

## Research Article

# Energy Distribution of a Regular Black Hole Solution in Einstein-Nonlinear Electrodynamics

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A study about the energy momentum of a new four-dimensional spherically symmetric, static and charged, regular black hole solution developed in the context of general relativity coupled to nonlinear electrodynamics is presented. Asymptotically, this new black hole solution behaves as the Reissner-Nordström solution only for the particular value  $\mu = 4$ , where  $\mu$  is a positive integer parameter appearing in the mass function of the solution. The calculations are performed by use of the Einstein, Landau-Lifshitz, Weinberg, and Møller energy momentum complexes. In all the aforementioned prescriptions, the expressions for the energy of the gravitating system considered depend on the mass  $M$  of the black hole, its charge  $q$ , a positive integer  $\alpha$ , and the radial coordinate  $r$ . In all these pseudotensorial prescriptions, the momenta are found to vanish, while the Landau-Lifshitz and Weinberg prescriptions give the same result for the energy distribution. In addition, the limiting behavior of the energy for the cases  $r \rightarrow \infty$ ,  $r \rightarrow 0$ , and  $q = 0$  is studied. The special case  $\mu = 4$  and  $\alpha = 3$  is also examined. We conclude that the Einstein and Møller energy momentum complexes can be considered as the most reliable tools for the study of the energy momentum localization of a gravitating system.

## 1. Introduction

Energy momentum localization plays an important role among the open issues which appeared over the years in general relativity. The difficulty which arises consists in constructing a properly defined energy density of gravitating systems. As a result, up today there is no generally accepted satisfactory description for the energy of the gravitational field.

A number of researchers have used different methods for the energy momentum localization. Standard research methods include the use of different tools such as super-energy tensors [1–4], quasi-local expressions [5–9], and the well-known energy momentum complexes of Einstein [10–12], Landau-Lifshitz [13], Papapetrou [14], Bergmann-Thomson [15], Møller [16], Weinberg [17], and Qadir-Sharif [18]. An agreement between, on one hand, Einstein, Landau-Lifshitz, Papapetrou, Bergmann-Thomson, Weinberg, and

Møller energy momentum complexes and, on the other hand, the quasi-local mass definition introduced by Penrose [19] and developed by Tod [20] for some gravitational backgrounds is worth noticing. The coordinate system dependence of these computational tools remains the main problem encountered. Indeed, only the Møller energy momentum complex is coordinate independent. “Cartesian coordinates” (also called quasi-Cartesian coordinates) and Kerr-Schild Cartesian coordinates are used to compute the energy momentum in the case of the other pseudotensorial prescriptions.

Despite the critique on energy momentum complexes, the 1990s have been proved to be a decade of rejuvenation of this field. This revival began in 1990 with the seminal research of Virbhadra [21, 22] who showed that the same energy distribution can be obtained for the (asymptotically Minkowskian) Kerr-Newman spacetime geometry by using different energy momentum prescriptions. In 1993, Rosen

and Virbhadra [23] studied the energy momentum distribution of cylindrical gravitational waves by use of the Einstein prescription and it was shown, for the first time, that it was possible to obtain meaningful results also for asymptotically non-Minkowskian spacetimes. Furthermore, in 1996, it was shown by Aguirregabiria et al. [24] that several coordinate dependent prescriptions yield the same results for any metric of the Kerr-Schild class, while, in 1999, Virbhadra [25] extended the research for metrics more generally than the Kerr-Schild class obtaining also the same energy for different prescriptions. This line of research was enforced by Xulu [26] who by applying one more prescription found results in agreement with those of Virbhadra.

Since then, the aforementioned revival gave a boost to the field of energy momentum complexes with their application yielding physically reasonable results for many spacetime geometries, in particular for geometries in  $(3 + 1)$ ,  $(2 + 1)$ , and  $(1 + 1)$  dimensions [27–48].

An alternative to avoid the dependence on coordinates is the teleparallel theory of gravitation [49–58] which has been used in many studies for the calculation of the energy and momentum distributions. Here, one can underline the similarity of the results obtained by this approach [59–63] with some of the results emerging by using the aforementioned standard energy momentum complexes.

One can also point out the ongoing attempts for the elaboration of the definition and application of energy momentum complexes as well as for their rehabilitation [64, 65].

Though the main motivation of knowing the nature of the spacetime itself by studying the energy momentum distributions remains, possible astrophysical implications also urge the research in this area. In his 1999 paper [25], Virbhadra studied the cosmic censorship hypothesis (CCH) and the Seifert conjecture by using several energy momentum complexes for investigating the “mass” of naked singularities. Building upon this work, gravitational lensing effects were attributed to naked singularities ([66] and references therein). A promising conjecture is that a positive energy region may act as a convergent gravitational lens, while a negative energy region may act as a divergent one (see, e.g., [67]). This line of research is still ongoing [68], whereas the determination of the effective gravitational mass of an object causing the curvature of spacetime, through energy momentum complexes, marks an independent line of research [69].

The present work has the following structure. In Section 2, we describe the new four-dimensional charged regular black hole solution under study. Section 3 contains the presentation of the Einstein, Landau-Lifshitz, Weinberg, and Møller energy momentum complexes utilized for the calculations. Section 4 contains the computations of the energy and momentum distributions. Finally, in the discussion given in Section 5, we present our results and examine some limiting and particular cases, while possible astrophysical perspectives are briefly mentioned. Throughout the paper we use geometrized units ( $c = G = 1$ ), while the signature chosen is  $(+, -, -, -)$ . The calculations for the Einstein, Landau-Lifshitz, and Weinberg energy momentum complexes are performed in Schwarzschild Cartesian coordinates. Greek indices range from 0 to 3, while Latin indices run from 1 to 3.

