Research Article

Adaptive Modified Function Projective Lag Synchronization of Memristor-Based Five-Order Chaotic Circuit Systems

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Received 27 October 2016; Revised 22 February 2017; Accepted 8 March 2017; Published 23 March 2017

Academic Editor: Zhi-Yuan Sun

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The modified function projective lag synchronization of the memristor-based five-order chaotic circuit system with unknown bounded disturbances is investigated. Based on the LMI approach and Lyapunov stability theorem, an adaptive control law is established to make the states of two different memristor-based five-order chaotic circuit systems asymptotically synchronized up to a desired scaling function matrix, while the parameter controlling strength update law is designed to estimate the parameters well. Finally, the simulation is put forward to demonstrate the correctness and effectiveness of the proposed methods. The control method involved is simple and practical.

1. Introduction

The memristor, an abbreviation for memory resistor studied by Chua in 1971 [1], is described as the missing 4th passive fundamental circuit element along with resistors, capacitors, and inductors. The memristor is a two-terminal element, either a charge-controlled memristor or a flux-controlled memristor. More than forty years later, on the first day of May 2008, the Hewlett-Packard (HP) research team proudly announced their realization of a memristor prototype, with an official publication in *Nature* [2, 3]. This new circuit element shares many properties of resistors and shares the same unit of measurement, ohm. Much attention has been attracted to this novel device for its resistance upon turning off the power source; in other words, it depends on the integral of its entire past current waveform. At present, research on chaotic system based on memristor becomes a focal topic [3–10]. Itoh and Chua proposed the possible nonlinear circuits, with a memristor which replaces Chua's diode in 2008 [4], showed a memristor-based four-order Chua's circuit which is derived from Chua's circuit using a PWL memristor. By replacing Chua's diode with an active flux-controlled memristor circuit, a memristor-based five-order chaotic circuit is derived from four-order Chua's oscillator By Bao et al. [10].

As is known to all, the synchronization of chaotic systems has been a subject of active research field due to its potential applications for secure communications and control. Up to now, many types of synchronization methods have been put forward in dynamical systems, such as complete synchronization (CS) [11, 12], antisynchronization (AS) [13], phase synchronization (PS) [14], lag synchronization (LS) [15], intermittent lag synchronization [16], generalized synchronization (GS) [17], intermittent generalized synchronization [18], time scale synchronization [19], projective synchronization [20, 21], modified projective synchronization (MPS) [22], and function projective synchronization (FPS) [23].

Recently a more general form of FPS called modified function projective synchronization (MFPS) [24–26] in which master and slave systems are synchronized up to a desired scaling function matrix has attracted attention of researchers as it can provide more security in secure communication. Therefore, the research on MFPS is more valuable in practice. Considering time-delays exist widely in engineering, recently, a general method called modified function projective lag synchronization (MFPLS) for chaotic systems has been proposed in [27].

To the best of our knowledge, the MFPLS of memristor-based five-order chaotic circuit system with unknown disturbances has not been reported yet. Motivated by the above

discussion, we will give a comprehensive study on this topic in this article. Based on the parameter modulation, the adaptive control technique, and Lyapunov stability theorem, the adaptive control laws are designed to make the states of two different memristor-based five-order chaotic circuit systems asymptotically synchronized up to a desired scaling function matrix.

2. Memristor-Based Five-Order Chaotic Circuit System

By replacing Chua's diode with an active flux-controlled memristor circuit, Bao derived a memristor-based five-order chaotic circuit from four-order Chua's oscillator. This new chaotic circuit can be shown in Figure 1 and it can be described by the following nonlinear equations:

$$\begin{aligned}\frac{dv_1(t)}{dt} &= \frac{1}{C_1} (i_3(t) - W(\varphi(t)) v_1(t)) \\ \frac{dv_2(t)}{dt} &= \frac{1}{C_2} (-i_3(t) + i_4(t)) \\ \frac{di_3(t)}{dt} &= \frac{1}{L_1} (v_2(t) - v_1(t) - Ri_3(t)) \\ \frac{di_4(t)}{dt} &= -\frac{1}{L_2} v_2(t) \\ \frac{d\varphi(t)}{dt} &= v_1(t),\end{aligned}\quad (1)$$

in which $W(\varphi(t)) = -a + 3b\varphi^2(t)$.

