Research Article

Existence of Electrically Charged Structures with Regular Center in Nonlinear Electrodynamics Minimally Coupled to Gravity

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We address the question of correct description of Lagrange dynamics for regular electrically charged structures in nonlinear electrodynamics coupled to gravity. Regular spherically symmetric configuration satisfying the weak energy condition has obligatory de Sitter center in which the electric field vanishes while the energy density of electromagnetic vacuum achieves its maximal value. The Maxwell weak field limit $\mathcal{L}(F) \rightarrow F$ as $r \rightarrow \infty$ requires vanishing electric field at infinity. A field invariant F evolves between two minus zero in the center and at infinity which makes a Lagrangian $\mathcal{L}(F)$ with nonequal asymptotic limits inevitably branching. We formulate the appropriate nonuniform variational problem including the proper boundary conditions and present the example of the spherically symmetric Lagrangian describing electrically charged structure with the regular center.

1. Introduction

The Abraham-Lorentz concept of a finite size electron which makes the total energy of the electron Coulomb field finite [1] encountered the problem of preventing the electron from flying apart under the Coulomb repulsion. Theories based on assumptions about the charge distribution were compelled to introduce cohesive forces of nonelectromagnetic origin (Poincaré stress), testifying that replacing a point charge with an extended one is impossible within the Maxwell electrodynamics since it demands introducing cohesive non-electromagnetic forces [2].

Further search for the electron models was motivated by discovery of the Kerr-Newman solution to the Einstein-Maxwell equations [3]:

$$ds^2 = -dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{(2mr - e^2)}{\Sigma} \cdot (dt - a \sin^2 \theta d\phi)^2 + (r^2 + a^2) \sin^2 \theta d\phi^2,$$

$$\begin{aligned} A_i &= -\frac{er}{\Sigma} [1; 0, 0, -a \sin^2 \theta], \\ \Sigma &= r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2mr + a^2 + e^2, \end{aligned} \quad (1)$$

where A_i is the associated electromagnetic potential.

Carter has found that the parameter a couples with the mass m and charge e independently, giving the angular momentum $J = ma$, an asymptotic magnetic momentum $\mu = ea$, and the same gyromagnetic ratio as predicted by the Dirac equation [4]. This suggested that the electron might be classically visualized as a massive charged source of the Kerr-Newman field [5].

Carter discovered also the nontrivial causality violation in the Kerr-Newman geometry in the case of a charged particle, when there are no Killing horizons, the manifold is geodesically complete (except for geodesics which reach the singularity), and any point can be connected to any other point by both a future and a past directed timelike curve. Closed timelike curves can extend over the whole space and cannot be removed by taking a covering space [4].

The Kerr-Newman solution belongs to the Kerr family of solutions to the source-free Einstein-Maxwell equations [4]. The source models for the Kerr-Newman exterior fields, involving a screening or covering causally dangerous region and Poincaré stress of different origins, can be roughly divided into disk-like [6–9], shell-like [5, 10–12], bag-like [13–20], and string-like ones ([21, 22] and references therein). The problem of matching the Kerr-Newman exterior to a rotating material source does not have a unique solution, since one is free to choose arbitrarily the boundary between the exterior and the interior [6] as well as an interior model.

On the other hand, this question can be addressed in the frame of the model-independent mathematical approach [23], which was applied for introducing a quasilocal energy as dictated by employing a Hamilton-Jacobi analysis of the action functional rather than postulating quasilocal energy properties and then searching for a suitable expression for it [24].

In the spirit of this approach one can study equations describing a dynamical system to get information about its basic features. In the case of an electromagnetically interacting structure the appropriate equations come from nonlinear electrodynamics coupled to gravity (NED-GR).

NED theories appear as low-energy effective limits in certain models of string/M-theories [25–27]. Nonlinear electrodynamics was proposed by Born and Infeld in 1934 as founded on two basic principles: to consider electromagnetic field and particles within the frame of one physical entity which is electromagnetic field and to avoid letting physical quantities become infinite [28]. Electromagnetic energy was obtained finite by introducing an upper limit on the electric field related to the electron finite radius, but geometry remained singular [28].

Two basic points formulated by Born and Infeld can be realized in nonlinear electrodynamics coupled to gravity. Source-free NED-GR equations admit the class of regular causally safe axially symmetric asymptotically Kerr-Newman solutions which describe electromagnetic solitons representing spinning particles [29].

Theorems of nonexistence in NED-GR of electrically charged spherically symmetric structures with a regular center impose the condition of the Maxwell weak field limit in the center [30]. However, regular electrically charged structures can exist without the Maxwell center. The key point is that for any gauge-invariant Lagrangian $\mathcal{L}(F)$ stress-energy tensor of a spherically symmetric electromagnetic field has the algebraic structure such that

$$T_t^t = T_r^r. \quad (2)$$

Regular spherically symmetric solutions with stress-energy tensors from the class (2) satisfying the weak energy condition (nonnegativity of density as measured by any local observer [31]) have obligatory de Sitter center [32–35]. In the NED-GR regular solutions [29, 36] interior de Sitter vacuum provides a needed cohesive force (an appearance of repulsive gravity was noted in the early electron models ([37] and references therein)) and proper cut-off on the field self-interaction divergent for a point charge [36, 38].

