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Research Article

CRE Solvability, Exact Soliton-Cnoidal Wave Interaction Solutions, and Nonlocal Symmetry for the Modified Boussinesq Equation

Wenguang Cheng[1] and Biao Li[2]

[1]*Department of Mathematics, Yuxi Normal University, Yuxi 653100, China*
[2]*Ningbo Collaborative Innovation Center of Nonlinear Hazard System of Ocean and Atmosphere, Ningbo University, Ningbo 315211, China*

Correspondence should be addressed to Biao Li; libiao@nbu.edu.cn

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It is proved that the modified Boussinesq equation is consistent Riccati expansion (CRE) solvable; two types of special soliton-cnoidal wave interaction solution of the equation are explicitly given, which is difficult to be found by other traditional methods. Moreover, the nonlocal symmetry related to the consistent tanh expansion (CTE) and the residual symmetry from the truncated Painlevé expansion, as well as the relationship between them, are obtained. The residual symmetry is localized after embedding the original system in an enlarged one. The symmetry group transformation of the enlarged system is derived by applying the Lie point symmetry approach.

1. Introduction

The exact solutions for nonlinear evolution equations (NLEEs) arising from many science fields are important because of their wide applications in explaining physical phenomena. Many powerful methods for obtaining the exact solutions of NLEEs have been presented, such as the inverse scattering transformation [1], the Darboux transformations (DT) [2], the Bäcklund transformation (BT) [3], the Lie group method [4, 5], Hirota's bilinear method [6], the Painlevé analysis [7], separated variable method [8], homogeneous balance method [9], and tanh function expansion method [10]. For these methods, the interaction solutions among different nonlinear excitations such as solitons on a cnoidal wave background are difficult to obtain. The soliton solutions on the background of the periodic solutions which are given by the elliptic theta function can be found with the help of Cauchy-Baker kernel [11, 12].

Recently, Lou found that the symmetry from the truncated Painlevé expansion is just the residue with respect to the singular manifold and proposed residual symmetry [13, 14]. Furthermore, by developing the truncated Painlevé expansion, Lou [15, 16] established the consistent Riccati expansion (CRE) method, which is a simple but effective method to construct interaction solutions among different nonlinear excitations. The method is valid for some nonlinear models like the bosonized supersymmetric Korteweg-de Vries (KdV) model [14], the nonlinear Schrödinger system [16], the Broer-Kaup System [17], and so on [18–22].

In this paper, we focus on investigating the CRE solvability and nonlocal symmetry of the modified Boussinesq equation:

$$3u^2u_x + 3u_x\int u_t dx - \frac{1}{2}u_{xxx} - \frac{3}{2}\int u_{tt}dx = 0, \qquad (1)$$

which was proposed by Hirota and Satsuma [23] from a BT of the Boussinesq equation:

$$u_{1tt} + \left(3u_1^2 + \frac{1}{3}u_{1xx}\right)_{xx} = 0. \qquad (2)$$

Equation (1) is called the modified Boussinesq equation since it is linked with the known Boussinesq equation (2) by the Miura transformation:

$$u_1 = \frac{1}{3}\left(\pm u_x - u^2 - \int u_t dx\right). \tag{3}$$

The generalized Hamiltonian form and the corresponding finite-dimensional integrable systems of (1) are obtained with the help of the nonlinearization approach of Lax pairs [24]. The Painlevé property [25] and similarity solutions [26, 27] have been studied. To our knowledge, the analytic interaction solutions among different nonlinear excitations for (1) have not been obtained up to now. Here we will study the interaction solution between the soliton and the cnoidal periodic wave of (1).

The paper is arranged as follows. Section 2 is devoted to the CRE method for the modified Boussinesq equation. As a result, two types of special explicit interaction solution between the soliton and the cnoidal periodic wave of this equation are given. In Section 3, for the modified Boussinesq equation, we derive the nonlocal symmetry related to the CTE and the nonlocal residual symmetry from the truncated Painlevé expansion. The relationship between them is presented. Then the corresponding symmetry group transformation is found in the process of the localization of residual symmetry. The last section is a short summary and discussion.

