Research Article

Existence of Exponential p -Stability Nonconstant Equilibrium of Markovian Jumping Nonlinear Diffusion Equations via Ekeland Variational Principle

Ruofeng Rao^{1,2} and Shouming Zhong³

¹Department of Mathematics, Chengdu Normal University, Chengdu, Sichuan 611130, China

²Institution of Mathematics, Yibin University, Yibin, Sichuan 644007, China

³School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

Correspondence should be addressed to Ruofeng Rao; ruofengrao@163.com

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The authors obtained a delay-dependent exponential p -stability criterion for a class of Markovian jumping nonlinear diffusion equations by employing the Lyapunov stability theory and some variational methods. As far as we know, it is the first time to apply Ekeland variational principle to obtain the existence of exponential stability equilibrium of p -Laplacian dynamic system so that some methods used in this paper are different from those methods of many previous related literatures. In addition, the obtained existence criterion is only involved in the activation functions so that the criterion is simpler and easier than other existence criteria to be verified in practical application. Moreover, a numerical example shows the effectiveness of the proposed methods owing to the large allowable variation range of time-delay.

1. Introduction

Nonlinear diffusion equations have been investigated extensively by many authors owing to their physics and biological engineering backgrounds, population dynamics, and so on (see [1–9] and references therein). In addition, Markovian jumping systems have attracted rapidly growing interest due to the fact that Markovian jumping parameters are useful in modeling abrupt phenomena, such as random failures, operating in different points of a nonlinear plant, and changing in the interconnections of subsystems (see [8, 10–12] and references therein). On the other hand, fuzzy logic theory has shown to be an appealing and efficient approach to deal with the analysis and synthesis problems for complex nonlinear systems. Among various kinds of fuzzy methods, Takagi-Sugeno (T-S) fuzzy models provide a successful method to describe certain complex nonlinear system using some local linear subsystems (see [13–15] and references therein). As pointed out in the above related literature, the Markovian jumping T-S fuzzy mathematical models have always found their extensive applications in

the real world. However, almost all the applications are greatly dependent on the stability of systems, which can often come down to the stability of the equilibrium solution for the corresponding mathematical models. So in this paper, we may consider the stability of the nonlinear p -Laplace ($p > 1$) diffusion fuzzy equations with Markovian jumping. Note that when $p = 2$, the so-called reaction-diffusion equations have been widely investigated (see [16–20] and references therein). For example, under Dirichlet boundary conditions, the existence result of μ -stability equilibrium solution in the sense of L^2 norm for a class of time-delay reaction-diffusion equations was obtained in [16]. Motivated by the abovementioned literature, in this paper, we will synthetically employ Ekeland variational principle, Sobolev imbedding theorem, and the Lyapunov functional method to study the existence of exponential p -stability nonconstant equilibrium solution in the sense of L^p norm ($p > 1$) for delayed Markovian jumping fuzzy equations with nonlinear p -Laplace diffusion ($p > 1$).

Let $\{r(t) : [0, +\infty) \rightarrow S\}$ be a right-continuous Markov process on the complete probability space $(\mathcal{G}, \mathcal{F}, \mathbb{P})$ with

a natural filtration $\{F_t\}_{t \geq 0}$ and take values in the finite space $S \triangleq \{1, 2, \dots, N\}$ with generator $\Pi = \{\pi_{ik}\}$ given by

$$\begin{aligned} \mathbb{P}(r(t+\delta) = k \mid r(t) = i) \\ = \begin{cases} \pi_{ik}\delta + o(\delta), & k \neq i, \\ 1 + \pi_{ii}\delta + o(\delta), & k = i, \end{cases} \end{aligned} \quad (1)$$

where $\pi_{ik} \geq 0$ is transition probability rate from i to k ($k \neq i$) and $\pi_{ii} = -\sum_{k \in S, k \neq i} \pi_{ik}$, $\delta > 0$, and $\lim_{\delta \rightarrow 0} (o(\delta)/\delta) = 0$.

Consider the following fuzzy T-S Markovian jumping p -Laplace partial dynamic equations.

Fuzzy rule j : IF ω_1 is ν_{j1} and $\dots$ ω_κ is $\nu_{j\kappa}$ THEN

$$\begin{aligned} \frac{du_i(t, x)}{dt} = \mathcal{D}_i \operatorname{div}(|\nabla u_i|^{p-2} \nabla u_i) - b_i(u_i) \\ + c_{ij}(r(t)) f_i(u_i(t, x)) \\ + d_{ij}(r(t)) g_i(u_i(t - \tau_i(t), x)) + I_i, \\ i \in \mathcal{N}, t > 0, x \in \Omega, \end{aligned} \quad (2)$$

$$u_i(t, x) = 0, \quad i \in \mathcal{N}, t \geq 0, x \in \partial\Omega,$$

$$u_i(s, x) = \phi_i(s, x),$$

$$-\tau_i(0) \leq s \leq 0, 0 \leq \tau_i(t) \leq \tau_i, \quad i \in \mathcal{N}, t \geq 0, x \in \Omega,$$

where $\omega_k(t)$ ($k = 1, 2, \dots, \kappa$) is the premise variable and ν_{jk} ($j = 1, 2, \dots, J; k = 1, 2, \dots, \kappa$) is the fuzzy set that is characterized by membership function. J is the number of the IF-THEN rules and κ is the number of the premise variables. Denote the premise variable vector $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_\kappa(t)]$ and $\rho_j(\omega(t)) = w_j(\omega(t)) / \sum_{k=1}^J w_k(\omega(t))$, where $w_j(\omega(t)) : R^\kappa \rightarrow [0, 1]$ ($j = 1, 2, \dots, J$) is the membership function of the system with respect to the fuzzy rule j . ρ_j can be regarded as the normalized weight of each IF-THEN rule, satisfying

$$\rho_j(\omega(t)) \geq 0, \quad \sum_{j=1}^J \rho_j(\omega(t)) = 1. \quad (3)$$

