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Research Article

Evaluation of Heat Irreversibility in a Thin Film Flow of Couple Stress Fluid on a Moving Belt

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This article addresses the inherent heat irreversibility in the flow of a couple stress thin film along a moving vertical belt subjected to free and adiabatic surface. Mathematical analysis for the fluid-governing-equations is performed in detail. For maximum thermal performance and efficiency, the present analysis follows the second law of thermodynamics approach for the evaluation of entropy generation rate in the moving film. With this thermodynamic process, the interconnectivity between variables responsible for energy wastage is accounted for in the thermo-fluid equipment. Results of the analysis revealed the fluid properties that contribute more to energy loss and how the exergy of the system can be restored.

1. Introduction

The past few years have witnessed a growing interest in studies related to thin film flow along a moving belt. This has been so due to its wide range of applications in the field of environmental sciences, chemical engineering, and physical sciences. For example, in the design of film evaporator, paper, coating production, etc., by a way of a short literature review on a thin film on moving belt, Siddiqui et al. [1] focused on the film flow of Sisko fluid and Oldroyd 6-constant fluids on the vertically moving belt. Alam et al. [2, 3] described the non-Newtonian Johnson–Segalman film in the case of hydrodynamics and magnetohydrodynamics flow conditions, respectively. In a study by Farooq et al. [4], the exact solution for the Jeffery film flow on a moving belt was obtained. Hameed and Ellahi [5] obtained a numerical approximation for the hydromagnetic flow of Oldroyd 6-constant film by using Chebyshev collocation method. By using the Optimal Homotopy analysis and Lie symmetry group classification both Ene et al. [6] and Hayat et al. [7] derived explicit solution for the Oldroyd 6-constant fluid film. The literature is inexhaustive when it comes to results on a thin film on a moving belt; these interesting results are not limited to [6, 8–13] and the references therein.

All the studies in [1–13] neglect the thermal effect; therefore, the energy transfer associated with heat flow in many mechanical and chemical engineering applications cannot be described. Motivated by Gul et al. [14–16], in which thermal effect on the moving belt is confined to the first law of thermodynamics. Also, Mahmud and Fraser [17] have done second law analysis to conserve exergy of a thermo-fluid setup. They argued that the useful energy destroyed accounts for underperformance and decreased the thermodynamic efficiency of a system. Therefore, the present study addresses both the energy conservation and management from a thermodynamics viewpoint; in other words, both first and second law of thermodynamics that governs energy conservation and transmission are incorporated in the vertically moving thin film for the very first time. Our focus here is to implement the energy evaluation technique as a valuable tool for identifying variables connected with the destruction of available energy for work and a measure of the efficiency of thermal equipment so as to reduce waste especially during fluid coupling in a conveyor belt. It is interesting to note that, a thorough literature search revealed that little or nothing has been done on couple stress fluid thin film on a moving belt despite the fact that the class of fluid is mostly used as lubricants since it accommodates

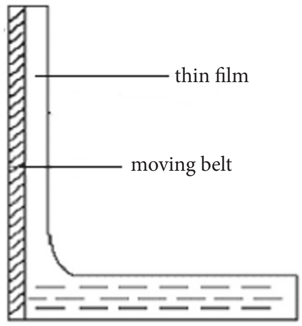

Figure 1: Flow geometry.

tiny microstructures of mechanical importance even in the lubrication of pulley bearing-system used for conveyor belts. The presence of these polymer additives in synthetic lubricants cannot be explained by the classical Newtonian model and this challenged was addresses by Stokes [18]. In the next section, the mathematical analysis is presented based on the steady, viscous, and incompressible unidirectional flow conditions. For the sake of brevity, the derivation of the couple stress model used here is described explicitly in ([19–21]) and not included. In Section 3 of the work, the solutions of the dimensionless boundary-value problems are obtained and used to generate the Bejan and entropy generation profiles.

2. Mathematical Analysis

The analysis is focused on a wide belt moving steadily through a container that is filled with couple stress fluid in an upward direction. A thin fluid layer is picked up as the belt passes through the container and the effect of gravitational force g on the thin is considered. The lamina film is assumed to be of uniform thickness δ. As shown in Figure 1, the (x', y') Cartesian coordinate is chosen along the substrate such that the x-axis lies in the transverse direction of the film flow while the y-axis is chosen in an upward direction.

