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Research Article

Guaranteed Cost Finite-Time Control of Fractional-Order Positive Switched Systems

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The problem of guaranteed cost finite-time control of fractional-order positive switched systems (FOPSS) is considered in this paper. Firstly, a new cost function is defined. Then, by constructing linear copositive Lyapunov functions and using the average dwell time (ADT) approach, a state feedback controller and a static output feedback controller are constructed, respectively, and sufficient conditions are derived to guarantee that the corresponding closed-loop systems are guaranteed cost finite-time stable (GCFTS). Such conditions can be easily solved by linear programming. Finally, two examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Positive switched systems are a class of hybrid systems consisting of a family of positive subsystems and a switching law that specifies which subsystem will be activated along the system trajectory at each instant of time. Many remarkable results related to positive switched systems have been presented; see [1–5] and references therein. These results mentioned above refer to the positive switched systems with integer order derivative. However, in many physical, chemical, and economic processes, such as fractional PID control [6, 7], fractional electrical networks [8], robotics [9], and electrical machines [10], fractional calculus is more feasible than integer calculations to model the behavior of such systems. Therefore, some researchers have investigated the fractional-order positive systems [11–14]. Particularly, very recently, only a few results about fractional-order positive switched systems have been presented [15, 16]. Reference [15] considered the controllability of FOPSS for fixed switching sequence. Reference [16] considered the problem of state-dependent switching control of FOPSS. But the above results are involved in asymptotic stability, which reflects the asymptotic behavior of the system in an infinite time interval.

By contrast, in many practical applications, one is more interested in what happens over a finite-time interval. Peter [17] has firstly defined finite-time stability (FTS) for linear deterministic systems, which means that, given a bound on the initial condition, the system state does not exceed a certain threshold during a specified time interval. There have been some meaningful results about FTS of positive switched systems [18–23]. Among these results, it should be pointed out that only one paper has considered the problem of FTS of FOPSS in [23]. Moreover, when controlling a real plant, it is also desirable to design a control system which is not only finite-time stable but also guarantees an adequate level of performance. One approach to this problem is the guaranteed cost finite-time control. It has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound [24, 25]. So it is necessary to study the design problem of guaranteed cost finite-time controller [26, 27]. In [26], robust finite-time guaranteed cost control for impulsive switched systems with time-varying delay was considered. In [27], guaranteed cost finite-time control was extended to positive switched linear systems with time-varying delays and a cost function for positive systems (or positive switched systems) was also proposed. As we know, fractional-order systems have some different features compared with ordinary systems, like the stability criterion. Then the research of fractional-order switched systems is more challenging than ordinary switched systems. Particularly, for FOPSS, the problem of guarantee cost finite-time control has been rarely reported.

In this paper, guarantee cost finite-time control of FOPSS is considered. A new cost function is firstly proposed, which can utilize the characteristics of nonnegative states of FOPSS. Then the definition of guaranteed cost finite-time stability is also given. By using the ADT approach and copositive Lyapunov functional method, two types of feedback controllers (state feedback controller and static output feedback controller, resp.) and a list of switching signals are designed. Some sufficient conditions are obtained to guarantee that the closed-loop systems are GCFTS. The rest of the paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. In Section 3, we obtained some sufficient conditions of GCFTS of FOPSS based on Lyapunov functional and ADT approach. In Section 4, two examples are given to illustrate the effective of two kinds of controllers, respectively. Section 5 is the conclusion.

Notations. Throughout this paper, $A \succ 0$ ($\succeq 0$, $\prec 0$, $\preceq 0$) means that $a_{ij} > 0$ (≥ 0, < 0, ≤ 0), which is applicable to a vector. a_{ij} is in the ith row and jth column of A. $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$). The symbols R, R^n, and $R^{n\times n}$ denote the set of real numbers, the space of the vectors of n-tuples of real numbers, and the space of $n \times n$ matrices with real numbers, respectively. R_+^n is the n-dimensional nonnegative (positive) vector space. $\Gamma(\cdot)$ denotes the Gamma function. A^T denotes the transpose of matrix A. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2. Preliminaries and Problem Statements

2.1. Fractional-Order Calculus. Fractional-order integrodifferential operator is the generalization of integer order integrodifferential operator. There are different definitions of the fractional-order integral or derivative. Given $0 < \alpha < 1$, the uniform formula of a fractional integral is defined as

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \quad (1) $$

For $0 < \alpha < 1$, Riemann-Liouville (RL) definition of fractional derivative is given as

$$ {}_{t_0}^{\mathrm{RL}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad (2) $$

and Caputo definition of fractional derivative is given as

$$ {}_{t_0}^{\mathrm{C}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad (3) $$

where $f(t)$ is an arbitrary integrable function, ${}_{t_0}D_t^{-\alpha}$ is the fractional integral of order α on $[t_0, t]$, and $\Gamma(\alpha)$ denotes the Gamma function with noninteger arguments, $\Gamma(\alpha) = \int_0^{\infty} e^{-t} t^{\alpha-1} dt$. ${}_{t_0}^{\mathrm{RL}}D_t^{\alpha}$ and ${}_{t_0}^{\mathrm{C}}D_t^{\alpha}$ represent Riemann-Liouville and Caputo fractional derivatives of order α of $f(t)$ on $[t_0, t]$, respectively. These two fractional-order operators are used in this paper. From the above two definitions, we can obtain the following relations between them:

$$ {}_{t_0}^{\mathrm{RL}}D_t^{\alpha} f(t) = {}_{t_0}^{\mathrm{C}}D_t^{\alpha} f(t) + \frac{t^{-\alpha}}{\Gamma(1-\alpha)} f(t_0). \quad (4) $$