Using a singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier, system (2) is inferred as follows:

$$\begin{aligned} \frac{du_i(t, x)}{dt} = \mathcal{D}_i \operatorname{div}(|\nabla u_i|^{p-2} \nabla u_i) - b_i(u_i) \\ + \sum_{j=1}^J \rho_j(\omega(t)) (c_{ij}(r(t)) f_i(u_i(t, x)) \\ + d_{ij}(r(t)) g_i(u_i(t - \tau_i(t), x))) + I_i, \\ i \in \mathcal{N}, t > 0, x \in \Omega, \end{aligned} \quad (4)$$

$$u_i(t, x) = 0, \quad i \in \mathcal{N}, t \geq 0, x \in \partial\Omega$$

with the initial condition

$$\begin{aligned} u_i(s, x) = \phi_i(s, x), \\ -\tau_i(0) \leq s \leq 0, 0 \leq \tau_i(t) \leq \tau_i, \quad i \in \mathcal{N}, t \geq 0, x \in \Omega, \end{aligned} \quad (4')$$

where Ω is a bounded subset in R^m with smooth boundary $\partial\Omega$. $p > 1$ is a scalar, and $\mathcal{N} = \{1, 2, \dots, n\}$ is a finite set. The divergence $\operatorname{div}(|\nabla u_i(t, x)|^{p-2} \nabla u_i(t, x)) = \Delta_p u_i(t, x)$ is the p -Laplacian (see [21] for details). In mode $r(t) = r$, we denote $c_{ij}(r(t)) = c_{ij}^r$ and $d_{ij}(r(t)) = d_{ij}^r$, which imply the connection strengths of the i th neuron on the j th neuron at the system mode $r(t) = r$, respectively. For any given $i \in \mathcal{N}$, $\mathcal{D}_i > 0$ is a corresponding constant, dependent on i .

Remark 1. Our methods employed in this paper are different from those of previous related literature. For example, homomorphic mapping theory was employed to obtain the existence of equilibrium of ordinary differential equations in [22]; topological degree theory was used to obtain the existence of equilibrium for fuzzy ordinary differential equations in [23] and of equilibrium for reaction-diffusion partial differential equations in [17]. In this paper, Ekeland variational principle is originally proposed to solve the existence of nonconstant equilibrium for nonlinear diffusion equations. Note that the abovementioned constant equilibrium point $u^* = (u_1^*, \dots, u_n^*)^T$ can actually be regarded as the special case of our nonconstant equilibrium point $u = u^*(x) = (u_1^*(x), \dots, u_n^*(x))^T$ with $u_i^*(x) \equiv \text{constant}$ for $i = 1, 2, \dots, n$. In addition, our criterion about existence is only involved in the activation functions while other more parameters need be considered in the proof of the existence of constant equilibrium point in those previous literatures (see Remark 8 below for details).

2. Preparation

Throughout this paper, we assume the following:

(A1) There exists a positive definition matrix $B = \operatorname{diag}(B_1, B_2, \dots, B_n)$ such that

$$\inf_{s \in R} b_i'(s) \geq B_i > 0, \quad \forall i \in \mathcal{N}. \quad (5)$$

(A2) There exist positive definite matrices $F = \operatorname{diag}(F_1, F_2, \dots, F_n)$ and $G = \operatorname{diag}(G_1, G_2, \dots, G_n)$ such that

$$\begin{aligned} |f_i(s) - f_i(t)| &\leq F_i |s - t|, \\ |g_i(s) - g_i(t)| &\leq G_i |s - t|, \end{aligned} \quad (6)$$

$$s, t \in R, i \in \mathcal{N}.$$

Definition 2. $u = u^*(x) = (u_1^*(x), u_2^*(x), \dots, u_n^*(x))^T$ is called a nonconstant equilibrium solution of PDEs (4) if and only if $u = u^*(x)$ satisfies (4). In addition, the nonconstant equilibrium solution $u^*(x)$ of PDEs (4) is called stochastically global exponential p -stability about L^p norm, if there are constants $M > 0$ and $\delta > 0$ for every stochastic field solution $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$ of PDEs (4) such that

$$E(\|u(t, x) - u^*\|_{L^p}) \leq M e^{-\delta(t-t_0)}, \quad (7)$$

where one denotes by $\|u\|_{L^p} = (\sum_{i=1}^n \|u_i\|_{L^p}^p)^{1/p}$ the L^p norm and denotes $\|u_i\|_{L^p} = (\int_\Omega |u_i| dx)^{1/p}$. Usually, $t_0 = 0$.

Definition 3. Let X be a Banach space, $\varphi \in C^1(X, R)$, and $c \in R$. φ satisfies the $(PS)_c$ condition if any sequence $\{u_n\} \subset X$, such that

$$\begin{aligned}\varphi(u_n) &\longrightarrow c, \\ \varphi'(u_n) &\longrightarrow 0,\end{aligned}\quad (8)$$

has a convergent subsequence. By the way, the above sequence $\{u_n\}$ with $\varphi(u_n) \rightarrow c$ and $\varphi'(u_n) \rightarrow 0$ is called the $(PS)_c$ sequence of φ for a given $c \in R$.

The following lemma originated from the famous Sobolev imbedding theorem.

Lemma 4. Let Ω be a bounded subset in R^m with smooth boundary $\partial\Omega$. For $1 < q < p$, there exist the corresponding positive constants c_1 and c_q such that, for any $\xi \in W_0^{1,p}(\Omega)$,

$$\begin{aligned}\int_{\Omega} |\xi| dx &\leq c_1 \|\xi\|, \\ \left(\int_{\Omega} |\xi|^q dx \right)^{1/q} &\leq c_q \|\xi\|,\end{aligned}\quad (9)$$

where the Sobolev space $W_0^{1,p}(\Omega)$ is the completion of $C_0^\infty(\Omega)$ with respect to the norm $\|\xi\| = (\int_{\Omega} |\nabla \xi|^p dx)^{1/p}$.

In 1979, Ekeland proposed the following famous Ekeland variational principle and its proof in [24]. As is well known, Ekeland variational principle has been the most important result in nonlinear analysis and has been applied to optimization theory, control theory, economic equilibrium theory, critical point theory, dynamic systems, and so forth. In this paper, we also need the following Ekeland variational principle.