## 2. A New Regular Black Hole Solution in Einstein-Nonlinear Electrodynamics

Attempts to avoid the central singularity in the presence of a horizon for a classical black hole have started already in the 1960s with Bardeen’s regular black hole solution [70]. In fact, the interest in this area was revived after, by coupling gravity to certain theories of nonlinear electrodynamics, a number of exact solutions describing regular black holes (with all curvature invariants being bounded everywhere) were found (see [71] for a review of existing models and many relevant references). In this context, it is worth mentioning that the application of the Born-Infeld Lagrangian of nonlinear electrodynamics does not give a static spherically symmetric regular black hole solution since it creates conical singularities at the origin [72]. In 1998, Ayón-Beato and García [73] presented a regular black hole solution obtained by coupling gravity to nonlinear electrodynamics, whereby the Einstein-Hilbert action of general relativity is augmented by the addition to the Lagrangian of a term depending nonlinearly on the electromagnetic field tensor. Since then, a number of very interesting works have been published on this subject [74–84]. In fact, it is worth noticing that Ayón-Beato and García were the first to establish a connection of Bardeen’s regular black hole solution with a nonlinear electrodynamics leading to the current interpretation of an exact solution supported by a nonlinear magnetic monopole [85].

The action for nonlinear electrodynamics minimally coupled to gravity used by Ayón-Beato and García is the following:

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi} R - \frac{1}{4\pi} \mathcal{L}(F) \right), \quad (1)$$

where  $R$  is the scalar curvature, while the gauge-invariant Lagrangian  $\mathcal{L}(F)$  is a nonlinear function of the invariant  $F = (1/4)F_{\mu\nu}F^{\mu\nu}$ ,  $F_{\mu\nu}$  being the electromagnetic field tensor. In the weak field limit,  $\mathcal{L}(F)$  gives Maxwell’s theory. Recently, Balart and Vagenas [86, 87] constructed a family of new spherical, static, and charged regular black hole solutions in the context of Einstein-nonlinear electrodynamics given above by imposing the following conditions: (i) the weak energy condition has to be satisfied, (ii) the energy momentum tensor has to respect the symmetry  $T_0^0 = T_1^1$  and, (iii) the metric must have an asymptotical behaviour as the Reissner-Nordström black hole solution. Starting with the general static and spherically symmetric line element,

$$ds^2 = B(r) dt^2 - B^{-1}(r) dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (2)$$

with  $B(r) = 1 - 2m(r)/r$  and, by relaxing the third condition, the authors have also constructed a more general family of charged regular black hole solutions which do not behave asymptotically as the Reissner-Nordström black hole metric.

In this case, the mass function depends on two integer constant parameters  $\mu \geq 4$  and  $\alpha \geq 1$ :

$$m(r) = \frac{r^3 q^2}{6} \cdot \frac{\Gamma(1/\alpha) \Gamma(\mu/\alpha)}{\Gamma(4/\alpha) \Gamma((\mu-3)/\alpha)} \left( \frac{6\Gamma(4/\alpha)}{\Gamma(1/\alpha) \Gamma((\alpha+3)/\alpha)} \frac{M}{q^2} \right)^4 \cdot {}_2F_1 \left( \frac{3}{\alpha}, \frac{\mu}{\alpha}, \frac{3+\alpha}{\alpha}; - \left( \frac{6\Gamma(4/\alpha)}{\Gamma(1/\alpha) \Gamma((\alpha+3)/\alpha)} \frac{M}{q^2} r \right)^\alpha \right), \quad (3)$$

where  $q$  is the electric charge,  $\Gamma$  is the Gamma function,  $M$  is the black hole mass, and  ${}_2F_1$  is the Gauss hypergeometric function. Particular cases such as  $\mu = 5$  and  $\alpha = 2$  and  $\mu = 6$  and  $\alpha = 3$  lead to the metrics obtained by Bardeen [70] and Hayward [88].

The line element (2) with the mass function (3) does not behave asymptotically as the Reissner-Nordström black hole solution unless  $\mu = 4$ . In fact, for  $\mu = 4$  and  $\alpha = 3$  in (3), Balart and Vagenas obtained the asymptotically Reissner-Nordström solution

$$B(r) = 1 - \frac{2M}{r} \left[ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{1/3}} \right]. \quad (4)$$

In the present paper, we will study the nonasymptotically Reissner-Nordström gravitational background obtained for the particular case  $\mu = 3$  and  $\alpha$  arbitrary. Consequently, the metric for the new charged regular black hole solution to be considered in what follows has the line element (2) with

$$B(r) = 1 - \frac{2M}{r} \left( 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{(\alpha-3)/3}} \right). \quad (5)$$

In fact, this regular black hole solution exhibits two event horizons. By choosing, for example, the numerical values  $M = 1$  and  $q = 0.9$ , we get Figure 1 showing the two horizons. One can see that the position of the inner horizon is shifted towards bigger values of  $r$  as the parameter  $\alpha$  decreases, while the position of the outer horizon remains unaffected.