2. CRE Solvability and Soliton-Cnoidal Wave Interaction Solutions

2.1. CRE Solvability. Based on the CRE method in [15], the possible truncated expansion expression for (1) has the form

$$u = u_1 R(w) + u_0, \tag{4}$$

where u_0, u_1, and w are undetermined functions of space-time x and t and $R(w)$ satisfies the Riccati equation

$$R_w = a_0 + a_1 R + a_2 R^2, \tag{5}$$

which includes $\tanh(w)$ as a special case. Differentiating with respect to x, (1) becomes

$$\begin{aligned} &12u_{xx}^2 u_t + 12u_x^3 u_{xx} + 36uu_x u_{xx}^2 + 6u_x u_{xx} u_{xt} \\ &\quad - u_{xx}u_{xxxxx} - 3u_{xx}u_{xtt} - 12uu_x^2 u_{xxx} \\ &\quad - 6u_x u_{xxx} u_t + u_{xxx}u_{xxxx} + 3u_{xxx}u_{tt} = 0. \end{aligned} \tag{6}$$

Substituting (4) with (5) into (6) and vanishing all the coefficients of the powers of $R(w)$, we obtain ten overdetermined equations for only three undetermined functions. Fortunately, these overdetermined equations are consistent and possess the following solution:

$$\begin{aligned} u_1 &= -a_2 w_x, \\ u_0 &= -\frac{1}{2}a_1 w_x - \frac{1}{2}\frac{w_{xx}}{w_x} - \frac{1}{2}\frac{w_t}{w_x}, \end{aligned} \tag{7}$$

and the function w satisfies a generalization of the Schwartzian form of (1):

$$3\mathscr{C}_t + 3\mathscr{C}\mathscr{C}_x + \mathscr{S}_x - \delta w_x w_{xx} = 0, \tag{8}$$

where

$$\begin{aligned} \mathscr{C} &= \frac{w_t}{w_x}, \\ \mathscr{S} &= \frac{w_{xxx}}{w_x} - \frac{3}{2}\frac{w_{xx}^2}{w_x^2}, \\ \delta &= a_1^2 - 4a_0 a_2. \end{aligned} \tag{9}$$

Due to consistency of the overdetermined system, we call the modified Boussinesq equation CRE solvable.

2.2. CTE Solvability and Soliton-Cnoidal Wave Interaction Solutions. We consider the function $R(w)$ in Riccati equation (5) as the following special solution:

$$R(w) = \tanh(w), \tag{10}$$

which means that a CRE solvable system must be consistent tanh expansion (CTE) solvable, and vice versa. Meanwhile, truncated expansion expression (4) is changed as

$$u = u_0 + u_1 \tanh(w), \tag{11}$$

where u_0, u_1, and w are undetermined by (7) and (8) with $a_0 = 1$, $a_1 = 0$, $a_2 = -1$, and $\delta = 4$. Thus, we have

$$\begin{aligned} u_1 &= w_x, \\ u_0 &= -\frac{1}{2}\frac{w_{xx}}{w_x} - \frac{1}{2}\frac{w_t}{w_x}, \end{aligned} \tag{12}$$

and the function w satisfies

$$3\mathscr{C}_t + 3\mathscr{C}\mathscr{C}_x + \mathscr{S}_x - 4w_x w_{xx} = 0. \tag{13}$$

In brief, we arrive at the following nonauto-BT theorem for (1).

Theorem 1. *If w is a solution of (13), then u, with*

$$u = w_x \tanh(w) - \frac{1}{2}\frac{w_{xx}}{w_x} - \frac{1}{2}\frac{w_t}{w_x}, \tag{14}$$

is a solution of modified Boussinesq equation (1).

