A Regime Switching Model of Schooling Choice as a Job Search Process

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1. Introduction

In the course of schooling as a job search process, the role of uncertainty that gives rise to an option was studied by Fan [1]. Fan [1] used an analytic formulation, in which the wage offer process \( w(t) \) was modeled as the following arithmetic Brownian motion:

\[
dw(t) = g dt + \sigma dB_t,
\]

\( w(0) = w > 0 \), \( (B_t)_{t \geq 0} \) is a standard Brownian motion under a given probability space \((\Omega, \mathcal{F}, \mathbb{P})\) and \((\Theta_t)_{t \geq 0} \) is a two-state Markov process defined on \((\Omega, \mathcal{F}, \mathbb{P})\) and can take value \(1\) or \(2\). We also assume that \((\Theta_t)_{t \geq 0}\) is independent of \((B_t)_{t \geq 0}\) and has the following generator:

\[
\Lambda = \begin{pmatrix}
-\lambda_1 & \lambda_1 \\
\lambda_2 & -\lambda_2
\end{pmatrix}, \quad \lambda_1 > 0, \quad \lambda_2 > 0.
\]

We propose a regime switching model of schooling choice as a job search process. We adopt a two-state Markov process and the derived coupled Bellman equations are solved by seeking the root of an auxiliary algebraic equation. Some numerical examples are also considered.
For each regime \(i, (i = 1, 2)\), \(\mu_i\) is a constant drift and \(\sigma_i\) is a constant volatility. We allow that schooling preference (net of education costs), denoted by \(Q_i = Q_{0i}\), is also regime switching.

We assume the following as in Fan [1] (see also verifications therein). An individual will stop schooling at time \(r\) to accept a wage offer, which will be his constant wage throughout the remainder of the individual’s infinite lifetime. Further, this stopping process is irreversible. Then an individual’s objective is to choose optimally the schooling duration \(\tau\) that maximizes the expected value \(V_i(w)\), which consists of the present value of lifetime earnings and schooling preference (net of education costs) for each regime \(i, (i = 1, 2)\).

\[
V_i(w) = \max_{i} \mathbb{E} \left[ \int_{0}^{\tau_i} e^{-\mu s} Q_{0i} ds + \int_{\tau_i}^{\infty} e^{-\mu s} w(\tau_i) ds \mid \Theta_0 = i, w(0) = w \right]
\]

where \(r\) is the discount rate and \(\tau_i\) can be considered as an optimal stopping time of education. We denote \(w(\tau_i) := \bar{w}_i\), \(w(\tau_i) := \bar{w}_2\) and we only consider the case where \(\bar{w}_1 < \bar{w}_2\).

\[\bar{w}_i\] is called the reservation wage in labor economics.

Remark 1. For each regime \(i, (i = 1, 2)\), we define the functions \(f_i(\cdot)\) and \(G(\cdot)\):

\[
f_i(m) = -\frac{1}{2} \sigma_i^2 m (m - 1) - \mu m + (r + \lambda_i),
\]

\[
G(m) = f_1(m) f_2(m) - \lambda_1 \lambda_2.
\]

Let \(n_1 < 0\) and \(n_2 > 0\) be two real roots of the quadratic equation \(f_2(n) = 0\). We also consider the quartic equation \(G(m) = 0\), which has four real roots \(m_1 < m_2 < 0 < m_3 < m_4\).

Theorem 2. Under assumption (6), for each regime \(i, (i = 1, 2)\), the value function \(V_i(\cdot)\) is derived as follows:

\[
V_1(w) = \begin{cases} 
\frac{A_3 w^{m_3} + A_4 w^{m_4} + (r + \lambda_2) Q_1 + \lambda_1 Q_2}{\lambda_1} & \text{for } 0 < w < \bar{w}_1, \\
\frac{f_1(m_3)}{\lambda_1} A_3 w^{m_3} + \frac{f_1(m_4)}{\lambda_2} A_4 w^{m_4} + \frac{Q_2}{r + \lambda_2} w & \text{for } \bar{w}_1 \leq w < \bar{w}_2,
\end{cases}
\]

\[
V_2(w) = \begin{cases} 
\frac{f_1(m_3)}{\lambda_1} A_3 w^{m_3} + \frac{f_1(m_4)}{\lambda_2} A_4 w^{m_4} + \frac{Q_2}{r + \lambda_2} w & \text{for } 0 < w < \bar{w}_1, \\
\frac{C_1 w^{m_1} + C_2 w^{m_2} + Q_2}{r - \mu_2 + \lambda_2} w & \text{for } \bar{w}_1 \leq w < \bar{w}_2,
\end{cases}
\]

The values of \(A_3, A_4, C_1, C_2, \bar{w}_1, \) and \(\bar{w}_2\) are expanded in the proof.