Lemma 5 (Ekeland variational principle [24, Theorem 1]). Let X be a complete metric space and let $\varphi : X \rightarrow (-\infty, +\infty]$ be a lower semicontinuous function, bounded from below and not identical to $+\infty$. Let $\varepsilon > 0$ be given and let $\xi \in X$ be such that

$$\varphi(\xi) \leq \inf_X \varphi + \varepsilon. \quad (10)$$

Then there exists $v \in X$ such that

$$\begin{aligned}\varphi(v) &\leq \varphi(\xi), \\ \text{dist}(\xi, v) &\leq 1,\end{aligned}\quad (11)$$

and, for each $w \neq v$ in X ,

$$\varphi(w) > \varphi(v) - \varepsilon \text{dist}(v, w). \quad (12)$$

3. Main Result

Theorem 6. Let $p > 1$. Assume that there exists a positive scalar $q = q_1/q_2$ with $1 < q < p$ such that

$$\begin{aligned}\lim_{|s| \rightarrow +\infty} \frac{b_i(s)}{s^{q-1}} &= \lim_{|s| \rightarrow +\infty} \frac{f_i(s)}{s^{q-1}} = \lim_{|s| \rightarrow +\infty} \frac{g_i(s)}{s^{q-1}} = 0 \\ &= b_i(0), \quad \forall i \in \mathcal{N},\end{aligned}\quad (13)$$

where q_1 is an odd number and so is q_2 . Assume, in addition, $|I_i| < M$.

If there exist a sequence of positive scalars $h_i(r(t))$ ($r \in S$, $i \in \mathcal{N}$) such that

$$-\mathfrak{A} + \frac{e^{\beta\tau}}{1-K} \mathfrak{B} < 0, \quad (14)$$

where $\tau = \max_{i \in \mathcal{N}} \tau_i$, $\tau_i'(t) \leq K < 1$ for all $i \in \mathcal{N}$, $\mathfrak{A} = \text{diag}(a_1, a_2, \dots, a_n)$, $\mathfrak{B} = \text{diag}(b_1, b_2, \dots, b_n)$, $a_i = \min_{r \in S} a_i(r)$, $b_i = \max_{r \in S} b_i(r)$, and

$$\begin{aligned}a_i(r) &= h_i(r(t)) \left[pB_i - \beta - pF_i \left(\sum_{j=1}^J |c_{ij}^r| \right) \right. \\ &\quad \left. - (p-1) G_i \left(\sum_{j=1}^J |d_{ij}^r| \right) \right], \\ b_i(r) &= h_i(r(t)) G_i \left(\sum_{j=1}^J |d_{ij}^r| \right),\end{aligned}\quad (15)$$

$r \in S$,

then there exists a nonconstant equilibrium solution for PDEs (4), which is stochastically global exponential p -stability about L^p norm.

Proof. The following proof may be divided into two big steps.

Step 1. Firstly, we need prove that there exists a nonconstant equilibrium solution for (4).

Consider the functional

$$\begin{aligned}\varphi_i(\eta) &= \int_{\Omega} \left[\frac{1}{p} \mathcal{D}_i |\nabla \eta|^p + \mathcal{B}_i(\eta) \right. \\ &\quad \left. - \sum_{j=1}^J \rho_j(\omega(t)) \left((c_{ij}^r \mathcal{F}_i(\eta) + d_{ij}^r \mathcal{G}_i(\eta)) - I_i \eta \right) \right] dx,\end{aligned}\quad (16)$$

where $\mathcal{B}_i(s) = \int_0^s b_i(\theta) d\theta$, $\mathcal{F}_i(s) = \int_0^s f_i(\theta) d\theta$, $\mathcal{G}_i(s) = \int_0^s g_i(\theta) d\theta$, and $\eta = \eta(t, x) \in W_0^{1,p}(\Omega)$, where the Sobolev space $W_0^{1,p}(\Omega)$ is the completion of $C_0^\infty(\Omega)$ with respect to the norm $\|\eta\| = (\int_{\Omega} |\nabla \eta|^p dx)^{1/p}$.

It is obvious that $\varphi_i \in C^1(W_0^{1,p}(\Omega), R)$, for all $i \in \mathcal{N}$. If its critical point $u_i^*(x)$ exists, then $u^*(x) = (u_1^*(x), u_2^*(x), \dots, u_n^*(x))^T$ must be a nonconstant equilibrium solution of (4). So we only need to prove the existence of the critical point of φ_i for all $i \in \mathcal{N}$.

Next, it follows from (13) that there exists a large enough $a_M > 0$ such that

$$|f_i(s)| + |g_i(s)| \leq |s|^{q-1}, \quad |s| > a_M. \quad (17)$$

Furthermore, we can conclude by the continuity of f_i and g_i that

$$|f_i(s)| + |g_i(s)| \leq b_M, \quad \forall |s| \leq a_M \quad (18)$$

and hence,

$$|f_i(s)| + |g_i(s)| \leq b_M + |s|^{q-1}, \quad \forall s \in R, \quad (19)$$

where $b_M > 0$ is a constant.

For $s \geq 0$, we can derive by $q = q_1/q_2$ and the restrictive conditions on q_1, q_2

$$|\mathcal{F}_i(s)| \leq \int_0^s |f_i(y)| dy \leq b_M s + \frac{1}{q} s^q = b_M |s| + \frac{1}{q} |s|^q, \quad (20)$$

where $b_M > 0$ is a constant.