Theorem 1 shows that the single soliton solution of (1) is only the straight line solution $w = k_0 x + \omega_0 t$ of (13); the interaction solutions between solitons and other nonlinear excitations of (1) can be constructed by solving (13). To find the interaction solutions between one soliton and other nonlinear waves of (1), we consider w in the form

$$w = k_0 x + \omega_0 t + g, \tag{15}$$

where g is a function with respect to x and t. In this study, we only discuss the solutions with the form

$$\begin{aligned} w &= k_0 x + \omega_0 t + W(X), \\ X &= k_1 x + \omega_1 t. \end{aligned} \tag{16}$$

Substituting (16) into (13), we can find that $W_1(X)$ satisfies

$$\begin{aligned} W_1(X)_X^2 = {} & 4W_1(X)^4 + 2a_1 W_1(X)^3 + 2a_2 W_1(X)^2 \\ & + 2a_3 W_1(X) + a_4, \end{aligned} \tag{17}$$

with

$$\begin{aligned} & W(X)_X = W_1(X), \\ & a_1 = \frac{4k_0 - C_1 k_1^3}{k_1}, \\ & a_2 = \frac{2k_0^2 - 3C_1 k_0 k_1^3 + C_2 k_1^3}{k_1^2}, \\ & a_3 = \frac{-3k_0\omega_1^2 + 3k_1\omega_0\omega_1 - 3C_1 k_0^2 k_1^5 + 2C_2 k_0 k_1^5}{k_1^5}, \\ & a_4 \\ & \quad = \frac{4k_0 k_1 \omega_0 \omega_1 - 5k_0^2\omega_1^2 + k_1^2\omega_0^2 - 2C_1 k_0^3 k_1^5 + 2C_2 k_0^2 k_1^5}{k_1^6}, \end{aligned} \tag{18}$$

and C_1, C_2 are arbitrary constants. Then (1) has the explicit solution expressed as

$$\begin{aligned} u = {} & (k_0 + k_1 W_1(X)) \tanh(k_0 x + \omega_0 t + W(X)) \\ & - \frac{\omega_0 + \omega_1 W_1(X) + k_1^2 W_1(X)_X}{2(k_0 + k_1 W_1(X))}. \end{aligned} \tag{19}$$

It is clear that (17) has abundant explicit solutions in terms of Jacobi elliptic functions. Hence, solution (19) exhibits the interactions between one soliton and cnoidal periodic waves. In what follows, only two nontrivial cases are considered in detail to obtain this kind of solution.

Case 1. A simple solution of (17) is given as

$$W(X) = cE_\pi(\mathrm{sn}(X, m), n, m), \tag{20}$$

which leads to the soliton-cnoidal wave interaction solution of (1) as follows:

$$\begin{aligned} u = {} & \left(k_0 - \frac{ck_1}{nS^2 - 1}\right) \tanh(k_0 x + \omega_0 t + cE_\pi(S, n, m)) \\ & + \frac{ck_1^2 nSCD}{(nS^2 - 1)[ck_1 - k_0(nS^2 - 1)]} \\ & - \frac{c\omega_1 - \omega_0(nS^2 - 1)}{2[ck_1 - k_0(nS^2 - 1)]}, \end{aligned} \tag{21}$$

where $\{k_0, k_1, m, n\}$ are arbitrary constant, $S \equiv \mathrm{sn}(k_1 x + \omega_1 t, m)$, $C \equiv \mathrm{cn}(k_1 x + \omega_1 t, m)$, $D \equiv \mathrm{dn}(k_1 x + \omega_1 t, m)$, and

$$\begin{aligned} c^2 &= -\frac{(n-1)(m^2 - n)}{n}, \\ \omega_1 &= -\frac{4ck_0^3 k_1 n + c^2 k_1^4 m^2 + 2ck_0 k_1^3 (3m^2 - m^2 n - n) + 3k_0^2 k_1^2 (3m^2 - 2m^2 n + n^2 - 2n)}{3\sqrt{ck_0 n[ck_0^3 n + c^2 k_1^3 m^2 + ck_0 k_1^2 (3m^2 - m^2 n - n) + k_0^2 k_1 (3m^2 - 2m^2 n + n^2 - 2n)]}}, \\ \omega_0 &= -\frac{2k_1^2 m^2 (3k_0 + ck_1)(3k_0 - 2k_0 n + ck_1)}{3c\omega_1 n} + \frac{k_0 (4ck_1^4 + 3c\omega_1^2 + 12k_0 k_1^3 - 6k_0 k_1^3 n - 8ck_0^2 k_1^2)}{3ck_1\omega_1}. \end{aligned} \tag{22}$$