Taking the thermal effect into consideration, the balanced equations for the thin film with constant physical properties are as follows.

Momentum equation:

$$0 = \mu \frac{d^2u'}{dx'^2} - \eta \frac{d^4u'}{dx'^4} - \rho g \tag{1}$$

The appropriate boundary conditions for the moving belt problem are as follows.

$$\begin{gathered} u'(0) = U, \\ 0 = \left(\frac{d^2u'}{dx'^2}\right)\bigg|_{x'=0} = \left(\frac{du'}{dx'} - \frac{\eta}{\mu}\frac{d^3u'}{dx'^3}\right)\bigg|_{x'=\delta} = \left(\frac{d^2u'}{dx'^2}\right)\bigg|_{x'=\delta} \end{gathered} \tag{2}$$

If the flow is accompanied with thermal effect, then the balanced energy equation for the fluid temperature T is given by

$$0 = k\frac{d^2T}{dx'^2} + \mu\left(\frac{du'}{dx'}\right)^2 + \eta\left(\frac{d^2u'}{dx'^2}\right)^2 \tag{3}$$

with thermal conditions

$$\begin{gathered} T(0) = T_a, \\ \left(\frac{dT}{dx'}\right)\bigg|_{x'=\delta} = 0. \end{gathered} \tag{4}$$

As a result of the inherent irreversibility in many heat transfer processes, the momentum and energy exchange on the couple stress thin film on the moving belt will ultimately lead to a nonequilibrium situation; therefore there will be a continuous generation of entropy, E_G, due to heat transfer and fluid friction within the moving film. The rate at which exergy is depleted which results in underperformance is given by

$$E_G = \frac{k}{T_a^2}\left(\frac{dT}{dx'}\right)^2 + \frac{\mu}{T_a}\left(\frac{du'}{dx'}\right)^2 + \frac{\eta}{T_a}\left(\frac{d^2u'}{dx'^2}\right)^2 \tag{5}$$

with

$$\begin{gathered} u = \frac{u'}{U}, \\ x = \frac{x'}{\delta}, \\ G = \frac{\rho g \delta^2}{\mu U}, \\ \kappa^2 = \frac{\eta}{\delta^2 \mu}, \\ \lambda = \frac{\mu U^2}{k T_a}, \\ Ns = \frac{\delta^2 E_G}{k}, \\ \theta = \frac{T - T_a}{T_a}. \end{gathered} \tag{6}$$

Using (6), (1)-(2) give the Boundary-Value Problem (BVP)

$$\begin{gathered} 0 = \frac{d^2u}{dx^2} - \kappa^2\frac{d^4u}{dx^4} - G; \\ u(0) = 1, \\ 0 = \left(\frac{d^2u}{dx^2}\right)\bigg|_{x=0} = \left(\frac{du}{dx} - \kappa^2\frac{d^3u}{dx^3}\right)\bigg|_{x=1} = \left(\frac{d^2u}{dx^2}\right)\bigg|_{x=1} \end{gathered} \tag{7}$$

and the dimensionless form (3)-(4) gives the following.

$$0 = \frac{d^2\theta}{dx^2} + \lambda\left(\left(\frac{du}{dx}\right)^2 + \kappa^2\left(\frac{d^2u}{dx^2}\right)^2\right);$$
$$\theta(0) = 0,$$
$$\frac{d\theta(1)}{dx} = 0 \tag{8}$$

Using the same analysis in (6), we have the dimensionless entropy generation rate (Ns) as follows.

$$Ns = \left(\frac{d\theta}{dx}\right)^2 + \lambda\left(\left(\frac{du}{dx}\right)^2 + \kappa^2\left(\frac{d^2u}{dx^2}\right)^2\right) \tag{9}$$

The first term on the right side of (9) is due to Heat Transfer Irreversibility (HTI) while the remaining terms are due to Fluid Friction Irreversibility (FFI) resulting from viscous dissipation. As a result, the Bejan number can be defined by the ratio of HTI with the total entropy generation, Ns, on the thin film. In other words

$$Be = \frac{(d\theta/dx)^2}{(d\theta/dx)^2 + \lambda\left((du/dx)^2 + \kappa^2(d^2u/dx^2)^2\right)}. \tag{10}$$