Denote

$$\begin{aligned}x(t) &= [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)]^T \\ &= [v_1(t), v_2(t), i_3(t), i_4(t), \varphi(t)]^T,\end{aligned}\quad (2)$$

and then system (2) can be written as

$$\begin{aligned}\dot{x}_1(t) &= \frac{1}{C_1} (x_3(t) - W(x_5(t)) x_1(t)) \\ \dot{x}_2(t) &= \frac{1}{C_2} (-x_3(t) + x_4(t)) \\ \dot{x}_3(t) &= \frac{1}{L_1} (x_2(t) - x_1(t) - Rx_3(t)) \\ \dot{x}_4(t) &= -\frac{1}{L_2} x_2(t) \\ \dot{x}_5(t) &= x_1(t).\end{aligned}\quad (3)$$

If we set

$$\begin{aligned}\frac{1}{C_1} &= 9, \\ C_2 &= 1, \\ \frac{1}{L_1} &= 30, \\ \frac{1}{L_2} &= 15, \\ R &= 1, \\ a &= 1.2, \\ b &= 0.4,\end{aligned}\quad (4)$$

and the initial value is chosen as

$$x(0) = [0, 0.2, 0, 0, 0]^T, \quad (5)$$

system (3) is chaotic and multiscroll attractor as shown in Figures 2–5.

3. MFPLS in Memristor-Based Five-Order Chaotic Circuit Systems

Taking into account the external disturbances, for the sake of convenience, we reexpress system (3) as

$$\dot{x}(t) = Ax(t) + f(x(t)) + w(t), \quad (6)$$

where

$$A = \begin{bmatrix} \frac{a}{C_1} & 0 & \frac{1}{C_1} & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2} & \frac{1}{C_2} & 0 \\ -\frac{1}{L_1} & \frac{1}{L_1} & -\frac{R}{L_1} & 0 & 0 \\ 0 & -\frac{1}{L_2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

$$f(x(t)) = \left[-\frac{b}{C_1} x_1(t) x_5^2(t), 0, 0, 0, 0 \right]^T,$$

$$w(t) = [w_1(t), w_2(t), w_3(t), w_4(t), w_5(t)]^T.$$

Taking system (6) as the drive system, the response system can be written as

$$\dot{y}(t) = By(t) + g(y(t)) + d(t) + u(t), \quad (8)$$

FIGURE 1: Memristor-based five-order chaotic circuit.

where

$$y(t) = [y_1(t), y_2(t), y_3(t), y_4(t), y_5(t)]^T,$$

$$B = \begin{bmatrix} \frac{\bar{a}}{\bar{C}_1} & 0 & \frac{1}{\bar{C}_1} & 0 & 0 \\ 0 & 0 & -\frac{1}{\bar{C}_2} & \frac{1}{\bar{C}_2} & 0 \\ -\frac{1}{\bar{L}_1} & \frac{1}{\bar{L}_1} & -\frac{\bar{R}}{\bar{L}_1} & 0 & 0 \\ 0 & -\frac{1}{\bar{L}_2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (9)$$

$$g(y(t)) = \left[-\frac{\bar{b}}{\bar{C}_1} y_1(t) y_5^2(t), 0, 0, 0, 0 \right]^T,$$

$$d(t) = [d_1(t), d_2(t), d_3(t), d_4(t), d_5(t)]^T.$$

Assumption 1. The unknown external time-varying disturbances $w(t)$ and $d(t)$ are bounded; in other words, there exist nonnegative constants l_1^i and l_2^i such that

$$\begin{aligned} |w_i(t)| &\leq l_1^i, \\ |d_i(t)| &\leq l_2^i. \end{aligned} \quad (10)$$

($i = 1, \dots, 5$)

Denoting $l_1 = \sum_{i=1}^5 l_1^i$, $l_2 = \sum_{i=1}^5 l_2^i$, we can further obtain that

$$\begin{aligned} \|w(t)\| &\leq l_1, \\ \|d(t)\| &\leq l_2, \end{aligned} \quad (11)$$

where $\|\cdot\|$ stands for the 1-norm.