### 3. Einstein, Landau-Lifshitz, Weinberg, and Møller Energy Momentum Complexes

The Einstein energy momentum complex [10, 11] for a (3 + 1) dimensional gravitational background is given by the expression

$$\theta_\nu^\mu = \frac{1}{16\pi} h_{\nu,\lambda}^{\mu\lambda}, \quad (6)$$

where the superpotentials  $h_\nu^{\mu\lambda}$  are given as

$$h_\nu^{\mu\lambda} = \frac{1}{\sqrt{-g}} g_{\nu\sigma} \left[ -g \left( g^{\mu\sigma} g^{\lambda\kappa} - g^{\lambda\sigma} g^{\mu\kappa} \right) \right]_{,\kappa} \quad (7)$$

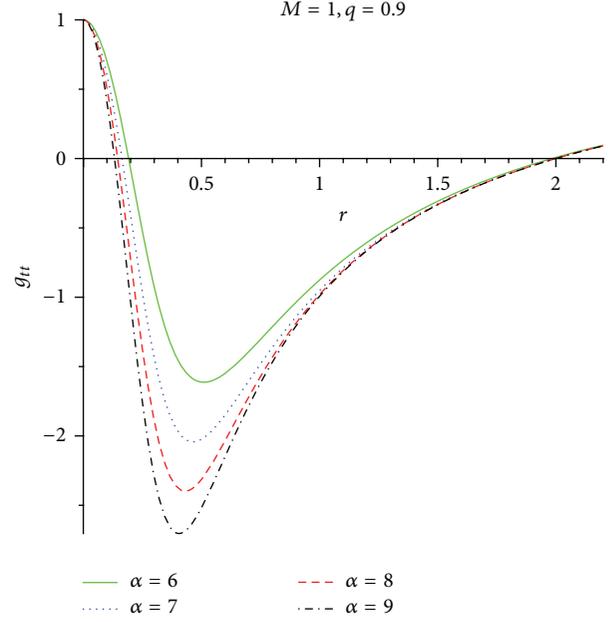


FIGURE 1: Two horizons exist for the charged regular black hole at the points where  $g_{tt}$  meets the  $r$ -axis. For different values of the parameter  $\alpha$ , the positions of the inner horizon assume different values, whereas the position of the outer horizon remains unaffected.

and satisfy the corresponding antisymmetric property

$$h_\nu^{\mu\lambda} = -h_\nu^{\lambda\mu}. \quad (8)$$

The components  $\theta_0^0$  and  $\theta_i^i$  represent the energy and the momentum densities, respectively. In this prescription, the local conservation law is respected:

$$\theta_{\nu,\mu}^\mu = 0. \quad (9)$$

Thus, the energy momentum can be calculated by

$$P_\mu = \iiint \theta_\mu^0 dx^1 dx^2 dx^3. \quad (10)$$

Applying Gauss's theorem, the energy momentum becomes

$$P_\mu = \frac{1}{16\pi} \iint h_\mu^{0i} n_i dS, \quad (11)$$

with  $n_i$  being the outward unit normal vector on the surface  $dS$ .

In the Landau-Lifshitz prescription, the corresponding energy momentum complex [13] is defined as

$$L^{\mu\nu} = \frac{1}{16\pi} S_{\rho\sigma}^{\mu\nu\rho\sigma}, \quad (12)$$

with the Landau-Lifshitz superpotentials given by

$$S^{\mu\nu\rho\sigma} = -g \left( g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} \right). \quad (13)$$

$L^{00}$  and  $L^{0i}$  components are the energy and the momentum densities, respectively. The local conservation law holds:

$$L_{,\nu}^{\mu\nu} = 0. \quad (14)$$

By integrating  $L^{\mu\nu}$  over the 3-space one gets for the energy momentum

$$P^\mu = \iiint L^{\mu 0} dx^1 dx^2 dx^3. \quad (15)$$

By using Gauss's theorem, we get

$$P^\mu = \frac{1}{16\pi} \iint S^{\mu 0 i \nu} n_i dS = \frac{1}{16\pi} \iint U^{\mu 0 i} n_i dS. \quad (16)$$

The Weinberg energy momentum complex [17] is given as

$$W^{\mu\nu} = \frac{1}{16\pi} D^{\lambda\mu\nu}, \quad (17)$$

where  $D^{\lambda\mu\nu}$  represents the corresponding superpotentials:

$$D^{\lambda\mu\nu} = \frac{\partial h_\kappa^\kappa}{\partial x_\lambda} \eta^{\mu\nu} - \frac{\partial h_\kappa^\kappa}{\partial x_\mu} \eta^{\lambda\nu} - \frac{\partial h^{\kappa\lambda}}{\partial x^\kappa} \eta^{\mu\nu} + \frac{\partial h^{\kappa\mu}}{\partial x^\kappa} \eta^{\lambda\nu} + \frac{\partial h^{\lambda\nu}}{\partial x_\mu} - \frac{\partial h^{\mu\nu}}{\partial x_\lambda}, \quad (18)$$

with

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}. \quad (19)$$

Here  $W^{00}$  and  $W^{0i}$  components represent the energy and the momentum densities, respectively. In the Weinberg prescription, the local conservation law is also respected:

$$W^{\mu\nu}_{, \nu} = 0. \quad (20)$$

The integration of  $W^{\mu\nu}$  over the 3-space gives the energy momentum

$$P^\mu = \iiint W^{\mu 0} dx^1 dx^2 dx^3. \quad (21)$$

Applying Gauss's theorem and integrating over the surface of a sphere of radius  $r$ , we get the following expression for the energy momentum distribution:

$$P^\mu = \frac{1}{16\pi} \iint D^{i0\mu} n_i dS. \quad (22)$$

The Møller energy momentum complex [16] is

$$\mathcal{F}_\nu^\mu = \frac{1}{8\pi} M_{\nu, \lambda}^{\mu\lambda}, \quad (23)$$

