Hindawi
Advances in Mathematical Physics
Volume 2017, Article ID 3138720, 8 pages
https://doi.org/10.1155/2017/3138720

Research Article

Analytic Calculation of Transmission Field in Homogeneously Layered Mediums Excited by EMP

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Received 18 July 2017; Accepted 29 August 2017; Published 9 October 2017

Academic Editor: Eugen Radu

This paper presents an analytic derivation for the time-domain transmission across layered mediums. The transmission coefficient and attenuation coefficient are obtained in the time-domain from general electromagnetic theory. The transmission electric field can be obtained within a few seconds by convolving the coefficients with incident EMP. The results are accordant with the FDTD method, and this approach can deal with the multilayer mediums problem. The limitations of this approach are discussed in this paper.

1. Introduction

Numerical simulation has become an indispensable tool for EMC studies, and these numerical methods can potentially simulate many physical problems [1]. Meanwhile, the analytic method has not been used widely, which is restricted by complex structure in practical problem. But analytic expressions for some ideal models can reveal the physical essential of these problems. And they are the origin of some numerical methods. It is the contention of this paper that the analytic derivation can offer an efficient method for some ideal physical problems in the time-domain.

Some installations such as subway or shelter are built under the ground. The depth increases the computational costs of numerical simulation about these installations excited by EMP (electromagnetic pulse) or other electromagnetic excitations over the ground. There are two steps for solving this kind of problem: first, establishing the time-domain transmission field across the ground; second, calculating the response of the installation excited by the transmission field using numerical simulation method. This paper addresses the first step.

The layered medium is a canonical structure in the electromagnetic field theory. The transmission in the frequency domain is determined by Snell's law of refraction. Several authors obtain the time-domain transmission field by performing a numerical FFT [2, 3]. Some improved time-domain methods are presented for reflected field, which do not refer to the transmission [4–7].

In this paper, the analytic derivation for the transmission coefficient and attenuation coefficient are presented, and the transmission electric field across layered medium is obtained by convolving the coefficients with incident EMP. The transmission electric field in monolayer medium is accordant with the FDTD method. The transmission electric field in dry soil and freshwater are compared, whose peculiarity is conforming to physical laws. This approach is also suitable for multilayer problem, and the limitations of this approach are discussed.

2. The Transmission Coefficient and Attenuation Coefficient in the Time-Domain

2.1. Transmission Coefficient in the Time-Domain. A plane wave obliquely striking a half-space interface from the optically thinner medium to optically denser medium in the frequency domain is considered. Figure 1 illustrates the geometry of this canonical problem, where the electric field

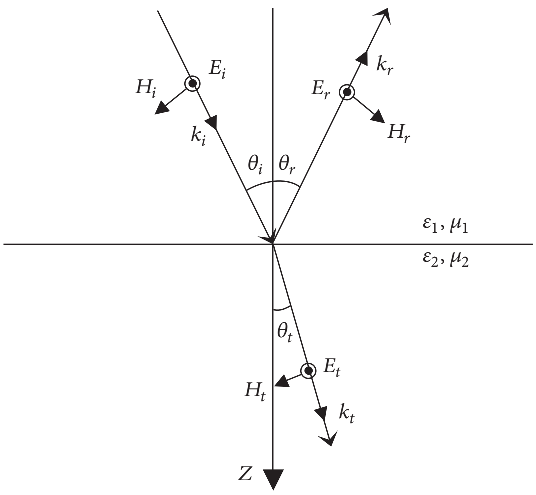

Figure 1: Plane wave obliquely strikes a half-space interface.

has the horizontal polarization with the strict Snell transmission coefficient $T_E(\omega)$ as shown in the following [8, 9]:

$$T_E(\omega) = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\varepsilon_2\mu_1/\varepsilon_1\mu_2}\sqrt{1-(\varepsilon_1\mu_1/\varepsilon_2\mu_2)\sin^2\theta_i}}. \quad (1)$$

The material permittivity is $\varepsilon_1 = \varepsilon_0\varepsilon_{r1} - j\sigma_1/\omega$ and $\varepsilon_2 = \varepsilon_0\varepsilon_{r2} - j\sigma_2/\omega$. The material permeability is $\mu_1 = \mu_0$ and $\mu_2 = \mu_0$. Here ε_0 is the permittivity of vacuum space, μ_0 is the permeability of vacuum space, ω is angle frequency, ε_{r1} and ε_{r2} are relative permittivity, σ_1 and σ_2 are electrical conductivity, θ_i is incident angle, and θ_t is refraction angle. Using the relation $s \leftrightarrow j\omega$ [10], the transmission coefficient is represented as follows:

$$T_E(s) = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{(\varepsilon_0\varepsilon_{r2}+\sigma_2/s)/(\varepsilon_0\varepsilon_{r1}+\sigma_1/s) - \sin^2\theta_i}}, \quad (2)$$

where s is the Laplace Transform variable.