The coefficients μ, u', η, ρ represent the dynamic viscosity, dimensional velocity, couple stress, and density, respectively, and λ, θ, k, T_a viscous heating parameter, dimensionless entropy generation rate, thermal conductivity of the fluid, and referenced temperature. The exact solution of the BVP (7) gives

$$u(x) = -\frac{e^{-x/\kappa}\left(-2e^{1/\kappa+x/\kappa} - 2e^{x/\kappa} + 2e^{1/\kappa+x/\kappa}Gx + 2e^{x/\kappa}Gx - e^{1/\kappa+x/\kappa}Gx^2 - e^{x/\kappa}Gx^2 - 2e^{1/\kappa+x/\kappa}G\kappa^2 + 2e^{1/\kappa}G\kappa^2 - 2e^{x/\kappa}G\kappa^2 + 2e^{2x/\kappa}G\kappa^2\right)}{2\left(1+e^{1/\kappa}\right)} \tag{11}$$

and also (8) admits

$$\begin{aligned}\theta(x) = {} & \frac{1}{12\left(1+e^{1/\kappa}\right)^2}e^{-2x/\kappa}G^2\left(8e^{1/\kappa+2x/\kappa}x\right. \\
& + 4e^{2/\kappa+2x/\kappa}x + 4e^{2x/\kappa}x - 12e^{1/\kappa+2x/\kappa}x^2 \\
& - 6e^{2/\kappa+2x/\kappa}x^2 - 6e^{2x/\kappa}x^2 + 8e^{1/\kappa+2x/\kappa}x^3 \\
& + 4e^{2/\kappa+2x/\kappa}x^3 + 4e^{2x/\kappa}x^3 - 2e^{1/\kappa+2x/\kappa}x^4 \\
& - e^{2/\kappa+2x/\kappa}x^4 - e^{2x/\kappa}x^4 + 24e^{1/\kappa+2x/\kappa}x\kappa^2 \\
& + 12e^{2/\kappa+2x/\kappa}x\kappa^2 + 12e^{2x/\kappa}x\kappa^2 - 12e^{1/\kappa+2x/\kappa}x^2\kappa^2 \\
& - 6e^{2/\kappa+2x/\kappa}x^2\kappa^2 - 6e^{2x/\kappa}x^2\kappa^2 + 24e^{1/\kappa+x/\kappa}\kappa^3 \\
& + 24e^{2/\kappa+x/\kappa}\kappa^3 - 24e^{2/\kappa+2x/\kappa}\kappa^3 - 24e^{1/\kappa+3x/\kappa}\kappa^3 \\
& + 24e^{2x/\kappa}\kappa^3 - 24e^{3x/\kappa}\kappa^3 - 24e^{1/\kappa+x/\kappa}x\kappa^3 \\
& - 24e^{2/\kappa+x/\kappa}x\kappa^3 + 12e^{2/\kappa+2x/\kappa}x\kappa^3 + 24e^{1/\kappa+3x/\kappa}x\kappa^3 \\
& - 12e^{2x/\kappa}x\kappa^3 + 24e^{3x/\kappa}x\kappa^3 - 24e^{1/\kappa+x/\kappa}\kappa^4 \\
& - 24e^{2/\kappa+x/\kappa}\kappa^4 + 48e^{1/\kappa+2x/\kappa}\kappa^4 + 30e^{2/\kappa+2x/\kappa}\kappa^4 \\
& - 24e^{1/\kappa+3x/\kappa}\kappa^4 - 6e^{2/\kappa}\kappa^4 + 30e^{2x/\kappa}\kappa^4 - 24e^{3x/\kappa}\kappa^4 \\
& \left.- 6e^{4x/\kappa}\kappa^4\right)\lambda.\end{aligned} \tag{12}$$

