_4 f_1(m_4)}{\lambda_1} A_4 \overline{w}_1^{m_4} = n_1 C_1 \overline{w}_1^{n_1} + n_2 C_2 \overline{w}_1^{n_2} + \frac{\lambda_2}{r(r-\mu_2+\lambda_2)} \overline{w}_1. \tag{18}$$

Plugging (15) into the LHS of (17) implies

$$\begin{aligned}
&\text{(LHS) of (17)} \\
&= \frac{f_1(m_3)}{\lambda_1} \frac{(1/r)(m_4-1)\overline{w}_1 - am_4}{(m_4-m_3)} + \frac{f_1(m_4)}{\lambda_1} \frac{(1/r)(1-m_3)\overline{w}_1 + am_3}{(m_4-m_3)} + b \\
&= \frac{(m_4-1) f_1(m_3) + (1-m_3) f_1(m_4)}{r\lambda_1(m_4-m_3)} \xi \overline{w}_2 + \frac{m_3 f_1(m_4) - m_4 f_1(m_3)}{\lambda_1(m_4-m_3)} a + b,
\end{aligned} \tag{19}$$

and plugging (16) into the RHS of (17) implies

$$\begin{aligned}
&\text{(RHS) of (17)} \\
&= \frac{((n_2-1)(r-\mu_2)/r(r-\mu_2+\lambda_2))\overline{w}_2 - n_2 Q_2/(r+\lambda_2)}{(n_2-n_1)} \cdot \xi^{n_1} + \frac{((1-n_1)(r-\mu_2)/r(r-\mu_2+\lambda_2))\overline{w}_2 + n_1 Q_2/(r+\lambda_2)}{(n_2-n_1)} \cdot \xi^{n_2} + \frac{\lambda_2}{r(r-\mu_2+\lambda_2)} \xi \overline{w}_2 + \frac{Q_2}{r+\lambda_2} \\
&= \frac{1}{r(r-\mu_2+\lambda_2)} \left[\frac{(r-\mu_2)\{(n_2-1)\xi^{n_1} + (1-n_1)\xi^{n_2}\}}{n_2-n_1} + \lambda_2 \xi \right] \overline{w}_2 + \frac{Q_2}{r+\lambda_2} \left[\frac{n_1 \xi^{n_2} - n_2 \xi^{n_1}}{n_2-n_1} + 1 \right],
\end{aligned} \tag{20}$$

with the relationship

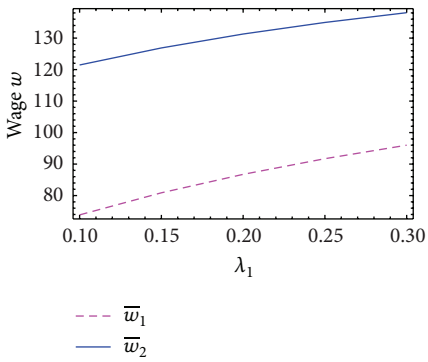
$$\overline{w}_1 = \xi \overline{w}_2, \quad \xi \in (0,1). \tag{21}$$

From (19) and (20), we obtain

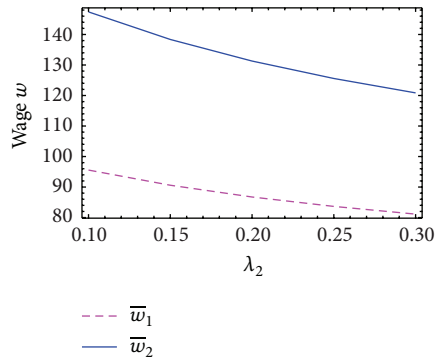
$$\overline{w}_2 = \frac{((m_3 f_1(m_4) - m_4 f_1(m_3))/\lambda_1(m_4-m_3))a + b - (Q_2/(r+\lambda_2))[(n_1\xi^{n_2} - n_2\xi^{n_1})/(n_2-n_1) + 1]}{(1/r(r-\mu_2+\lambda_2))[(r-\mu_2)\{(n_2-1)\xi^{n_1} + (1-n_1)\xi^{n_2}\}/(n_2-n_1) + \lambda_2\xi] - (((m_4-1) f_1(m_3) + (1-m_3) f_1(m_4))/r\lambda_1(m_4-m_3))\xi}. \tag{22}$$