Equation (2) can be transformed to (3) through some cumbersome transformations.

$$T_E(s-\gamma) = a + \frac{bc}{d}\sum_{n=1}^{\infty} S^n, \quad (3)$$

where $\gamma = (\gamma_1+\gamma_2)/2$, $\gamma_1 = \sigma_1/\varepsilon_0\varepsilon_{r1}$, $\gamma_2 = (\sigma_2 - \sigma_1\sin^2\theta_i)/(\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i)$, $a = 2\chi_1/(1+\chi_1)$, $b = -4\chi_1/(1+\chi_1)^2$, $c = (\gamma_2-\gamma_1)/2$, $d = c((\chi_1-1)/(\chi_1+1))$, $\chi_1 = \cos\theta_i\sqrt{\varepsilon_{r1}/(\varepsilon_{r2}-\varepsilon_{r1}\sin^2\theta_i)}$, and $S = d/(s+\sqrt{s^2-c^2})$.

The inverse Laplace Transform of (2) is

$$T_E(t) = a\delta(t)e^{-\gamma t} + \frac{bc}{d}\frac{e^{-\gamma t}}{t}\sum_{n=1}^{\infty} n\left(\frac{d}{c}\right)^n I_n(ct), \quad (4)$$

where $I_n(ct) = \sum_{k=0}^{\infty}(1/k!\Gamma(k+n+1))(ct/2)^{n+2k}$ is deformation Bessel function and $\delta(t)$ is Dirac function.

The derivation for the transmission coefficient is cumbersome, whose details are recorded in Appendix A at the end of the paper. The transmission coefficient of vertical polarization wave is not listed, which can be obtained using similar approach.

2.2. Attenuation Coefficient in the Time-Domain. The attenuation coefficient of electric wave propagation in lossy medium is calculated by an exponential function as shown in the following [11]:

$$K_E(\omega) = \exp[-\mathbf{k}_I \cdot \mathbf{r}], \quad (5)$$

where $\mathbf{k}_I$ is the attenuation wave vector, whose scalar value is determined by (6); $\mathbf{r}$ is propagation distance vector, whose scalar value is determined by (7):

$$\mathbf{k}_I(\omega) = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}, \quad (6)$$

$$\mathbf{r} = \frac{z}{\cos\theta_t}, \quad (7)$$

where z is vertical depth.

In (6), $\mu = \mu_2 = \mu_0$, $\varepsilon = \varepsilon_2 = \varepsilon_0\varepsilon_{r2} - j\sigma_2/\omega$, $s \leftrightarrow j\omega$; using the power series expansion of binomial expression (see (8)), the $k_I(\omega)$ can be represented as (9) where high order terms are ignored:

$$\sqrt{1-a} = 1 - \frac{1}{2}a - \frac{1}{8}a^2 - \frac{1}{16}a^3 - \cdots, \quad (8)$$

$$k_I(s) = \frac{\sigma_2}{2}\sqrt{\frac{s\mu_0}{s\varepsilon_0\varepsilon_{r2}+\sigma_2}} = \frac{\sigma_2}{2}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{r2}+\sigma_2/s}}. \quad (9)$$

The magnitude of the $k_I(s)$ is proportional to s and s represents the frequency. This relationship means that high frequency wave attenuates more rapidly than low frequency wave in lossy medium.

Similarly, (7) needs to be represented by the Laplace Transform variable s. Snell's Law of refraction is

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}} = \sqrt{\frac{\varepsilon_0\varepsilon_{r1}+\sigma_1/s}{\varepsilon_0\varepsilon_{r2}+\sigma_2/s}}. \quad (10)$$

The refraction angle θ_t is determined by the incident angle, the medium parameters, and the transmission wave frequency s (or ω):

$$\cos\theta_t = \sqrt{1-\sin^2\theta_i\frac{s\varepsilon_0\varepsilon_{r1}+\sigma_1}{s\varepsilon_0\varepsilon_{r2}+\sigma_2}}. \quad (11)$$

Combining with the results of upper derivation and using the series expansion, the attenuation coefficient is represented as follows:

$$K_E(s) = \exp\left(-x\sqrt{\frac{s}{s+y}}\right) = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}\left(\frac{s}{s+y}\right)^{n/2}, \quad (12)$$

where $x = z\sigma_2\sqrt{\mu_0}/2\sqrt{\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i}$ and $y = (\sigma_2 - \sin^2\theta_i\sigma_1)/(\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i)$.