Lemma 1 (see [23]). *Let $\alpha \in (0,1)$; if $f(0) \geq 0$, then ${}_{t_0}^{\mathrm{RL}}D_t^{\alpha} f(t) \leq {}_{t_0}^{\mathrm{C}}D_t^{\alpha} f(t)$.*

2.2. Fractional-Order Positive Switched Systems. Consider the following FOPSS:

$$ \begin{aligned} {}_{t_0}^{\mathrm{C}}D_t^{\alpha} x(t) &= A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t), \\ y(t) &= C_{\sigma(t)} x(t), \end{aligned} \quad (5) $$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^z$ represent the system state, control input, and control output, respectively. ${}_{t_0}^{\mathrm{C}}D_t^{\alpha}$ denotes Caputo fractional-order derivative. $\sigma(t) : [0, \infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is the switching signal, where N is the number of subsystems; $\forall p \in \underline{N}$, A_p, B_p, and C_p are constant matrices with appropriate dimensions, p denotes the pth systems and t_q denotes the qth switching instant.

Next, we will present some definitions, lemmas, and inequalities for the FOPSS (5) for our further study.

Definition 2 (see [23]). System (5) is said to be positive if, for any switching signals $\sigma(t)$ and any initial conditions $x(t_0) \succeq 0$, the corresponding trajectory satisfies $x(t) \succeq 0$ and $y(t) \succeq 0$ for all $t \geq 0$.

Definition 3 (see [1]). A matrix A is called a Metzler matrix if the off-diagonal entries of matrix A are nonnegative.

Lemma 4 (see [3]). *A matrix is a Metzler matrix if and only if there exists a positive constant ς such that $A + \varsigma I_n \succeq 0$.*

Lemma 5 (see [23]). *System (5) is positive if and only if A_p, $\forall p \in \underline{N}$, are Metzler matrices and $\forall p \in \underline{N}$, $B_p \succeq 0$ and $C_p \succeq 0$.*

Definition 6 (see [21]). For any switching signals $\sigma(t)$ and any $T_2 \geq T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the switching numbers of $\sigma(t)$ over the interval $[T_1, T_2)$. If there exist $N_0 \geq 0$ and $T_\alpha > 0$ such that

$$ N_\sigma(T_1, T_2) \leq N_0 + \frac{T_2 - T_1}{T_\alpha}, \quad (6) $$

then T_α and N_0 are called ADT and chattering bound, respectively. Generally speaking, we choose $N_0 = 0$ in this paper.

Definition 7 (FTS). For given time constant T_f and vectors $\delta \succ \varepsilon \succ 0$, system (5) is said to be FTS with respect to (δ, ε, T_f, $\sigma(t)$), if

$$ \begin{aligned} & x^T(t_0)\delta \leq 1 \Longrightarrow \\ & x^T(t)\varepsilon \leq 1, \\ & \forall t \in [0, T_f]. \end{aligned} \quad (7) $$

Remark 8. This description of FTS is different from the one in [23], but they are the same definitions in nature.

Definition 9. Define the cost function of system (5) as follows:

$$J = {}_{t_0}D_t^{-\alpha}\left(x^T(s)R_1 + u^T(s)R_2\right) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left(x^T(s)R_1 + u^T(s)R_2\right)ds, \tag{8}$$

where $R_1 \succ 0$ and $R_2 \succ 0$ are two given vectors.

Remark 10. It should be noted that the proposed cost function is different from the nonpositive systems [24–26]. Particularly, if $\alpha = 1$, this definition is turned into the definition of cost function in positive switched system [27]. This proposed definition provides a more useful description, because it takes full advantage of the characteristics of nonnegative states of FOPSS.

Now we give the definition of GCFTS for the FOPSS (5).

Definition 11 (GCFTS). For given time constant T_f and vectors $\delta \succ \varepsilon \succ 0$, consider system (5); if there exists a feedback control law $u(t)$ and a positive scalar J^* such that the closed-loop system is finite-time stable with the respect to $(\delta, \varepsilon, T_f, \sigma(t))$ and the cost function satisfies $J \le J^*$, then the closed-loop FOPSS is called GCFTS, where J^* is a guaranteed cost value and $u(t)$ is a guaranteed cost finite-time controller.

Lemma 12 (Gronwall-Bellman inequality). *Let $a(t)$, $b(t)$, and $g(t)$ be continuous real-valued functions. If $a(t)$ is nonnegative and $g(t)$ satisfies the integral inequality*

$$g(t) \le a(t) + \int_0^t b(s)g(s)\,ds, \tag{9}$$

then

$$g(t) \le a(t) + \int_0^t a(s)b(s)\exp\left(\int_s^t b(r)\,dr\right)ds. \tag{10}$$

In addition, if $a(t)$ is a constant, then

$$g(t) \le a(t)\exp\left(\int_0^t b(s)\,ds\right). \tag{11}$$

Lemma 13 (Young's inequality). *If a and b are nonnegative real numbers and p and q are positive real numbers such that $1/p + 1/q = 1$, then*

$$ab \le \frac{a^p}{p} + \frac{b^q}{q}. \tag{12}$$

Lemma 14 (C_p inequality). *For $0 < a < 1$ and any positive real numbers $x_1, x_2, \ldots, x_k$,*

$$\sum_{k=1}^{n} x_k^a \le n^{1-a}\left(\sum_{k=1}^{n} x_k\right)^a. \tag{13}$$

The aim of this paper is to design a state feedback controller $u(t) = K_{1\sigma(t)}x(t)$ and a static output feedback controller $u(t) = K_{2\sigma(t)}y(t)$ and find a class of switching signals $\sigma(t)$ for FOPSS (5) such that the corresponding closed-loop systems are GCFTS.