The regular spherical solutions generated by (2) belong to the Kerr-Schild class [19, 39, 40] and can be transformed by the Gürses-Gürsey algorithm [41] into regular axially symmetric asymptotically Kerr-Newman solutions describing electrically charged black holes and solitons. De Sitter center transforms into the interior de Sitter disk which behaves like perfect conductor and ideal diamagnetic and displays superconducting behaviour within a single spinning charged structure [29]. The mass of an object is the finite positive electromagnetic mass generically related to interior de Sitter vacuum and breaking of space-time symmetry from the de Sitter group in the origin [29, 34, 36, 42, 43].

The price for such a promising road is unavoidable branching of a spherically symmetric Lagrangian for regular electrically charged NED-GR configurations.

The generic feature of any regular electromagnetic spherically symmetric structure satisfying the weak energy condition is the de Sitter center where Lagrangian takes the value $\mathcal{L} = 2\rho(0)$ as $r \rightarrow 0$ [36]. Regularity requires $F \rightarrow -0$ as $r \rightarrow 0$ [30, 36]. In the limit $r \rightarrow \infty$ Lagrangian follows the Maxwell weak field limit, $\mathcal{L}(F) \rightarrow F \rightarrow -0$. Electromagnetic invariant F evolves thus between minus zeros at the center and at infinity with at least one minimum in between. Nonmonotonic behaviour of the invariant F leads unavoidably to branching of any spherically symmetric gauge-invariant Lagrangian $\mathcal{L}(F)$ [30, 36].

Regular spherically symmetric solutions with the nonzero electric charge [36, 44–47] were found with using the alternative P -form of nonlinear electrodynamics obtained from the standard Lagrangian F -form by the Legendre transformation [48]. F - P duality turns into electric-magnetic duality in the Maxwell limit but in general case it connects different theories [30] which is clearly manifested by branching of a Lagrangian in the F frame.

The aim of this paper is to formulate the correct variational problem of the Lagrange dynamics for the regular electrically charged spherically symmetric NED-GR configurations.

The paper is organized as follows. In Section 2 we present the basic equations and in Section 3 we outline the generic features of regular electrically charged configurations. Section 4 is devoted to the variational problem of the Lagrange dynamics. In Section 5 we present the particular Lagrangian responsible for the regular electrically charged solution [36]. In Section 6 we summarize and discuss the results. Appendix contains the detailed consideration concerning definition of the stress-energy tensor for an electromagnetic field.

2. Basic Equations

In nonlinear electrodynamics minimally coupled to gravity, the action is given (in geometrical units $c = G = 1$) by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R - \mathcal{L}(F)]; \quad F = F_{\mu\nu} F^{\mu\nu}. \quad (3)$$

Here R is the scalar curvature, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field. The gauge-invariant electromagnetic

Lagrangian $\mathcal{L}(F)$ is an arbitrary function of F which should have the Maxwell limit, $\mathcal{L} \rightarrow F$, $\mathcal{L}_F = d\mathcal{L}/dF \rightarrow 1$ in the weak field regime.

Action (3) gives the dynamic field equations

$$\nabla_\mu (\mathcal{L}_F F^{\mu\nu}) = 0. \quad (4)$$

The Bianchi identities yield

$$\nabla_\mu {}^* F^{\mu\nu} = 0. \quad (5)$$

An asterisk denotes the Hodge dual [49]; the antisymmetric unit tensor is chosen in such a way that $\eta_{0123} = \sqrt{-g}$.

The Einstein equations can be written in the form [30]

$$\begin{aligned} G^\mu_\nu &= -T^\mu_\nu = 2\mathcal{L}_F F_{\nu\beta} F^{\mu\beta} - \frac{1}{2}\delta^\mu_\nu \mathcal{L} \\ &= \frac{1}{2} \text{diag}(\mathcal{L} - 2F\mathcal{L}_F, \mathcal{L} - 2F\mathcal{L}_F, \mathcal{L}, \mathcal{L}). \end{aligned} \quad (6)$$

The definition of $T_{\mu\nu}$ in (6) differs from the standard definition (see, e.g., [49, 50]) by 8π , so that $T_{t(\text{here})}^t = 8\pi\rho$ and so forth.

The density and pressures for electrically charged structures are given by

$$\rho = -p_r = \left[\frac{1}{2}\mathcal{L} - F\mathcal{L}_F \right]; \quad p_\perp = -\frac{1}{2}\mathcal{L}. \quad (7)$$

It follows from (7) that a spherically symmetric electromagnetic field with an arbitrary gauge-invariant Lagrangian $\mathcal{L}(F)$ has stress-energy tensor with the algebraic structure (2).