with the Møller superpotentials  $M_{\nu, \lambda}^{\mu\lambda}$ ,

$$M_{\nu, \lambda}^{\mu\lambda} = \sqrt{-g} \left( \frac{\partial g_{\nu\sigma}}{\partial x^\kappa} - \frac{\partial g_{\nu\kappa}}{\partial x^\sigma} \right) g^{\mu\kappa} g^{\lambda\sigma}, \quad (24)$$

being antisymmetric

$$M_{\nu, \lambda}^{\mu\lambda} = -M_{\nu, \lambda}^{\lambda\mu}. \quad (25)$$

Møller's energy momentum complex also satisfies the local conservation law:

$$\frac{\partial \mathcal{F}_\nu^\mu}{\partial x^\mu} = 0, \quad (26)$$

while  $\mathcal{F}_0^0$  and  $\mathcal{F}_i^0$  are the energy and the momentum densities, respectively. Thus, the energy momentum is given by

$$P_\mu = \iiint \mathcal{F}_\mu^0 dx^1 dx^2 dx^3. \quad (27)$$

With the use of Gauss's theorem, one gets

$$P_\mu = \frac{1}{8\pi} \iint M_\mu^{0i} n_i dS. \quad (28)$$

#### 4. Energy and Momentum Distributions for the Charged Regular Black Hole

In order to perform the calculations using the Einstein energy momentum complex, it is necessary to transform the metric given by the line element (2) in Schwarzschild Cartesian coordinates. Thus, we obtain the line element in the following form:

$$ds^2 = B(r) dt^2 - (dx^2 + dy^2 + dz^2) - \frac{B^{-1}(r) - 1}{r^2} (xdx + ydy + zdz)^2. \quad (29)$$

Now, after computing the superpotentials in the Einstein prescription, we get the following nonvanishing components:

$$\begin{aligned} h_0^{01} &= \frac{2x}{r^2} \frac{2M}{r} \left\{ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{(\alpha-3)/3}} \right\}, \\ h_0^{02} &= \frac{2y}{r^2} \frac{2M}{r} \left\{ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{(\alpha-3)/3}} \right\}, \\ h_0^{03} &= \frac{2z}{r^2} \frac{2M}{r} \left\{ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{(\alpha-3)/3}} \right\}. \end{aligned} \quad (30)$$

Using the line element (2), with the metric coefficient (5), (11) for the energy and the expressions, and (30) for the superpotentials, we obtain the energy distribution for the nonasymptotically Reissner-Nordström regular charged black hole in the Einstein prescription:

$$E_E = M \left\{ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{(\alpha-3)/3}} \right\}. \quad (31)$$

Furthermore, all the momenta are zero.

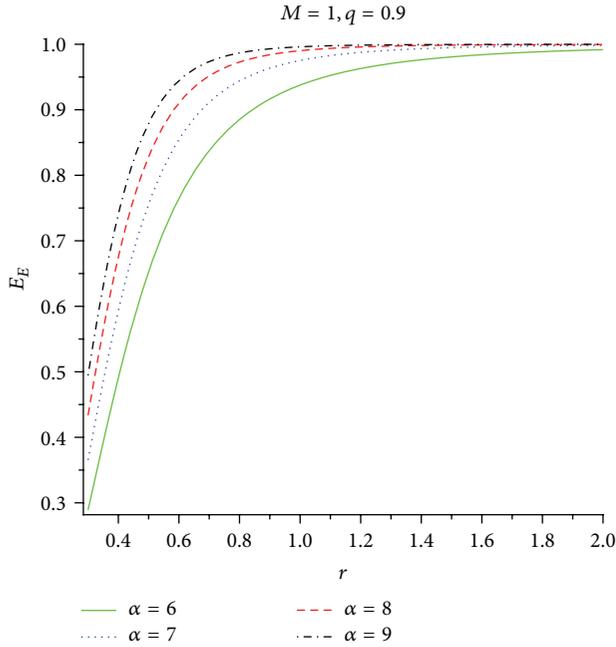


FIGURE 2: Energy distribution calculated by the Einstein prescription between the inner and outer horizons for different values of the parameter  $\alpha$ .

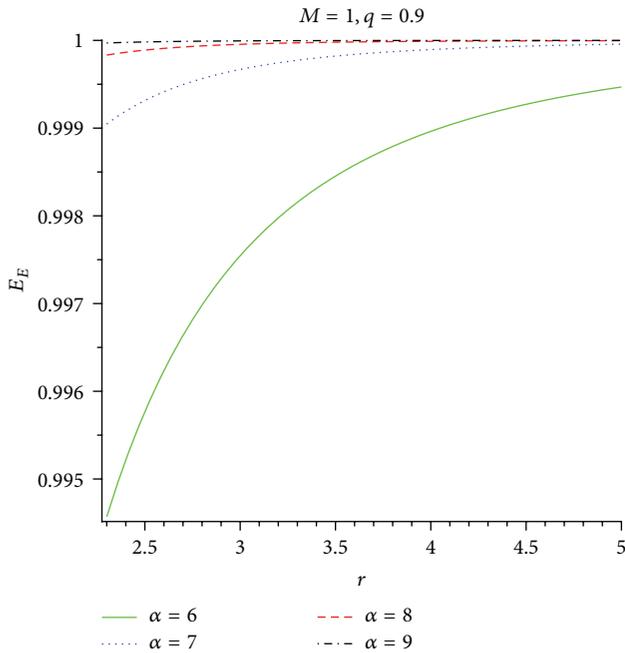


FIGURE 3: Energy distribution calculated by the Einstein prescription outside the outer horizon for different values of the parameter  $\alpha$ .

