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Research Article

Effect of Internal Heat Source on the Onset of Double-Diffusive Convection in a Rotating Nanofluid Layer with Feedback Control Strategy

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A linear stability analysis has been carried out to examine the effect of internal heat source on the onset of Rayleigh–Bénard convection in a rotating nanofluid layer with double diffusive coefficients, namely, Soret and Dufour, in the presence of feedback control. The system is heated from below and the model used for the nanofluid layer incorporates the effects of thermophoresis and Brownian motion. Three types of bounding systems of the model have been considered which are as follows: both the lower and upper bounding surfaces are free, the lower is rigid and the upper is free, and both of them are rigid. The eigenvalue equations of the perturbed state were obtained from a normal mode analysis and solved using the Galerkin method. It is found that the effect of internal heat source and Soret parameter destabilizes the nanofluid layer system while increasing the Coriolis force, feedback control, and Dufour parameter helps to postpone the onset of convection. Elevating the modified density ratio hastens the instability in the system and there is no significant effect of modified particle density in a nanofluid system.

1. Introduction

The importance of understanding convective heat transfer in nanofluids has been a topic of interest for the last few years. A nanofluid, that is, a colloidal mixture of nanosized particles (1–100 nm), and a base fluid (nanoparticle fluid suspension) was first introduced by Choi [1]. Nanofluids possess their stability due to the small size of their particles, low weight, and less chance of sedimentation. Subsequently, there have been tremendous attempts to observe the enhancement of thermal conductivity in a nanofluid. The earliest investigation was reported by Masuda et al. [2], followed by Xuan and Li [3], Eastman et al. [4], and Das et al. [5]. Buongiorno [6] discussed the seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid. These are inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling. He concluded that the effects of Brownian diffusion and thermophoresis are important for convective transport in nanofluids and his model is the basis for the present study. Tzou [7, 8] employed Buongiorno's model to investigate the thermal instability problem and found that nanofluids are less stable than regular fluids. Alloui et al. [9] studied the natural convection of nanofluids in a shallow cavity heated from below. Nield and Kuznetsov [10] studied the parameters involved in the onset of convection in a horizontal nanofluid layer of finite depth. A linear analysis of the Rayleigh–Bénard instability for a nanofluid is performed by Yadav et al. [11]. Haddad et al. [12] reported that the thermophoresis and Brownian motion are significant in the thermal enhancement of the natural convection in a nanofluid layer. Recently, Gupta et al. [13] investigated the thermosolutal convection in a horizontal nanofluid layer heated from below.

Soret diffusion, or also known as thermal diffusion, and Dufour diffusion, also known as thermodiffusion [14], are

important in both Newtonian and non-Newtonian convective heat and mass transfer, and they are often encountered in chemical process engineering and in high-speed aerodynamics. Such effects are significant for gases of intermediate molecular weight in the coupled heat and mass transfer in binary systems. Hurle and Jakeman [15] demonstrated the Soret-driven thermosolutal convection both theoretically and experimentally using a water-methanol mixture. Then, Platten and Chavepeyer [16] continued using a water-ethanol mixture while Caldwell [17] extended the investigation using a salt solution. The linear stability of experimental Soret convection in a water-ethanol mixture under various boundary conditions had been investigated by Knobloch and Moore [18] with an emphasis on the Biot number. The thermocapillary instability in a binary fluid on the onset of convection with Soret effect and other physical influences also had been studied [19–21]. Later, Nield and Kuznetsov [22] extended [10] and used the linear instability theory to study the onset of double-diffusive convection in a horizontal layer of a nanofluid. Kuznetsov and Nield [23], Yadav et al. [24, 25], and Agarwal et al. [26] investigated double-diffusive convection in a porous medium permeated by a nanofluid layer. The authors in [27–29] added other effects to the system of double-diffusive convection in a nanofluid layer.

The thermal instability induced by uniform internal heat generation arising in a horizontal fluid layer has attracted the attention of many researchers [30–38]. It is found that an internal heat source decreases the stability of the fluid layer. Yadav et al. [39, 40] and Nield and Kuznetsov [41] included the effect of an internal heat source on the thermal instability in a nanofluid layer and found that the heat source advances the onset of convection. Recently, Wakif et al. [42] studied the combined effects of rotation and internal heating on a radiating nanofluid layer using the power series method (PSM) solely at both the upper and lower rigid boundaries.

The Coriolis force which is due to rotation in a system has an important effect on convective instability. Chandrasekhar [43] discussed Rayleigh–Bénard convection in a regular fluid with a linear temperature profile, both with and without the effect of rotation. In 1966, Vidal and Acrivos [44] included the effect of Coriolis force on the thermocapillary type of convection. Later, McConaghy and Finlayson [45], Takashima and Namikawa [46], Friedrich and Rudraiah [47], and Douiebe et al. [48] extended the previous analysis with other aspects of the problem. Meanwhile, several authors [49–52] have investigated the convective instability in a rotating fluid layer induced by buoyancy and thermocapillary, which has received profound attention from the engineering industry. Yadav et al. [53–56] studied the effect of rotation in a nanofluid layer and found that an increase in the Taylor number delays the onset of convection.

The use of feedback control in stabilizing the thermal convection was by Wang et al. [57] and they managed to inhibit the chaotic behaviour in the fluid layer by applying proportional control in a thermal convection loop. Later, Tang [58] and Tang and Bau [59, 60] showed that, with the use of a feedback controller, the critical Rayleigh number for the onset of convection can be significantly increased. Tang and Bau [61–64] and Howle [65–67] demonstrated

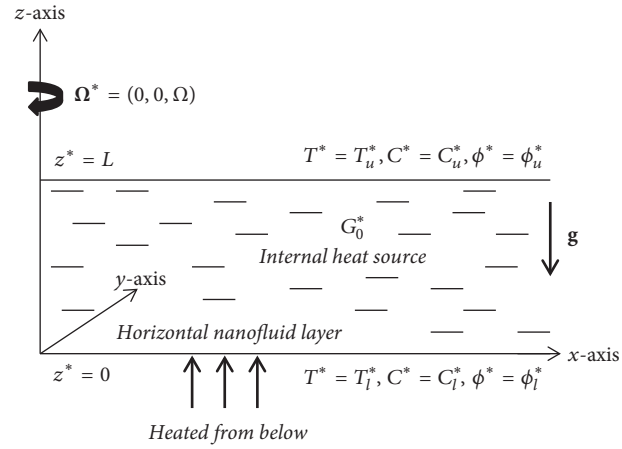

Figure 1: Physical configuration and coordinate system.

experimentally that feedback control can be used to stable the system through the use of the control strategy. It is interesting to note that similar control strategies can be used to modify the flow patterns of shear flows [68] and surface-tension driven flows [69]. Further, many researchers have attempted to include other effects with the feedback control [70–75] on convection due to buoyancy and surface-tension.