In solution (21), $E_\pi(\zeta, n, m)$ is the third type of incomplete elliptic integral.

The dynamic behavior of soliton-cnoidal wave interaction solution (21) with the parameters $k_0 = 1$, $k_1 = 1.2$, $m = 0.9$, and $n = 0.5$ is illustrated in Figure 1. It can be seen from Figure 1 that a soliton moves on a cnoidal wave background instead of moving on the plane continuous wave background. This kind of solution can be applicable to describe some interesting physical phenomena, such as the Fermionic quantum plasma [28].

Case 2. Another special solution of (17) reads as follows:

$$W(X) = A \operatorname{arctanh}(\mathrm{sn}(X, m)), \tag{23}$$

which yields the soliton-cnoidal wave interaction solution of (1):

$$\begin{aligned} u = {} & \left(k_0 - \frac{k_1 CD}{2(S^2 - 1)}\right) \\ & \cdot \tanh\left(k_0 x + \omega_0 t + \frac{1}{2}\operatorname{arctanh}(S)\right) \\ & + \frac{k_1^2 SC^2 D^2}{(S^2 - 1)[k_1 CD - 2k_0(S^2 - 1)]} \\ & + \frac{k_1^2 S(m^2 C^2 + D^2) + 2\omega_0(S^2 - 1) - \omega_1 CD}{2[k_1 CD - 2k_0(S^2 - 1)]}, \end{aligned} \tag{24}$$

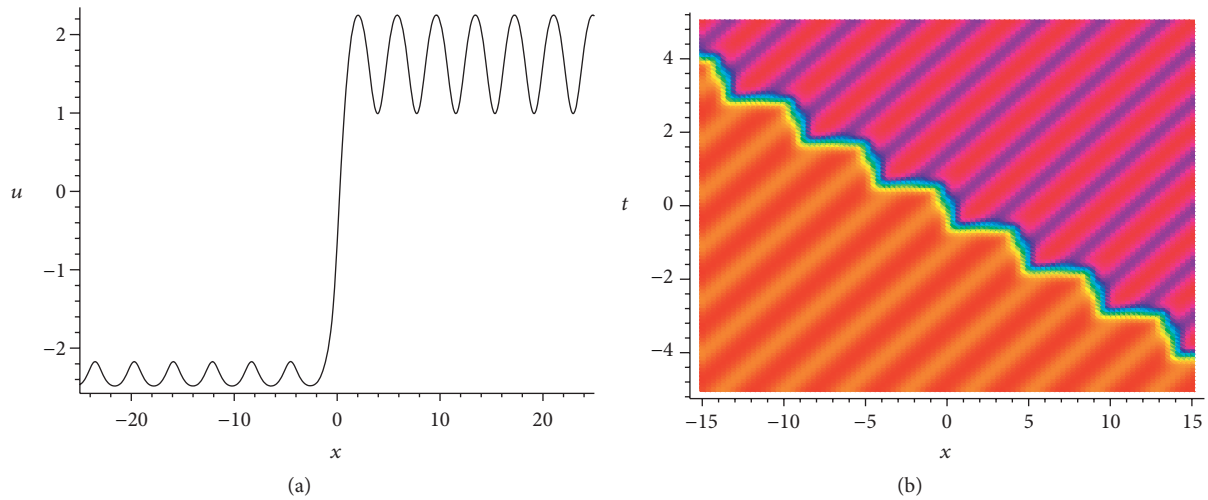

Figure 1: Interaction between the soliton and the cnoidal periodic wave via solution (21) with the parameters $k_0 = 1$, $k_1 = 1.2$, $m = 0.9$, and $n = 0.5$. (a) The profile of the special structure at $t = 0$. (b) Overhead view of the wave.