3. Main Results

3.1. Guaranteed Cost Finite-Time Stability Analysis. In this subsection, we will focus on the problem of GCFTS for FOPSS (5) with $u(t) \equiv 0$.

Theorem 15. *Consider system (5) with $u(t) \equiv 0$. Given positive constants T_f and λ $(\lambda > 1)$ and vectors $\delta \succ \varepsilon \succ 0$ and $R_1 \succ 0$, if there exist positive constants ξ_1, ξ_2, and μ $(\mu \ge 1)$ and positive vectors v_p and $p \in \underline{N}$, such that the following inequalities hold:*

$$A_p^T v_p + R_1 \preceq \lambda v_p, \tag{14}$$

$$\xi_1\varepsilon \prec v_p \prec \xi_2\delta, \tag{15}$$

$$v_p \preceq \mu v_q, \tag{16}$$

$$v_p \prec R_1, \tag{17}$$

$$\frac{\left[(1-\alpha) + \alpha T_f\right](\lambda - 1)}{\Gamma(\alpha+1)} < \ln\frac{\xi_1}{\xi_2} \tag{18}$$

then under the following ADT scheme

$$T_\alpha > T_\alpha^* = \frac{T_f\left(\ln\mu + (1-\alpha)(\lambda-1)/\Gamma(\alpha+1)\right)}{\left(\ln(\xi_1/\xi_2) - \left[(1-\alpha)+\alpha T_f\right](\lambda-1)/\Gamma(\alpha+1)\right)} \tag{19}$$

the FOPSS (5) is GCFTS with respect to $(\delta, \varepsilon, T_f, \sigma(t))$ and the guaranteed cost value of system (5) with $u(t) = 0$ is given by

$$\begin{aligned} J &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left(x^T(s)R_1 + u^T(s)R_2\right)ds \le J^* \\ &= \xi_2\mu^{T_f/T_\alpha} + \lambda\xi_2\mu^{T_f/T_\alpha}\frac{1}{\Gamma(\alpha)} \\ &\cdot \exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right)\right. \\ &\left.+ (\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\frac{1}{\alpha}\left(T_f\right)^\alpha. \end{aligned} \tag{20}$$

Proof. Construct the multiple linear type Lyapunov-Krasovskii functional for system (5) as follows:

$$V_{\sigma(t)}(t, x(t)) = x^T(t)v_{\sigma(t)}, \tag{21}$$

where $v_p \in R_+^n$, $\forall p \in \underline{N}$.

Denote $t_0, t_1, \ldots, t_k$ as the switching instants over the interval $[0, T_f]$. Along the trajectory of system (5) with $u(t) \equiv 0$, we have

$${}_{t_0}^{C}D_t^\alpha V_{\sigma(t)}(t, x(t)) = x^T(t)A_{\sigma(t)}^T v_{\sigma(t)}. \tag{22}$$

From (14), we obtain

$$\begin{aligned} &{}_{t_0}^{C}D_t^\alpha V_{\sigma(t)}(t, x(t)) + x^T(t)R_1 \\ &\quad = x^T(t)\left(A_{\sigma(t)}^T v_{\sigma(t)} + R_1\right) \le \lambda V_{\sigma(t)}(t, x(t)). \end{aligned} \tag{23}$$

Taking the fractional integral ${}_{t_0}^{C}D_t^{-\alpha}$ to both sides of (23) during the period $[t_k, t)$ for $t \in [t_k, t_{k+1})$ leads to

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\quad \leq V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\qquad + \frac{\lambda}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} V_{\sigma(t)}(s, x(s))\, ds \\ &\qquad - \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} x^T(s) R_1 ds. \end{aligned} \tag{24}$$

From (17) and (21), (24) can be rewritten as

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\quad \leq V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\qquad + \frac{\lambda}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} V_{\sigma(t)}(s, x(s))\, ds \\ &\qquad - \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} x^T(s) v_{\sigma(t)} ds \\ &\quad = V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\qquad + \frac{\lambda-1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} V_{\sigma(t)}(s, x(s))\, ds. \end{aligned} \tag{25}$$

According to Lemma 12 and the properties of Gamma function $\Gamma(\alpha)$, for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\quad \leq V_{\sigma(t_k)}(t_k, x(t_k)) \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} ds \right\} \\ &\quad = V_{\sigma(t_k)}(t_k, x(t_k)) \exp\left\{ \frac{\lambda-1}{\alpha\Gamma(\alpha)} (t-t_k)^{\alpha} \right\} \\ &\quad = V_{\sigma(t_k)}(t_k, x(t_k)) \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} (t-t_k)^{\alpha} \right\}. \end{aligned} \tag{26}$$

For $t \in [t_k, t_{k+1})$, $V_{\sigma(t_k)} \leq \mu_{\sigma(t_k)} V_{\sigma(t_k^-)}(t_k^-, x(t_k^-))$ is easily obtained from (16) and (21). Noting that $\exp\{((\lambda-1)/\Gamma(\alpha+1))(t-t_k)^{\alpha}\} > 0$, we have

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\quad \leq \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-)) \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} (t-t_k)^{\alpha} \right\}. \end{aligned} \tag{27}$$