Symmetry of source term (2) leads to the metric

$$ds^2 = f(r) dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega^2, \quad (8)$$

where $d\Omega^2$ is the line element on a unit sphere. The metric function and mass function are given by

$$f(r) = 1 - \frac{2\mathcal{M}(r)}{r}; \quad \mathcal{M}(r) = \frac{1}{2} \int_0^r \rho(x) x^2 dx. \quad (9)$$

Radial pressure satisfies, in accordance with (2), $p_r = -\rho$, and tangential pressure is given by

$$p_\perp = -\rho - \frac{r}{2}\rho'. \quad (10)$$

Metric (9) has the de Sitter asymptotic for $r \rightarrow 0$ which directly follows from (2)

$$f(r) = 1 - \frac{\Lambda}{3}r^2; \quad \Lambda = \rho(0); \quad p_\perp = -\rho \quad (11)$$

and the Reissner-Nordström asymptotic for $r \rightarrow \infty$ dictated by the Maxwell limit

$$f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}. \quad (12)$$

In the spherically symmetric case the only essential component of $F_{\mu\nu}$ describing a radial electric field is $F_{01} = -E(r)$. Dynamical equations (4) yield [30]

$$r^2 \mathcal{L}_F F^{01} = q, \quad (13)$$

where q is constant of integration identified as an electric charge by asymptotic behavior in the weak field limit. The field invariant F is given by

$$F = 2F_{01}F^{01} = -\frac{2q^2}{\mathcal{L}_F^2 r^4}. \quad (14)$$

Equation (7) implies

$$p_\perp + \rho = -F\mathcal{L}_F. \quad (15)$$

The weak energy condition requires $\rho \geq 0$ and $p_\perp + \rho \geq 0$ [31] which imposes two general constraints on the Lagrangian $\mathcal{L}(F)$. Equations (7) and (15) lead to

$$\mathcal{L}_F \geq 0, \quad (16)$$

$$\mathcal{L}(F) \geq 2F\mathcal{L}_F. \quad (17)$$

3. Generic Features of Field Dynamics

The theorem of nonexistence in NED-GR of electrically charged configurations with the regular center requires the Maxwell behavior at the regular center, $\mathcal{L} \rightarrow F$, $\mathcal{L}_F \rightarrow 1$ as $F \rightarrow 0$ [30]. The proof is that regularity of stress-energy tensor requires $|F\mathcal{L}_F| < \infty$ as $r \rightarrow 0$, while $F\mathcal{L}_F^2 \rightarrow -\infty$ by virtue of (14); it follows that $\mathcal{L}_F \rightarrow \infty$, while $F \rightarrow 0$, which is strongly non-Maxwell behavior.

Strictly speaking, this theorem tells us that a regular electrically charged structure cannot exist with the Maxwell weak field limit $\mathcal{L} \rightarrow F$ as $r \rightarrow 0$.

However, a regular electromagnetic structure can exist without the Maxwell limit in the centre [36]. Regularity of a stress-energy tensor of an electromagnetic field requires $T_t^t = \rho < \infty$. Algebraic structure of a stress-energy tensor (2) leads to the existence of de Sitter center in regular solutions [34], where electromagnetic density ρ achieves its maximum, since the weak energy condition requires, by virtue of (10), $\rho' \leq 0$, and one cannot expect validity of the weak field limit in the region of the maximal density. As a result one obtains a regular electrically charged spherically symmetric structure with the de Sitter center obligatory for all solutions of class (2), in which field tension must go to zero (by requirement of regularity), while the energy density of the electromagnetic vacuum T_t^t achieves its maximal finite value which represents the de Sitter cut-off on the self-energy density [36].

Requirement of regularity demands inevitably $F \rightarrow -0$ and $F\mathcal{L}_F^2 \rightarrow -\infty$ [30, 36]. De Sitter asymptotic implies $p_\perp + \rho = 0$ which leads to $F\mathcal{L}_F = 0$ at $r = 0$. It follows, with taking into account (16) and (14), $\mathcal{L}_F \rightarrow +\infty$ when $r \rightarrow 0$. Conditions for a Lagrangian in the regular center where $F \rightarrow -0$ are thus

$$\mathcal{L}(0) = 2\rho(0), \quad (18)$$

$$\mathcal{L}_F \rightarrow \infty \quad \text{as } r \rightarrow 0.$$

FIGURE 1: Typical behavior of a Lagrangian. Cusp corresponds to $F' = 0$.

The Maxwell asymptotic at $r \rightarrow \infty$ imposes two conditions on Lagrangian in the limit $F \rightarrow -0$:

$$\mathcal{L} \rightarrow F \rightarrow -0, \quad (19)$$

$$\mathcal{L}_F \rightarrow 1. \quad (20)$$

Regularity requires $F \rightarrow -0$ as $r \rightarrow 0$, while the Maxwell limit requires $F \rightarrow -0$ as $r \rightarrow \infty$. The invariant F is not a monotonic function of r evolving between two minus zeros and having at least one minimum in between. It leads unavoidably to branching of a Lagrangian [30, 36]. Lagrangian on its way from (18) to (19) must change its sign. Additional branching would require additional change of the sign. According to (7), the sign of a Lagrangian is opposite to the sign of tangential pressure which can vanish only once for the case of one de Sitter vacuum scale [34] (for the case of several vacuum scales see [51]); so in the considered case we deal with two branches of a Lagrangian. Equation (14) defines in the first approximation the derivative $d\mathcal{L}_F/dF$ in the minimum $r = r_c$ of the invariant F by

$$\mathcal{L}_{FF} = -\frac{2\mathcal{L}_F}{(F'r_c)}. \quad (21)$$

In accordance with (16), $\mathcal{L}_F$ has the same finite limit as $F \rightarrow F_c + 0$ and $F \rightarrow F_c - 0$, while F' changes its sign in the minimum, so that $\mathcal{L}_{FF}$ tends to infinities of opposite signs. Therefore a Lagrangian $\mathcal{L}(F)$ has the cusp at $F = F_c$.