The exact solutions are used to determine the entropy generation rate (9) and heat irreversibility ratio in (10). The skin friction and the rate of heat transfer along the belt are defined as

$$\tau_w = \left.\left(\frac{du'}{dx'} - \frac{\eta}{\mu}\frac{d^3u'}{dx'^3}\right)\right|_{x=0},$$
$$q = -k\left.\frac{dT}{dx}\right|_{x=0} \tag{13}$$

and the dimensionless form becomes

$$Sf = \frac{\delta\tau_w}{U} = \left.\left(\frac{du}{dx} - \kappa^2\frac{d^3u}{dx^3}\right)\right|_{x=0},$$
$$Nu = \frac{q\delta}{kT_a} = -\left.\frac{d\theta}{dx}\right|_{x=0} \tag{14}$$

3. Results and Discussion

In the section, the responses of the exact solution to a variation of parameters for the velocity, temperature, and entropy generation are presented graphically as a function of the spatial variable. The two limiting cases of the couple stress parameter are shown in Figure 2; evidently, as the couple stress parameter approaches zero, the Newtonian class is fully recovered, while as it becomes infinite, the fluid behaves like plastic. Therefore in this work, the couple stress parameter is carefully chosen in between the limiting cases. In Figures 3(a)–3(d), the drainage problem is presented. In Figure 3(a), the flow is observed to increase with increasing values of the couple stress parameter, since the increase in the couple stress parameter implies a decrease in the dynamic viscosity of the fluid and thus an increase in the molecular distance between the fluid particles. Figure 3(b) displayed an asymmetrical heat flow from the isothermal wall to the adiabatic surface; this

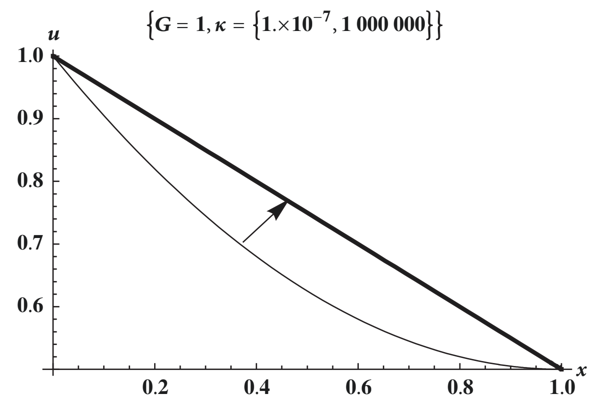

FIGURE 2: Drainage velocity against the limiting cases of the couple stress parameter.