Then, for $t \in [t_0, T_f)$, we can obtain from (26) and (27) that

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \leq V_{\sigma(t_k)}(t_k, x(t_k)) \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} (t \right. \\ &\quad \left. - t_k)^{\alpha} \right\} \leq \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-)) \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} (t \right. \\ &\quad \left. - t_k)^{\alpha} \right\} \leq \mu V_{\sigma(t_{k-1})}(t_{k-1}, x(t_{k-1})) \\ &\quad \cdot \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} \left[(t-t_k)^{\alpha} + (t_k - t_{k-1})^{\alpha} \right] \right\} \\ &\quad \leq \mu^2 V_{\sigma(t_{k-1}^-)}(t_{k-1}^-, x(t_{k-1}^-)) \\ &\quad \cdot \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} \left[(t-t_k)^{\alpha} + (t_k - t_{k-1})^{\alpha} \right] \right\} \leq \cdots \\ &\quad \leq \mu^{N_\sigma(t_0, T_f)} V_{\sigma(t_0)}(t_0, x(t_0)) \\ &\quad \cdot \exp\left\{ \frac{\lambda-1}{\Gamma(\alpha+1)} \left[(t-t_k)^{\alpha} + (t_k - t_{k-1})^{\alpha} + \cdots \right. \right. \\ &\quad \left. \left. + (t_1 - t_0)^{\alpha} \right] \right\}. \end{aligned} \tag{28}$$

By Lemma 14 and $\mu > 1$, we have

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \leq V_{\sigma(t_0)}(t_0, x(t_0)) \exp\left\{ \frac{T_f - t_0}{T_\alpha} \ln \mu \right. \\ &\quad \left. + \frac{\left(N_\sigma(t_0, T_f) + 1\right)^{1-\alpha} \left(T_f - t_0\right)^{\alpha} (\lambda-1)}{\Gamma(\alpha+1)} \right\}. \end{aligned} \tag{29}$$

According to Lemma 13, (29) can be rewritten as

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \leq V_{\sigma(t_0)}(t_0, x(t_0)) \exp\left\{ \frac{T_f - t_0}{T_\alpha} \ln \mu \right. \\ &\quad + (\lambda-1) \\ &\quad \left. \cdot \frac{(1-\alpha)\left(N_\sigma(t_0, T_f) + 1\right) + \alpha\left(T_f - t_0\right)}{\Gamma(\alpha+1)} \right\} \\ &\quad = V_{\sigma(t_0)}(t_0, x(t_0)) \exp\left\{ \frac{T_f - t_0}{T_\alpha} \ln \mu \right. \\ &\quad + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)} \frac{T_f - t_0}{T_\alpha} + (\lambda-1) \\ &\quad \left. \cdot \frac{(1-\alpha) + \alpha\left(T_f - t_0\right)}{\Gamma(\alpha+1)} \right\}. \end{aligned} \tag{30}$$

From (15) and (21), for $t \in [0, T_f)$, we have

$$V_{\sigma(t)}(t, x(t)) \geq \xi_1 x^T(t)\varepsilon, \tag{31}$$

$$V_{\sigma(t)}(t, x(t)) \leq \xi_2 x^T(t_0)\delta \exp\left\{\frac{T_f}{T_\alpha}\ln\mu + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\frac{T_f}{T_\alpha} + (\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}. \tag{32}$$

From (7), (31), and (32), we obtain

$$\begin{aligned} x^T(t)\varepsilon &\leq \frac{\xi_2}{\xi_1}\left\{x^T(t_0)\delta\right\}\exp\left\{\frac{T_f}{T_\alpha}\ln\mu + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\frac{T_f}{T_\alpha} + (\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\} \\ &\leq \frac{\xi_2}{\xi_1}\exp\left\{\frac{T_f}{T_\alpha}\ln\mu + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\frac{T_f}{T_\alpha} + (\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}. \end{aligned} \tag{33}$$

Substituting (19) into (33), one has

$$x^T(t)\varepsilon < 1. \tag{34}$$

From Definition 11, we conclude that system (5) is GCFTS with respect to $(\delta, \varepsilon, T_f, \sigma(t))$.

Next, we will give the fractional-order guaranteed cost value of system (5).

According to (24) and $V_{\sigma(t_k)} \leq \mu_{\sigma(t_k)} V_{\sigma(t_k^-)}(t_k^-, x(t_k^-))$, for $t \in [t_k, t_{k+1})$, we can obtain

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\leq V_{\sigma(t_k)}(t_k, x(t_k)) + \frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}x^T(s)R_1 ds \\ &\leq \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-)) + \frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}x^T(s)R_1 ds \\ &\leq \mu V_{\sigma(t_{k-1})}(t_{k-1}, x(t_{k-1})) + \frac{\lambda\mu}{\Gamma(\alpha)}\int_{t_{k-1}}^{t_k}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{\mu}{\Gamma(\alpha)}\int_{t_{k-1}}^{t_k}(t-s)^{\alpha-1}x^T(s)R_1 ds \\ &\quad + \frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}x^T(s)R_1 ds \leq \cdots \\ &\leq \mu^{N_\sigma(t_0,t)}V_{\sigma(t_0)}(t_0, x(t_0)) + \frac{\lambda\mu^{N_\sigma(t_0,t)}}{\Gamma(\alpha)}\int_{t_0}^{t_1}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{\mu^{N_\sigma(t_0,t)}}{\Gamma(\alpha)}\int_{t_0}^{t_1}(t-s)^{\alpha-1}x^T(s)R_1 ds \\ &\quad + \frac{\lambda\mu^{N_\sigma(t_0,t_k)}}{\Gamma(\alpha)}\int_{t_1}^{t_2}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{\mu^{N_\sigma(t_0,t_k)}}{\Gamma(\alpha)}\int_{t_1}^{t_2}(t-s)^{\alpha-1}x^T(s)R_1 ds + \cdots \\ &\quad + \frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds - \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}x^T(s)R_1 ds. \end{aligned} \tag{35}$$