Definition 2 (MFPLS, [27]). For the drive system (6) and the response system (8), it is said that these two systems are modified function projective lag synchronization (MFPLS), if there exist a delay time τ and a scaling function matrix $\Lambda(t)$ such that

$$\lim_{t \rightarrow \infty} \|x(t - \tau) - \Lambda(t) y(t)\| = 0, \quad (12)$$

FIGURE 2: Attractor of the memristor-based five-order chaotic circuit (a).

FIGURE 3: Attractor of the memristor-based five-order chaotic circuit (b).

or

$$\lim_{t \rightarrow \infty} \|y(t) - \Lambda(t) x(t - \tau)\| = 0, \quad (13)$$

where $\Lambda(t) = \text{diag}\{\lambda_1(t), \dots, \lambda_5(t)\}$ is reversible and differentiable and $\lambda_i(t) \neq 0$ is a bounded continuously differentiable function.

Remark 3. It is clear that (12) and (13) are equivalent in the form.

Remark 4. When $\tau = 0$, $\lambda_1(t) = \lambda_2(t) = \dots = \lambda_5(t)$, or $\lambda_1(t) = \lambda_2(t) = \dots = \lambda_5(t) = 1$, MFPLS is simplified to MFPS, FPS, or complete synchronization, respectively.

Denote $\Lambda_1(t) = \text{diag}\{|\lambda_1(t)|, \dots, |\lambda_5(t)|\}$, $D = \text{diag}\{\text{sign}(\lambda_1(t)), \dots, \text{sign}(\lambda_5(t))\}$, and then the diagonal matrix $\Lambda(t)$ can be decomposed as

$$\Lambda(t) = \Lambda_1(t) \cdot D. \quad (14)$$

It is obvious that

$$D \cdot D = I, \quad (15)$$

in which I is an unit matrix.

Noted that $\lambda_i(t) \neq 0$ is a continuously differentiable function with bound, we further pose the following assumption.

FIGURE 4: Attractor of the memristor-based five-order chaotic circuit (c).

FIGURE 5: Attractor of the memristor-based five-order chaotic circuit (d).

Assumption 5. There exist three nonnegative constants α , β , and γ such that

$$\alpha \leq \|\Lambda_1^{-1}(t)\| \leq \beta, \quad (16)$$

$$\|(\dot{\Lambda}_1^{-1})(t)\| \leq \gamma.$$

The main purpose of this paper is to design an appropriate controller $u(t)$ to ensure systems (6) and (8) are modified function projective lag synchronization.

Let us define the MFPLS error vector

$$e(t) = x(t - \tau) - \Lambda(t)y(t). \quad (17)$$

Combining systems (6) and (8) with the MFPLS error (17), the following error dynamical system can be obtained:

$$\begin{aligned} \dot{e}(t) = & Ae(t) + h(x, y) + w(t - \tau) - \Lambda(t)d(t) \\ & - \Lambda(t)u(t), \end{aligned} \quad (18)$$

where

$$\begin{aligned} h(x, y) = & [A\Lambda(t) - \Lambda(t)B - \dot{\Lambda}(t)]y(t) \\ & + f(x(t - \tau)) - \Lambda(t)g(y(t)). \end{aligned} \quad (19)$$

Furthermore, we can obtain

$$\begin{aligned} \Lambda_1^{-1}(t)\dot{e}(t) = & \Lambda_1^{-1}(t)Ae(t) + \Lambda_1^{-1}(t)h(x, y) \\ & + \Lambda_1^{-1}(t)w(t - \tau) - Dd(t) - Du(t). \end{aligned} \quad (20)$$

It followed by designing an adaptive controller to achieve MFPLS of systems (6) and (8).

4. Design of the Adaptive Controller

4.1. Case 1. We start with a simple case in which the bounds l_1 and l_2 are known. In order to achieve the MFPLS, the control law is given by

$$\begin{aligned} u(t) = & D[\Lambda_1^{-1}(t)Ae(t) + \Lambda_1^{-1}(t)h(x, y) + Ke(t)] \\ & + (\beta l_1 + l_2)D\text{sign}(e), \end{aligned} \quad (21)$$

where $K = \text{diag}\{k_1, \dots, k_5\}$ is the control gain matrix and $\text{sign}(e) = [\text{sign}(e_1(t)), \dots, \text{sign}(e_5(t))]^T$.