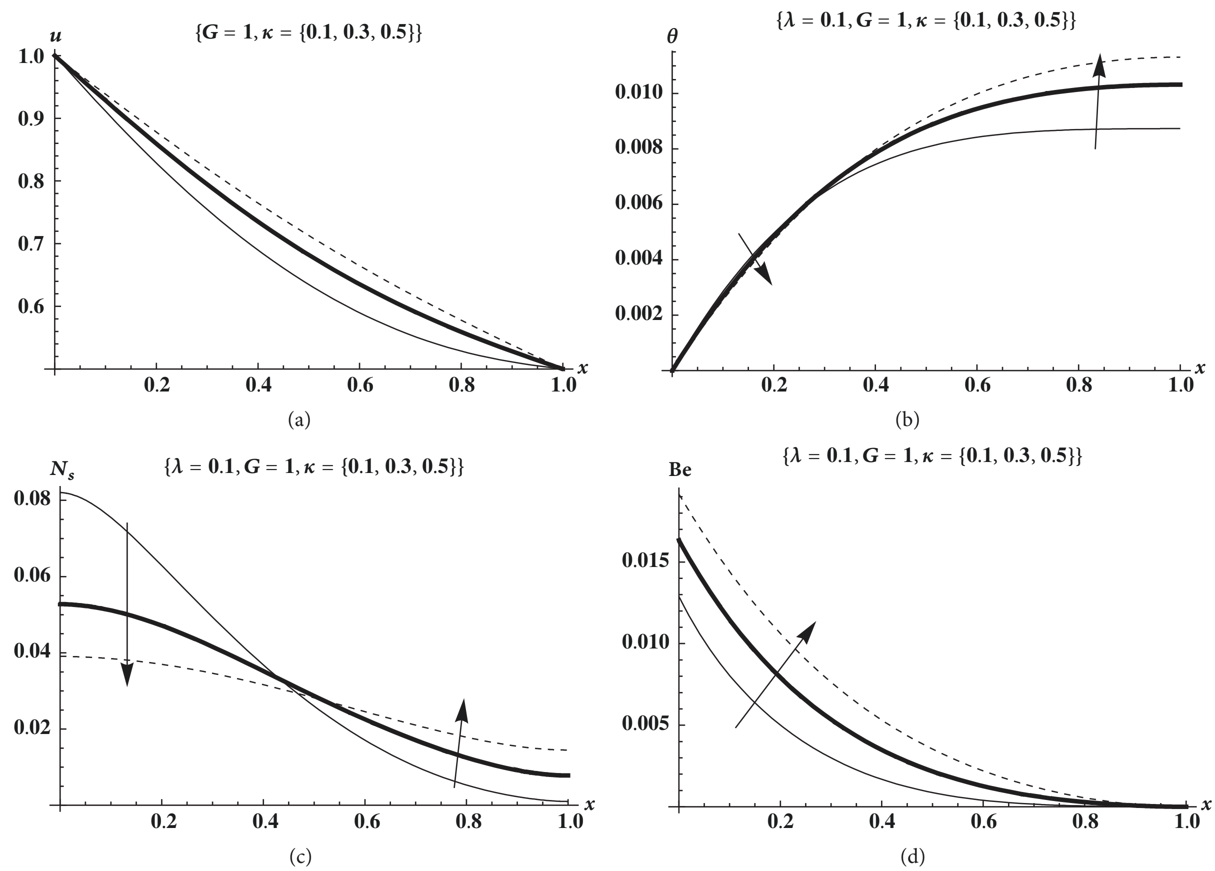

FIGURE 3: (a) drainage velocity against couple stress parameter, (b) temperature against couple stress parameter, (c) entropy generation against couple stress parameter, (d) Bejan number against couple stress parameter.

shows that the temperature profile is well behaved. Moreover, the rate of heat transfer at the free end increases significantly due to the migration of heat from cold region to the cold ambient air. Figure 3(c) revealed the contribution of the couple stress parameter to the entropy generation profile in the moving thin film flow. The result here shows that entropy at the cold isothermal vertical wall is decreasing with increasing values of the couple stress parameter while the reverse trend is observed at the free surface. Figure 3(d) represents the nonequilibrium in the irreversible process; at small values of the couple stress parameter, FFI is seen to contribute more to the heat irreversibility; this dominance declines across the vertical length as seen in the plot. Interestingly as the couple stress parameter increases, the dynamic viscosity of the fluid declines and the impact of the HTI is seen. Result in Figure 4(a) depicts the lifting effect when the fluid moves against gravitational force. Naturally, the gravitational pull decreases the flow velocity as the dynamic