Let $t = T_f$; then (35) can be turned into

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\leq \mu^{N_\sigma(t_0,T_f)}V_{\sigma(t_0)}(t_0, x(t_0)) + \frac{\lambda\mu^{N_\sigma(t_0,T_f)}}{\Gamma(\alpha)}\int_{t_0}^{T_f}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds \\ &\quad - \frac{1}{\Gamma(\alpha)}\int_{t_0}^{T_f}(t-s)^{\alpha-1}x^T(s)R_1 ds. \end{aligned} \tag{36}$$

By $V_{\sigma(t)}(t, x(t)) \geq 0$, we can get

$$\begin{aligned} \frac{1}{\Gamma(\alpha)}\int_{t_0}^{T_f}(t-s)^{\alpha-1}x^T(s)R_1 ds &\leq \mu^{N_\sigma(t_0,T_f)}V_{\sigma(t_0)}(t_0, x(t_0)) \\ &\quad + \frac{\lambda\mu^{N_\sigma(t_0,T_f)}}{\Gamma(\alpha)}\int_{t_0}^{T_f}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))\,ds. \end{aligned} \tag{37}$$

From (32) and (37), we have

$$\begin{aligned}
&\frac{1}{\Gamma(\alpha)}\int_{t_0}^{T_f}(t-s)^{\alpha-1}x^T(s)R_1 ds \\
&\le \mu^{N_\sigma(t_0,T_f)}\xi_2 x^T(t_0)\delta \\
&\quad + \frac{\lambda\mu^{N_\sigma(t_0,T_f)}}{\Gamma(\alpha)}\int_{t_0}^{T_f}\left\{(t-s)^{\alpha-1}\xi_2 x^T(t_0)\delta\exp\left\{\frac{T_f}{T_\alpha}\ln\mu+\frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\frac{T_f}{T_\alpha}+(\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\right\}ds \\
&\le \xi_2\mu^{T_f/T_\alpha}+\lambda\xi_2\frac{\mu^{T_f/T_\alpha}}{\Gamma(\alpha)}\exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu+\frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right)+(\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\int_{t_0}^{T_f}(t-s)^{\alpha-1}ds \\
&\le \xi_2\mu^{T_f/T_\alpha}+\lambda\xi_2\frac{\mu^{T_f/T_\alpha}}{\Gamma(\alpha)}\exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu+\frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right)+(\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\frac{1}{\alpha}\left(T_f\right)^\alpha.
\end{aligned} \tag{38}$$

Then, the guaranteed cost value of system (5) is given by

$$\begin{aligned}
J &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}x^T(s)R_1 ds \le J^* = \xi_2\mu^{T_f/T_\alpha} \\
&\quad + \lambda\xi_2\mu^{T_f/T_\alpha}\frac{1}{\Gamma(\alpha)} \\
&\quad \cdot\exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu+\frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right)\right. \\
&\quad \left.+(\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\frac{1}{\alpha}\left(T_f\right)^\alpha.
\end{aligned} \tag{39}$$

According to Definition 11, we can conclude that system (5) is GCFTS with respect to $(\delta,\varepsilon,T_f,\sigma(t))$. Thus, the proof is completed. □

Corollary 16. *Replace ${}_{t_0}^{C}D_t^\alpha x(t)$ by ${}_{t_0}^{RL}D_t^\alpha x(t)$ in Theorem 15. If the conditions in Theorem 15 are held, then the FOPSS (5) is GCFTS with respect to $(\delta,\varepsilon,T_f,\sigma(t))$, and the guaranteed cost value is (20).*

Proof. According to (4) and Lemma 1, we can obtain

$$\begin{aligned}
&{}_{t_0}^{C}D_t^\alpha V_{\sigma(t)}(t,x(t)) + x^T(t)R_1 \\
&\le {}_{t_0}^{RL}D_t^\alpha V_{\sigma(t)}(t,x(t)) + x^T(t)R_1 \\
&\le x^T(t)\left(A_{\sigma(t)}^T v_{\sigma(t)}+R_1\right)\le\lambda V_{\sigma(t)(t,x(t))}.
\end{aligned} \tag{40}$$

Similar to the proof process of Theorem 15, we can obtain the same results and the proved process is omitted. □

3.2. Guaranteed Cost Finite-Time Controller Design. In this section, we focus on the problem of guaranteed cost finite-time controller design of system (5). The state feedback controller and static output feedback controller will be designed to ensure the corresponding close-loop system is GCFTS, respectively.

3.2.1. State Feedback Controller Design. Consider system (5); under the controller $u(t) = K_{1\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$$\begin{aligned}
{}_{t_0}^{C}D_t^\alpha x(t) &= \left(A_{\sigma(t)}+B_{\sigma(t)}K_{1\sigma(t)}\right)x(t), \\
y(t) &= C_{\sigma(t)}x(t), \quad 0<\alpha<1.
\end{aligned} \tag{41}$$

According to Lemma 4, to guarantee the positivity of system (41), $A_p + B_pK_{1p}$ should be Metzler matrices, $\forall p \in \underline{N}$. Theorem 17 gives some sufficient conditions to guarantee that the closed-loop system (41) is GCFTS.