The Lagrangian $\mathcal{L}(F)$ is a monotonic function of F ; when the weak energy condition is satisfied, $\mathcal{L}_F \geq 0$ according to (16). A monotonic function $\mathcal{L}(F)$ first decreases smoothly along the first branch from its maximal value $\mathcal{L}(0) = 2\rho(0)$ to its value in the cusp $\mathcal{L}_c$ as F decreases from $F = -0$ at $r = 0$ to $F_{\min} = F_c$; then the Lagrangian increases along the second branch from its minimal value $\mathcal{L}_c < 0$ to its Maxwell limit $\mathcal{L} \rightarrow F \rightarrow -0$ as F increases from F_c to $F \rightarrow -0$ as $r \rightarrow \infty$. Typical behavior of the Lagrangian as a function of F is shown in Figure 1.

On the other hand, a dynamical system represents a physical object existing in the space within the range $0 \leq r \leq \infty$.

So we can present a Lagrangian in the parametric form combining $\mathcal{L}(F)$ with $F(r)$. Behaviour of Lagrangian as a function of r , according to (7), follows the behavior of tangential pressure up to the opposite sign, $\mathcal{L} = -2p_\perp$. The Lagrangian r -derivative $\mathcal{L}' = \mathcal{L}_F F'$ is finite everywhere and vanishes in the minimum of the invariant $F(r)$. Value $\mathcal{L}_F$ is finite and nonzero there by (15), since $p_\perp + \rho$ vanishes only at the center and infinity.

Second derivative is given by

$$\frac{d^2 \mathcal{L}}{dr^2} = \frac{d\mathcal{L}_F}{dr} F' + \mathcal{L}_F F''. \quad (22)$$

It is finite and positive in the extremum at $r = r_c$; hence Lagrangian achieves its minimum in the minimum of the invariant F . At the same time it is the maximum of the tangential pressure. We find that the invariant F and Lagrangian $\mathcal{L}(r)$ achieve their minimum at the surface $r = r_c$ where $p_\perp(r)$ has its maximum.

As a function of r , Lagrangian does not demonstrate a branching. Its r -derivative is negative in the region $0 \leq r < r_c$ and positive in the region $r_c < r < \infty$. Starting from its maximal value $\mathcal{L}(0)$, the Lagrangian decreases to its minimum $\mathcal{L}_c$ and then increases to $\mathcal{L} = -0$ as $r \rightarrow \infty$.

Parametric presentation of a Lagrangian is admissible but not enough for studying field dynamics described by the dependence $\mathcal{L}(F)$. Branching of Lagrangian as a function of F enforces introduction of two Lagrangians. In terms of the variational calculus we deal with nonuniform variational problem typically applied for the case of two media. In the next section we apply this approach for actually one medium, nonlinear electromagnetic field, divided in two parts by nonmonotonic behaviour of the field invariant F and derive proper conditions on their boundary $r = r_c$ where F has the minimum.

4. Variational Problem

Nonuniform variational problem in our case is presented by the action

$$\begin{aligned} S &= S_{\text{int}} + S_{\text{ext}} \\ &= \frac{1}{16\pi} \left[\int_{\Omega_{\text{int}}} (R - \mathcal{L}_{\text{int}}(F)) \sqrt{-g} d^4x \right. \\ &\quad \left. + \int_{\Omega_{\text{ext}}} (R - \mathcal{L}_{\text{ext}}(F)) \sqrt{-g} d^4x \right]. \end{aligned} \quad (23)$$

Asymptotic behavior of the electromagnetic Lagrangians is given by

$$\begin{aligned} \mathcal{L}_{\text{int}}(F) &\rightarrow 2\rho_0; \quad \mathcal{L}_{F(\text{int})} \rightarrow +\infty \\ &\text{as } F \rightarrow -0 \quad \text{at } r \rightarrow 0, \\ \mathcal{L}_{\text{ext}}(F) &\rightarrow F; \quad \mathcal{L}_{F(\text{ext})} \rightarrow 1 \\ &\text{as } F \rightarrow -0 \quad \text{at } r \rightarrow \infty. \end{aligned} \quad (24)$$

Each part of the action includes the Hilbert action S_G and the matter (electromagnetic) action S_M . In the case of a minimal coupling, S_G does not depend on a matter field. Variation

of the total action with respect to electromagnetic variables yields the same dynamical field equations as variation of S_M only.