In Figures 2 and 3, we can see the graphs of the energy distribution as a function of the distance for different values of  $\alpha$  between the two horizons and outside the outer horizon, respectively, while Figure 4 presents the energy distribution as a function of charge and mass.

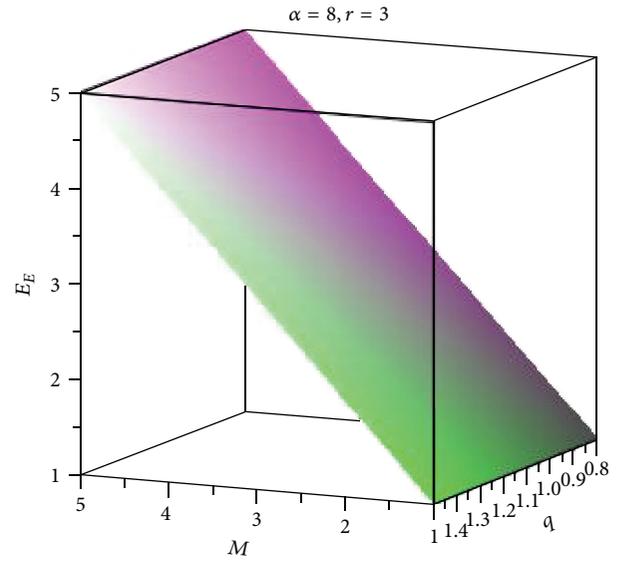


FIGURE 4: Variation of the energy distribution calculated by the Einstein prescription with respect to charge and mass outside the outer horizon.

In order to apply the Landau-Lifshitz prescription, we use  $U^{\mu 0i}$  quantities defined in (16) to compute the energy momentum distribution. The nonvanishing components of  $U^{\mu 0i}$  quantities are

$$\begin{aligned}
 U^{ttx} &= \frac{2x}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}, \\
 U^{ttx} &= \frac{2y}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}, \\
 U^{ttz} &= \frac{2z}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left( 1 + (2Mr/q^2)^3 \right)^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}.
 \end{aligned} \tag{32}$$

Substituting (32) in (16), we get the energy distribution

$$E_{LL} = \frac{M \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}. \tag{33}$$

Also, for this prescription, all the momenta are found to vanish.

In the case of the Weinberg prescription, we calculate the nonvanishing components:

$$\begin{aligned}
 D^{xtt} &= \frac{2x}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}, \\
 D^{ytt} &= \frac{2y}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}, \\
 D^{ztt} &= \frac{2z}{r^2} \frac{(2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}.
 \end{aligned} \tag{34}$$

Inserting the expressions obtained in (34) into (22), we obtain for the energy distribution inside a 2-sphere of radius  $r$

$$\begin{aligned}
 E_W &= M \frac{\left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1 / \left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]},
 \end{aligned} \tag{35}$$

while all the momenta vanish. As it can be seen, the energy in the Weinberg prescription (35) is identical with the energy in the Landau-Lifshitz prescription (33).

In Figures 5 and 6, we can see the graphs of the energy distribution as a function of the distance for different values of  $\alpha$  between the two horizons and outside the outer horizon, respectively, while Figure 7 presents the energy distribution as a function of charge and mass.

In the Møller prescription, the only nonvanishing superpotential is

$$\begin{aligned}
 M_0^{01} &= \left\{ \frac{2M \left[ 1 - 1 / \left( 1 + 8M^3 r^3 / q^6 \right)^{(\alpha/3-1)} \right]}{r^2} \right. \\
 &\quad \left. - \frac{48M^4 (\alpha/3 - 1) r}{\left[ 1 + 8M^3 r^3 / q^6 \right]^{(\alpha/3-1)} q^6 \left[ 1 + 8M^3 r^3 / q^6 \right]} \right\} r^2 \\
 &\quad \cdot \sin \theta.
 \end{aligned} \tag{36}$$

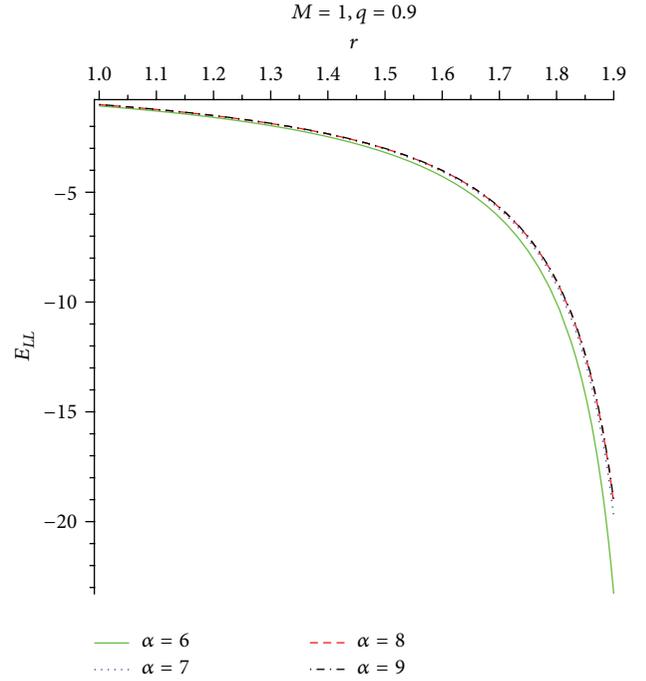


FIGURE 5: Energy distribution calculated by the Landau-Lifshitz prescription between the inner and outer horizons for different values of the parameter  $\alpha$ .