On the other hand, for $s < 0$, we can get by $q = q_1/q_2$ and the restrictive conditions on q_1, q_2

$$\begin{aligned} |\mathcal{F}_i(s)| &\leq \int_s^0 |f_i(y)| dy \leq \int_s^0 (b_M + |y|^{q-1}) dy \\ &\leq b_M |s| + \frac{1}{q} |s|^q. \end{aligned} \quad (21)$$

Hence,

$$|\mathcal{F}_i(s)| \leq b_M |s| + \frac{1}{q} |s|^q, \quad \forall s \in R. \quad (22)$$

Similarly, we can also deduce that

$$|\mathcal{G}_i(s)| \leq b_M |s| + \frac{1}{q} |s|^q, \quad \forall s \in R. \quad (23)$$

Denote $|c_{ij}| = \max_{r \in S} |c_{ij}^r|$ and $|d_{ij}| = \max_{r \in S} |d_{ij}^r|$; then $|c_{ij}| \geq 0$ and $|d_{ij}| \geq 0$ are constants, independent of r . Similarly, we can prove that there exist $a_{i1} > 0$ and $a_{i2} > 0$ such that

$$\begin{aligned} &\sum_{j=1}^J \rho_j(\omega(t)) (c_{ij}^r \mathcal{F}_i(s) + d_{ij}^r \mathcal{G}_i(s)) \\ &\leq \left(\sum_{j=1}^J |c_{ij}| \right) |\mathcal{F}_i(s)| + \left(\sum_{j=1}^J |d_{ij}| \right) |\mathcal{G}_i(s)| \\ &\leq a_{i1} |s| + a_{i2} |s|^q, \quad \forall s \in R, \end{aligned} \quad (24)$$

where $a_{i1} = b_M (\sum_{j=1}^J |c_{ij}| + \sum_{j=1}^J |d_{ij}|)$ and $a_{i2} = (1/q) (\sum_{j=1}^J |c_{ij}| + \sum_{j=1}^J |d_{ij}|)$ are the constants, independent of r .

Thereby, we have

$$\begin{aligned} &\left[\sum_{j=1}^J \rho_j(\omega(t)) (c_{ij}^r \mathcal{F}_i(s) + d_{ij}^r \mathcal{G}_i(s)) \right] + I_i s \\ &\leq (a_{i1} + M) |s| + a_{i2} |s|^q, \quad \forall s \in R. \end{aligned} \quad (25)$$

Since $b'_i(s) > 0$, we know that $b_i(s) > b_i(0) = 0$ if $s > 0$ and $b_i(s) < b_i(0) = 0$ if $s < 0$. And hence $\mathcal{B}_i(s) = \int_0^s b_i(\theta) d\theta \geq 0$ for all $s \in R$. Besides, we know from the above analysis and the Sobolev imbedding theorem (Lemma 4) that there

exist positive constants $b_{i1} = c_1(a_{i1} + M)$ and $b_{i2} = c_q^q a_{i2}$, independent of r , such that

$$\begin{aligned} \varphi_i(\eta) &\geq \frac{1}{p} \mathcal{D}_i \|\eta\|^p - (a_{i1} + M) \int_{\Omega} |\eta| dx \\ &\quad - a_{i2} \int_{\Omega} |\eta|^q dx \\ &\geq \frac{1}{p} \mathcal{D}_i \|\eta\|^p - b_{i2} \|\eta\|^q - b_{i1} \|\eta\|, \quad i \in \mathcal{N}. \end{aligned} \quad (26)$$

Denote $\mu(s) = (1/p) \mathcal{D}_i s^p - b_{i2} s^q - b_{i1} s$. Owing to $p > q > 1$, there exists a large enough constant $s_0 > 0$ such that $\mu(s) > 0$ for all $|s| > s_0$. And hence

$$\mu(s) \geq \min \left\{ 0, \min_{s \in [-s_0, s_0]} \mu(s) \right\}, \quad s \in R, \quad (27)$$

which implies that φ_i is bounded below. And the infimum may be defined as $c_i = \inf_X \varphi_i$, where we denote $X = W_0^{1,p}(\Omega)$ for convenience and denote by X^* its dual space. Define the operators $\mathbb{A}_i, \mathbb{B}_i : X \rightarrow X^*$ as follows:

$$\begin{aligned} \langle \mathbb{A}_i(\eta), \xi \rangle &= \mathcal{D}_i \int_{\Omega} |\nabla \eta|^{p-2} \nabla \eta \nabla \xi dx, \\ \langle \mathbb{B}_i(\eta), \xi \rangle &= \int_{\Omega} \left(b_i(\eta) \right. \\ &\quad \left. - \sum_{j=1}^J \rho_j(\omega(t)) (c_{ij}^r f_i(\eta) + d_{ij}^r g_i(\eta)) - I_i \right) \xi dx, \end{aligned} \quad (28)$$

where $\eta, \xi \in X$, $\langle \cdot, \cdot \rangle$ denotes the adjoint pair for X and X^* . It follows by (A1) and (A2) that all b_i, f_i, g_i are continuous. Similarly to (19), we can get

$$|b_i(s)| + |f_i(s)| + |g_i(s)| \leq c_M + |s|^{q-1}, \quad \forall s \in R, \quad (29)$$

where c_M is a constant. we can conclude from (13) and (29) that there exist positive constants a_{i3} and a_{i4} , independent of r , such that

$$\begin{aligned} &\left| b_i(s) - \sum_{j=1}^J \rho_j(\omega(t)) (c_{ij}^r f_i(s) + d_{ij}^r g_i(s)) - I_i \right| \\ &\leq c_M + |s|^{q-1} + \sum_{j=1}^J |c_{ij}| \cdot |f_i(s)| + \left(\sum_{j=1}^J |d_{ij}^r| \right) \\ &\quad \cdot |g_i(s)| + M \leq a_{i3} + a_{i4} |s|^{q-1}, \quad s \in R. \end{aligned} \quad (30)$$

Here, $a_{i3} = M + c_M + c_M (\sum_{j=1}^J |c_{ij}| + \sum_{j=1}^J |d_{ij}|)$ and $a_{i4} = 1 + \sum_{j=1}^J |c_{ij}| + \sum_{j=1}^J |d_{ij}|$.

So we know from [25, 26] that both $\mathbb{A}_i$ and $\mathbb{A}_i^{-1}$ are continuous, and $\mathbb{B}_i$ is compact.

Next, we claim that, for each $i \in \mathcal{N}$, the functional φ_i must satisfy the $(PS)_c$ condition if only every $(PS)_c$ sequence of φ_i is bounded.