The $n/2$ increases the difficulty of inverse Laplace Transform for (12). The typical items are

$$\kappa_1(s) = \sqrt{\frac{s}{s+y}}, \qquad \kappa_2(s) = \frac{s}{s+y}. \tag{13}$$

The inverse Laplace Transforms of the typical items listed in (13) are

$$\kappa_1(t) = \delta(t) + \frac{1}{2}ye^{-(y/2)t}\left[I_1\left(\frac{y}{2}t\right) - I_0\left(\frac{y}{2}t\right)\right], \qquad \kappa_2(t) = \delta(t) - ye^{-yt}. \tag{14}$$

The derivation details are recorded in Appendix B at the end of the paper. The inverse Laplace Transforms of other items in (12) can be obtained by several convolutions. Therefore the attenuation coefficient expression in time-domain is

$$\begin{aligned} K_E(t) = {} & \delta(t) - x\kappa_1(t) + \frac{x^2}{2!}\kappa_2(t) - \frac{x^3}{3!}\kappa_1(t) * \kappa_2(t) \\ & + \frac{x^4}{4!}\kappa_2(t) * \kappa_2(t) - \frac{x^5}{5!}\kappa_1(t) * \kappa_2(t) \\ & * \kappa_2(t) + \cdots . \end{aligned} \tag{15}$$

In order to obtain the transmission field in the time-domain, the transmission coefficient and the attenuation coefficient are represented in the time-domain. The transmission electric field across lossy medium is

$$E_T(t) = E_i(t) * T_E(t) * K_E(t). \tag{16}$$

The analytic expression of (16) is too cumbersome to simplify. In order to obtain the final transmission electric field $E_T(t)$, a numerical convolution is chosen to manipulate (16) by MATLAB software; meanwhile the high order terms in (15) are ignored for simplification.

2.3. The Limitations of Coefficients. Firstly, the parameter y in attenuation coefficient has a large value from its expression. The exponential function e^{-yt} attenuates rapidly if σ_2 is not too smaller than ε_{r2} such as seawater ($\varepsilon_{r2} = 81$, $\sigma_2 = 4$) [11]; ye^{-yt} attenuates to zero within one nanosecond as shown in Figure 2; therefore $\kappa_1(t)$ and $\kappa_2(t)$ attenuate to zero within one nanosecond ($I_1(t) < I_0(t)$).

The attenuation coefficient of seawater is ineffectual for some transient electromagnetic wave; for example, the duration of EMP is tens of nanoseconds; that is, it is ineffectual to good conductor. Otherwise the attenuation coefficient is effectual to some other usual media like wet soil ($\varepsilon_{r2} = 10$, $\sigma_2 = 10^{-3}$), dry soil ($\varepsilon_{r2} = 4$, $\sigma_2 = 10^{-5}$), freshwater ($\varepsilon_{r2} = 81$, $\sigma_2 = 10^{-3}$), and so on. Figure 2 illustrates the duration of e^{-yt} about seawater and wet soil for contrast.

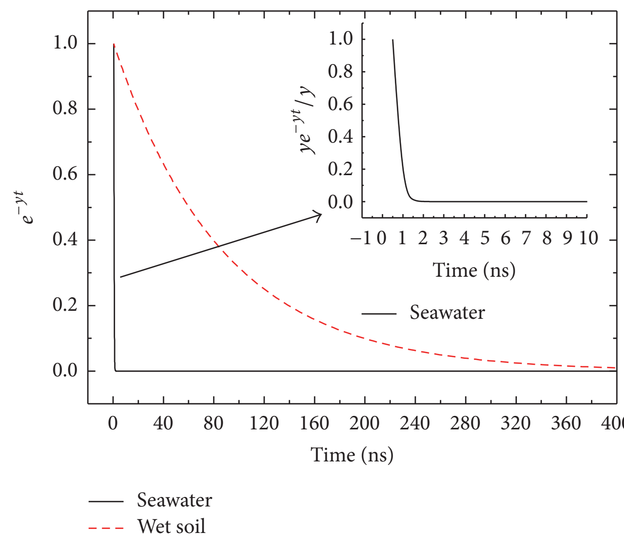

Figure 2: e^{-yt} about seawater and wet soil.