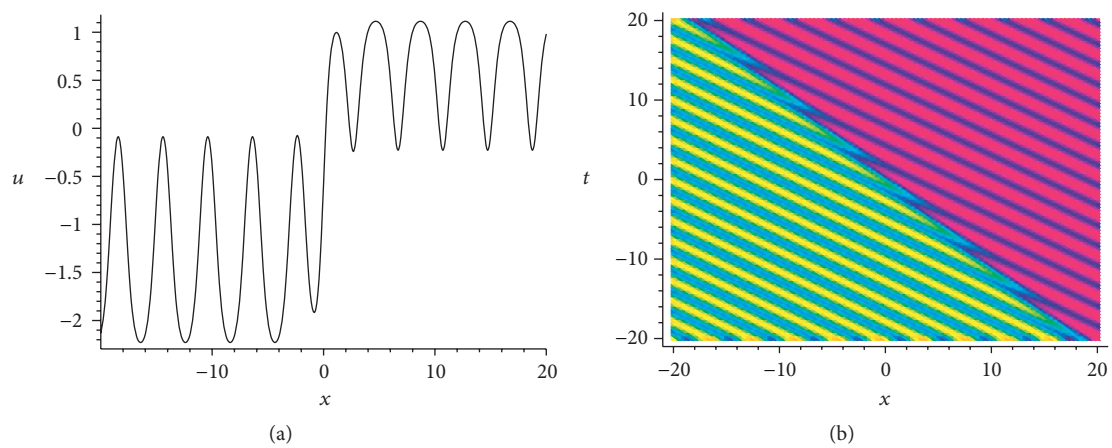

Figure 2: Soliton-cnoidal wave interaction solution for (1) expressed by (24) with the parameters selected as $k_0 = 1$, $k_1 = 1.2$, and $m = 1.5$. (a) One-dimensional image at $t = 0$. (b) Overhead view of the wave.

where $\{k_0, k_1, m\}$ are arbitrary constant, $S \equiv \mathrm{sn}(k_1 x + \omega_1 t, m)$, $C \equiv \mathrm{cn}(k_1 x + \omega_1 t, m)$, $D \equiv \mathrm{dn}(k_1 x + \omega_1 t, m)$, and

$$A = \frac{1}{2},$$

$$\omega_0 = -\frac{8k_0^2\left(2k_0^2 - k_1^2\right) - k_1^2\left(8k_0^2 - 3k_1^2\right)}{6\sqrt{\left(2k_0 - k_1\right)\left(2k_0 + k_1\right)\left(2k_0 - k_1 m\right)\left(2k_0 + k_1 m\right)}}, \quad (25)$$

$$\omega_1 = \frac{2k_0 k_1\left(8k_0^2 - k_1^2 - k_1^2 m^2\right)}{3\sqrt{\left(2k_0 - k_1\right)\left(2k_0 + k_1\right)\left(2k_0 - k_1 m\right)\left(2k_0 + k_1 m\right)}}.$$

In Figure 2, we plot soliton-cnoidal wave interaction solution (24) with the parameters selected as $k_0 = 1$, $k_1 = 1.2$, and $m = 1.5$. It presents that a soliton propagates on a cnoidal wave background instead of propagating on the plane continuous wave background.

3. Nonlocal Symmetry and Its Localization

3.1. Nonlocal Symmetry. To derive the nonlocal symmetry related to the CTE, we give a nonauto-BT theorem for modified Boussinesq equation (1) as follows.

Theorem 2. *If w is a solution of (13), then u, with*

$$u = -w_x - \frac{1}{2}\frac{w_{xx}}{w_x} - \frac{1}{2}\frac{w_t}{w_x}, \quad (26)$$

is a solution of modified Boussinesq equation (1).