In this paper, we intend to scrutinize the effect of internal heat source on the thermal instability of double-diffusive convection in a rotating nanofluid layer with feedback control. As in industrial heat transfer, heat needs to be efficiently controlled whether being added, removed, or transferred from one process stream to another. It is reported that between 20% and 50% of industrial energy input is lost as waste heat [76], which has attracted numerous studies on energy efficiency, recovering waste heat, and so on. This motivated us to contribute mathematically by providing a mathematical model that can simulate the influence of an internal heat source and feedback control in a nanofluid layer with other crucial parameters. We assume the nanofluid layer is heated from below and the lower-upper boundary conditions are considered to be free-free, rigid-free, and rigid-rigid. For systems that have thermal gradient and energy flux due to a mass concentration, Soret and Dufour coefficients are vital. To understand deeply the reaction of these two coefficients in a rotating nanofluid layer with internal heat source and feedback controller, we considered the interdiffusion parameter in the system. A linear stability analysis is performed, and the eigenvalue is obtained by employing the Galerkin technique. Numerical computations of the various relevant parameters are presented graphically.

2. Mathematical Formulation

Cartesian coordinates (x, y, z) are used, where the z-axis points vertically upward. From now on, we denote the dimensional (nondimensional) variables with (without) asterisks. Consider a horizontal layer of a rotating incompressible nanofluid of thickness L confined between the planes $z^* \in [0, L]$ and subjected to a uniform internal heat source G_0^* is heated from below as shown in Figure 1. The nanofluid layer

rotates about the vertical axis at a constant angular velocity, $\mathbf{\Omega}^* = (0, 0, \Omega)$. The stability of a horizontal rotating nanofluid layer in the presence of internal heat source is examined. The temperature, solute concentration, and nanoparticle volume fraction at the lower and upper walls are denoted by T_l^*, C_l^*, and ϕ_l^* at $z = 0$ and T_u^*, C_u^*, and ϕ_u^* at $z = L$, respectively. Following Nield and Kuznetsov [22], the governing equations that describe the Boussinesq flow under this model with the presence of the effect of rotation and internal heat source are

$$\begin{aligned}
&\nabla^* \cdot \mathbf{U}^* = 0,\\
&\rho_l \left[\frac{\partial \mathbf{U}^*}{\partial t^*} + \left(\mathbf{U}^* \cdot \nabla^*\right)\mathbf{U}^*\right] = -\nabla^* p^* + \mu \nabla^{*2}\mathbf{U}^*\\
&\quad + \left(2\rho_l \mathbf{U}^* \times \mathbf{\Omega}^*\right) + \mathbf{g}\left[\phi^* \rho_p\right.\\
&\quad \left. + \left(1 - \phi^*\right)\rho_l\left[1 - \alpha_T\left(T^* - T_l^*\right) - \alpha_C\left(C^* - C_l^*\right)\right]\right],\\
&\rho c\left[\frac{\partial T^*}{\partial t^*} + \left(\mathbf{U}^* \cdot \nabla^*\right)T^*\right] = \kappa \nabla^{*2} T^* + G_0^*\\
&\quad + \rho c\left[D_{TC}\nabla^{*2} C^*\right] + \rho_p c_p\left[D_B \nabla^* \phi^* \cdot \nabla^* T^*\right.\\
&\quad \left. + \left(\frac{D_T}{T_u^*}\right)\nabla^* T^* \cdot \nabla^* T^*\right],\\
&\left[\frac{\partial C^*}{\partial t^*} + \left(\mathbf{U}^* \cdot \nabla^*\right)C^*\right] = D_S \nabla^{*2} C^* + D_{CT}\nabla^{*2} T^*,\\
&\left[\frac{\partial \phi^*}{\partial t^*} + \left(\mathbf{U}^* \cdot \nabla^*\right)\phi^*\right] = D_B \nabla^{*2}\phi^* + \frac{D_T}{T_u^*}\nabla^{*2} T^*,
\end{aligned} \tag{1}$$

where $\mathbf{U}^* = (u, v, w)$ is the velocity, ρ is the density, ρ_l is the nanofluid density at the reference temperature T_l^*, ρ_p is the nanoparticle mass density, t^* is time, p^* is the pressure, μ is the viscosity, $\mathbf{g}$ is the gravitational force, ϕ^* is the nanoparticle volume fraction, G_0^* is the uniform internal heat source, α_T is the thermal volumetric coefficient, T^* is the temperature, α_C is the solutal volumetric coefficient, C^* is the solute concentration, c is the specific heat, c_p is the specific heat of the nanoparticles, κ is the nanofluid's thermal conductivity, D_{TC} is the Dufour diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, D_S is the solutal diffusivity, and D_{CT} is the Soret diffusivity.

In the steady state, the upper surface of the nanofluid layer is flat and stationary. Following Char and Chiang [31], the pressure and temperature fields are

$$\begin{aligned}
p^*(z) &= p_l^* - \rho_l\left[1 + \frac{\alpha z^* \Delta T^*}{2L} - \frac{\alpha L \Delta T^*}{2L}\right]\mathbf{g}\left(z^* - L\right),\\
T^*(z) &= T_l^* - \frac{G_0^*}{2\kappa}z^{*2} + \frac{G_0^* L}{2\kappa}z^* - \frac{\Delta T^*}{L}z^*,
\end{aligned} \tag{2}$$

where p_l^* is the reference pressure and ΔT^* is the temperature difference across the fluid layer. Then, (1) are nondimensionalised using the following definitions:

$$\begin{aligned}
(x, y, z) &= \frac{x^*, y^*, z^*}{L},\\
p &= L^2 \frac{p^*}{\mu \alpha_f},\\
t &= \alpha_f \frac{t^*}{L^2},\\
\psi &= L^2 \frac{\psi^*}{\alpha_f},\\
\phi &= \frac{\phi^* - \phi_l^*}{\phi_u^* - \phi_l^*},\\
(u, v, w) &= L\frac{u^*, v^*, w^*}{\alpha_f},\\
T &= \frac{T^* - T_l^*}{\Delta T^*},\\
C &= \frac{C^* - C_l^*}{\Delta C^*},
\end{aligned} \tag{3}$$

where $\alpha_f = \kappa/\rho c$ is the thermal diffusivity and ψ is the vorticity due to rotation in the z component.

By substituting (3) into (1), the following nondimensional variables are obtained:

$$\begin{aligned}
&\nabla \cdot \mathbf{U} = 0,\\
&\frac{1}{\mathrm{Pr}}\left[\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right] = -\nabla p + \nabla^2 \mathbf{U} - \mathrm{Rm}\hat{\mathbf{e}}_\mathbf{z} + \mathrm{Ra}T\hat{\mathbf{e}}_\mathbf{z}\\
&\quad + \frac{\mathrm{Rs}}{\mathrm{Le}}C\hat{\mathbf{e}}_\mathbf{z} - \mathrm{Rn}\phi\hat{\mathbf{e}}_\mathbf{z}\\
&\quad + \sqrt{\mathrm{Ta}}\left(\mathbf{U} \times \hat{\mathbf{e}}_\mathbf{z}\right),\\
&\left[\frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T\right] = \nabla^2 T + \frac{N_B}{\mathrm{Ln}}\nabla\phi \cdot \nabla T + \frac{N_A N_B}{\mathrm{Ln}}\nabla T\\
&\quad \cdot \nabla T + \mathrm{Df}\nabla^2 C\\
&\quad + \left[Q\left(1 - 2z\right) - 1\right]w,\\
&\left[\frac{\partial C}{\partial t} + \mathbf{U} \cdot \nabla C\right] = \frac{1}{\mathrm{Le}}\nabla^2 C + \mathrm{Sr}\nabla^2 T,\\
&\left[\frac{\partial \phi}{\partial t} + \mathbf{U} \cdot \nabla \phi\right] = \frac{1}{\mathrm{Ln}}\nabla^2 \phi + \frac{N_A}{\mathrm{Ln}}\nabla^2 T,
\end{aligned} \tag{4}$$