Secondly, the item x^n within the sum from n equals zero to $+\infty$ in (12) is unharmonious; when x is greater than one, the attenuation coefficient may be vertiginous. There is a similar restriction in (4), but the parameter d is less than the parameter c from their expressions; the transmission coefficient is effectual naturally. By the restriction, the effectual depth of attenuation coefficient is restricted by (17) and is listed in Table 1 for some usual mediums:

$$z < \frac{2\sqrt{\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i}}{\sigma_2\sqrt{\mu_0}}. \tag{17}$$

3. The Transmission Electric Field Excited by EMP

3.1. Verification by the FDTD Method. The incident EMP from air ($\varepsilon_{r1} = 1$, $\sigma_1 = 0$) to layered medium can be approximated by a double exponential function in time-domain [12]:

$$E_i(t) = k_0E_0\left(e^{-\alpha t} - e^{-\beta t}\right), \tag{18}$$

where $k_0 = 1.3$, $E_0 = 50$ (kV/m), $\alpha = 4 \times 10^7$ (1/s), and $\beta = 6 \times 10^8$ (1/s); Figure 3 illustrates the waveform of the EMP.

Convolving the coefficients with the incident EMP, the transmission electric fields are obtained when $\theta_i = 0$ in freshwater and dry soil. Use the FDTD method to calculate the transmission electric fields excited by the same EMP. Two kinds of results are compared, shown as in Figure 4.

The analytic calculation result is accordant with the FDTD method. The analytic calculation is finished within a few seconds by MATLAB software performing the convolution operation.

3.2. The Transmission Electric Field in Lossy Medium. Considering the attenuation of the lossy medium, convolving the two coefficients with the incident EMP, the transmission electric

Table 1: The effectual depth about attenuation coefficient.

Medium	Freshwater	Wet soil	Dry soil
Effectual depth (m) when $\theta_i = 0^\circ$	47.8	16.8	1062
Effectual depth (m) when $\theta_i = 90^\circ$	47.5	15.9	920

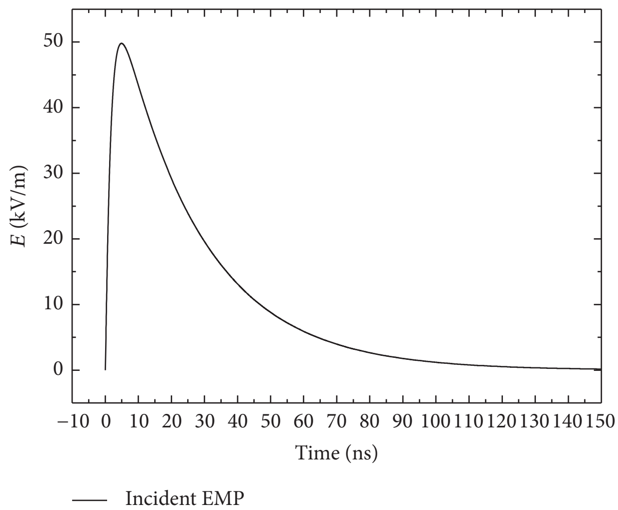

Figure 3: Incident EMP (IEC61000-2-9).

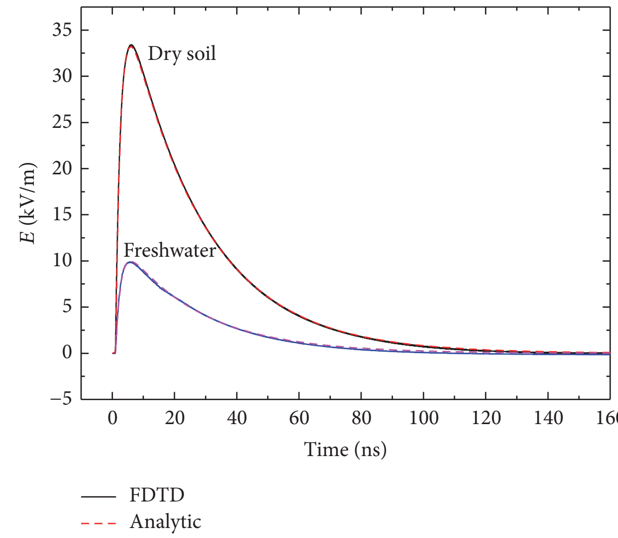

Figure 4: Analytical result is compared with the FDTD method. Black solid line refers to transmission electric field in dry soil by FDTD. Red dashed line refers to transmission electric field in dry soil by analytic. Blue solid line refers to transmission electric field in freshwater by FDTD. Pink dashed line refers to transmission electric field in freshwater by analytic.

field is obtained in different depth when θ_i equal $\pi/6$. The results in freshwater and dry soil are relatively illustrated in Figures 5 and 6.

Without considering the attenuation coefficient, the transmission electric field does not change about z. The value of transmission electric field in freshwater is less and attenuates more rapidly than the value in dry soil if considering the attenuation coefficient. Change the incident angle from 0° to 90° by 10°; the peak values of transmission electric field in dry soil and freshwater are relatively illustrated in Figure 7 when z equal 10 m.