Proof. By direct substitution, a symmetry σ^u of (6) is defined as a solution of its linearized equation, which means

(6) is form invariant under the infinitesimal transformation

$$u \longrightarrow u + \epsilon\sigma^u, \tag{27}$$

with the infinitesimal parameter ϵ. □

Proposition 3. *Modified Boussinesq equation (1) has a nonlocal symmetry given by*

$$\sigma^u = w_x e^{2w}, \tag{28}$$

where w satisfies (13).

Proof. By direct calculation, we substitute (28) into the linearized equation of (6) with the help of the nonauto-BT (26) in Theorem 2 and the w equation (13). □

Now, applying the following transformation:

$$\phi = \frac{1}{1 - \tanh(w)}, \tag{29}$$

to the nonlocal symmetry (28), we find

$$\sigma^u = \phi_x, \tag{30}$$

which is the residual symmetry of (1).

Here we can obtain residual symmetry (30) from the truncated Painlevé expansion. Balancing the nonlinear and dispersive terms in (1), the truncated Painlevé expansion can be written as

$$u = \frac{u_0}{\phi} + u_1, \tag{31}$$

where ϕ is the singular manifold and u_0 and u_1 are functions of (x, t) to be determined later.

Substituting (31) into (6) and vanishing coefficients of the each powers of $1/\phi$, we obtain

$$\begin{aligned} u_0 &= \phi_x, \\ u_1 &= -\frac{1}{2}\frac{\phi_{xx}}{\phi_x} - \frac{1}{2}\frac{\phi_t}{\phi_x}. \end{aligned} \tag{32}$$

Consequently,

$$u = \frac{\phi_x}{\phi} - \frac{1}{2}\frac{\phi_{xx}}{\phi_x} - \frac{1}{2}\frac{\phi_t}{\phi_x} \tag{33}$$

is a solution of (1) with ϕ satisfying Schwartzian form of (1):

$$3C_t + 3CC_x + S_x = 0, \tag{34}$$

where the notations C and S are defined as

$$\begin{aligned} C &= \frac{\phi_t}{\phi_x}, \\ S &= \frac{\phi_{xxx}}{\phi_x} - \frac{3}{2}\frac{\phi_{xx}^2}{\phi_x^2}. \end{aligned} \tag{35}$$

Schwarzian equation (34) is invariant under the Möbious transformation

$$\phi \longrightarrow \frac{a + b\phi}{c + d\phi}, \quad (ad \neq bc), \tag{36}$$

which means (34) possesses the following Lie point symmetries in the form of

$$\sigma^\phi = a_1 + b_1\phi + c_1\phi^2, \tag{37}$$

with a_1, b_1, and c_1 being arbitrary constants. From the above standard truncated Painlevé expansion, we have the BT theorem as follow.

Theorem 4. *If ϕ is a solution of the Schwartzian equation (34), then*

$$u = -\frac{1}{2}\frac{\phi_{xx}}{\phi_x} - \frac{1}{2}\frac{\phi_t}{\phi_x} \tag{38}$$

is a nonauto-BT between ϕ and the solution u of modified Boussinesq equation (1).

The residual of truncated Painlevé expansion (31) with the singular manifold ϕ, that is, u_0, is a symmetry of (1) with the solution u_1. Thus, (30) is the residual symmetry of (1). It should be emphasized that the above residual symmetry is just related to the Möbious transformation symmetry (37) by the linearized equation of nonauto-BT (38) in Theorem 4.

3.2. Localization of Residual Symmetry. To find out the symmetry group transformation of nonlocal residual symmetry, we should solve the following initial value problem:

$$\begin{aligned} \frac{d\widehat{u}(\varepsilon)}{d\varepsilon} &= \widehat{\phi_x}(\varepsilon), \\ \widehat{u}(0) &= u, \end{aligned} \tag{39}$$

where ε is an arbitrary group parameter.