Proof. From (5), for \(0 < w < \bar{w}_1\), we obtain the following coupled Bellman equations:

\[
\mathcal{L}_1 V_1(w) - Q_1 = \lambda_1 V_1(w),
\]

\[
\mathcal{L}_2 V_2(w) - Q_2 = \lambda_2 V_1(w),
\]

where the differential operator \(\mathcal{L}_i\) is given by

\[
\mathcal{L}_i = -\frac{1}{2} \sigma_i^2 w^2 \frac{\partial^2}{\partial w^2} - \mu_i w \frac{\partial}{\partial w} + (r + \lambda_i) w + \lambda_i w^2.
\]

for each regime \(i, (i = 1, 2)\). Thus the solutions to (9) are of the forms

\[
V_1(w) = A_3 w^{m_3} + A_4 w^{m_4} + a, \\
V_2(w) = \frac{f_1(m_3)}{\lambda_1} A_3 w^{m_3} + \frac{f_1(m_4)}{\lambda_2} A_4 w^{m_4} + b,
\]

where

\[
a = \frac{(r + \lambda_2) Q_1 + \lambda_1 Q_2}{(r + \lambda_1)(r + \lambda_2) - \lambda_1 \lambda_2},
\]

\[
b = \frac{(r + \lambda_1) Q_2 + \lambda_2 Q_1}{(r + \lambda_1)(r + \lambda_2) - \lambda_1 \lambda_2}.
\]

For \(\bar{w}_1 \leq w < \bar{w}_2\), we have the equation

\[
\mathcal{L}_2 V_2(w) - Q_2 = \lambda_2 w,
\]
since $V_1(w) = w/r$. Thus the solution to (13) is of the form

$$V_2(w) = C_1 w^{n_1} + C_2 w^{n_2} + \frac{\lambda_2}{r (r - \mu_2 + \lambda_2)} w$$

(14)

Now we determine $A_1, A_2, C_1, C_2, \overline{w}_1,$ and $\overline{w}_2$ using the $C^1$-conditions of $V_1$ and $V_2$ at $w = \overline{w}_1$ and $w = \overline{w}_2$. The $C^1$-condition of $V_1(w)$ at $w = \overline{w}_1$ implies

$$A_3 = \frac{(1/r) (m_4 - 1) \overline{w}_1 - am_4}{(m_4 - m_3) \overline{w}_1^m},$$

$$A_4 = \frac{(1/r) (1 - m_3) \overline{w}_1 + am_3}{(m_4 - m_3) \overline{w}_1^m}.$$  

(15)

The $C^1$-condition of $V_2(w)$ at $w = \overline{w}_2$ yields

$$C_1 = \frac{((n_2 - 1) (r - \mu_2) / r (r - \mu_2 + \lambda_2)) \overline{w}_2 - n_2 Q_2 / (r + \lambda_2)}{(n_2 - n_1) \overline{w}_2^m},$$

$$C_2 = \frac{((1 - n_1) (r - \mu_2) / r (r - \mu_2 + \lambda_2)) \overline{w}_2 + n_1 Q_2 / (r + \lambda_2)}{(n_2 - n_1) \overline{w}_2^m}.$$  

(16)

From the $C^1$-condition of $V_2(w)$ at $w = \overline{w}_1$ we obtain the equations

$$\frac{f_i (m_3)}{\lambda_1} A_3 \overline{w}_1^m + \frac{f_i (m_4)}{\lambda_1} A_4 \overline{w}_1^m + b = C_1 \overline{w}_1 + C_2 \overline{w}_1 + \frac{\lambda_2}{r (r - \mu_2 + \lambda_2)} \overline{w}_1 + \frac{Q_2}{r + \lambda_2},$$

$$\frac{m_3 f_i (m_3)}{\lambda_1} A_3 \overline{w}_1^m + \frac{m_4 f_i (m_4)}{\lambda_1} A_4 \overline{w}_1^m = n_1 C_1 \overline{w}_1 + n_2 C_2 \overline{w}_1 + \frac{\lambda_2}{r (r - \mu_2 + \lambda_2)} \overline{w}_1.$$  