Indeed, let $\{\eta_n\}$ be the $(PS)_c$ sequence of φ_i for any given c . It is obvious that

$$\langle \varphi'_i(\eta_n), \xi \rangle = \langle \mathbb{A}_i(\eta_n) - \mathbb{B}_i(\eta_n), \xi \rangle, \quad \forall \xi \in X. \quad (31)$$

Owing to $\varphi'_i(\eta_n) \rightarrow 0$, we have $(\mathbb{A}_i(\eta_n) - \mathbb{B}_i(\eta_n)) \rightarrow 0$. If $\{\eta_n\}$ is bounded, we know from the reflexivity of the Banach space $W_0^{1,p}(\Omega)$ that there exists a weakly convergent subsequence (say, $\{\eta_n\}$). Since $\mathbb{B}_i$ is compact, $\{\mathbb{B}_i(\eta_n)\}$ must own a convergent subsequence, which implies that $\mathbb{A}_i(\eta_n)$ must own a convergent subsequence. And furthermore, the continuity of $\mathbb{A}_i^{-1}$ yields that $\{\eta_n\}$ owns a convergent subsequence.

Below, we only need to prove that every $(PS)_c$ sequence (say, $\{\eta_n\}$) of φ_i is bounded.

Indeed, similarly to (26), we have

$$\varphi_i(\eta_n) \geq \frac{1}{p} \mathcal{D}_i \|\eta_n\|^p - b_{i2} \|\eta_n\|^q - b_{i1} \|\eta_n\|. \quad (32)$$

Owing to the boundedness of $\{\varphi_i(\eta_n)\}$, it is not difficult to prove by the application of reduction to absurdity that $\{\eta_n\}$ must be bounded in $W_0^{1,p}(\Omega)$.

Now we may define the metric for the space X as follows:

$$\text{dist}(\eta, \xi) = \|\eta - \xi\|. \quad (33)$$

Then X is a complete metric space with the above metric. From the continuity of φ_i and the above analysis, we know that $\varphi : W_0^{1,p}(\Omega) \rightarrow (-\infty, +\infty]$ is a lower semicontinuous function and bounded from below. Owing to $0 \in W_0^{1,p}(\Omega)$, we can compute and deduce that $|\varphi_i(0)| \leq \sum_{j=1}^J (|c_{ij}| |\mathcal{F}_i(0)| + |d_{ij}| |\mathcal{G}_i(0)|)$, which implies $\varphi_i \not\equiv +\infty, \forall i \in \mathcal{N}$.

According to Ekeland variational principle, for given $\varepsilon = 1/n$, there exists $\eta_n \in X$ such that

$$\varphi_i(\eta) > \varphi_i(\eta_n) - \frac{1}{n} \|\eta - \eta_n\|, \quad \forall \eta \neq \eta_n, \quad (34)$$

$$\varphi_i(\eta_n) < c_i + \frac{1}{n}. \quad (35)$$

So we can deduce from (34)

$$\|\varphi'_i(\eta_n)\| = \sup_{\|\xi\|=1} |d\varphi_i(\eta_n, \xi)| \leq \frac{1}{n}. \quad (36)$$

Here, $d\varphi_i(\eta_n, \xi)$ is Gâteaux derivative of φ_i at η_n , and $\varphi'_i(\eta_n)$ is Fréchet derivative of φ_i at η_n . Besides, (35) yields $\varphi_i(\eta_n) \rightarrow c_i$. Then we can conclude from the $(PS)_{c_i}$ condition that there exist a convergent subsequence $\{\eta_{n_j}\} \subset \{\eta_n\}$ and $u_i^* \in W_0^{1,p}(\Omega)$ such that $\eta_{n_j} \rightarrow u_i^*$. Moreover, the continuity of φ_i yields $\varphi_i(u_i^*) = c_i$. Hence, φ_i has a critical point $u_i^*(x)$ in X for all $i \in \mathcal{N}$. That is, there exists an equilibrium $u^* = (u_1^*, u_2^*, \dots, u_n^*)^T$ for (4).

Step 2. Below, we will prove the exponential p -stability for the equilibrium point u^* .

Consider the Lyapunov-Krasovskii functional as

$$V_i(t, r(t)) = V_{1i}(t, r(t)) + V_{2i}(t, r(t)) \quad i \in \mathcal{N}, \quad (37)$$

where

$$\begin{aligned} V_{1i}(t, r(t)) &= e^{\beta t} \int_{\Omega} h_i(r(t)) |u_i(t, x) - u_i^*(x)|^p dx, \\ V_{2i}(t, r(t)) &= \frac{b_i}{1-K} e^{\beta t} \int_{\Omega} \int_{t-\tau_i(t)}^t e^{\beta s} |u_i(s, x) - u_i^*(x)|^p ds dx. \end{aligned} \quad (38)$$

Let $\mathfrak{L}$ be the weak infinitesimal operator; then for any given mode $r(t) = r \in S$, taking the derivative of $V_{1i}(t, r)$ with respect to t along the trajectory of (4) yields

$$\begin{aligned} \mathfrak{L}V_{1i}(t, r) &= \lim_{\delta \rightarrow 0^+} \frac{1}{\delta} \left\{ \mathbb{E} \left[\int_{\Omega} e^{\beta(t+\delta)} h_i(r(t+\delta)) |u_i(t+\delta, x) - u_i^*(x)|^p dx \mid \right. \right. \\ &\quad \left. \left. r(t) = r \right] - e^{\beta t} \int_{\Omega} h_i(r(t)) |u_i(t, x) - u_i^*(x)|^p dx \right\} \\ &= e^{\beta t} \left\{ \int_{\Omega} h_i(r(t)) p |u_i - u_i^*|^{p-2} (u_i - u_i^*) \right. \\ &\quad \cdot \left[\mathcal{D}_i \sum_{k=1}^m \left(\frac{\partial}{\partial x_k} \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} \right) - \frac{\partial}{\partial x_k} \left(|\nabla u_i^*|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \right) \right. \\ &\quad \left. - (b_i(u_i) - b_i(u_i^*)) + \sum_{j=1}^J \rho_j(\omega(t)) c_{ij}^r (f_i(u_i) - f_i(u_i^*)) \right. \\ &\quad \left. + \sum_{j=1}^J \rho_j(\omega(t)) d_{ij}^r (g_i(u_i(t - \tau_i(t), x)) - g_i(u_i^*)) \right] dx \\ &\quad \left. + \int_{\Omega} \sum_{k \in S} \pi_{ik} h_i(k) |u_i(t, x) - u_i^*|^p dx + \beta h_i(r(t)) \int_{\Omega} |u_i - u_i^*|^p dx \right\}. \end{aligned} \quad (39)$$