Theorem 17. *Consider the FOPSS (41). For given constants T_f and λ $(\lambda > 1)$ and vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$, and $R_2 \succ 0$, if there exist constants ξ_1, ξ_2, and μ $(\mu \ge 1)$ and positive vectors v_p and $p \in \underline{N}$, such that (15), (16), (18), and the following conditions hold:*

$$\begin{gathered}
A_p + B_pK_{1p} \quad \text{are Metzler matrices}, \\
A_p^T v_p + R_1 + f_p + K_{1p}^T R_2 \preceq \lambda v_p, \\
v_p \prec R_1 + K_{1p}^T R_2,
\end{gathered} \tag{42}$$

where $f_p = K_{1p}^T B_p^T v_p$, then, under the ADT scheme (19), the resulting closed-loop system (41) is GCFTS with respect to $(\delta,\varepsilon,T_f,\sigma(t))$, and the cost value is given by

$$\begin{aligned}
J &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left(x^T(s)R_1\right. \\
&\quad \left.+x^T(s)K_{1p}^T(s)R_2\right)ds \le J^* = \xi_2\mu^{T_f/T_\alpha} \\
&\quad + \lambda\xi_2\mu^{T_f/T_\alpha}\frac{1}{\Gamma(\alpha)} \\
&\quad \cdot\exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu+\frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right)\right. \\
&\quad \left.+(\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\frac{1}{\alpha}\left(T_f\right)^\alpha.
\end{aligned} \tag{43}$$

Proof. By Lemma 4, we get that $A_p + G_pK_{1p}$ are Metzler matrices for each $p \in \underline{N}$. According to Lemma 5, system (41) is positive if $\forall p \in \underline{N}$, B_p are all nonnegative. Replacing A_p in (14) with $A_p + B_pK_{1p}$ and letting $f_p = K_{1p}^TB_p^Tv_p$, μ ($\mu \geq 1$) satisfy (16), then, under the ADT scheme (19), we easily obtain that the resulting closed-loop system (41) is GCFTS with respect to $(\delta, \varepsilon, T_f, \sigma(t))$ and the guaranteed cost value is given by (43).

The proof is completed. □

3.2.2. Static Output Feedback Controller Design. Consider system (5), under the controller $u(t) = K_{2\sigma(t)}y(t)$, the corresponding closed-loop system is given by

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha}x(t) &= \left(A_{\sigma(t)} + B_{\sigma(t)}K_{2\sigma(t)}C_{\sigma(t)}\right)x(t), \\ y(t) &= C_{\sigma(t)}x(t), \quad 0 < \alpha < 1. \end{aligned} \tag{44}$$

According to Lemma 4, to guarantee the positivity of system (44), $A_p + B_pK_{2p}C_p$ should be Metzler matrices, $\forall p \in \underline{N}$. Theorem 18 gives some sufficient conditions to guarantee that the closed-loop system (44) is GCFTS.

Theorem 18. *Consider the FOPSS (44). For given constants T_f and λ ($\lambda > 1$) and vectors $\delta \succ \varepsilon \succ 0$, $R_1 \succ 0$, and $R_2 \succ 0$, if there exist constants ξ_1, ξ_2, and μ ($\mu \geq 1$) and positive vectors v_p and $p \in \underline{N}$, such that (15), (16), (18), and the following conditions hold:*

$$\begin{gathered} A_p + B_pK_{2p}C_p \quad \text{are Metzler matrices}, \\ A_p^Tv_p + R_1 + f_p + C_p^TK_{2p}^TR_2 \preceq \lambda v_p, \\ v_p \prec R_1 + C_p^TK_{2p}^TR_2, \end{gathered} \tag{45}$$

where $f_p = C_p^TK_{2p}^TB_p^Tv_p$, then, under the ADT scheme (19), the resulting closed-loop system (44) is GCFTS with respect to $(\delta, \varepsilon, T_f, \sigma(t))$, and the cost value is given by

$$\begin{aligned} J &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left(x^T(s)R_1 + x^T(s)C_p^TK_{2\sigma}^TR_2\right)ds \leq J^* = \xi_2\mu^{T_f/T_\alpha} \\ &+ \lambda\xi_2\mu^{T_f/T_\alpha}\frac{1}{\Gamma(\alpha)} \\ &\cdot \exp\left\{\frac{T_f}{T_\alpha}\left(\ln\mu + \frac{(1-\alpha)(\lambda-1)}{\Gamma(\alpha+1)}\right) + (\lambda-1)\frac{(1-\alpha)+\alpha T_f}{\Gamma(\alpha+1)}\right\}\frac{1}{\alpha}\left(T_f\right)^{\alpha}. \end{aligned} \tag{46}$$

Proof. By Lemma 4, we get that $A_p + G_pK_{2p}C_p$ are Metzler matrices for each $p \in \underline{N}$. According to Lemma 5, system (44) is positive if $\forall p \in \underline{N}$, B_p and C_p are all nonnegative. Replacing A_p in (14) with $A_p + B_pK_{2p}C_p$ and letting $f_p = C_p^TK_{2p}^TB_p^Tv_p$, μ ($\mu \geq 1$) satisfy (16), then, under the ADT scheme (19), we easily obtain that the resulting closed-loop system (44) is GCFTS with respect to $(\delta, \varepsilon, T_f, \sigma(t))$ and the guaranteed cost value is given by (46).

The proof is completed. □

Next, an algorithm is presented to obtain the feedback gain matrices K_{1p} (or K_{2p}) and $p \in \underline{N}$.

Algorithm 19.

Step 1. Input the matrices A_p, B_p, C_p, R_1, and R_2.

Step 2. By adjusting the parameter λ and solving the inequalities in Theorem 17 (or Theorem 18) via linear programming, we can get the solutions v_p, K_{1p} (or K_{2p}), and f_p.

Step 3. Then, $\tilde{f}_p = K_{1p}^TB_p^Tv_p$ (or $\tilde{f}_p = C_p^TK_{2p}^TB_p^Tv_p$) is obtained. If $f_p - \tilde{f}_p \geq 0$ and $A_p + B_pK_{1p}$ (or $A_p + B_pK_{2p}C_p$) are Metzler matrices, then K_{1p} (or K_{2p}) are admissible. Otherwise, return to Step 2.

4. Numerical Examples

In this section, two examples are given to illustrate the effectiveness of the two controllers proposed above.