Also plugging (15) into the LHS of (18) implies

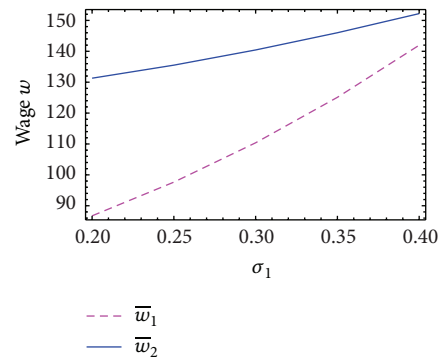
$$\begin{aligned}
&\text{(LHS) of (18)} \\
&= \frac{m_3 f_1(m_3)}{\lambda_1} \frac{(1/r)(m_4-1)\overline{w}_1 - am_4}{(m_4-m_3)} + \frac{m_4 f_1(m_4)}{\lambda_1} \frac{(1/r)(1-m_3)\overline{w}_1 + am_3}{(m_4-m_3)} \\
&= \frac{m_3(m_4-1) f_1(m_3) + m_4(1-m_3) f_1(m_4)}{r\lambda_1(m_4-m_3)} \xi \overline{w}_2 + \frac{m_3 m_4 \{f_1(m_4) - f_1(m_3)\}}{\lambda_1(m_4-m_3)} a,
\end{aligned} \tag{23}$$



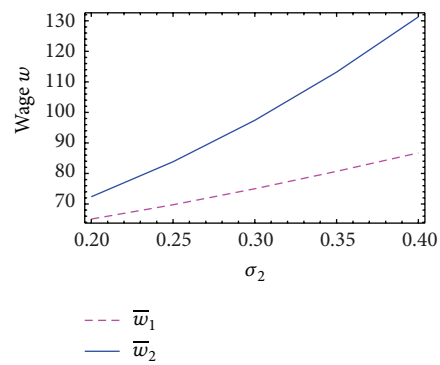
(a) Reservation wage with respect to λ_1 if $r = 0.03$, $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $Q_1 = 10$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $Q_2 = 15$, $\lambda_2 = 0.2$



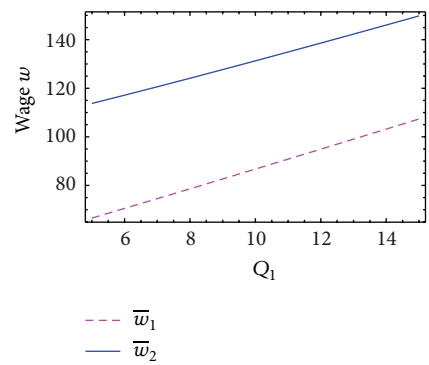
(b) Reservation wage with respect to λ_2 if $r = 0.03$, $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $Q_1 = 10$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $Q_2 = 15$



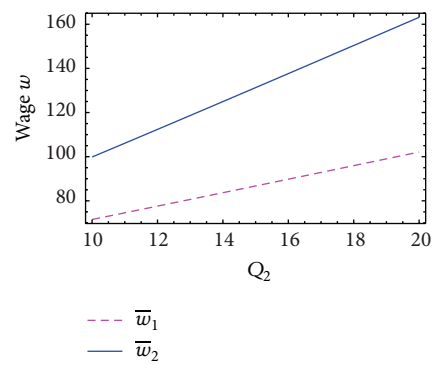
(c) Reservation wage with respect to σ_1 if $r = 0.03$, $\mu_1 = 0.02$, $Q_1 = 10$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $Q_2 = 15$, $\lambda_2 = 0.2$