where $\hat{\mathbf{e}}_\mathbf{z} = (0, 0, 1)$ is the unit vector in the z direction, $\mathrm{Pr} = \mu/\rho\alpha_f$ is the Prandtl number, $\mathrm{Rm} = [\rho_p\phi_l^* + \rho(1 - \phi_l^*)]gL^3/\mu\alpha_f$ is the basic-density Rayleigh number, $\mathrm{Ra} = \rho g \alpha_T L^3 \Delta T^*/\mu\alpha_f$ is the thermal Rayleigh number, $\mathrm{Rs} = \rho g \alpha_C L^3 \Delta C^*/\mu D_s$ is the solutal Rayleigh number, $\mathrm{Le} = \alpha_f/D_s$ is the Lewis number, $\mathrm{Rn} = (\rho_p - \rho)(\phi_u^* - \phi_l^*)gL^3/\mu\alpha_f$

is the nanoparticle concentration Rayleigh number, $\mathrm{Ta} = (4\Omega^{*2}/\mu^2)L^4$ is the Taylor number, $N_B = ((\rho c)_p/(\rho c)_f)(\phi_u^* - \phi_l^*)$ is the modified particle-density increment, $\mathrm{Ln} = \alpha_f/D_B$ is the nanofluid Lewis number, $N_A = D_T\Delta T^*/D_B T_u^*(\phi_u^* - \phi_l^*)$ is the modified density ratio, $\mathrm{Df} = D_{TC}\Delta C^*/\alpha_f\Delta T^*$ is the Dufour parameter, $\mathrm{Sr} = D_{CT}\Delta T^*/\alpha_f\Delta C^*$ is the Soret parameter, and $Q = G_0^* L^2/2\kappa\Delta T^*$ represents the internal heat generation.

We seek a time-independent quiescent solution for (4) with the temperature, concentration, and nanoparticle volume fraction varying only in the z direction, that is, a basic state solution of the form

$$\begin{aligned}(u, v, w) &= (0, 0, 0),\\ p &= p_b(z),\\ T &= T_b(z),\\ C &= C_b(z),\\ \phi &= \phi_b(z).\end{aligned} \tag{5}$$

Suppose that the basic state is disturbed by an infinitesimal thermal perturbation. We now superimpose perturbations on the basic solution. We write

$$\begin{aligned}&(u, v, w, p, T, C, \phi, \psi)\\ &\quad = [0, 0, 0, p_b(z), T_b(z), C_b(z), \phi_b(z), \psi_b(z)]\\ &\qquad + \left[u', v', w', p', T', C', \phi', \psi'\right].\end{aligned} \tag{6}$$

We substitute (6) into (4) and linearise them by neglecting the products of primed quantities and obtain

$$\nabla \cdot \mathbf{U}' = 0, \tag{7}$$

$$\begin{aligned}&\frac{1}{\mathrm{Pr}}\frac{\partial \mathbf{U}'}{\partial t}\\ &\quad = -\nabla p' + \nabla^2 \mathbf{U}' + \mathrm{Ra}T'\hat{\mathbf{e}}_{\mathbf{z}} + \frac{\mathrm{Rs}}{\mathrm{Le}}C'\hat{\mathbf{e}}_{\mathbf{z}} - \mathrm{Rn}\phi'\hat{\mathbf{e}}_{\mathbf{z}}\\ &\qquad + \sqrt{\mathrm{Ta}}\left(\mathbf{U}' \times \hat{\mathbf{e}}_{\mathbf{z}}\right),\end{aligned} \tag{8}$$

$$\begin{aligned}&\frac{\partial T'}{\partial t} - [Q(1 - 2z) - 1]w'\\ &\quad = \nabla^2 T' + \frac{N_B}{\mathrm{Ln}}\left[\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right] - \frac{2N_A N_B}{\mathrm{Ln}}\frac{\partial T'}{\partial z}\\ &\qquad + \mathrm{Df}\nabla^2 C',\end{aligned} \tag{9}$$

$$\left[\frac{\partial C'}{\partial t} - w'\right] = \frac{1}{\mathrm{Le}}\nabla^2 C' + \mathrm{Sr}\nabla^2 T', \tag{10}$$

$$\left[\frac{\partial \phi'}{\partial t} + w'\right] = \frac{1}{\mathrm{Ln}}\nabla^2 \phi' + \frac{N_A}{\mathrm{Ln}}\nabla^2 T'. \tag{11}$$

The parameter Rm is not included in these and subsequent equations. It is just a measure of the basic static pressure gradient.

Operating curl twice to (8) and using the curl identity together with (7), we obtain

$$\begin{aligned}&\left[\nabla^4 - \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\nabla^2\right]w' + \frac{\mathrm{Rs}}{\mathrm{Le}}\nabla_P^2 C' - \mathrm{Rn}\nabla_P^2\phi' + \mathrm{Ra}\nabla_P^2 T'\\ &\quad - \sqrt{\mathrm{Ta}}\frac{\partial \psi'}{\partial z} = 0,\\ &\left[\nabla^2 - \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\right]\psi' + \sqrt{\mathrm{Ta}}\frac{\partial w'}{\partial z} = 0,\end{aligned} \tag{12}$$

where ∇_P^2 is the horizontal two-dimensional Laplacian operator.

The proposed normal mode representation is

$$\begin{aligned}&\left(w', T', C', \phi', \psi'\right)\\ &\quad = [W(z), \Theta(z), \eta(z), \phi(z), \Psi(z)]\, e^{[i(a_x x + a_y y) + \sigma t]},\end{aligned} \tag{13}$$

where (a_x, a_y) is the wavevector in the (x, y) plane, $a^2 = a_x^2 + a_y^2$ is the square of the wavenumber, and σ is the growth parameter.

Substituting (13) into (9)–(12) and neglecting the terms of the second and higher orders in the perturbations, we obtain

$$\begin{aligned}&\left(D^2 - a^2\right)^2 W - a^2\mathrm{Ra}\Theta - a^2\frac{\mathrm{Rs}}{\mathrm{Le}}\eta + a^2\mathrm{Rn}\phi\\ &\quad - \sqrt{\mathrm{Ta}}D\Psi = 0,\\ &[1 - Q(1 - 2z)]W\\ &\quad + \left[D^2 - a^2 + \frac{N_B}{\mathrm{Ln}}D - 2\frac{N_A N_B}{\mathrm{Ln}}D\right]\Theta - \frac{N_B}{\mathrm{Ln}}D\phi\\ &\quad + \mathrm{Df}\left(D^2 - a^2\right)\eta = 0,\\ &W + \mathrm{Sr}\left(D^2 - a^2\right)\Theta + \frac{1}{\mathrm{Le}}\left(D^2 - a^2\right)\eta = 0,\\ &W - \frac{N_A}{\mathrm{Ln}}\left(D^2 - a^2\right)\Theta - \frac{1}{\mathrm{Ln}}\left(D^2 - a^2\right)\phi = 0,\\ &\sqrt{\mathrm{Ta}}DW + \left(D^2 - a^2\right)\Psi = 0,\end{aligned} \tag{14}$$

where $a = \sqrt{a_x^2 + a_y^2}$ is the wavenumber and $D = d/dz$.