Next, we claim that

$$\begin{aligned} \sum_{k=1}^m \int_{\Omega} \mathcal{D}_i |u_i - u_i^*|^{p-2} (u_i - u_i^*) \left(\frac{\partial}{\partial x_k} \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} \right) \right. \\ \left. - \frac{\partial}{\partial x_k} \left(|\nabla u_i^*|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \right) dx \leq 0. \end{aligned} \quad (40)$$

To verify (40), we have to prove firstly the following proposition by the Yang inequality.

Proposition 7. For $\eta, \psi \in W_0^{1,p}(\Omega)$, one has

$$(|\nabla \eta|^{p-2} \nabla \eta - |\nabla \psi|^{p-2} \nabla \psi) (\nabla \eta - \nabla \psi) \geq 0. \quad (41)$$

Proof. In fact, the Yang inequality yields

$$\begin{aligned} |\nabla \eta|^{p-2} \nabla \eta \nabla \psi &\leq |\nabla \eta|^{p-1} |\nabla \psi| \\ &\leq \frac{p-1}{p} (|\nabla \eta|^{p-1})^{p/(p-1)} + \frac{|\nabla \psi|^p}{p}, \quad (42) \\ |\nabla \psi|^{p-2} \nabla \psi \nabla \eta &\leq \frac{p-1}{p} (|\nabla \psi|^{p-1})^{p/(p-1)} + \frac{|\nabla \eta|^p}{p}. \end{aligned}$$

Synthesizing the above two inequalities results in (41).

So we can get by Gauss formula, the Dirichlet zero-boundary value, and Proposition 7

$$\begin{aligned} &\sum_{k=1}^m \int_{\Omega} \mathcal{D}_i |u_i - u_i^*|^{p-2} (u_i - u_i^*) \\ &\cdot \left(\frac{\partial}{\partial x_k} \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} \right) - \frac{\partial}{\partial x_k} \left(|\nabla u_i^*|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \right) dx \\ &= - \sum_{k=1}^m \int_{\Omega} \mathcal{D}_i \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} - |\nabla u_i^*|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \\ &\cdot \frac{\partial}{\partial x_k} (|u_i - u_i^*|^{p-2} (u_i - u_i^*)) dx = - \int_{\Omega} \mathcal{D}_i (p \\ &- 1) |u_i - u_i^*|^{p-2} [(|\nabla u_i|^{p-2} \nabla u_i - |\nabla u_i^*|^{p-2} \nabla u_i^*) \\ &\cdot (\nabla u_i - \nabla u_i^*)] dx \leq 0, \end{aligned} \quad (43)$$

which proves (40). $\square$

In addition, we get by (A1)

$$\begin{aligned} &\int_{\Omega} h_i(r(t)) p |u_i - u_i^*|^{p-2} (u_i - u_i^*) \\ &\cdot (b_i(u_i) - b_i(u_i^*)) dx \geq h_i(r(t)) \\ &\cdot p B_i \int_{\Omega} |u_i - u_i^*|^p dx. \end{aligned} \quad (44)$$

Further, we can derive by (A2)

$$\begin{aligned} &\int_{\Omega} h_i(r(t)) p |u_i - u_i^*|^{p-2} (u_i - u_i^*) \\ &\cdot \sum_{j=1}^J \rho_j(\omega(t)) c_{ij}^r (f_i(u_i) - f_i(u_i^*)) dx \leq h_i(r(t)) \\ &\cdot p F_i \left(\sum_{j=1}^J |c_{ij}^r| \right) \int_{\Omega} |u_i - u_i^*|^p dx. \end{aligned} \quad (45)$$

In addition, (A2) and the Yang inequality yield

$$\begin{aligned} &\int_{\Omega} h_i(r(t)) p |u_i - u_i^*|^{p-2} (u_i - u_i^*) \\ &\cdot \sum_{j=1}^J \rho_j(\omega(t)) d_{ij}^r (g_i(u_i(t - \tau_i(t), x)) - g_i(u_i^*)) \\ &\leq h_i(r(t)) G_i \left(\sum_{j=1}^J |d_{ij}^r| \right) \left[(p-1) \int_{\Omega} |u_i - u_i^*|^p dx \right. \\ &\left. + \int_{\Omega} |u_i(t - \tau_i(t), x) - u_i^*|^p dx \right]. \end{aligned} \quad (46)$$

Combining the above analyses results in

$$\begin{aligned} \mathfrak{L}V_{1i}(t, r) &\leq e^{\beta t} \left[-a_i(r) \int_{\Omega} |u_i - u_i^*|^p dx \right. \\ &\left. + b_i(r) \int_{\Omega} |u_i(t - \tau_i(t), x) - u_i^*|^p dx \right], \quad \forall r \in S. \end{aligned} \quad (47)$$

On the other hand, we have

$$\begin{aligned} \mathfrak{L}V_{2i} &\leq \frac{b_i}{1-K} e^{\beta \tau} \int_{\Omega} e^{\beta t} |u_i(t, x) - u_i^*(x)|^p dx \\ &- b_i \int_{\Omega} e^{\beta t} |u_i(t - \tau_i(t), x) - u_i^*(x)|^p dx. \end{aligned} \quad (48)$$