Applying the last result for the line element (2) with the metric coefficient (5) and using the expression (28), we get the energy distribution in the Møller prescription:

$$\begin{aligned}
 E_M &= \frac{r^2}{2} \left\{ \frac{2M \left[ 1 - 1 / \left( 1 + 8M^3 r^3 / q^6 \right)^{(\alpha/3-1)} \right]}{r^2} \right. \\
 &\quad \left. - \frac{48M^4 (\alpha/3 - 1) r}{\left[ 1 + 8M^3 r^3 / q^6 \right]^{(\alpha/3-1)} q^6 \left[ 1 + 8M^3 r^3 / q^6 \right]} \right\}.
 \end{aligned} \tag{37}$$

Also, in this prescription, it is found that all the momenta vanish.

In Figures 8 and 9, we can see the graphs of the energy distribution as a function of the distance for different values of  $\alpha$  between the two horizons and outside the outer horizon, respectively, while Figure 10 presents the energy distribution as a function of charge and mass.

It is pointed out, according to Figure 8, that for  $\alpha \leq 6$  the Møller energy of the regular black hole is negative and decreases monotonically for the whole range of the values of  $r$  considered.

Finally, in Figure 11, we give a comparison of the energy distributions as obtained by the different prescriptions for  $\alpha = 8$ .

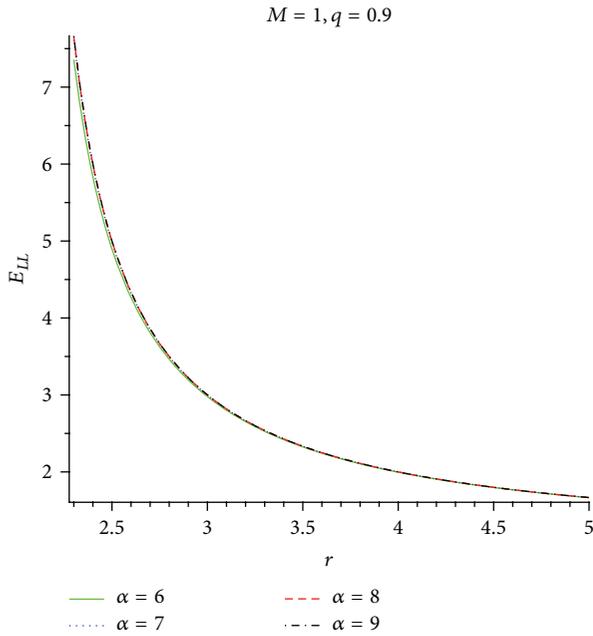


FIGURE 6: Energy distribution calculated by the Landau-Lifshitz prescription outside the outer horizon for different values of the parameter  $\alpha$ .

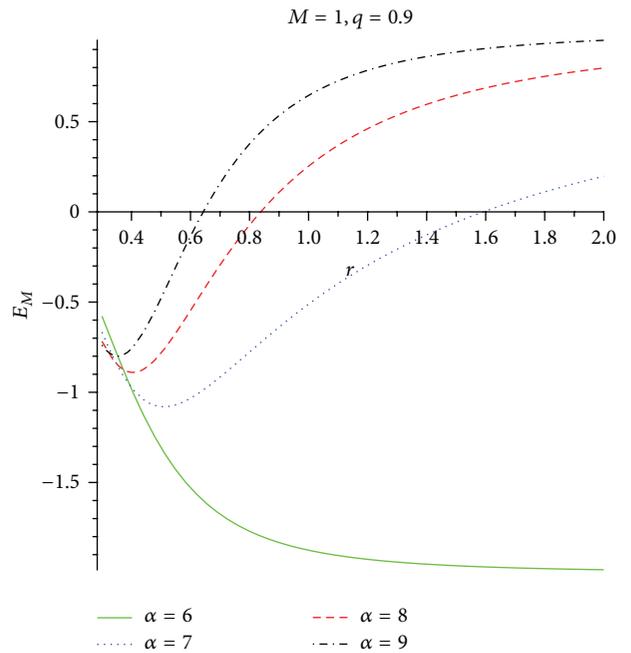


FIGURE 8: Energy distribution calculated by the Møller prescription between the inner and outer horizons for different values of the parameter  $\alpha$ .

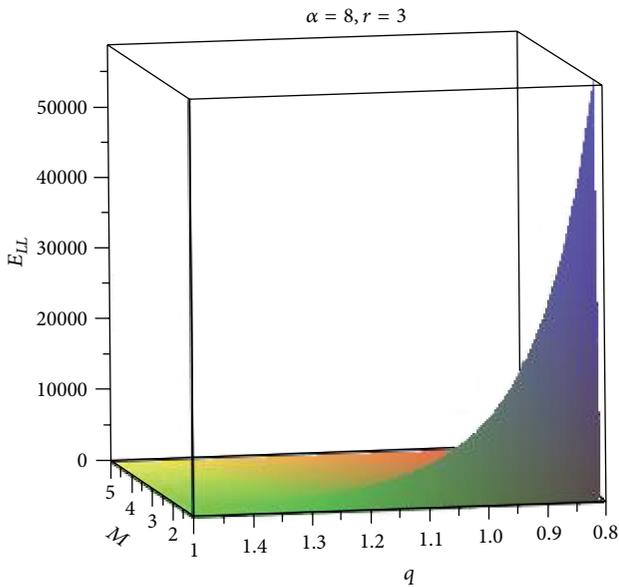


FIGURE 7: Variation of the energy distribution calculated by the Landau-Lifshitz prescription with respect to charge and mass outside the outer horizon.