Following the proportional feedback control [69], the continuously distributed actuators and sensors are arranged in a way that, for every sensor, there is an actuator positioned directly beneath it. The determination of a control $q(t)$ can be accomplished using the proportional-integral-differential (PID) controller of the form

$$q(t) = r + K[e(t)], \tag{15}$$

where r is the calibration of the control, $e(t) = \widehat{m}(t) + m(t)$ is an error or deviation from the measurement, $\widehat{m}(t)$, from some desired reference value, $m(t)$, K is the scalar controller gain where $K = K_P + K_D(d/dt) + K_L\int_0^t dt$, K_P is the proportional gain, K_D is the differential gain, and K_L is the integral gain.

Based on (15), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relation between the upper, $z = 1$, and the lower, $z = 0$, thermal boundaries for the perturbation field

$$T'(x, y, 0, t) = -KT'(x, y, 1, t), \tag{16}$$

where T' denotes the deviation of the temperature of fluid from its conductive state.

Equations (14) are solved subject to the appropriate boundary conditions. Considering the proportional controller, K positioned at the lower boundary of nanofluid layer, we will have

$$W = DW = \Theta(0) + K\Theta(1) = \phi = \eta = \Psi = D\Psi = 0 \quad \text{at } z = 0. \tag{17}$$

We assumed that the upper boundary is nondeformable and insulating to temperature perturbations. The suitable upper boundary conditions are as follows.

For the upper free boundary, which is at $z = 1$, we have

$$W = D^2W = D\Theta = \phi = \eta = D\Psi = 0 \quad \text{at } z = 1. \tag{18}$$

For the upper rigid boundary, which is at $z = 1$, we have

$$W = DW = D\Theta = \phi = \eta = D\Psi = 0 \quad \text{at } z = 1. \tag{19}$$

The Galerkin-type weighted residuals method is applied to find an approximate solution to the system. The variables are written in a series of basis functions:

$$\begin{aligned} W &= \sum_{i=1}^{n} A_i W_i, \\ \Theta &= \sum_{i=1}^{n} B_i \Theta_i, \\ \eta &= \sum_{i=1}^{n} C_i \eta_i, \\ \phi &= \sum_{i=1}^{n} D_i \phi_i, \\ \Psi &= \sum_{i=1}^{n} E_i \Psi_i, \end{aligned} \tag{20}$$

where A_i, B_i, C_i, D_i, and E_i are constants and the basis functions W_i, Θ_i, η_i, ϕ_i, and Ψ_i where $i = 1, 2, 3, \ldots$ will be chosen corresponding to the free-free, rigid-free, and rigid-rigid lower-upper boundary conditions:

$$\begin{aligned} W = \eta = \phi &= \sin(z\pi), \\ \Theta &= z(2 - z), \\ \Psi &= z\left(3z - 2z^2\right), \\ W &= z^2(1 - z)(3 - 2z), \\ \Theta &= z(2 - z), \\ \eta = \phi &= z(1 - z), \\ \Psi &= z\left(3z - 2z^2\right), \\ W &= z^2(1 - z)^2, \\ \Theta &= z(2 - z), \\ \eta = \phi &= z(1 - z), \\ \Psi &= z\left(3z - 2z^2\right). \end{aligned} \tag{21}$$

Substitute (20) into (14) and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of $5N$ linear algebraic equations in the $5N$ unknowns. The vanishing of the determinant of the coefficients produces the eigenvalue equation for the system. One can regard Ra as the eigenvalue and thus Ra is found in terms of the other parameters.

Perform an integration by parts with respect to z between $z \in [0, 1]$. By using boundary conditions (21), we obtain the system of linear homogeneous algebraic equations

$$\begin{aligned} a_{ji}W_i + b_{ji}\Theta_i + c_{ji}\eta_i + d_{ji}\phi_i + e_{ji}\Psi_i &= 0, \\ f_{ji}W_i + g_{ji}\Theta_i + h_{ji}\eta_i + i_{ji}\phi_i + j_{ji}\Psi_i &= 0, \\ k_{ji}W_i + l_{ji}\Theta_i + m_{ji}\eta_i + n_{ji}\phi_i + o_{ji}\Psi_i &= 0, \\ p_{ji}W_i + q_{ji}\Theta_i + r_{ji}\eta_i + s_{ji}\phi_i + t_{ji}\Psi_i &= 0, \\ u_{ji}W_i + v_{ji}\Theta_i + w_{ji}\eta_i + x_{ji}\phi_i + y_{ji}\Psi_i &= 0. \end{aligned} \tag{22}$$

The above set of homogeneous algebraic equations can have a nontrivial solution if and only if

$$\begin{vmatrix} a_{ji} & b_{ji} & c_{ji} & d_{ji} & e_{ji} \\ f_{ji} & g_{ji} & h_{ji} & i_{ji} & j_{ji} \\ k_{ji} & l_{ji} & m_{ji} & n_{ji} & o_{ji} \\ p_{ji} & q_{ji} & r_{ji} & s_{ji} & t_{ji} \\ u_{ji} & v_{ji} & w_{ji} & x_{ji} & y_{ji} \end{vmatrix} = 0. \tag{23}$$

The eigenvalue has to be obtained from the characteristic (23).

The obtained eigenvalues of the Rayleigh number, Ra, for the lower-upper rigid-rigid boundary conditions are

$$\mathrm{Ra} = \frac{42\left[(1/140)\,a^2\mathrm{Rn}\Lambda_v - \Lambda_w - (1/140\mathrm{Le})\,a^2\mathrm{Rs}\Lambda_x + (3/70)\sqrt{\mathrm{Ta}}\Lambda_y\right]}{a^2\Lambda_z}, \tag{24}$$