Theorem 6. If there exists symmetric positive definite matrix Q , such that the following LMI holds:

$$-K + \gamma I = -Q, \quad (22)$$

then systems (6) and (8) are MFPLS.

Proof. Substituting the control law (21) into (20), we can obtain

$$\begin{aligned} \Lambda_1^{-1}(t)\dot{e}(t) = & \Lambda_1^{-1}(t)w(t - \tau) - Dd(t) - Ke(t) \\ & - (\beta l_1 + l_2)\text{sign}(e). \end{aligned} \quad (23)$$

Since $\Lambda_1^{-1}(t)$ is positive definite matrix, we design the following Lyapunov function:

$$V(t) = \frac{1}{2}e^T(t)\Lambda_1^{-1}(t)e(t). \quad (24)$$

Calculating the time derivative of $V(t)$ along the trajectory of the error system (18), it can be found that

$$\begin{aligned} \dot{V}(t) = & e^T\Lambda_1^{-1}\dot{e} + \frac{1}{2}e^T(\dot{\Lambda}_1^{-1})e = e^T[\Lambda_1^{-1}w(t - \tau) \\ & - Dd(t) - Ke - (\beta l_1 + l_2)\text{sign}(e)] + \frac{1}{2}e^T(\dot{\Lambda}_1^{-1})e \\ \leq & (\|\Lambda_1^{-1}w(t - \tau)\| + \|d(t)\|)\|e\| - (\beta l_1 + l_2)\|e\| \\ & - e^TKe + e^T(t)(\gamma I)e \leq -e^T(K - \gamma I)e = -e^TQe \\ < & 0. \end{aligned} \quad (25)$$

Utilizing Lyapunov stability theorem, we get

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0, \quad (26)$$

which means that systems (6) and (8) are MFPLS. This completes the proof. $\square$

4.2. Case 2. We now consider the general case in which the bounds l_1 and l_2 are unknown; denote $\rho = \beta l_1 + l_2$ and $\hat{\rho}$ stands

for the estimated value of ρ . Based on the adaptive control method, the controller is designed as

$$u(t) = D \left[\Lambda_1^{-1}(t) A e(t) + \Lambda_1^{-1}(t) h(x, y) + K e(t) \right] + q \hat{\rho} D \text{sign}(e), \quad (27)$$

while the parameter adaptive law is given by

$$\dot{\hat{\rho}} = q \|e\|, \quad (28)$$

where constant $q > 0$ is the adjustable gain.

Theorem 7. *If there exist a positive constant η and a symmetric positive definite matrix Q such that the following LMIs hold:*

$$\begin{aligned} \beta l_1 + l_2 - \rho q &= -\eta, \\ -K + \gamma I &= -Q, \end{aligned} \quad (29)$$

then the two systems (6) and (8) are MFPLS.

Proof. Substituting the controller (27) and the adaptive update law (28) into (20), we can obtain

$$\begin{aligned} \Lambda_1^{-1}(t) \dot{e}(t) &= \Lambda_1^{-1}(t) w(t - \tau) - D d(t) - K e(t) \\ &\quad - q \hat{\rho} \text{sign}(e). \end{aligned} \quad (30)$$

Choose the following Lyapunov function:

$$V(t) = \frac{1}{2} e^T(t) \Lambda_1^{-1}(t) e(t) + \frac{1}{2} (\hat{\rho} - \rho)^2. \quad (31)$$