Thus, we get by (14)

$$\begin{aligned} \mathfrak{L}V_i(t, r(t)) &\leq e^{\beta t} \left(-a_i + \frac{b_i}{1-K} e^{\beta \tau} \right) \int_{\Omega} |u_i - u_i^*|^p dx \\ &\leq 0, \quad \forall t \geq 0. \end{aligned} \quad (49)$$

So we can obtain by the Dynkin formula

$$\begin{aligned} EV(t, r(t)) - EV(0, r(0)) &= E \int_0^t \mathfrak{L}V(s, r(s)) ds \\ &\leq 0, \quad t \geq 0. \end{aligned} \quad (50)$$

Hence, we have

$$\begin{aligned} &\left(\min_{r \in S} h_i(r(t)) \right) E \left(e^{\beta t} \int_{\Omega} |u_i(t, x) - u_i^*(x)|^p dx \right) \\ &\leq EV(t, r(t)) \leq EV(0, r(0)) \\ &\leq E \left(\max_{r \in S} h_i(r(t)) \int_{\Omega} |u_i(0, x) - u_i^*(x)|^p dx \right. \\ &\left. + \frac{b_i}{1-K} e^{\beta \tau} \int_{\Omega} \int_{-\tau}^0 e^{\beta s} |u_i(s, x) - u_i^*(x)|^p ds dx \right) \\ &\leq \left(\max_{r \in S} h_i(r(t)) + \frac{b_i}{1-K} e^{\beta \tau} \right) \\ &\cdot \sup_{-\tau \leq s \leq 0} E \|\phi_i(s) - u_i^*\|_{L^p}^p, \end{aligned} \quad (51)$$

which implies

$$\begin{aligned} E \left(\int_{\Omega} |u_i(t, x) - u_i^*(x)|^p dx \right) \\ \leq h_i e^{-\beta t} \sup_{-\tau \leq s \leq 0} E \|\phi_i(s) - u_i^*\|_{L^p}^p, \quad i \in \mathcal{N}, \end{aligned} \quad (52)$$

where $h_i = (\max_{r \in S} h_i(r(t)) + (b_i/(1-K))e^{\beta \tau}) / \min_{r \in S} h_i(r(t)) > 0$, $\forall i \in \mathcal{N}$. Then we have

$$\begin{aligned} E \|u - u^*\|_{L^p}^p &= \sum_{i \in \mathcal{N}} E \left(\int_{\Omega} |u_i(t, x) - u_i^*(x)|^p dx \right) \\ &\leq \left(\max_{i \in \mathcal{N}} h_i \right) e^{-\beta t} E \|\phi - u^*\|_{\tau}^p, \end{aligned} \quad (53)$$

where we denote $E \|\phi - u^*\|_{\tau}^p = \sum_{i \in \mathcal{N}} \sup_{-\tau \leq s \leq 0} (E \|\phi_i(s) - u_i^*\|_{L^p}^p)$.

Now, we can conclude from Definition 2 that the nonconstant equilibrium solution of (4) is stochastically exponentially p -stable about L^p norm. And that completes the proof of Theorem 6.

Remark 8. In [27], existence theorems of stochastic differential equations on $t \in [t_0, T)$ were given under some conditions on activation functions, where $T > 0$ is a constant. And in [28, 29], existence theorems of stochastic differential equations were presented under some conditions on function $V \in C^{1,2}([t_0 - \tau, T) \times R^n; R_+)$. Motivated by [27], we proposed some conditions on activation functions to set up existence criterion for the equilibrium solution of system (4). In [22, 23], the constant equilibrium solution $u = u^*$ for all $t \in [t_0, +\infty)$ was obtained by homomorphic mapping theory and matrix theory, or matrix theory and homotopy invariance theorem, where $u^* = (u_1^*, u_2^*, \dots, u_i^*, \dots, u_n^*)$, and each u_i^* is a constant. In this paper, we also need to consider the equilibrium solution of (4) defined on $[t_0, +\infty)$. Different from [22, 23], we consider the nonconstant equilibrium solution $u = u^*(x) = (u_1^*(x), u_2^*(x), \dots, u_i^*(x), \dots, u_n^*(x))$ for all $t \in [t_0, +\infty)$. This equilibrium solution is a solution for a nonlinear p -Laplacian elliptic partial differential equation whose space frame may be considered as infinite dimension function space $W_0^{1,p}(\Omega)$. And variational method is always a powerful tool to solve the problem. Although the variational method is more complicated than homomorphic mapping method, M -matrix method, or homotopy invariance theorem, our criterion about existence is only involved in the activation functions (remark: condition (14) is not used in the proof of existence) and hence is simpler and more effective than other criteria, such as M -matrix criteria and LMI-based criteria, because LMI-based criteria or M -matrix criteria always involve the computer MATLAB programming in practical application while our condition (13) is easy to verify. So our existence criterion is actually simpler and more effective than LMI-based criteria and other criteria, which is the main contribution in this paper.

Remark 9. LMI-based stability criteria or M -matrix stability criteria are always proposed in many literatures related to

the mean square stability (see, e.g., [30–33] and references therein). However, when $p > 1$ and $p \neq 2$, p -stability criteria always involve more complicated mathematical method and mathematical deduction. For example, the stability criteria in [34] are not simpler than our stability criterion in Theorem 6. Similar phenomena exposed in many literatures related to p -stability (see [15, 34–38]). Besides, the nonlinear p -Laplacian ($p > 1$) operator produces great difficulties in p -stability proof. However, our condition (14) is still a LMI condition, which can be computed and verified by computer MATLAB LMI Toolbox in practical application.

4. Numerical Example

Example 1. Consider the 5-Laplace fuzzy T-S dynamical equations as follows.