The peak values decrease as the attenuation coefficient as long as the incident angle is increasing.

3.3. The Transmission Electric Field in Multilayer Mediums. The transient plane wave strikes multilayer mediums always from one optically thinner medium to optically denser medium; the transmission coefficient obtained above is effectual. Consider the geometry of the multilayer mediums as shown in Figure 8 when θ_i equal 0°; the transmission electric fields in three mediums are illustrated in Figure 9.

The results do not refer to attenuation and the incident angle equals 0°. So the results in Figure 9 may be the maximal transmission electric field. It is apparent that if considering the attenuation or increasing the incident angle, a lesser transmission electric field ought to be gained.

There is a question here: how can these incident angles be determined potentially in different interfaces when the first incident angle is not 0°?

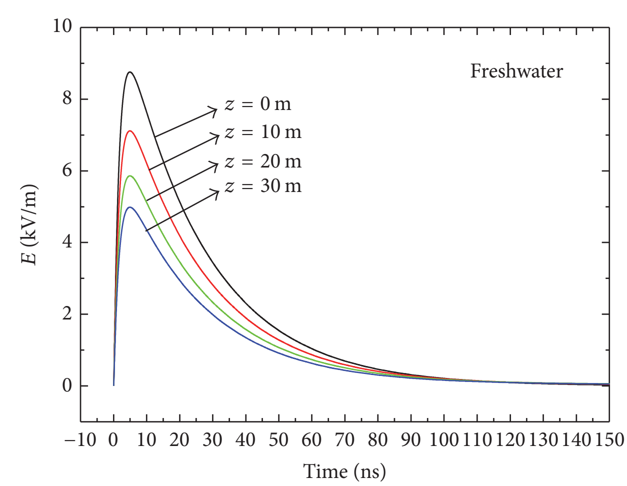

Figure 5: Transmission electric fields in freshwater.

4. Conclusion

For the purpose of increasing the computational efficiency in the transient electromagnetics, the transmission issue in layered mediums is addressed with analytic method. The transmission coefficient and attenuation coefficient are

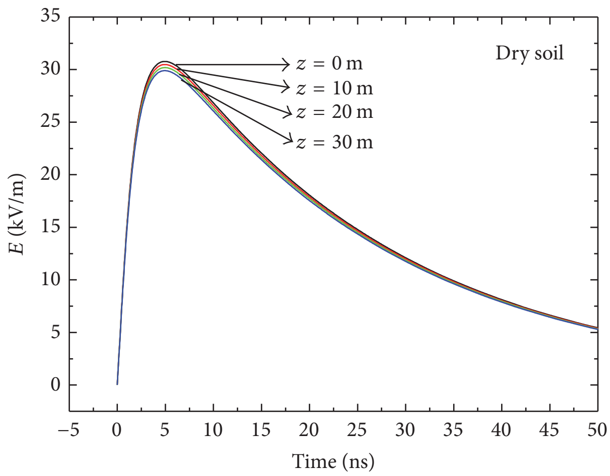

FIGURE 6: Transmission electric fields in dry soil.

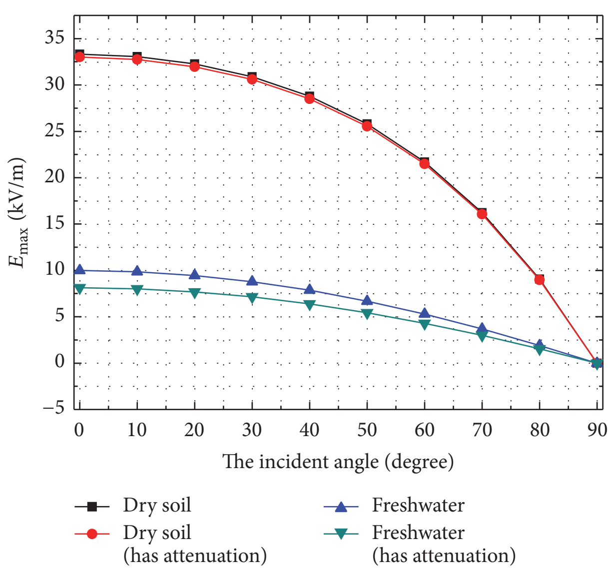

FIGURE 7: Peak values with different incident angles.

precise in frequency domain, performing inverse Laplace Transform, and convolving them with transient incident wave; the transmission field can be obtained. There are some complicated steps in performing inverse Laplace Transform; the atenuation coefficient in time-domain has a restricted depth and is ineffectual for good conductor such as seawater. The truncation of the infinite sum before performing convolution degrades the accuracy.