where

$$\begin{aligned}
\Lambda_v &= \left(\lambda_6\lambda_1\lambda_2\lambda_3 + \frac{1}{140}\lambda_4\lambda_1\lambda_3 + \frac{1}{140}\lambda_5\lambda_2\lambda_3 - \frac{1}{140}\lambda_5\lambda_7\lambda_3\right),\\
\Lambda_w &= \lambda_9\left(\lambda_4\lambda_1\lambda_8\lambda_3 - \lambda_5\lambda_7\lambda_8\lambda_3 - \frac{1}{6}\frac{N_B\lambda_1\lambda_2\lambda_3}{\mathrm{Ln}}\right),\\
\Lambda_x &= \left(\lambda_6\lambda_7\lambda_8\lambda_3 - \frac{1}{140}\lambda_4\lambda_8\lambda_3 + \frac{1}{840}\frac{N_B\lambda_2\lambda_3}{\mathrm{Ln}} - \frac{1}{840}\frac{N_B\lambda_7\lambda_3}{\mathrm{Ln}}\right),\\
\Lambda_y &= \left(-\frac{3}{70}\lambda_4\lambda_1\sqrt{\mathrm{Ta}}\lambda_8 + \frac{3}{70}\lambda_5\lambda_7\sqrt{\mathrm{Ta}}\lambda_8 + \frac{1}{140}\frac{N_B\lambda_1\sqrt{\mathrm{Ta}}\lambda_2}{\mathrm{Ln}}\right),\\
\Lambda_z &= \left(-\lambda_6\lambda_1\lambda_8\lambda_3 - \frac{1}{140}\lambda_5\lambda_8\lambda_3 - \frac{1}{840}\frac{N_B\lambda_1\lambda_3}{\mathrm{Ln}}\right)
\end{aligned} \tag{25}$$

is defined as

$$\begin{aligned}
\lambda_1 &= \frac{3}{10}\frac{a^2}{\mathrm{Le}} - \frac{1}{3}\frac{4+a^2}{\mathrm{Le}} + \frac{1}{\mathrm{Le}},\\
\lambda_2 &= -\frac{11}{20}\frac{N_A a^2}{\mathrm{Ln}} + \frac{1}{3}\frac{N_A\left(4+2a^2\right)}{\mathrm{Ln}} - \frac{N_A}{\mathrm{Ln}},\\
\lambda_3 &= \frac{13}{35}a^2 + \frac{6}{5},\\
\lambda_4 &= -\frac{8}{15}a^2 - \frac{4}{3}\frac{N_B}{\mathrm{Ln}} - \frac{4}{3} - \frac{8}{3}\frac{N_B N_A}{\mathrm{Ln}} - K,\\
\lambda_5 &= \frac{11}{20}\mathrm{Df}a^2 - \frac{1}{3}\mathrm{Df}\left(4+2a^2\right) + \mathrm{Df},\\
\lambda_6 &= \frac{1}{420}Q + \frac{1}{42},\\
\lambda_7 &= \frac{11}{20}\mathrm{Sr}a^2 - \frac{1}{3}\mathrm{Sr}\left(4+2a^2\right) + \mathrm{Sr},\\
\lambda_8 &= -\frac{3}{10}\frac{a^2}{\mathrm{Ln}} + \frac{1}{3}\frac{4+a^2}{\mathrm{Ln}} - \frac{1}{\mathrm{Ln}},\\
\lambda_9 &= \frac{4}{5} + \frac{1}{630}a^4 + \frac{4}{105}a^2.
\end{aligned} \tag{26}$$

3. Results and Discussion

In this paper, the resulting eigenvalue problem in double-diffusive convection on a horizontal rotating nanofluid layer was solved analytically using Galerkin method. A linear stability analysis has been employed, and three types of lower-upper boundary conditions have been considered which are free-free, rigid-free, and rigid-rigid. The values of the various relevant parameters were chosen according to the range of parameter values proposed by Chand and Rana [27]. The obtained results have been presented graphically to illustrate the impact of various parameters on the Rayleigh number Ra as well as critical Rayleigh number Ra_c.

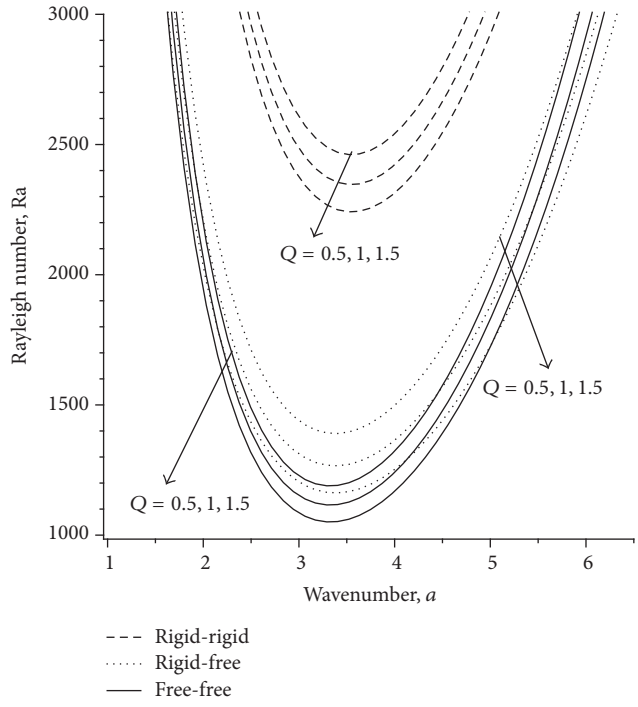

Figure 2: Impact of Q on Ra against a.

Test computations have been performed and we compared our results with Chandrasekhar [43] and Yadav et al. [55] for the limiting case of nanofluid (regular fluids) in the absence of internal heat source, feedback control, Soret parameter, and Dufour parameter. The comparison results of critical Rayleigh number Ra_c for the rigid-rigid boundary conditions are presented in Table 1. As can be seen in the table, our results are in good agreement with those reported in [43, 55] and thus verify the accuracy of our analysis.

The influence of internal heat source, Q, on the onset of Rayleigh–Bénard convection in a nanofluid layer is illustrated in Figure 2. It can be seen clearly that an increase of Q decreases the Rayleigh number spontaneously. This indicates that the system becomes destabilized because of an increase in the energy supply to the system that leads to an increase in the rate of disturbances in the nanofluid layer. As for the type of boundaries, the curve that represents rigid-rigid boundaries dominates the upper part of the graph and this reveals that the onset of convection can be delayed using this type of boundary.

Figure 3 shows the Rayleigh number, Ra, versus the wavenumber a for different values of Taylor number, Ta, with various horizontal boundary conditions. Ra increases with increasing of the Taylor number Ta, indicating that the Coriolis force due to a rotation inhibits the onset of Rayleigh–Bénard convection in a nanofluid layer. The fluid moves to the horizontal plane with higher velocity because of

Table 1: Comparison of Ra_c in rigid-rigid boundary conditions for the limiting case of nanofluid.