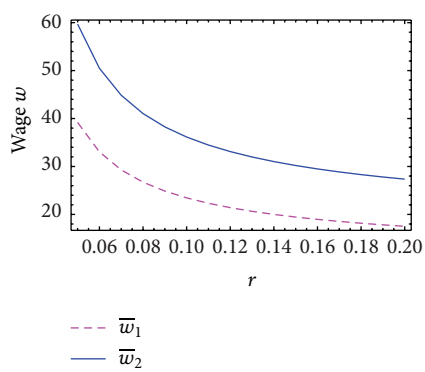
(d) Reservation wage with respect to σ_2 if $r = 0.03$, $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $Q_1 = 10$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $Q_2 = 15$, $\lambda_2 = 0.2$



(e) Reservation wage with respect to Q_1 if $r = 0.03$, $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $Q_2 = 15$, $\lambda_2 = 0.2$



(f) Reservation wage with respect to Q_2 if $r = 0.03$, $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $Q_1 = 10$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $\lambda_2 = 0.2$



(g) Reservation wage with respect to r if $\mu_1 = 0.02$, $\sigma_1 = 0.2$, $Q_1 = 10$, $\lambda_1 = 0.2$, $\mu_2 = 0.02$, $\sigma_2 = 0.4$, $Q_2 = 15$, $\lambda_2 = 0.2$

Figure 1: Numerical implications.

and plugging (16) into the RHS of (18) implies

$$\begin{aligned}
&(\text{RHS) of (18)} = n_1 \\
&\quad \cdot \frac{\left(\left(n_2-1\right)\left(r-\mu_2\right)/r\left(r-\mu_2+\lambda_2\right)\right)\overline{w}_2-n_2Q_2/\left(r+\lambda_2\right)}{\left(n_2-n_1\right)}\xi^{n_1} \\
&\quad + n_2\frac{\left(\left(1-n_1\right)\left(r-\mu_2\right)/r\left(r-\mu_2+\lambda_2\right)\right)\overline{w}_2+n_1Q_2/\left(r+\lambda_2\right)}{\left(n_2-n_1\right)} \\
&\quad \cdot \xi^{n_2} + \frac{\lambda_2}{r\left(r-\mu_2+\lambda_2\right)}\xi\overline{w}_2 \\
&\quad = \frac{1}{r\left(r-\mu_2+\lambda_2\right)}\left[\frac{\left(r-\mu_2\right)\left\{n_1\left(n_2-1\right)\xi^{n_1}+n_2\left(1-n_1\right)\xi^{n_2}\right\}}{n_2-n_1}\right. \\
&\quad \left. + \lambda_2\xi\right]\overline{w}_2 + \frac{n_1n_2Q_2\left(\xi^{n_2}-\xi^{n_1}\right)}{\left(r+\lambda_2\right)\left(n_2-n_1\right)},
\end{aligned} \tag{24}$$

with condition (21). From (23) and (24), we obtain

$$\overline{w}_2 = \frac{(m_3 m_4 \{f_1(m_4) - f_1(m_3)\}/\lambda_1(m_4 - m_3))a - (n_1 n_2 Q_2(\xi^{n_2} - \xi^{n_1})/(r+\lambda_2)(n_2 - n_1))}{(1/r(r-\mu_2+\lambda_2))[((r-\mu_2)\{n_1(n_2-1)\xi^{n_1} + n_2(1-n_1)\xi^{n_2}\}/(n_2-n_1)) + \lambda_2\xi] - ((m_3(m_4-1)f_1(m_3) + m_4(1-m_3)f_1(m_4))/r\lambda_1(m_4-m_3))\xi}. \quad (25)$$