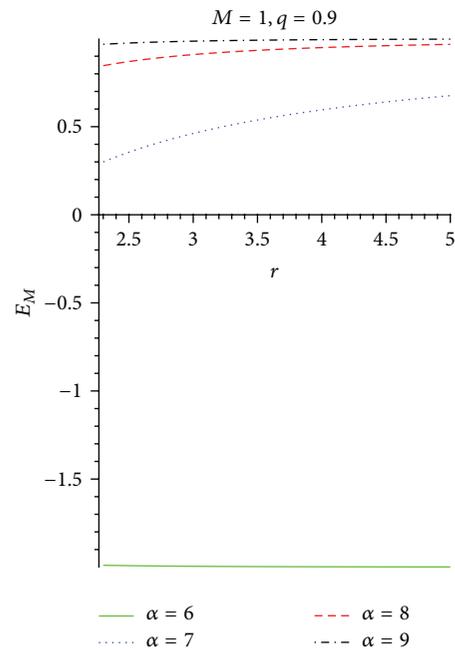


FIGURE 9: Energy distribution calculated by the Møller prescription outside the outer horizon for different values of the parameter  $\alpha$ .

### 5. Discussion

This paper is focused on the study of the energy momentum for the gravitational field of a new four-dimensional, spherically symmetric, static and charged, and regular black hole solution developed in the context of a particular Einstein-nonlinear electrodynamics coupling, by applying the Einstein, Landau-Lifshitz, Weinberg, and Møller energy

momentum complexes. The metric considered, which has two horizons, does not asymptotically behave as the Reissner-Nordström black hole solution unless  $\mu = 4$ , where  $\mu$  is a positive integer parameter. The expressions for the energy are well defined in all the aforementioned prescriptions and depend on the mass  $M$  of the black hole, its charge  $q$ , a

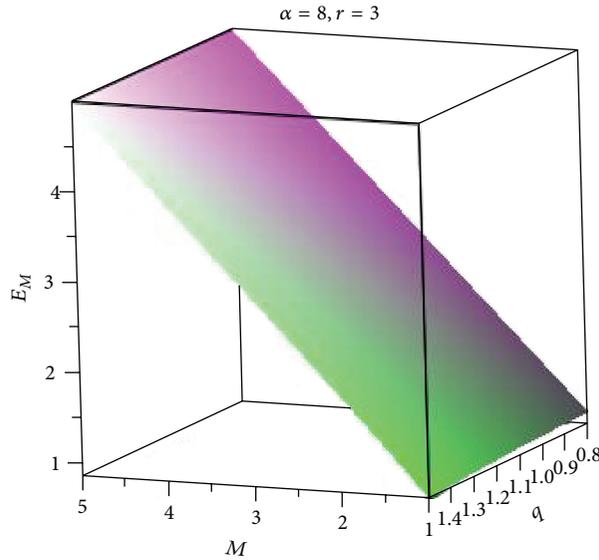


FIGURE 10: Variation of the energy distribution calculated by the Møller prescription with respect to charge and mass outside the outer horizon.

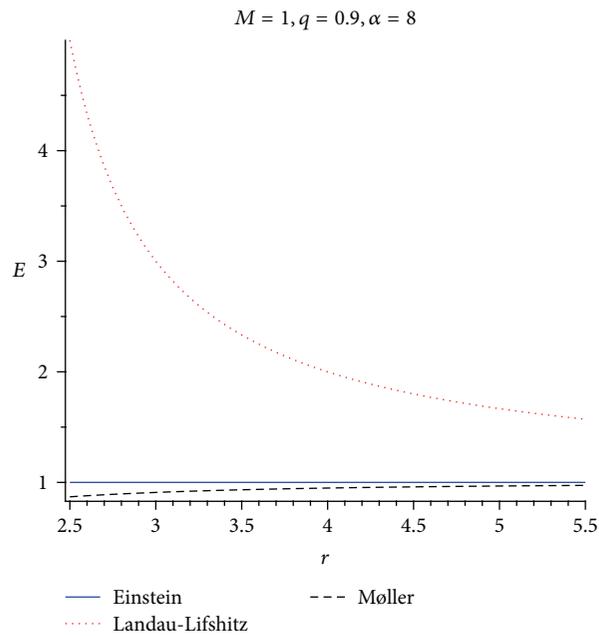


FIGURE 11: Comparison of the energy distributions calculated by the Einstein, Landau-Lifshitz, and Møller prescriptions outside the outer horizon.

positive integer  $\alpha$ , and the radial coordinate  $r$ . Both the Landau-Lifshitz and Weinberg prescriptions give the same result for the energy distribution, while all the momenta are zero in all four prescriptions used. In Table 1, we present the results for the energy as obtained in the four different prescriptions for the nonasymptotically Reissner-Nordström regular charged black hole.

In order to examine the physical meaning of the results obtained, we study the limiting behavior of the energy for

$r \rightarrow \infty, r \rightarrow 0$ , and  $q = 0$  in three cases: (i)  $1 \leq \alpha < 3$ , (ii)  $\alpha = 3$ , and (iii)  $\alpha > 3$ . However, since cases (i) and (ii) do not yield any physically meaningful results, we present only the results for case (iii) in Table 2.

For  $\mu = 4$  and  $\alpha = 3$ , we calculate the energy momentum distribution for the asymptotically Reissner-Nordström metric with the metric coefficient (4). For this black hole solution, the energy distribution obtained by using the Einstein, Landau-Lifshitz, Weinberg, and Møller energy

TABLE 1: Energy calculated for the nonasymptotically Reissner-Nordström regular charged black hole ( $\mu = 3$ ,  $\alpha$  arbitrary) in the different prescriptions.