Ta	Chandrasekhar [43] Rigid-rigid Ra_c	Yadav et al. [55] Rigid-rigid Ra_c	Present study Rigid-rigid Ra_c
0	1707.8	1707.83	1707.80
10	1713.0	1712.74	1712.85
100	1756.60	1756.41	1756.98
500	1940.30	1940.26	1931.03
1000	2151.70	2151.39	2116.38

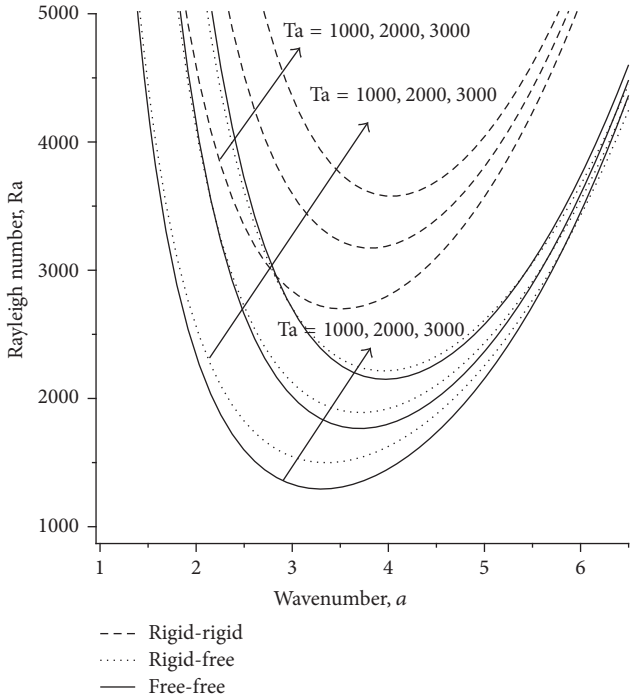

Figure 3: Impact of Ta on Ra against a.

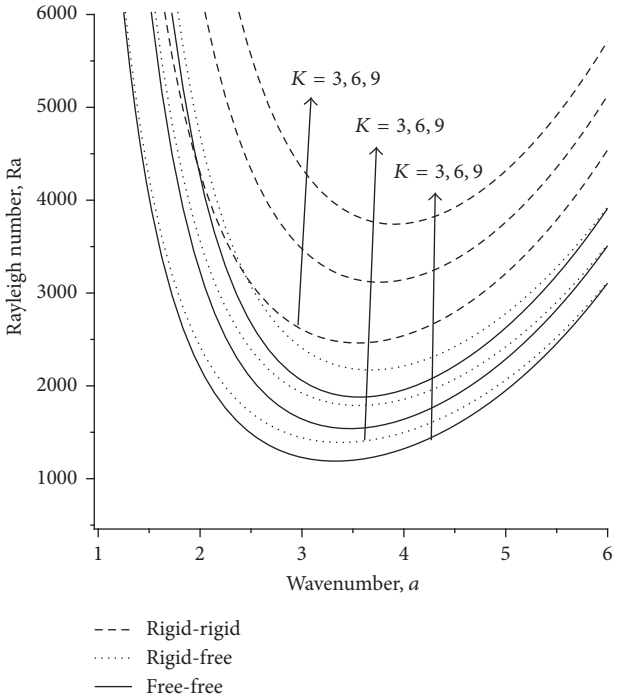

Figure 4: Impact of K on Ra against a.

the vorticity introduced by the rotation mechanism. Therefore, the velocity of the fluid in the vertical plane is reduced, thereby minimising the amount of thermal convection.

Figure 4 shows the response of the Rayleigh number with different values of proportional feedback control, K, on the convection instability in a nanofluid layer. From the graph, it can be seen that the values of Ra elevate proportionally as the controller K increases with all types of boundary conditions considered. This reveals that the controller is capable for delaying the onset of steady convection in a nanofluid layer. Physically, the sensors detect the departure of the fluid from its conductive state and then they direct the actuators to take action so as to suppress any disturbances [69].

The impacts of two important types of interdiffusion which are Soret (Sr) and Dufour (Df) parameters that arise due to the combination of temperature and concentration gradients in a nanofluid system are depicted in Figures 5 and 6, respectively. In Figure 5, the values of Ra decrease as the parameter Sr increases and thus promotes the rate of convection in the system. This occurs because of the increase in the temperature flux, which contributes to the acceleration of the onset of convection. In contrast with the effect of Sr, and the influence of the Dufour parameter increases the Rayleigh number and stabilizes the system as can be seen in Figure 6. It is observed that this is due to the energy flux from the lower to the higher solute concentration driven by the mass gradient in the system.

The linear instability thresholds of the modified diffusivity ratio $N_A = D_T \Delta T^* / D_B T_u^* (\phi_u^* - \phi_l^*)$ on the Rayleigh number Ra are plotted in Figure 7 for $N_A = 2$, 6, and 10. Ra decreases slightly with an increase of N_A for three types of lower-upper boundary conditions. This phenomenon occurs because the parameter N_A is directly proportional to the thermophoretic diffusion coefficient D_T, where an increasing D_T indicates an instability in the temperature difference within the nanofluid layer. Therefore, as N_A increases the thermal instability also increases and destabilizes the system. It is interesting to note that the modified particle density, N_B, has no significant effect on the nanofluid system. An attempt has been made to scrutinize the effect of N_B in this study but

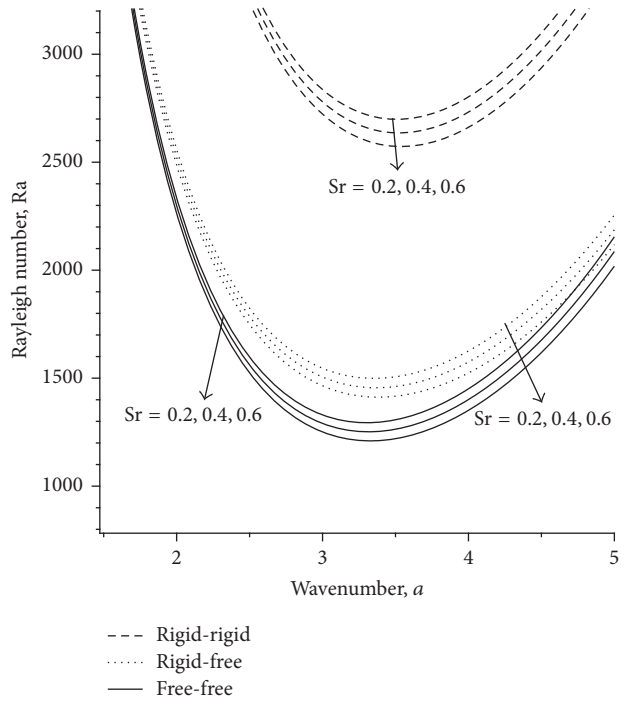

Figure 5: Impact of Sr on Ra against a.

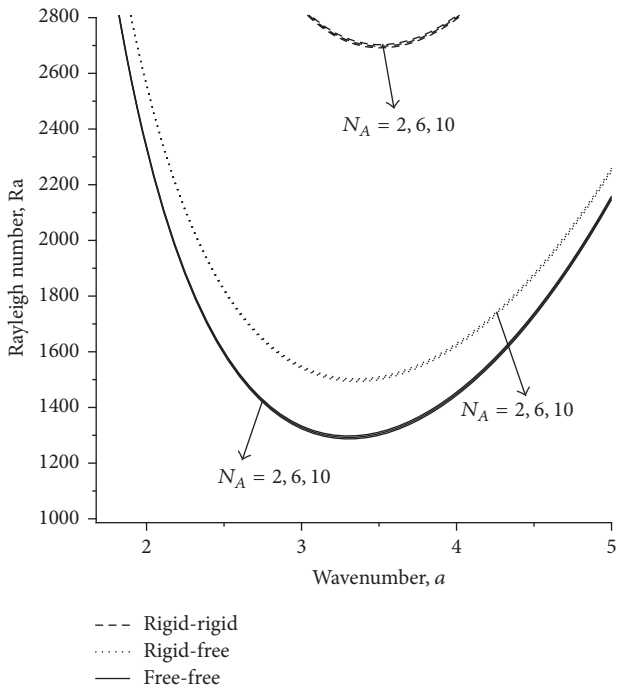

Figure 7: Impact of N_A on Ra against a.