(17)

(18)

Plugging (15) into the LHS of (17) implies

$$\text{(LHS of (17)}$$

$$= \frac{f_i (m_3) (1/r) (m_4 - 1) \overline{w}_1 - am_4}{(m_4 - m_3)}$$

$$+ \frac{f_i (m_4) (1/r) (1 - m_3) \overline{w}_1 + am_3 + b}{(m_4 - m_3)}$$

$$= \frac{(m_4 - 1) f_i (m_3) + (1 - m_3) f_i (m_4) \xi \overline{w}_2}{r \lambda_1 (m_4 - m_3)}$$

$$+ \frac{m_3 f_i (m_4) - m_4 f_i (m_3)}{\lambda_1 (m_4 - m_3)} a + b,$$  

(19)

and plugging (16) into the RHS of (17) implies

$$\text{(RHS of (17)}$$

$$= \frac{((n_2 - 1) (r - \mu_2) / r (r - \mu_2 + \lambda_2)) \overline{w}_2 - n_2 Q_2 / (r + \lambda_2)}{(n_2 - n_1)},$$

$$+ \frac{((1 - n_1) (r - \mu_2) / r (r - \mu_2 + \lambda_2)) \overline{w}_2 + n_1 Q_2 / (r + \lambda_2)}{(n_2 - n_1)}$$

$$+ \frac{\lambda_2}{r (r - \mu_2 + \lambda_2)} \xi \overline{w}_2 + \frac{Q_2}{r + \lambda_2}$$

$$= \frac{1}{r (r - \mu_2 + \lambda_2)} \left[ \frac{(r - \mu_2) [(n_2 - 1) \xi^{n_2} + (1 - n_1) \xi^{n_1}]}{n_2 - n_1} + \right.$$

$$+ \left. \lambda_2 \xi \right] \overline{w}_2 + \frac{Q_2}{r + \lambda_2} \left[ \frac{n_1 \xi^{n_1} - n_2 \xi^{n_2}}{n_2 - n_1} + 1 \right],$$

with the relationship

$$\overline{w}_1 = \xi \overline{w}_2, \quad \xi \in (0, 1).$$  

(20)

From (19) and (20), we obtain

$$\overline{w}_2 = \frac{((m_3 f_i (m_4) - m_4 f_i (m_3)) / \lambda_1 (m_4 - m_3)) a + b - (Q_2 / (r + \lambda_2)) [(n_2 \xi^{n_2} - n_1 \xi^{n_1}) / (n_2 - n_1) + 1]}

(21)

$$= \frac{1 / r (r - \mu_2 + \lambda_2)) [(r - \mu_2) [(n_2 - 1) \xi^{n_2} + (1 - n_1) \xi^{n_1}] / (n_2 - n_1) + \lambda_2 \xi] - (((m_4 - 1) f_i (m_3) + (1 - m_3) f_i (m_4)) / r \lambda_1 (m_4 - m_3)) \xi -}

+ \frac{m_3 f_i (m_4) (1/r) (1 - m_3) \overline{w}_1 + am_3}{(m_4 - m_3)}$$

$$= \frac{m_3 (m_4 - 1) f_i (m_3) + m_4 (1 - m_3) f_i (m_4) \xi \overline{w}_2}{r \lambda_1 (m_4 - m_3)}$$

$$+ \frac{m_3 m_4 \left[ f_i (m_4) - f_i (m_3) \right]}{\lambda_1 (m_4 - m_3)} a,$$  

(22)

(23)
and plugging (16) into the RHS of (18) implies

\[(\text{RHS of (18)} = n_1 \]

\[
(\frac{(n_2-1) (r-\mu_2) / (r-\mu_2 + \lambda_2)}{(n_2-n_1)}) \overline{w_2} - n_2 Q_2 / (r + \lambda_2) \xi_{n_1} + n_2 \left[ (1-n_1) (r-\mu_2) / (r-\mu_2 + \lambda_2) \overline{w_2} + n_1 Q_2 / (r+\lambda_2) \right] + \overline{w_2} + \frac{n_1 n_2 Q_2 (\xi_{n_1} - \xi_{n_2})}{(r + \lambda_2) (n_2 - n_1)}, \]