Several examples of this approach are presented. The transmission electric field calculated by analytic method in monolayer medium is accordant with the FDTD method. The transmission electric field in freshwater is less than dry soil and attenuates more rapidly. The peak values of electric field and the attenuation coefficient decrease as long as the incident angle is increasing as expected. The maximal transmissions in multilayer mediums are calculated finally.

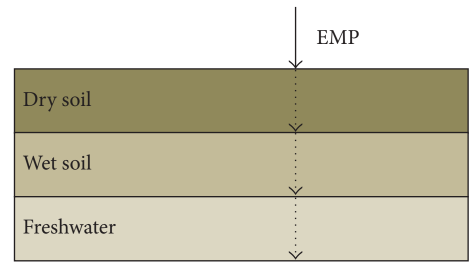

FIGURE 8: Normal incidence at multilayer mediums interfaces.

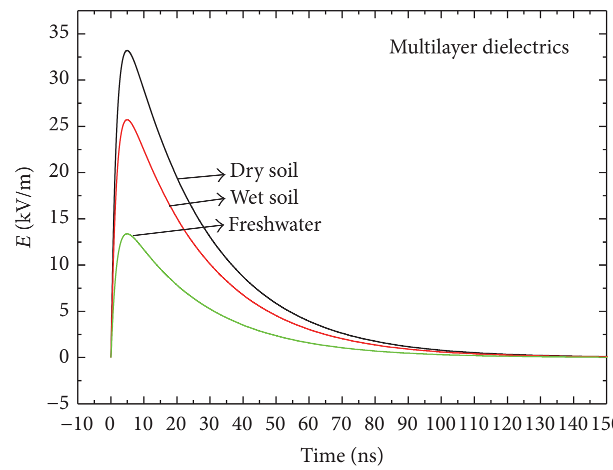

FIGURE 9: Transmission electric fields in multiple mediums.

Appendix

A. The Derivation Details of $T_E(t)$

The reflection coefficient in frequency domain is as follows [8]:

$$\Gamma_E(\omega) = \frac{\cos\theta_i - \sqrt{\varepsilon_2\mu_1/\varepsilon_1\mu_2}\sqrt{1-(\varepsilon_1\mu_1/\varepsilon_2\mu_2)\sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_2\mu_1/\varepsilon_1\mu_2}\sqrt{1-(\varepsilon_1\mu_1/\varepsilon_2\mu_2)\sin^2\theta_i}}. \tag{A.1}$$

The material permittivity and permeability are

$$\begin{aligned} \varepsilon_1 &= \varepsilon_0\varepsilon_{r1} - \frac{j\sigma_1}{\omega}, \quad \mu_1 = \mu_0, \\ \varepsilon_2 &= \varepsilon_0\varepsilon_{r2} - \frac{j\sigma_2}{\omega}, \quad \mu_2 = \mu_0. \end{aligned} \tag{A.2}$$

The reflection coefficient is represented as follows:

$$\Gamma_E(\omega) = \frac{\cos\theta_i - \sqrt{(\varepsilon_0\varepsilon_{r2} - j\sigma_2/\omega)/(\varepsilon_0\varepsilon_{r1} - j\sigma_1/\omega) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_0\varepsilon_{r2} - j\sigma_2/\omega)/(\varepsilon_0\varepsilon_{r1} - j\sigma_1/\omega) - \sin^2\theta_i}}. \quad (A.3)$$

Using the relation $s \leftrightarrow j\omega$, the reflection coefficient is

$$\Gamma_E(s) = \frac{\cos\theta_i - \sqrt{(\varepsilon_0\varepsilon_{r2} + \sigma_2/s)/(\varepsilon_0\varepsilon_{r1} + \sigma_1/s) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_0\varepsilon_{r2} + \sigma_2/s)/(\varepsilon_0\varepsilon_{r1} + \sigma_1/s) - \sin^2\theta_i}}. \quad (A.4)$$

The transmission coefficient is

$$T_E(s) = 1 + \Gamma_E(s) = 1 + \frac{\cos\theta_i - \sqrt{(\varepsilon_0\varepsilon_{r2} + \sigma_2/s)/(\varepsilon_0\varepsilon_{r1} + \sigma_1/s) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_0\varepsilon_{r2} + \sigma_2/s)/(\varepsilon_0\varepsilon_{r1} + \sigma_1/s) - \sin^2\theta_i}}. \quad (A.5)$$