Fuzzy Ruler 1. IF $\omega_1(t)$ is $1/e^{-2\omega_1(t)}$, THEN

$$\begin{aligned} \frac{du_1(t, x)}{dt} &= \mathcal{D}_1 \operatorname{div} (|\nabla u_1|^3 \nabla u_1) - b_1(u_1) \\ &\quad + (c_{11}^r f_1(u_1(t, x)) + d_{11}^r g_1(u_1(t - \tau_1(t), x))) \\ &\quad + I_1, \quad t > 0, \quad x \in \Omega, \\ \frac{du_2(t, x)}{dt} &= \mathcal{D}_2 \operatorname{div} (|\nabla u_2|^3 \nabla u_2) - b_2(u_2) \\ &\quad + (c_{21}^r f_2(u_2(t, x)) + d_{21}^r g_2(u_2(t - \tau_2(t), x))) \\ &\quad + I_2, \quad t > 0, \quad x \in \Omega, \\ u_i(t, x) &= 0, \quad i \in \mathcal{N}, \quad t \geq 0, \quad x \in \partial\Omega. \end{aligned} \quad (54a)$$

Fuzzy Ruler 2. IF $\omega_2(t)$ is $1 - 1/e^{-2\omega_1(t)}$, THEN

$$\begin{aligned} \frac{du_1(t, x)}{dt} &= \mathcal{D}_1 \operatorname{div} (|\nabla u_1|^3 \nabla u_1) - b_1(u_1) \\ &\quad + (c_{12}^r f_1(u_1(t, x)) + d_{12}^r g_1(u_1(t - \tau_1(t), x))) \\ &\quad + I_1, \quad t > 0, \quad x \in \Omega, \\ \frac{du_2(t, x)}{dt} &= \mathcal{D}_2 \operatorname{div} (|\nabla u_2|^3 \nabla u_2) - b_2(u_2) \\ &\quad + (c_{22}^r f_2(u_2(t, x)) + d_{22}^r g_2(u_2(t - \tau_2(t), x))) \\ &\quad + I_2, \quad t > 0, \quad x \in \Omega, \\ u_i(t, x) &= 0, \quad i \in \mathcal{N}, \quad t \geq 0, \quad x \in \partial\Omega \end{aligned} \quad (54b)$$

FIGURE 1: The state variable $u_1(t, x)$.

equipped with the initial value

$$\phi(s, x) = \begin{pmatrix} 0.2 \cos(2\pi x(1-x))^2 \sin(\pi s)^2 \\ 0.25 \sin(6x(1-x))^2 \cos(3\pi s)^2 \end{pmatrix}, \quad (55)$$

$$-83 \leq s \leq 0,$$

where $x \in \Omega \triangleq \{(x_1, x_2)^T \in \mathbb{R}^2 : |x_i| < 1, i \in \mathcal{N}\}$, $N = \{1, 2\}$, $r \in S \triangleq \{1, 2\}$, $c_{11}^1 = c_{21}^1 = 0.1$, $c_{11}^2 = c_{21}^2 = 0.11$, $c_{12}^1 = c_{22}^1 = 0.15$, $c_{12}^2 = 0.12 = c_{22}^2$, $d_{11}^1 = d_{21}^1 = 0.1$, $d_{11}^2 = d_{21}^2 = 0.11$, $d_{12}^1 = d_{22}^1 = 0.15$, $d_{12}^2 = 0.12 = d_{22}^2$, $b_1(u_1) = u_1^3 + (1/2)u_1$, $b_2(u_2) = u_2^3 + (1/3)u_2$, and $B_1 = 1/2$, and $B_2 = 1/3$. Consider $f_1(u_1) = 0.2u_1 \sin u_1 = g_1(u_1)$, $f_2(u_2) = 0.3u_2 + 0.2 \cos u_2 = g_2(u_2)$, and $F_1 = G_1 = 0.4$, $F_2 = G_2 = 0.5$; let $\tau_i(t) \equiv 83$, $i \in \mathcal{N}$, and $\tau = 83$, $K = 0$; denote $h_i(r) = h_i^r$, $r \in S$. Let $h_1^1 = 1$, $h_2^1 = 2$, $h_1^2 = 3$, $h_2^2 = 4$, and $\beta = 0.01$. Denote $a_i(r) = a_i^r$ and $b_i(r) = b_i^r$; then we can compute by computer MATLAB that $a_1^1 = 1.5900$, $a_2^1 = 4.9860$, $a_1^2 = 1.0633$, $a_2^2 = 2.4867$, $b_1^1 = 0.1000$, $b_2^1 = 0.2760$, $b_1^2 = 0.2500$, and $b_2^2 = 0.4600$, and hence $a_1 = 1.5900$, $a_2 = 1.0633$, $b_1 = 0.2760$, $b_2 = 0.4600$, and

$$-a_1 + \frac{b_1}{1-K} e^{\beta\tau} = -0.9570 < 0,$$

$$-a_2 + \frac{b_2}{1-K} e^{\beta\tau} = -0.0084 < 0, \quad (56)$$

which imply that condition (14) is satisfied. In addition, condition (13) is obviously satisfied. Therefore, there exists a nonconstant equilibrium solution for PDEs (54a)-(54b), which is stochastically global exponential p -stability about L^p norm (see Figures 1 and 2).

5. Conclusions

The nonlinear p -Laplace ($p > 1$, $p \neq 2$) brings great difficulties to the proof of the existence of the p -stability nonconstant equilibrium solution, for p -Laplace ($p > 1$, $p \neq 2$) partial differential equations (PDEs) always need be considered in Banach space $W^{1,p}(\Omega)$ while the common linear Laplace ($p = 2$) PDEs can be studied in the setting of the special Hilbert space H^1 that can be orthogonally decomposed into the direct sum of the eigenfunction spaces. However, by applying Ekeland variational principle and the Yang inequality and constructing the suitable Lyapunov functional, we overcome those difficulties to obtain the existence of exponential p -stability nonconstant equilibrium solution for Markovian jumping p -Laplace ($p > 1$) partial differential equations (PDEs) (4) under Dirichlet boundary condition. As far as we know, it is the first time to apply the Ekeland variational principle to solve the above problem so that some methods used in this paper are different from those of many previous related literatures (see Remark 1). In addition, the obtained existence criterion is only involved in the activation functions so that the criterion is simpler and easier than other existence criteria to be verified in practical application. Moreover, a numerical example shows the effectiveness of the proposed methods owing to the large allowable variation range of time-delay.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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FIGURE 2: The state variable $u_2(t, x)$.

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