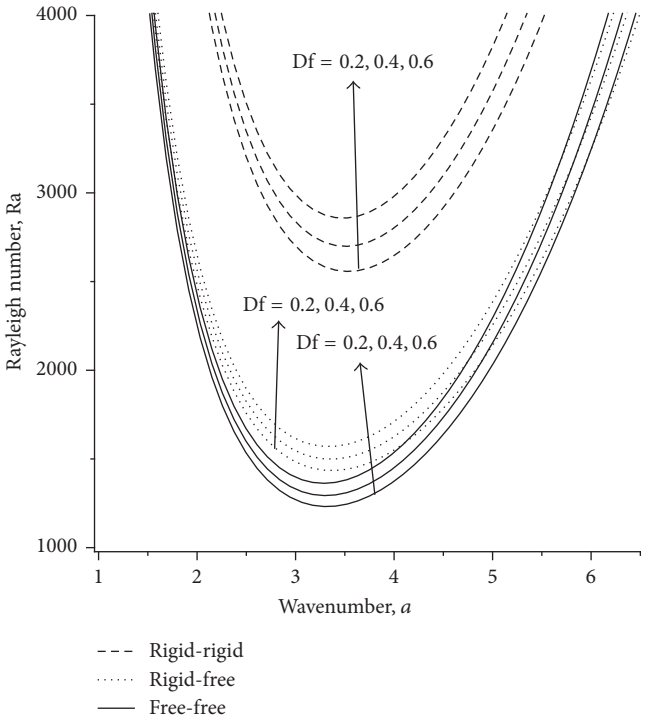

Figure 6: Impact of Df on Ra against a.

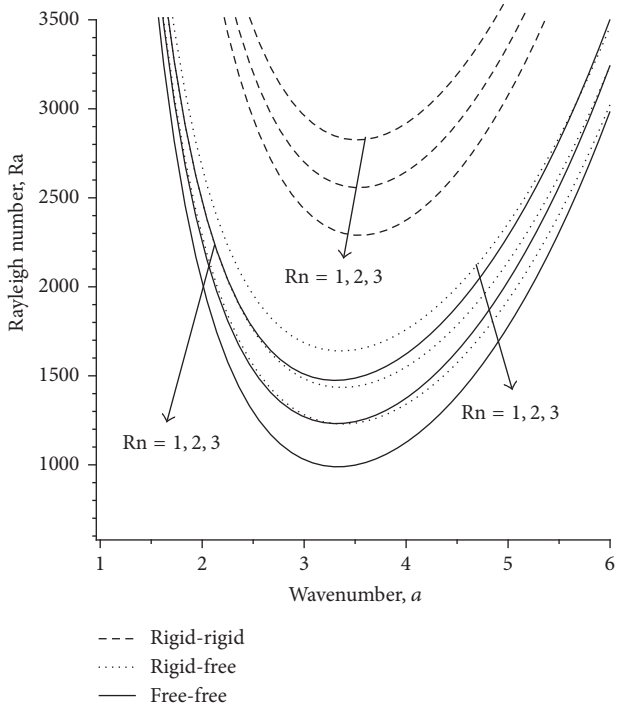

Figure 8: Impact of Rn on Ra against a.

there are no apparent results. This finding agrees well with those reported in [53, 54, 56] that the impact of N_B is so small that can be omitted.

Figure 8 shows the Rayleigh number Ra versus the wavenumber a for various positive values of the nanoparticle concentration Rayleigh number with various boundary conditions. Theoretically, an increase of Rn will increase the nanoparticle volume fraction, ϕ^*, because Rn is directly proportional to ϕ^* according to the definition $\mathrm{Rn} = (\rho_p - \rho)(\phi_u^* - \phi_l^*)gL^3/\mu\alpha_f$. The increase of ϕ^* results in a combination of Brownian motion and thermophoresis diffusion within the nanofluid layer and thus destabilizes the system. As can be

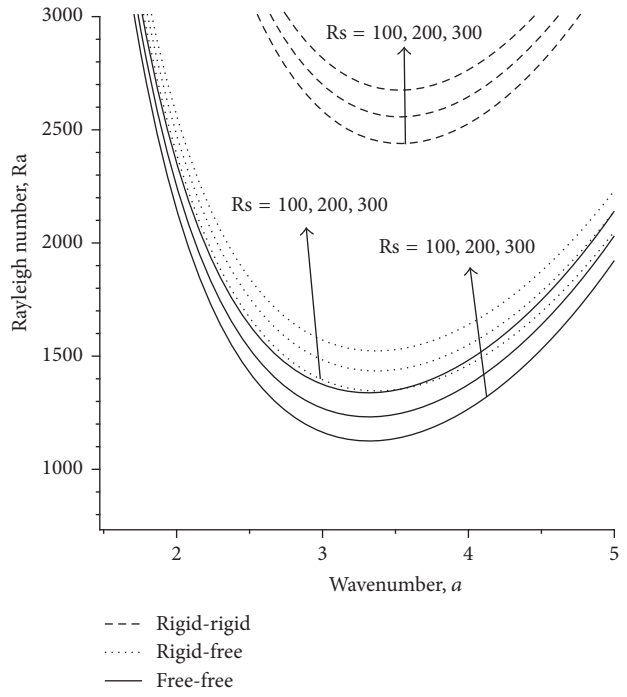

Figure 9: Impact of Rs on Ra against a.

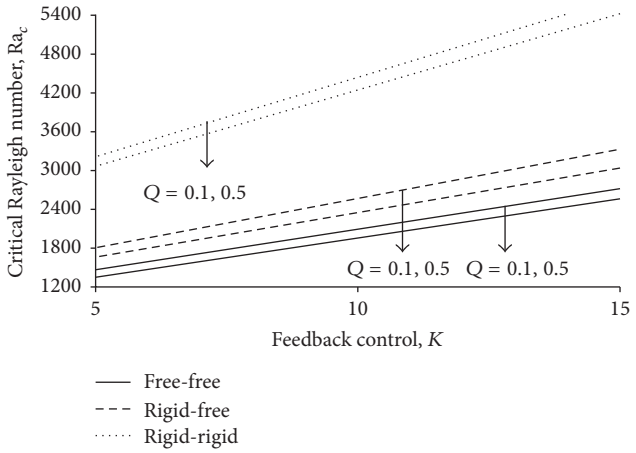

Figure 10: Plot of Ra_c with K for selected values of Q.