Due to the intrusion of the function $\widehat{\phi}(\varepsilon)$ and its differentiations, it is difficult to solve the above initial value problem according to the Lie first theorem. Thus, one may extend the original system such that nonlocal residual symmetry becomes the local Lie point symmetry of a closed prolonged system.

The nonlocal residual symmetry of (1) can be localized to the Lie point symmetry

$$\begin{aligned} \sigma^u &= g, \\ \sigma^g &= -2\phi g, \\ \sigma^\phi &= -\phi^2, \end{aligned} \tag{40}$$

for the prolonged system

$$\begin{aligned} &3u^2u_x + 3u_x\int u_t dx - \frac{1}{2}u_{xxx} - \frac{3}{2}\int u_{tt}dx = 0, \\ &u = -\frac{1}{2}\frac{\phi_{xx}}{\phi_x} - \frac{1}{2}\frac{\phi_t}{\phi_x}, \\ &g = \phi_x. \end{aligned} \tag{41}$$

Prolonged system (41) is closed after covering dependent variables u, g, and ϕ with the vector form

$$V = g\frac{\partial}{\partial u} - 2\phi g\frac{\partial}{\partial g} - \phi^2\frac{\partial}{\partial \phi}. \tag{42}$$

Correspondingly, initial value problem (39) is changed as

$$\begin{aligned} \frac{d\widehat{u}(\varepsilon)}{d\varepsilon} &= \widehat{g}(\varepsilon), \quad \widehat{u}(0) = u, \\ \frac{d\widehat{g}(\varepsilon)}{d\varepsilon} &= -2\widehat{\phi}(\varepsilon)\widehat{g}(\varepsilon), \quad \widehat{g}(0) = g, \\ \frac{d\widehat{\phi}(\varepsilon)}{d\varepsilon} &= -\widehat{\phi}(\varepsilon)^2, \quad \widehat{\phi}(0) = \phi. \end{aligned} \tag{43}$$

By solving the above initial value problem, we arrive at the symmetry group transformation theorem as follow.

Theorem 5. *If $\{u, g, \phi\}$ is a solution of prolonged system (41), then so is $\{\widehat{u}, \widehat{g}, \widehat{\phi}\}$ with*

$$\begin{aligned} \widehat{u}(\varepsilon) &= u + \frac{g\varepsilon}{1+\phi\varepsilon}, \\ \widehat{g}(\varepsilon) &= \frac{g}{(1+\phi\varepsilon)^2}, \\ \widehat{\phi}(\varepsilon) &= \frac{\phi}{1+\phi\varepsilon}. \end{aligned} \tag{44}$$

Finite transformation (44) provides a way to generate a new solution from old one. It is necessary to point out that the nonlocal residual symmetry is nothing but the infinitesimal form of the finite transformation. Actually, if we set

$$\begin{aligned} 1 + \phi\varepsilon &= \phi, \\ g\varepsilon &= \phi_x, \end{aligned} \tag{45}$$

the first equation of (44) is just the truncated Painlevé expansion (31) with (32).

4. Summary and Discussion

In conclusion, the modified Boussinesq equation is proved to be CRE integrable. The CTE method which is a special simplified form leads to a nonauto-BT, which strengthens a single soliton to a straight line solution. Two types of special interaction solution between the soliton and the cnoidal periodic wave are derived by means of the nonauto-BT.

On the other hand, for the modified Boussinesq equation, the nonlocal symmetry related to the CTE is derived. Under the transformation $\phi = 1/(1 - \tanh(w))$, this nonlocal symmetry is changed as the residual symmetry which can be obtained from the truncated Painlevé expansion. To solve the initial value problem related to the residual symmetry, we extend the original system such that residual symmetry becomes the local Lie point symmetry of a closed prolonged system. The symmetry group transformation of the prolonged system is derived by using the Lie first theorem.

More about further integrable properties from CRE will be investigated in detail in our future research work.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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