From (22) and (25), we derive the following algebraic equation with respect to ξ

$$\frac{((m_3 f_1(m_4) - m_4 f_1(m_3))/\lambda_1(m_4-m_3))a + b - (Q_2/(r+\lambda_2))[((n_1\xi^{n_2} - n_2\xi^{n_1})/(n_2-n_1)) + 1]}{(1/r(r-\mu_2+\lambda_2))[((r-\mu_2)\{(n_2-1)\xi^{n_1} + (1-n_1)\xi^{n_2}\}/(n_2-n_1)) + \lambda_2\xi] - (((m_4-1)f_1(m_3) + (1-m_3)f_1(m_4))/r\lambda_1(m_4-m_3))\xi}$$
$$= \frac{(m_3 m_4 \{f_1(m_4) - f_1(m_3)\}/\lambda_1(m_4-m_3))a - (n_1 n_2 Q_2(\xi^{n_2} - \xi^{n_1})/(r+\lambda_2)(n_2-n_1))}{(1/r(r-\mu_2+\lambda_2))[((r-\mu_2)\{n_1(n_2-1)\xi^{n_1} + n_2(1-n_1)\xi^{n_2}\}/(n_2-n_1)) + \lambda_2\xi] - ((m_3(m_4-1)f_1(m_3) + m_4(1-m_3)f_1(m_4))/r\lambda_1(m_4-m_3))\xi}, \quad (26)$$

and if there exists a unique solution $\xi \in (0,1)$ to (26), we can sequentially determine $\overline{w}_2$, $\overline{w}_1$, A_3, A_4, C_1, and C_2. □

3. Numerical Examples

The drift term μ_{Θ_t} in the wage offer dynamics in (3) is the exponential growth rate of the wage offer and can be regarded as depending on an individual's skill and ability but not on labor market conditions or macroeconomic conditions. Therefore, we set $\mu_1 = \mu_2$ in our numerical examples. On the other hand, we use $\sigma_1 \leq \sigma_2$ and $Q_1 \leq Q_2$. That is, we assume that, in regime 2, schooling preference (net of education costs) and wage offer volatility are higher than those in regime 1.

In Figures 1(a) and 1(b), we plot the reservation wage $\overline{w}_i$ against the transition intensity λ_i. Figures 1(c) and 1(d) state that a higher wage offer volatility σ_i yields a higher reservation wage $\overline{w}_i$. These results are consistent with those of Fan [1] in the sense that the wage offer volatility yields an option value to schooling. An interesting result, however, is that σ_1 (resp., σ_2) has an impact on $\overline{w}_2$ (resp., $\overline{w}_1$) and the schooling decision in a regime is dependent on the wage offer volatility in the other regime. The role of schooling preference (net of education costs) is illustrated in Figures 1(e) and 1(f). We see that more preference in education provides an individual more incentive to postpone starting work. Again, the schooling decision in a regime is dependent on schooling preference (net of education costs) in the other regime. Lastly, an individual's opportunity cost of schooling increases with the discount rate and this is explored in Figure 1(g).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] C. Fan, "Schooling as a job search process," *Economics Letters*, vol. 41, no. 1, pp. 85–91, 1993.

[2] M. Binder, "Schooling indicators during Mexico's 'Lost decade,'" *Economics of Education Review*, vol. 18, no. 2, pp. 183–199, 1999.

[3] R. Gronau, "Information and frictional unemployment," *The American Economic Review*, vol. 61, pp. 290–301, 1971.

[4] A. Dixit and R. S. Pindyck, *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ, USA, 1994.

[5] X. Guo, J. Miao, and E. Morellec, "Irreversible investment with regime shifts," *Journal of Economic Theory*, vol. 122, no. 1, pp. 37–59, 2005.

[6] A. Bensoussan, Z. Yan, and G. Yin, "Threshold-type policies for real options using regime-switching models," *SIAM Journal on Financial Mathematics*, vol. 3, no. 1, pp. 667–689, 2012.