Example 1. Under the state feedback controller $u(t) = K_{1\sigma(t)}x(t)$, consider the FOPSS (5) with the parameters as follows:

$$A_1 = \begin{bmatrix} -6 & 2 & 1 \\ 2 & -5 & 1 \\ 1 & 2 & -5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 2.5 \\ 3 \\ 2 \end{bmatrix}^T,$$

$$R_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

$$\varepsilon = \begin{bmatrix} 0.4 \\ 0.2 \\ 0.3 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -4 & 1 & 1.5 \\ 1 & -4 & 2 \\ 2 & 1.8 & -5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.2 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}^T,$$
$$R_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix},$$
$$\delta = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}. \tag{47}$$

Let $\alpha = 0.9$, $\mu = 1.1$, and $\lambda = 1.12$. Solving the inequalities in Theorem 17 by linear programming, we have

$$\begin{gathered} v_1 = \begin{bmatrix} 2.0188 \\ 1.3213 \\ 1.5412 \end{bmatrix}, \\ v_2 = \begin{bmatrix} 2.0883 \\ 1.2623 \\ 1.5862 \end{bmatrix}, \\ \xi_1 = 5.2301, \\ \xi_2 = 1.1471, \\ K_{11} = \begin{bmatrix} 1.0390 & 0.1754 & 2.0385 \end{bmatrix}, \\ K_{12} = \begin{bmatrix} 1.9433 & 0.2481 & 0.9922 \end{bmatrix}. \end{gathered} \tag{48}$$

It is easy to verify that $A_p + B_p K_{1p}$ are Metzler matrices, $\forall p \in \underline{N}$. Then, according to (19), we can obtain $T_\alpha^* = 2.8233$ s. Choosing $T_\alpha = 2.9$ s $> T_\alpha^*$. Under the state feedback controller, the simulation results are shown in Figures 1–3, the initial conditions of system (5) are $x(0) = [0.2 \quad 0.3 \quad 0.1]^T$, which satisfy $x^T(0)\delta \leq 1$. The state trajectories of the closed-loop system with ADT are shown in Figure 1. The switching signal $\sigma(t)$ with ADT is depicted in Figure 2. Figure 3 plots the evolution of $x^T(t)\varepsilon$ of system (5). The cost value is $J^* = 62.2654$, which can be obtained by (43).

Example 2. A fractional-order positive electrical circuit system was modeled in [28]. Accordingly, a switching type positive fractional-order electrical circuit system could be modeled by (5). Under the static output feedback controller

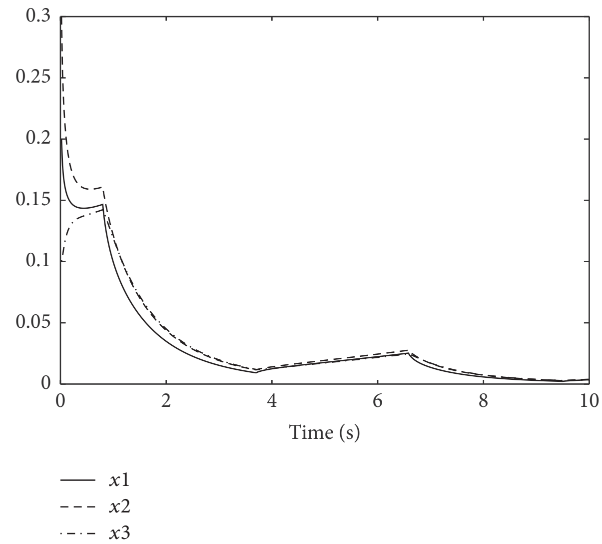

Figure 1: State trajectories of closed-loop system (5).

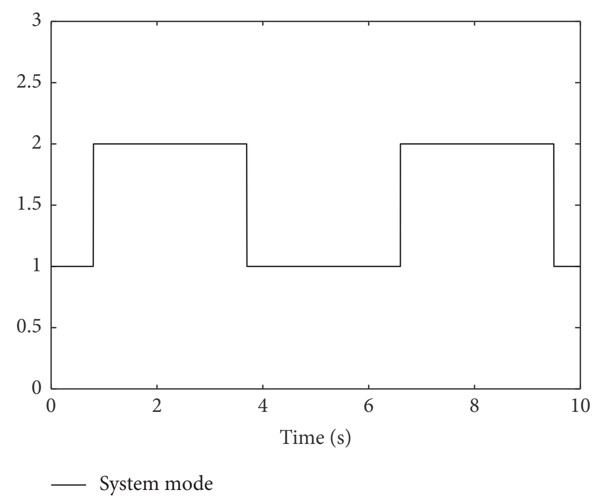

Figure 2: Switching signal of system (5) with ADT.

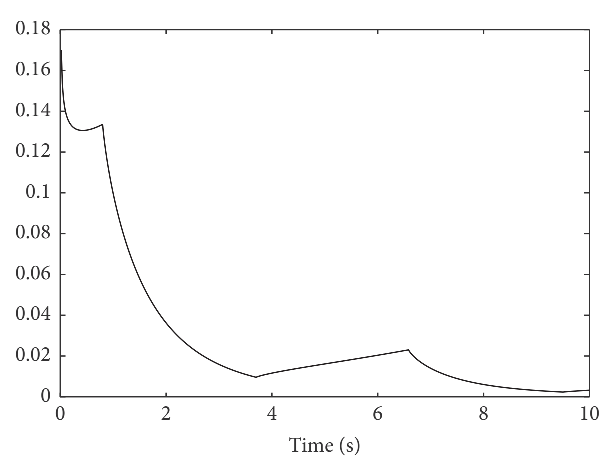

Figure 3: The evolution of $x^T(t)\varepsilon$ of system (5).