Each part of the manifold, Ω_{int} and Ω_{ext} is confined by the initial and final spacelike hypersurfaces t_{in} and t_{fin} , by the timelike 3-surface Σ_3 extended to infinity where electromagnetic field vanishes and by the timelike internal common boundary between Ω_{int} and Ω_{ext} denoted as Σ_c . This surface is not arbitrary but distinguished by the extremum of the field invariant F . Our task is to find the proper conditions on the internal boundary surface Σ_c to guarantee satisfying the dynamical and the Einstein equations.

Dynamical variables in the Lagrangian $\mathcal{L}(F)$ are electromagnetic potentials A_μ . At the surfaces t_{in} and t_{fin} their variations should satisfy $\delta A_\mu = 0$.

Electromagnetic tensor $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = -F_{\nu\mu}$ satisfies $F_{\mu\nu}\delta F^{\mu\nu} = F^{\mu\nu}\delta F_{\mu\nu}$. Variation of S_M reduces to

$$\begin{aligned} \delta S_M = & \frac{1}{4\pi} \int_{\Omega_{\text{int}}} \mathcal{L}_{F(\text{int})} F_{\text{int}}^{\mu\nu} \frac{\partial}{\partial x^\nu} \delta A_{\mu(\text{int})} \sqrt{-g} d\Omega \\ & + \frac{1}{4\pi} \int_{\Omega_{\text{ext}}} \mathcal{L}_{F(\text{ext})} F_{\text{ext}}^{\mu\nu} \frac{\partial}{\partial x^\nu} \delta A_{\mu(\text{ext})} \sqrt{-g} d\Omega. \end{aligned} \quad (25)$$

Integrating by parts with using the Gauss theorem we get

$$\begin{aligned} \delta S_M = & \frac{1}{4\pi} \int_{\Sigma_{\text{int}}} \mathcal{L}_{F(\text{int})} F_{\text{int}}^{\mu\nu} \delta A_{\mu(\text{int})} \sqrt{-g} d\sigma \\ & + \frac{1}{4\pi} \int_{\Sigma_{\text{ext}}} \mathcal{L}_{F(\text{ext})} F_{\text{ext}}^{\mu\nu} \delta A_{\mu(\text{ext})} \sqrt{-g} d\sigma \\ & + \frac{1}{4\pi} \int_{\Omega_{\text{int}}} (\mathcal{L}_{F(\text{int})} \nabla_\nu F_{\text{int}}^{\mu\nu}) \delta A_{\mu(\text{int})} \sqrt{-g} d\Omega \\ & + \frac{1}{4\pi} \int_{\Omega_{\text{ext}}} (\mathcal{L}_{F(\text{ext})} \nabla_\nu F_{\text{ext}}^{\mu\nu}) \delta A_{\mu(\text{ext})} \sqrt{-g} d\Omega. \end{aligned} \quad (26)$$

$\delta S_M = 0$ requires fulfilment of the dynamical Euler equations $\nabla_\nu F^{\mu\nu} = 0$ in both parts of the manifold and vanishing surface terms. The choice of the external boundaries t_{in} , t_{fin} , and Σ_3 guarantees vanishing of surface contributions. On the internal boundary Σ_c we should have

$$\int_{\Sigma_c} [\mathcal{L}_{F(\text{int})} F_{\text{int}}^{\mu\nu} (\delta A_\mu)_{\text{int}} - \mathcal{L}_{F(\text{ext})} F_{\text{ext}}^{\mu\nu} (\delta A_\mu)_{\text{ext}}] \sqrt{-g} d\sigma_\nu = 0. \quad (27)$$

The surface Σ_c is specified by the condition of continuity of the dynamical variables

$$A_{\text{int}}^\mu = A_{\text{ext}}^\mu. \quad (28)$$

As a result we get

$$\int_{\Sigma_c} (\mathcal{L}_{F(\text{int})} F_{\mu\nu(\text{int})} - \mathcal{L}_{F(\text{ext})} F_{\mu\nu(\text{ext})}) \sqrt{-g} \delta A^\mu d\sigma^\nu = 0. \quad (29)$$

In the spherically symmetric static case the only nonzero field component $F_{01} = -\partial A_0 / \partial r$. Introducing $A^0 = \phi$ and $F_{01} = -\partial\phi / \partial r = -E(r)$, we write (28) in the explicit form

$\phi_{\text{int}} = \phi_{\text{ext}}$, and taking into account that variation $\delta\phi$ is arbitrary, we obtain the condition imposed on the electromagnetic variables on the boundary Σ_c :

$$\mathcal{L}_{F(\text{int})} E_{\text{int}} = \mathcal{L}_{F(\text{ext})} E_{\text{ext}}. \quad (30)$$