Taking the time derivative of $V(t)$ along the error system leads to

$$\begin{aligned} \dot{V}(t) &= e^T \Lambda_1^{-1} \dot{e} + \frac{1}{2} e^T (\dot{\Lambda}_1^{-1}) e + (\hat{\rho} - \rho) \dot{\hat{\rho}} \\ &= e^T \left[\Lambda_1^{-1} w(t - \tau) - D d(t) - K e - q \hat{\rho} \text{sign}(e) \right] \\ &\quad + \frac{1}{2} e^T (\dot{\Lambda}_1^{-1}) e + (\hat{\rho} - \rho) \dot{\hat{\rho}} \\ &\leq \left(\left\| \Lambda_1^{-1} w(t - \tau) \right\| + \|d(t)\| \right) \|e\| - q \rho \|e\| \\ &\quad - e^T (K - \gamma I) e \\ &\leq (\beta l_1 + l_2) \|e\| - q \rho \|e\| - e^T (K - \gamma I) e \\ &= (\beta l_1 + l_2 - q \rho) \|e\| - e^T (K - \gamma I) e \\ &= -\eta \|e\| - e^T Q e < 0. \end{aligned} \quad (32)$$

Applying Lyapunov stability theorem, we can obtain

$$\lim_{t \rightarrow \infty} \|e\| = 0, \quad (33)$$

which means that systems (6) and (8) are MFPLS. Hence, the proof is completed. $\square$

4.3. Case 3. More generally, if all the bounds l_1^i and l_2^i ($i = 1, 2, \dots, 5$) are unknown, let us denote

$$\begin{aligned} \rho &= (\rho_1, \dots, \rho_5)^T, \\ \rho_i &= \beta l_1^i + l_2^i, \\ &\quad (i = 1, 2, \dots, 5) \end{aligned} \quad (34)$$

with the vector $\hat{\rho} = (\hat{\rho}_1, \dots, \hat{\rho}_5)^T$ standing for the estimated value of ρ .

For this case, the adaptive control law and parameter update rule is chosen by

$$\begin{aligned} u(t) &= D \left[\Lambda_1^{-1}(t) A e(t) + \Lambda_1^{-1}(t) h(x, y) + K e(t) \right] \\ &\quad + D \left[q_1 \hat{\rho}_1 \text{sign}(e_1), \dots, q_5 \hat{\rho}_5 \text{sign}(e_5) \right]^T, \end{aligned} \quad (35)$$

with

$$\dot{\hat{\rho}}_i = q_i \|e\|, \quad (i = 1, \dots, 5), \quad (36)$$

where q_i is adjustable gain.

Theorem 8. *If there exist positive constants η_i and a symmetric positive definite matrix Q , such that the following LMIs hold:*

$$\begin{aligned} \beta l_1^i + l_2^i - \rho_i q_i &= -\eta_i, \quad (i = 1, \dots, 5) \\ -K + \gamma I &= -Q, \end{aligned} \quad (37)$$

then systems (6) and (8) are MFPLS.

Proof. Substituting the controller (35) and the adaptive update law (36) into (20), we obtain

$$\begin{aligned} \Lambda_1^{-1}(t) \dot{e}(t) &= \Lambda_1^{-1}(t) w(t - \tau) - D d(t) - K e(t) \\ &\quad - [q_1 \hat{\rho}_1 \text{sign}(e_1), \dots, q_5 \hat{\rho}_5 \text{sign}(e_5)]^T. \end{aligned} \quad (38)$$

The Lyapunov function is designed as

$$V(t) = \frac{1}{2} e^T \Lambda_1^{-1}(t) e + \frac{1}{2} (\hat{\rho} - \rho)^T (\hat{\rho} - \rho). \quad (39)$$

The time derivative of $V(t)$ is given by

$$\dot{V}(t) = e^T \Lambda_1^{-1} \dot{e} + \frac{1}{2} e^T (\dot{\Lambda}_1^{-1}) e + (\hat{\rho} - \rho)^T \dot{\hat{\rho}}. \quad (40)$$