Prescription	Energy
Einstein	$M \left[ 1 - \frac{1}{\left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3}} \right]$
Landau-Lifshitz	$\frac{M \left[ 1 - 1/\left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1/\left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}$
Weinberg	$\frac{M \left[ 1 - 1/\left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}{1 - (2M/r) \left[ 1 - 1/\left[ 1 + (2Mr/q^2)^3 \right]^{(\alpha-3)/3} \right]}$
Møller	$\frac{r^2}{2} \left[ \frac{2M \left[ 1 - 1/\left( 1 + 8M^3 r^3/q^6 \right)^{(a/3-1)} \right]}{r^2} - \frac{48M^4 (a/3 - 1)r}{\left[ 1 + 8M^3 r^3/q^6 \right]^{(a/3-1)} q^6 \left[ 1 + 8M^3 r^3/q^6 \right]} \right]$

 TABLE 2: Energy for  $\alpha > 3$  in the different prescriptions at limiting cases.

Prescription	$r \rightarrow \infty$	$r \rightarrow 0$	$q = 0$
Einstein	$M$	$0$	$M$
Landau-Lifshitz	$M$	$0$	$M \left( 1 - \frac{2M}{r} \right)^{-1}$
Weinberg	$M$	$0$	$M \left( 1 - \frac{2M}{r} \right)^{-1}$
Møller	$M$	$0$	$M$

momentum complexes is given in Table 3, along with the limiting cases for  $r \rightarrow \infty$ ,  $r \rightarrow 0$ , and  $q = 0$ . As we can see, the limiting behavior of the energy for the asymptotically Reissner-Nordström solution ( $\mu = 4$ ,  $\alpha = 3$ ) is the same as in the case of the nonasymptotically Reissner-Nordström solution given in Table 2.

Thus, by comparing the results obtained for the energy with all the energy momentum complexes applied, we see that, for  $q = 0$  and at infinity, the Einstein prescription and the Møller prescription give the same results (namely, the ADM mass  $M$ ) as those obtained for the Schwarzschild black hole solution. On the other hand, the Landau-Lifshitz and the Weinberg prescriptions, while at infinity lead to the same result (the ADM mass  $M$ ), in the chargeless case  $q = 0$  give the energy distribution  $E_{LL} = E_W = M(1 - 2M/r)^{-1}$  which is in total agreement with the expression obtained for the Schwarzschild black hole using Schwarzschild Cartesian coordinates (see, [25]).

Finally, all the four prescriptions used yield a vanishing energy as  $r \rightarrow 0$ .

Based on the present study, as well as on the aforementioned agreement with previous results obtained in the case of the Schwarzschild metric, we conclude that the Einstein

and Møller energy momentum complexes give physically meaningful results and, thus, can be considered as the most reliable tools among the energy momentum complexes for the study of the energy momentum localization of gravitational systems.

At this point, we would like to mention a possible astrophysical implication of our results. Identifying [89] the energy distribution at radial distance  $r$  with the effective gravitational mass of the astrophysical object considered inside the region determined by this distance  $r$ , it would be possible to investigate whether this effective gravitational mass is positive or negative and decide accordingly if the astrophysical object could act as a convergent or divergent gravitational lens. This investigation, though, lies beyond the scope of the present work.

Another interesting future perspective might be the calculation of the energy momentum of the new regular black hole solution considered in the present work by applying other energy momentum complexes and/or the teleparallel equivalent to general relativity (TEGR).

A final remark is deemed proper. Recently, it has been shown that the Maxwell-modified  $F(R)$  theory of gravity with an action of the form  $S = \int d^4x \sqrt{-g} [F(R) - (1/4)F_{\mu\nu}F^{\mu\nu}]$  allows the occurrence of instability and antievaporation effects for charged (in particular, extremal Reissner-Nordström) black holes [90]. Since Einstein-nonlinear electrodynamics can also be considered as a kind of modified gravity theory, similar instability and antievaporation effects can in principle be expected to occur. Their study will shed new light on the behavior of classical black holes.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

TABLE 3: Energy of the asymptotically Reissner-Nordström regular solution ( $\mu = 4$  and  $\alpha = 3$ ) in the different prescriptions as well as in limiting cases and in the uncharged case.

Prescription	Energy	$r \rightarrow \infty$	$r \rightarrow 0$	$q = 0$
Einstein	$M \left\{ 1 - \frac{1}{[1 + (2Mr/q^2)^3]^{1/3}} \right\}$	$M$	$0$	$M$
Landau-Lifshitz	$\frac{M \left[ 1 - 1/[1 + (2Mr/q^2)^3]^{1/3} \right]}{1 - (2M/r) \left[ 1 - 1/[1 + (2Mr/q^2)^3]^{1/3} \right]}$	$M$	$0$	$M \left( 1 - \frac{2M}{r} \right)^{-1}$
Weinberg	$\frac{M \left[ 1 - 1/[1 + (2Mr/q^2)^3]^{1/3} \right]}{1 - (2M/r) \left[ 1 - 1/[1 + (2Mr/q^2)^3]^{1/3} \right]}$	$M$	$0$	$M \left( 1 - \frac{2M}{r} \right)^{-1}$
Møller	$\frac{r^2}{2} \left\{ -\frac{2M \left[ (8M^3 r^3/q^6 + 1)^{-1/3} - 1 \right]}{r^2} - \frac{16M^4 r}{q^6 [8M^3 r^3/q^6 + 1]^{4/3}} \right\}$	$M$	$0$	$M$

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