For spherically symmetric nonlinear electromagnetic field dielectric permeability $\epsilon = \mathcal{L}_F$ [48], condition (30) requires thus continuity of the electric induction $D = \epsilon E$ on Σ_c :

$$D_{\text{int}} = D_{\text{ext}}. \quad (31)$$

Variation of the total action $S = S_G + S_M$ in each part of the manifold with respect to $g_{\mu\nu}$ yields the Einstein equations $G_{\mu\nu} = R_{\mu\nu} - (1/2)Rg_{\mu\nu} = -T_{\mu\nu}$, where

$$T_{\mu\nu} = \frac{4\pi}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} \quad (32)$$

is the stress-energy tensor. For a diffeomorphism invariant action, the conservation equation $\nabla^\mu T_{\mu\nu} = 0$ is fulfilled by virtue of the dynamic field equations (detailed consideration in Appendix). The conservation equation $\nabla_\mu T^{\mu\nu} = 0$ imposes additional conditions on the boundary surface Σ_c . Vanishing a surface term under variation with respect to $g_{\mu\nu}$ requires $T_{\mu\nu}$ to satisfy on the boundary Σ_c

$$\int_{\Sigma_c} [T_{\nu(\text{int})}^\mu - T_{\nu(\text{ext})}^\mu] \sqrt{-g} w^\nu d\sigma_\mu = 0, \quad (33)$$

where w^ν is an arbitrary vector field. In our case Σ_c is the sphere, T_ν^μ satisfies (2) and (7), and condition (33) reduces to

$$\int_{\Sigma_c} [T_{1(\text{int})}^1 - T_{1(\text{ext})}^1] w^1 \sqrt{-g} d\sigma_1 = 0 \quad (34)$$

which results, due to arbitrariness of w^μ , in

$$\mathcal{L}_{(\text{int})} - 2\mathcal{L}_{F(\text{int})} F_{\text{int}} = \mathcal{L}_{(\text{ext})} - 2\mathcal{L}_{F(\text{ext})} F_{\text{ext}}. \quad (35)$$

Condition (35) requires, by virtue of (7), continuity of density on the surface Σ_c :

$$\rho_{\text{int}} = \rho_{\text{ext}}. \quad (36)$$

Conditions (30) and (35) are the Erdmann-Weierstrass conditions obtained here for the Lagrange density $\mathcal{L}(F)$. (In the textbooks on variational calculus they are typically derived for a function $\Phi(x, y_i, y'_i)$ (corresponding to a Lagrange function L) for the cases of discontinuity of derivatives of dynamical variables y_i .) In the case of continuous invariant F , considered here, conditions (30) and (35) are satisfied automatically and lead, with taking into account generic behavior of a Lagrangian, $\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{ext}}$ on Σ_c , to $E_{\text{ext}} = E_{\text{int}}$, $\mathcal{L}_{F(\text{int})} = \mathcal{L}_{F(\text{ext})}$, and $p_{\perp(\text{int})} = p_{\perp(\text{ext})}$ on Σ_c . For a discontinuous invariant F the situation will be different [52].

Now let us consider conditions arising on Σ_c under variation with respect to $g_{\mu\nu}$. The boundary terms coming from the integration by parts and applying the Gauss theorem have the form [49, 50]

$$\int_{\Omega} g^{\mu\nu} \delta R_{\mu\nu} \sqrt{-g} d^4x = \int_{\Sigma} \sqrt{-g} v^\mu d\sigma_\mu, \quad (37)$$

where

$$v^\nu = g^{\alpha\beta} \delta\Gamma_{\alpha\beta}^\nu - g^{\nu\mu} \delta\Gamma_{\mu\beta}^\beta. \quad (38)$$

Each boundary term in (37) vanishes if both metric tensor $g^{\mu\nu}$ and its first derivatives are held fixed on the boundary; that is, vanishing of boundary contributions requires vanishing $\delta\Gamma_{\mu\nu}^\beta$ on the surface Σ [49]. Less restrictive and more informative conditions on the metric tensor at the boundary Σ require only $\delta g_{\mu\nu} = 0$. The first integral in (37) can be written as [50]

$$\int_{\Omega} g^{\mu\nu} \delta R_{\mu\nu} \sqrt{-g} d^4x = \int_{\Sigma} v_\mu n^\mu, \quad (39)$$

where n^μ is the unit normal to the boundary Σ . With using (38) we have on Σ [50]

$$v_\alpha n^\alpha = n^\alpha h^{\mu\nu} [\nabla_\nu (\delta g_{\alpha\mu}) - \nabla_\alpha (\delta g_{\mu\nu})], \quad (40)$$

$$h_{\mu\nu} = g_{\mu\nu} \pm n_\mu n_\nu,$$

where $h_{\mu\nu}$ is the induced metric on Σ . The first term in (40) vanishes by virtue of $\delta g_{\mu\nu} = 0$ on Σ . The second term is related to the trace of the extrinsic curvature of the boundary $K = K_\alpha^\alpha = h_\beta^\alpha \nabla_\alpha n^\beta$, its variation $2\delta K = n^\alpha h^{\mu\nu} \nabla_\alpha (\delta g_{\mu\nu})$ [50]. It follows the appearance of the additional term in the action

$$\delta S_G = \int_{\Omega} G_{\alpha\beta} \delta g^{\alpha\beta} \sqrt{-g} d^4x - 2 \int_{\Sigma} \delta K \quad (41)$$

and the unwanted contribution can be excluded by modifying S_G to $S_G + 2 \int_{\Sigma} K$ [50].