Try to normalize the coefficient of s under the radical

$$T_E(s) = 1 + \frac{\cos\theta_i\sqrt{\varepsilon_{r1}/(\varepsilon_{r2} - \varepsilon_{r1}\sin^2\theta_i)}\sqrt{s + \sigma_1/\varepsilon_0\varepsilon_{r1}} - \sqrt{s + (\sigma_2 - \sigma_1\sin^2\theta_i)/(\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i)}}{\cos\theta_i\sqrt{\varepsilon_{r1}/(\varepsilon_{r2} - \varepsilon_{r1}\sin^2\theta_i)}\sqrt{s + \sigma_1/\varepsilon_0\varepsilon_{r1}} + \sqrt{s + (\sigma_2 - \sigma_1\sin^2\theta_i)/(\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i)}}. \quad (A.6)$$

Define three parameters to simplify the expression:

$$T_E(s) = \frac{2\chi_1\sqrt{s+\gamma_1}}{\chi_1\sqrt{s+\gamma_1} + \sqrt{s+\gamma_2}}, \quad (A.7)$$

where $\chi_1 = \cos\theta_i\sqrt{\varepsilon_{r1}/(\varepsilon_{r2} - \varepsilon_{r1}\sin^2\theta_i)}$, $\gamma_2 = (\sigma_2 - \sigma_1\sin^2\theta_i)/(\varepsilon_0\varepsilon_{r2} - \varepsilon_0\varepsilon_{r1}\sin^2\theta_i)$, and $\gamma_1 = \sigma_1/\varepsilon_0\varepsilon_{r1}$. Define $\gamma = (\gamma_1 + \gamma_2)/2$ and $c = (\gamma_2 - \gamma_1)/2$; then

$$\begin{aligned} T_E(s-\gamma) &= \frac{2\chi_1\sqrt{s - (\gamma_2 - \gamma_1)/2}}{\chi_1\sqrt{s - (\gamma_2 - \gamma_1)/2} + \sqrt{s + (\gamma_2 - \gamma_1)/2}} \\ &= \frac{2\chi_1\sqrt{s-c}}{\chi_1\sqrt{s-c} + \sqrt{s+c}}. \end{aligned} \quad (A.8)$$

Use $\sqrt{s+c} + \sqrt{s-c}$ to multiply the denominator and numerator of equation (A.8):

$$\begin{aligned} T_E(s-\gamma) &= \frac{2\chi_1\sqrt{s-c}\left(\sqrt{s+c} + \sqrt{s-c}\right)}{\left(\chi_1\sqrt{s-c} + \sqrt{s+c}\right)\left(\sqrt{s+c} + \sqrt{s-c}\right)} = \frac{2\chi_1\left(\sqrt{s^2-c^2} + s - c\right)}{\left(\chi_1\sqrt{s^2-c^2} + s + c\right) + \chi_1(s-c) + \sqrt{s^2-c^2}} \\ &= \frac{2\chi_1}{\chi_1+1}\frac{\sqrt{s^2-c^2} + s - c}{\sqrt{s^2-c^2} + s - ((\chi_1-1)/(\chi_1+1))c} \\ &= \frac{2\chi_1}{\chi_1+1}\frac{\sqrt{s^2-c^2} + s - ((\chi_1-1)/(\chi_1+1))c + ((\chi_1-1)/(\chi_1+1))c - c}{\sqrt{s^2-c^2} + s - ((\chi_1-1)/(\chi_1+1))c} \\ &= \frac{2\chi_1}{\chi_1+1}\left(1 - \frac{(2/(\chi_1+1))c}{\sqrt{s^2-c^2} + s - ((\chi_1-1)/(\chi_1+1))c}\right) \\ &= \left(\frac{2\chi_1}{\chi_1+1} - \frac{2\chi_1}{\chi_1+1}\frac{2}{\chi_1+1}c\frac{1}{\sqrt{s^2-c^2} + s - ((\chi_1-1)/(\chi_1+1))c}\right). \end{aligned} \quad (A.9)$$

Simplify the expression as follows:

$$T_E(s-\gamma) = a + \frac{bc}{d}\frac{S}{1-S} = \left(a + \frac{bc}{d}\sum_{n=1}^{\infty} S^n\right), \quad (A.10)$$

where $a = 2\chi_1/(\chi_1+1)$, $b = -(2\chi_1/(\chi_1+1))(2/(\chi_1+1))$, $d = ((\chi_1-1)/(\chi_1+1))c$, and $S = d/(\sqrt{s^2-c^2} + s)$.