(24)
with condition (21). From (23) and (24), we obtain
\[ w^2 = \frac{\left( m_3 m_4 f_1(m_4) - f_1(m_3) \right) a - \left( n_1 n_2 Q_2 (\xi^n - \xi^n) \right) / (r + \lambda_1 (m_4 - m_3))}{\left( 1 / r - \mu_2 + \lambda_3 \right) \left( \left[ (r - \mu_2) / (n_1 (n_2 - 1) \xi^n + n_2 (1 - n_1) \xi^n) / (n_2 - n_1) \right] + \lambda_2 \xi \right)} - \left( 1 / r - \mu_2 + \lambda_3 \right) \left[ \left( m_1 m_2 f_1(m_3) / \lambda_1 (m_4 - m_3) \right) a - \left( n_1 n_2 Q_2 (\xi^m - \xi^m) \right) / (r + \lambda_1 (m_4 - m_3)) \right] \xi \]
\[ (25) \]

From (22) and (25), we derive the following algebraic equation with respect to \( \xi \)
\[ ((m_3 f_1(m_4) - m_3 f_1(m_3)) / \lambda_1 (m_4 - m_3)) a + b - (Q_2 / (r + \lambda_3)) \left[ \left( n_1 n_2 Q_2 (\xi^m - \xi^m) / (n_2 - n_1) \right) + 1 \right] \]
\[ (1 / r - \mu_2 + \lambda_3) \left[ (r - \mu_2) / (n_2 - 1) \xi^n + (1 - n_1) \xi^n) / (n_2 - n_1) \right] + \lambda_2 \xi \right) - \left( (m_1 m_2 f_1(m_3) / \lambda_1 (m_4 - m_3)) a - (n_1 n_2 Q_2 (\xi^m - \xi^m) / (r + \lambda_1 (m_4 - m_3)) ) \right] \xi \]
\[ = \left( 1 / r - \mu_2 + \lambda_3 \right) \left( (r - \mu_2) / (n_1 (n_2 - 1) \xi^n + n_2 (1 - n_1) \xi^n) / (n_2 - n_1) \right) + \lambda_2 \xi \right) - \left( (m_1 (m_4 - 1) f_1(m_4) + m_4 (1 - m_4) f_1(m_4) / \lambda_1 (m_4 - m_3)) \xi \right) \]
\[ (26) \]

and if there exists a unique solution \( \xi \in (0, 1) \) to (26), we can sequentially determine \( \bar{w}_2, \bar{w}_1, A_3, A_4, C_1, \) and \( C_2 \).

3. Numerical Examples

The drift term \( \mu_0 \) in the wage offer dynamics in (3) is the exponential growth rate of the wage offer and can be regarded as depending on an individual’s skill and ability but not on labor market conditions or macroeconomic conditions. Therefore, we set \( \mu_1 = \mu_2 \) in our numerical examples. On the other hand, we use \( \sigma_1 \leq \sigma_2 \) and \( Q_1 \leq Q_2 \). That is, we assume that, in regime 2, schooling preference (net of education costs) and wage offer volatility are higher than those in regime 1.

In Figures 1(a) and 1(b), we plot the reservation wage \( \bar{w}_1 \) against the transition intensity \( \lambda_1 \). Figures 1(c) and 1(d) state that a higher wage offer volatility \( \sigma_1 \) yields a higher reservation wage \( \bar{w}_1 \). These results are consistent with those of Fan [1] in the sense that the wage offer volatility yields an option value to schooling. An interesting result, however, is that \( \sigma_1 \) (resp., \( \sigma_2 \)) has an impact on \( \bar{w}_2 \) (resp., \( \bar{w}_1 \)) and the schooling decision in a regime is dependent on the wage offer volatility in the other regime. The role of schooling preference (net of education costs) is illustrated in Figures 1(e) and 1(f). We see that more preference in education provides an individual more incentive to postpone starting work. Again, the schooling decision in a regime is dependent on schooling preference (net of education costs) in the other regime. Lastly, an individual’s opportunity cost of schooling increases with the discount rate and this is explored in Figure 1(g).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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