On the internal boundary surface Σ_c we have two contributions from (40). The first of them vanishes since the metric function (9) is regular (the surface $r = r_c$ is not distinguished in the metric), and continuity of the function $\rho(x)$ in the integral in (9) is guaranteed by condition (36). The second term in (40) vanishes since the condition for the extrinsic curvature $K_{\text{int}} = K_{\text{ext}}$ on Σ_c is satisfied in this case.

5. Example

The regular spherically symmetric solution with the regular center presented in [36] was obtained in the P -frame of NED-GR with the Hamiltonian-like function $\mathcal{H}(P) = P(1 + \alpha\sqrt{-P})^2$, where $P = -2q^2/r^4$, which gives the regularized Coulomb profile [36]

$$\rho(r) = \frac{q^2}{(r^2 + r_q^2)^2}. \quad (42)$$

Its value in the center gives the proper expression for a cut-off on self-energy density.

The metric function [36]

$$f(r) = 1 - \frac{4m}{\pi r} \left(\arctan \frac{r}{r_q} - \frac{rr_q}{r^2 + r_q^2} \right) \quad (43)$$

has the de Sitter asymptotic $f(r) = 1 - r^2/r_0^2$ and $r_0^2 = 3/\rho(0)$ as $r \rightarrow 0$ and the Reissner-Nordström asymptotic (12) as $r \rightarrow \infty$.

The characteristic parameter $\eta = 2r_g/(\pi r_q)$, where $r_g = 2m$ is the Schwarzschild radius, defines the type of a spherically symmetric structure with the regular de Sitter center: a regular electrically charged black hole for $\eta \geq \eta_{\text{crit}} = 2.816$ with at most two horizons, the double horizon $r_\pm = 1.825r_q$ for $\eta = \eta_{\text{crit}}$, and electrically charged soliton for $\eta < \eta_{\text{crit}}$ [36].

The invariant $F(r)$ and Lagrangian $\mathcal{L}(r)$ are given by [36]

$$F(r) = -\frac{2q^2 r^8}{(r^2 + r_q^2)^6}; \quad \mathcal{L}(r) = \frac{2q^2 (r_q^2 - r^2)}{(r^2 + r_q^2)^3}. \quad (44)$$

The minimum of F is achieved at $r_c = \sqrt{2}r_q$. Normalizing r on r_q and F on q^2/r_q^4 , we write equations in the dimensionless form and resolve them with respect to $z = (r/r_q)^2$ to obtain $\mathcal{L}_{\text{int}}$ and $\mathcal{L}_{\text{ext}}$ as the functions F .

We find that the regular solution presented in [36] can be obtained in the F frame of the Lagrange dynamics with the Lagrangians

$$\begin{aligned} \mathcal{L}_{\text{int}}(F) &= \frac{2(1 - z_2(\beta))}{(z_2(\beta) + 1)^3}, \\ z_2 &= -\frac{2}{3} \frac{\sqrt{1 - 6\beta}}{\beta} \cos\left(\frac{\alpha}{3} + \frac{\pi}{3}\right) - 1 + \frac{1}{3\beta}, \\ \mathcal{L}_{\text{ext}}(F) &= \frac{2(1 - z_1(\beta))}{(z_1(\beta) + 1)^3}, \\ z_1 &= \frac{2}{3} \frac{\sqrt{1 - 6\beta}}{\beta} \cos\frac{\alpha}{3} - 1 + \frac{1}{3\beta}, \\ \beta &= \sqrt{-\frac{F}{2}}, \quad \beta \in \left(0, \frac{4}{27}\right), \\ \alpha &= \arccos\left(\frac{2 - 18\beta + 27\beta^2}{2(1 - 6\beta)^{3/2}}\right). \end{aligned} \quad (45)$$

For $\mathcal{L}_{\text{int}}$ parameter z_2 changes within the range $0 \leq z_2 \leq 2$ and for $\mathcal{L}_{\text{ext}}$ we have $2 \leq z_1 < \infty$. Minimal value of the invariant F is $F_{\text{min}} = -2^5 \cdot 3^{-6}$. In the minimum of the invariant $\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{ext}} = -2/27$, and $d\mathcal{L}_{\text{int}}/dF = d\mathcal{L}_{\text{ext}}/dF = 3.395$. Second derivatives go to infinities of the opposite signs: $d^2\mathcal{L}_{\text{ext}}/dF^2 \rightarrow -\infty$, while $d^2\mathcal{L}_{\text{int}}/dF^2 \rightarrow +\infty$.

For $F \rightarrow -0$, we have $\beta \rightarrow 0$, and the parameter $\alpha \rightarrow 0$. Then $z_2 \rightarrow 0$ and $\mathcal{L}_{\text{int}} \rightarrow 2$, while $\mathcal{L}_{\text{ext}} \rightarrow F \rightarrow -0$. The Lagrangian $\mathcal{L}_{\text{int}}$ evolves from $\mathcal{L}_{\text{int}}(0) = 2$ to $\mathcal{L}_{\text{int}}(F_{\text{min}}) = -2/27$. The external Lagrangian starts from $\mathcal{L}_{\text{ext}}(F_{\text{min}}) = -2/27$ and evolves to $\mathcal{L}_{\text{ext}}(r \rightarrow \infty) = -0$.