Define $\varphi(s) = \sum_{n=1}^{\infty} S^n = \sum_{n=1}^{\infty}(d/(\sqrt{s^2-c^2}+s))^n$, whose first derivative is

$$\varphi'(s) = -\sum_{n=1}^{\infty} n\left(\frac{d}{c}\right)^n \frac{\left(s - \sqrt{s^2-c^2}\right)^n}{c^n\sqrt{s^2-c^2}}. \quad (A.11)$$

There are some known Laplace Transform pairs in mathematics handbook [13]:

$$L^{-}[F(s-a)] = e^{at}L^{-}[F(s)],$$
$$L^{-}\left[F'(s)\right] = -tL^{-}[F(s)],$$
$$L^{-}\left[\frac{\left(s-\sqrt{s^2-a^2}\right)^n}{a^n\sqrt{s^2-a^2}}\right] = I_n(at), \tag{A.12}$$
$$L^{-}[1] = \delta(t).$$

Homologous transformation relations used in this paper are

$$L^{-}[T_E(s)] = e^{-\gamma t}L^{-}[T_E(s-\gamma)],$$
$$L^{-}[\varphi(s)] = -\frac{1}{t}L^{-}\left[\varphi'(s)\right],$$
$$L^{-}\left[\varphi'(s)\right] = -\sum_{n=1}^{\infty}n\left(\frac{d}{c}\right)^n I_n(ct), \tag{A.13}$$
$$L^{-}[a] = a\delta(t).$$

Combined with the expressions of upper derivation, the final transmission coefficient in time-domain is

$$T_E(t) = a\delta(t)e^{-\gamma t} + \frac{bc}{d}\frac{e^{-\gamma t}}{t}\sum_{n=1}^{\infty}n\left(\frac{d}{c}\right)^n I_n(ct). \tag{A.14}$$

B. The Derivation Details of $\kappa_1(t)$ and $\kappa_2(t)$

There are some known Laplace Transform pairs in mathematics handbook [13]:

$$L^{-1}(s) = \delta'(t),$$
$$L^{-1}\left(\frac{1}{s+y}\right) = e^{-yt}, \tag{B.1}$$
$$L^{-1}\left(\frac{1}{\sqrt{s}\sqrt{s+y}}\right) = e^{-(y/2)t}I_0\left(\frac{y}{2}t\right).$$

There are two approaches to obtain $\kappa_2(t)$ from $\kappa_2(s)$. The first approach is relatively simple:

$$\kappa_2(t) = L^{-1}\left(\frac{s}{s+y}\right) = L^{-1}\left(1-\frac{y}{s+y}\right)$$
$$= \delta(t) - ye^{-yt}. \tag{B.2}$$

The second approach is to use partial integration.

$$\kappa_2(t) = L^{-1}\left(\frac{s}{s+y}\right) = \delta'(t) * e^{-yt}$$
$$= \int_0^t \delta'(\tau)e^{-y(t-\tau)}d\tau \tag{B.3}$$
$$= \delta(\tau)e^{-y(t-\tau)}\Big|_{0_+}^{t} - \int_0^t \delta(\tau)ye^{-y(t-\tau)}d\tau$$
$$= \delta(t) - ye^{-yt}.$$

The approach to obtaining $\kappa_1(t)$ is to use partial integration. In partial integral calculation, the derivative of one-order deformation Bessel function $I_0'(x) = I_1(x)$ is used:

$$\kappa_1(t) = L^{-1}\left(\frac{\sqrt{s}}{\sqrt{s+y}}\right) = \delta'(t) * e^{-(y/2)t}I_0\left(\frac{y}{2}t\right)$$
$$= \int_0^t \delta'(\tau)e^{-(y/2)(t-\tau)}I_0\left[\frac{y}{2}(t-\tau)\right]d\tau = \delta(\tau)$$
$$\cdot e^{-y(t-\tau)}\Big|_{0_+}^{t} - \int_0^t \delta(\tau)$$
$$\cdot\left\{e^{-(y/2)(t-\tau)}I_0\left[\frac{y}{2}(t-\tau)\right]\right\}'d\tau = \delta(t)$$
$$-\int_0^t \delta(\tau)\left\{\frac{y}{2}e^{-(y/2)(t-\tau)}I_0\left[\frac{y}{2}(t-\tau)\right]\right. \tag{B.4}$$
$$\left.-\frac{y}{2}e^{-(y/2)(t-\tau)}I_1\left[\frac{y}{2}(t-\tau)\right]\right\}d\tau = \delta(t)$$
$$-\frac{y}{2}e^{-(y/2)(t)}I_0\left(\frac{y}{2}t\right) + \frac{y}{2}e^{-(y/2)(t)}I_1\left(\frac{y}{2}t\right) = \delta(t)$$
$$+\frac{1}{2}ye^{-(y/2)t}\left[I_1\left(\frac{y}{2}t\right) - I_0\left(\frac{y}{2}t\right)\right].$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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