$u(t) = K_{2\sigma(t)} y(t)$, consider system (5) with the parameters as follows:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
R_1 &= \begin{bmatrix} 2 \\ 1.6 \end{bmatrix}, \\
\varepsilon &= \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
R_2 &= \begin{bmatrix} 1.2 \\ 1 \end{bmatrix}, \\
\delta &= \begin{bmatrix} 0.8 \\ 1 \end{bmatrix}.
\end{aligned} \tag{49}
$$

Let $\alpha = 0.9$, $\mu = 1.08$, and $\lambda = 1.1$. Solving the inequalities in Theorem 18 by linear programming, we have

$$
\begin{aligned}
v_1 &= \begin{bmatrix} 1.0156 \\ 1.2353 \end{bmatrix}, \\
v_2 &= \begin{bmatrix} 1.0765 \\ 1.2241 \end{bmatrix}, \\
\xi_1 &= 6.8058, \\
\xi_2 &= 1.3769, \\
K_{21} &= \begin{bmatrix} 0.2823 & 0.3684 \\ 0.2823 & 0.3684 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} 0.2009 & 0.3174 \\ 0.2009 & 0.3174 \end{bmatrix}.
\end{aligned} \tag{50}
$$

It is easy to verify that $A_p + C_p B_p K_{2p}$ are Metzler matrices, $\forall p \in \underline{N}$. Then, according to (19), we can obtain $T_\alpha^* = 1.3591$ s. Choose $T_\alpha = 1.4$ s $> T_\alpha^*$. Under the static output feedback controller, the simulation results are shown in Figures 4–6, and the initial conditions of system (5) are

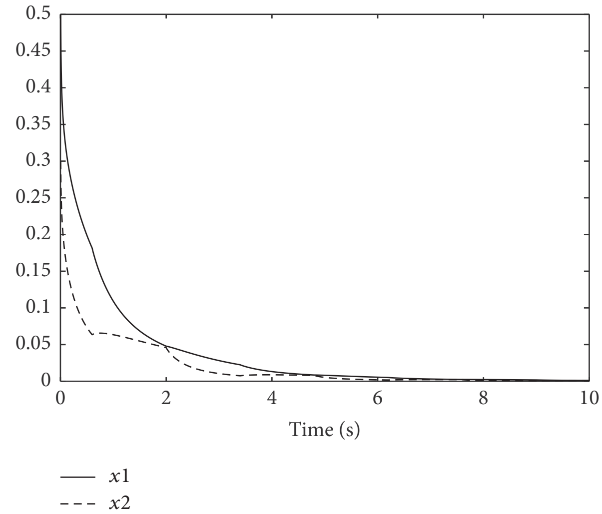

Figure 4: State trajectories of closed-loop system (5).

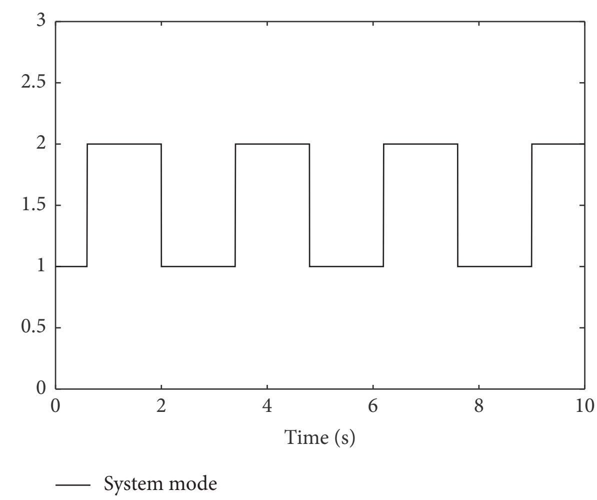

Figure 5: Switching signal of system (5) with ADT.

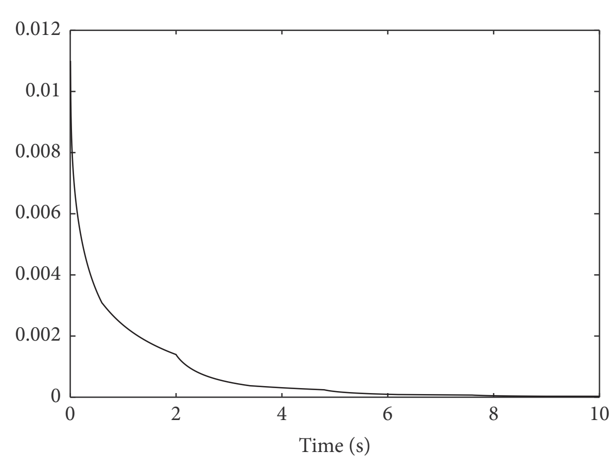

Figure 6: The evolution of $x^T(t)\varepsilon$ of system (5).

$x(0) = [0.5 \quad 0.3]^T$, which satisfy $x^T(0)\delta \leq 1$. The state trajectories of the closed-loop system (5) with ADT are shown in Figure 4. The switching signal $\sigma(t)$ with ADT is depicted in Figure 5. Figure 6 plots the evolution of $x^T(t)\varepsilon$ of system (5). The cost value is $J^* = 107.2803$, which can be obtained by (46).

5. Conclusions

This paper has dealt with the problem of guaranteed cost finite-time control for FOPSS. A novel fractional-order cost function is introduced. By using ADT approach and constructing multiple linear copositive Lyapunov functions, two kinds of controllers are designed, and some sufficient conditions are obtained to guarantee that the closed-loop system is GCFTS. Such sufficient conditions can be solved by linear programming. Finally, two examples are provided to show the effectiveness of the proposed method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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