6. Summary and Discussion

We have proved the existence of spherically symmetric electrically charged structures with the regular center in non-linear electrodynamics coupled to gravity. They are described

in the frame of the Lagrange dynamics by the nonuniform variational problem with the action

$$S = \frac{1}{16\pi} \left[\int_{\Omega_{\text{int}}} (R - \mathcal{L}_{\text{int}}(F)) \sqrt{-g} d^4x + \int_{\Omega_{\text{ext}}} (R - \mathcal{L}_{\text{ext}}(F)) \sqrt{-g} d^4x \right]. \quad (46)$$

Introduction of two Lagrangians is enforced by behaviour of the field invariant F which must vanish at $r = 0$ to guarantee regularity and at infinity to satisfy the Maxwell weak field limit.

In the case of continuous invariant F the nonuniform variational problem (46) describes nonlinear electromagnetic field as a medium with amazingly uniform properties: equality not only of electric induction but also of electric field tension on the internal boundary surface Σ_c and in consequence equality of the dielectric permeability. The situation can be essentially different for a rotating electrically charged regular structure.

Introduction of two Lagrangians for one electromagnetic medium suggests nontrivial field dynamics. It could, for example, involve a phase transition related to interior de Sitter vacuum [53, 54]. In the case of a charged regular black hole or soliton a phase transition can be related to dual electrodynamics [19]. A particular example is regular structure of a hybrid type [55] in which an electrically charged exterior region is matched to a magnetically charged core.

Regular spherically symmetric solutions of the Einstein equations satisfying the weak energy condition and specified by $T_t^t = T_r^r$ have obligatory de Sitter center [32, 34], belong to the Kerr-Schild class [40], and can be transformed into regular spinning solutions by the Gürses-Gürsey algorithm [41] which transforms the de Sitter center to the de Sitter vacuum disk. It has properties of a perfect conductor and ideal diamagnetic and displays superconducting behavior within a single spinning object [29].

Interior de Sitter vacuum supplies a structure with the finite positive electromagnetic mass related to breaking of space-time symmetry from the de Sitter group in the origin, which can explain existence of negative mass-squared differences for neutrino and predict a TeV scale for gravitoelectroweak unification [42].

Many modern theories propose two key modifications on the way to unify gravity with standard model of particle physics: the existence of additional space dimensions and the existence of a minimal length scale related to maximal resolution.

Existence of minimal length was observed in the annihilation reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ which puts the limit on maximal resolution at the scale $E = 1.253 \text{ TeV}$ with the 5σ reliability. Related minimal length $l_e = 1.57 \times 10^{-17} \text{ cm}$ appears naturally in the NED-GR context as a distance of the closest approach of annihilating particles at which the electromagnetic attraction is balanced by the gravitational repulsion of the interior de Sitter vacuum [38].

Appendix

Stress-Energy Tensor

The matter action $S_M(\psi)$ should be invariant under diffeomorphisms [50] which gives

$$\int \frac{\delta S_M}{\delta g^{\mu\nu}} \delta g^{\mu\nu} + \int \frac{\delta S_M}{\delta \psi} \delta \psi = 0 \quad (A.1)$$

with

$$\delta g^{\mu\nu} = 2\nabla^{(\mu} w^{\nu)}, \quad (A.2)$$

where w^μ is an arbitrary vector field. When matter field ψ satisfies the dynamical field equations, then $\delta S_M / \delta \psi = 0$, and this leads, with taking into account (32), to [49]

$$\begin{aligned} & \int \sqrt{-g} T_{\mu\nu} \nabla^{(\mu} w^{\nu)} d^4x \\ &= 0 = - \int \nabla^\mu T_{\mu\nu} w^\nu \sqrt{-g} d^4x + \int_\Sigma \sqrt{-g} T_{\mu\nu} w^\nu d\sigma^\mu. \end{aligned} \quad (A.3)$$

The second term on the right-hand side vanishes for all smooth w^μ of compact support which results in $\nabla^\mu T_{\mu\nu} = 0$. Therefore for a diffeomorphism invariant action, $T_{\mu\nu}$ is conserved by virtue of the dynamic field equations and (32) may be used to define the stress-energy tensor of the field ψ . The above reasoning applied to S_G gives $\nabla^\mu G_{\mu\nu} = 0$ independently on any field equations, so that the contracted Bianchi identities follow from the invariance of the Hilbert action under diffeomorphisms [50].

The stress-energy tensor given by (32) is the metric stress-energy tensor for the case when $\mathcal{L}_M$ depends only on metric tensor $g_{\mu\nu}$. In general case it can depend also on its derivatives $g_{\mu\nu,\beta}$, and in this case it differs from the canonical stress-energy tensor introduced by variation of $\mathcal{L}_M$ with respect to dynamical variables describing the matter field. For electromagnetic field the Lagrangian $\mathcal{L}(F)$ does not depend on the derivatives of the metric tensor and can be reduced to the metric stress-energy tensor (32) by symmetrization [49, 50].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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