Substituting (38) into (40), it follows that

$$\begin{aligned}
 \dot{V}(t) &= e^T \left\{ \Lambda_1^{-1} w(t - \tau) - Dd(t) \right. \\
 &\quad \left. - [q_1 \hat{\rho}_1 \operatorname{sign}(e_1), \dots, q_5 \hat{\rho}_5 \operatorname{sign}(e_5)]^T \right\} \\
 &\quad + \sum_{i=1}^5 (q_i \hat{\rho}_i |e_i| - q_i \rho_i |e_i|) - e^T K e + \frac{1}{2} e^T (\Lambda_1^{-1}) e \\
 &\leq \sum_{i=1}^5 ((\beta l_1^i + l_2^i) |e_i| - q_i \hat{\rho}_i |e_i|) \\
 &\quad + \sum_{i=1}^5 (q_i \hat{\rho}_i |e_i| - q_i \rho_i |e_i|) - e^T (K - \gamma I) e \\
 &= \sum_{i=1}^5 (((\beta l_1^i + l_2^i) - q_i \rho_i) |e_i|) - e^T (K - \gamma I) e \\
 &= -\sum_{i=1}^5 \eta_i |e_i| - e^T Q e < 0.
 \end{aligned} \tag{41}$$

According to Lyapunov stability theorem, we can get

$$\lim_{t \rightarrow \infty} \|e\| = 0, \tag{42}$$

which means that the two systems (6) and (8) are MFPLS. Hence, the proof is completed. $\square$

5. Simulation

In this section, two different memristor-based five-order chaotic circuit systems with unknown bounded disturbances are considered as the master system and the slave system, respectively, which can be described by

$$\begin{aligned}
 \dot{x}(t) &= Ax(t) + f(x(t)) + w(t), \\
 \dot{y}(t) &= By(t) + g(y(t)) + d(t) + u(t),
 \end{aligned} \tag{43}$$

where

$$A = \begin{bmatrix} 10.8 & 0 & 9 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -30 & 30 & -30 & 0 & 0 \\ 0 & -15 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 9 & 0 & 10 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -28 & 28 & -33.6 & 0 & 0 \\ 0 & -11 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

FIGURE 6: Time response of MFPLS error $e_i(t)$.

$$\begin{aligned}
 f(x(t)) &= [-10.8x_1x_5^2, 0, 0, 0, 0]^T, \\
 g(y(t)) &= [-9y_1y_5^2, 0, 0, 0, 0]^T, \\
 w(t) &= [\sin 10t, 0, 0, 0, 0]^T, \\
 d(t) &= [0.1 \sin t, 0.2 \sin t, 0.5 \cos 2t, 0.2 \\
 &\quad - 0.1 \sin t, 0.1 + 0.2 \cos t]^T.
 \end{aligned} \tag{44}$$

The delay time is chosen as $\tau = 1$, and the scaling function matrix is given by

$$\Lambda(t) = \operatorname{diag} \{3.1 + \cos t, 3.1 + \sin t, 4.2 - \sin t, 3.2 + \sin 5t, 4.5 - 0.5 \sin t\}, \tag{45}$$

with the control gain

$$\begin{aligned}
 K &= \operatorname{diag} \{10, 10, 10, 10, 10\}, \\
 q_1 &= \dots = q_5 = 10.
 \end{aligned} \tag{46}$$

The drive system is initialized with

$$x(0) = (0, 0, 1, 0, 0), \tag{47}$$

and the response system is started with

$$y(0) = (0.5, 0, -1, 0, 1). \tag{48}$$

Using the control method proposed in Theorem 8, the MFPLS error state trajectories are depicted in Figure 6, which illustrate that the error can quickly approach zero while the controller $u(t)$ is maintained in a reasonable range which is shown in Figure 7. At the meantime, as is shown in Figure 8, all of the unknown parameters can be tracked well under the parameter adaptive update law.

FIGURE 7: Time response of the input controller $u_i(t)$. FIGURE 8: Time response of the estimated value of $\rho_i(t)$.

6. Conclusion

In this paper, the problem of MFPLS of memristor-based five-order chaotic circuit systems with unknown bounded disturbances has been addressed. Combining the LMI approach with Lyapunov stability theory, an adaptive control law is designed to make the states of two different memristor-based five-order chaotic circuit systems asymptotically synchronized up to a desired scaling function matrix and the unknown parameters can be estimated accurately. At the end of the paper, the corresponding numerical simulations have

been given to verify the effectiveness of the proposed control techniques. The proposed method is also suitable for the MFPLS of other chaotic systems and has broad application in secure communication, image processing, and other fields.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China (61373174 and 11301409), and thanks are due to all the references authors.

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