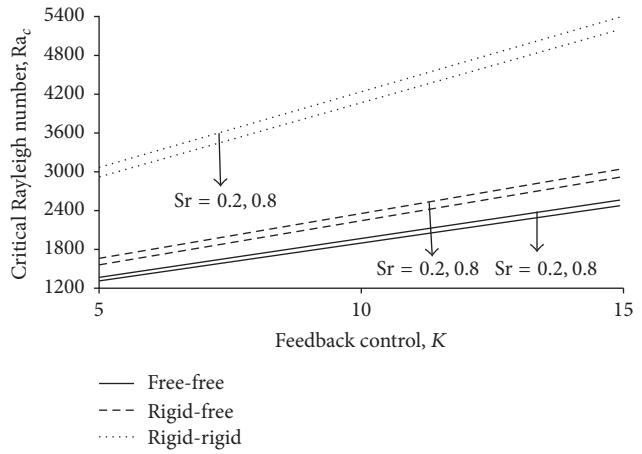

Figure 11: Plot of Ra_c with K for selected values of Sr.

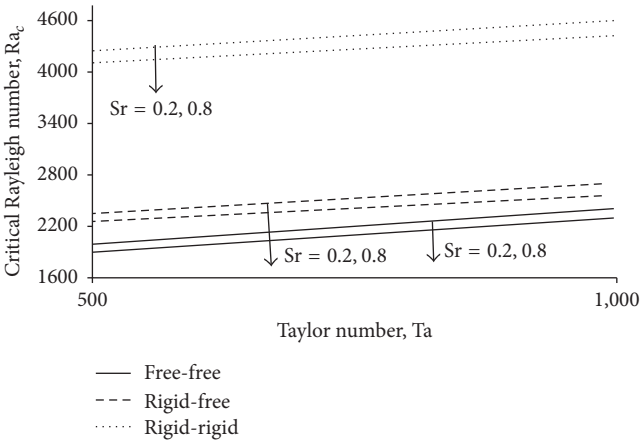

Figure 12: Plot of Ra_c with Ta for selected values of Sr.

seen clearly in the graph, the Rayleigh number decreases as Rn increase and thus hastens the convection in the system.

The marginal stability curves for various solutal Rayleigh numbers are displayed in Figure 9 for a double-diffusive rotating nanofluid layer. The chosen values of Rs have positive and significant impact on the stability of the nanofluid layer, where it is found that Ra increases monotonically with an increase of Rs. Therefore, the onset of convection can be postponed within the nanofluid layer because the amount of solute concentration is greater than that of the solvent, leading to a decrease in the overall temperature within the system.

The graphs of critical Rayleigh number against feedback control for different values of internal heat source and Soret parameter are depicted in Figures 10 and 11, respectively. As illustrated earlier, the impact of increasing both internal heat source and Soret parameter suppressed the critical Rayleigh number monotonically. However, elevating the feedback control, K, is to delay the onset of double-diffusive convection in rotating nanofluid layer and stabilize the system.

The influences of the Taylor number, Ta, and Soret parameter are presented in Figure 12. Obviously, increasing the Coriolis force in double-diffusive nanofluid layer helps to reduce the rate of disturbance caused by Sr and thus promotes stability within the nanofluid layer. Scrutinizing the critical Rayleigh number with different boundary conditions, the rigid-rigid boundaries maintain gaining the highest values of Ra_c compared to the free-free and rigid-free boundaries.

The combined effects of the Dufour and Soret inter-diffusions on the critical Rayleigh number Ra_c are shown in Figure 13. From the graph, the stability in a horizontal nanofluid layer on the onset of double-diffusive convection is significantly altered by both effects. Ra_c decreases slightly with increasing of Sr for Df = 0.4 and 0.8, and thus it hastens the onset of convection. Increasing the value of Df stabilizes the system.

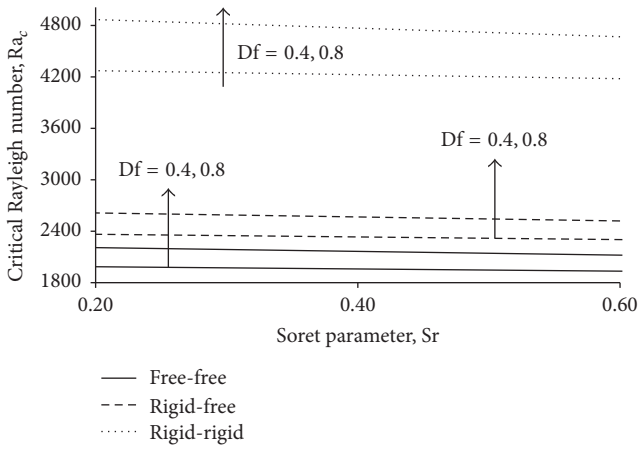

Figure 13: Plot of Ra$_c$ with Sr for selected values of Df.

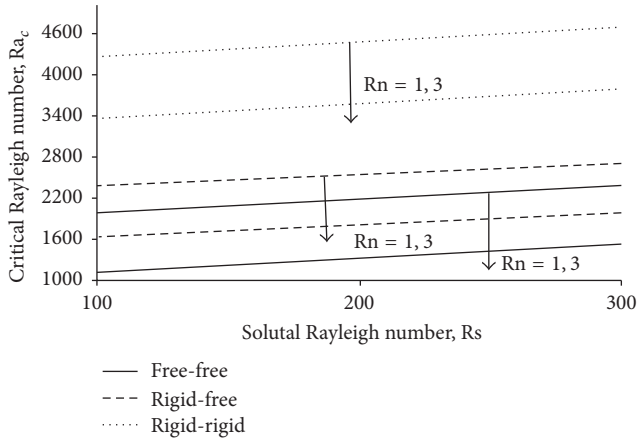

Figure 14: Plot of Ra$_c$ with Rs for selected values of Rn.

Lastly, Figure 14 shows the effect of the nanoparticle concentration Rayleigh number, Rn, with various values of the solutal Rayleigh numbers, Rs. Rn always has a destabilizing effect on the system and this is in contrast to the effect of Rs, which stabilize the nanofluid layer.

4. Conclusion

The effect of internal heat source on the double-diffusive convection in a rotating nanofluid layer with feedback control strategy is analyzed theoretically. Linear stability theory is applied and the eigenvalue solution is obtained numerically using the Galerkin technique. Focusing on the stationary convection, the influence of increasing the internal heat source is found to suppress the critical Rayleigh number in a nanofluid layer and thus destabilizes the system. However, the onset of convection in a double-diffusive rotating nanofluid layer can be delayed with the use of feedback control. The controller retards the loss of stability, which means helping to reduce the intensity of the Rayleigh–Bénard convection thus sustaining the stability of the system. As for the double-diffusive coefficients, an increase of the Dufour parameter helps to slow down the destabilizing process and the opposite outcome can be seen when increasing the values of the Soret parameter, from which one can conclude that Soret parameter drives a destabilization effect within the system. It has also been seen that the influence of increasing the effect of Rs is to cause a lag in the onset of heat transfer, and an increase in Rn and N_A advances the onset of convection while the modified particle-density increment N_B has no significant impact on the system. It is worth mentioning that the critical Rayleigh number recorded for rigid-rigid horizontal boundaries is the highest and this indicates that the rigid-rigid horizontal boundaries is the most stable compared to the free-free and rigid-free horizontal boundaries.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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