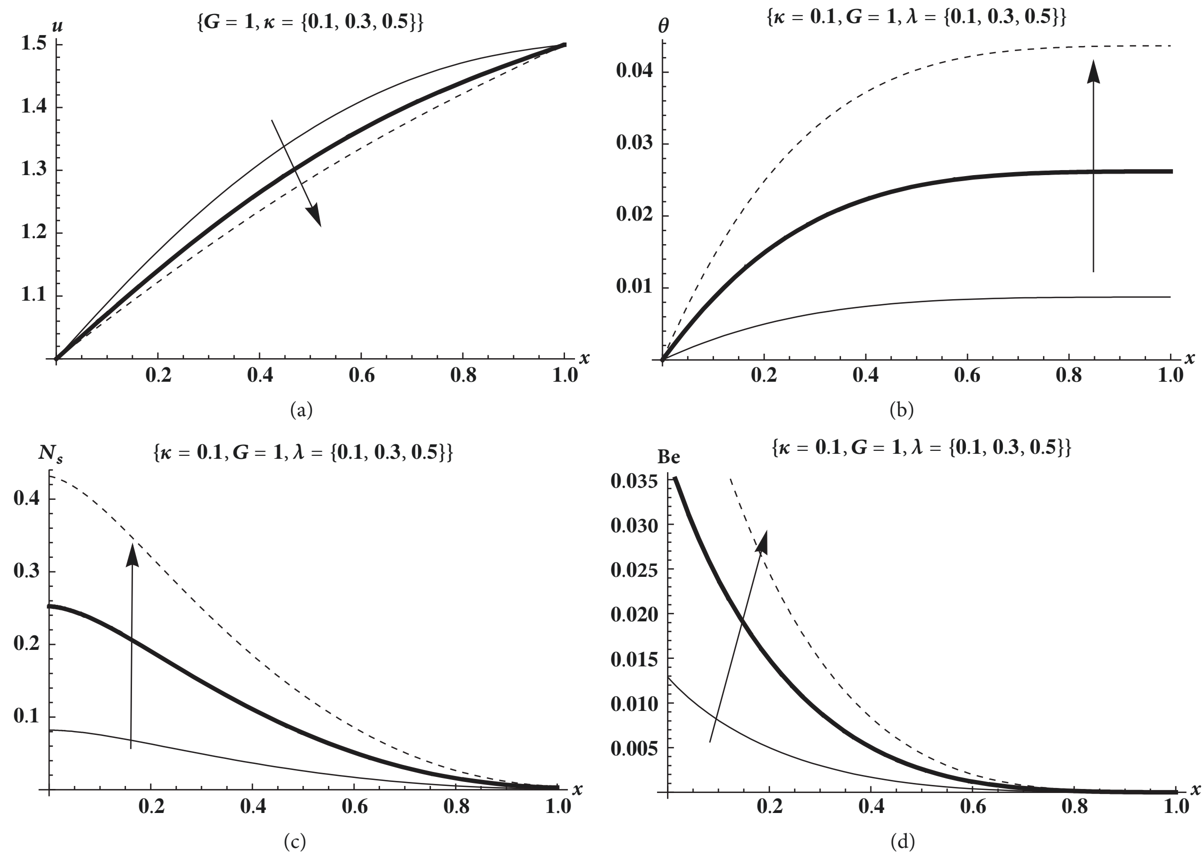

Figure 4: (a) lifting flow velocity against couple stress parameter, (b) temperature against viscous heating parameter, (c) entropy generation against viscous heating parameter, (d) Bejan number against viscous heating parameter.

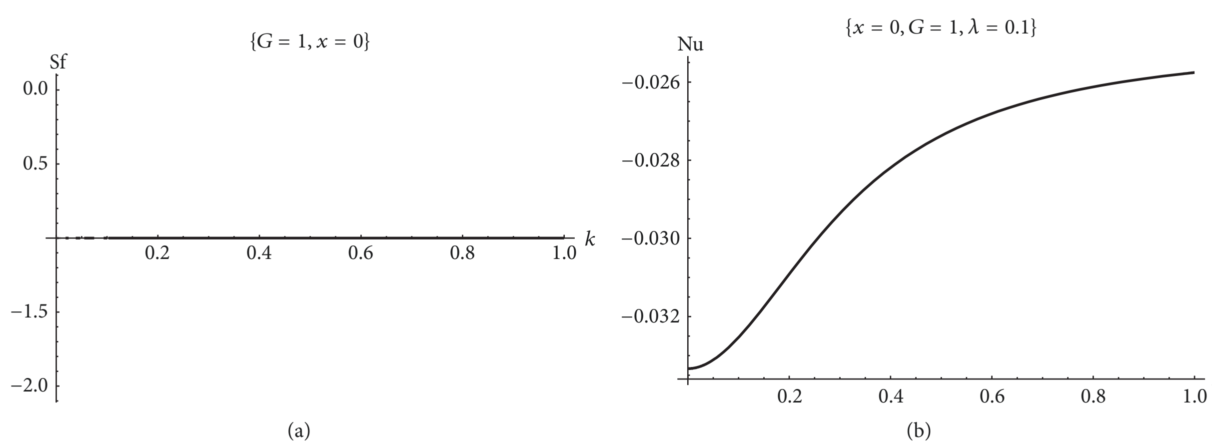

Figure 5: (a) Skin friction for lifting flow against couple stress parameter, (b) heat transfer rate for lifting flow against couple stress parameter.

viscosity of the fluid decreases. Figure 4(b) confirmed that, as the viscous heating increases, the fluid temperature is expected to increase as shown in the plot. This contributes to the total heat energy, and entropy is expected to increase the kinetic energy of the fluid particles as presented in Figure 4(c) and the HTI dominates over FFI in the moving film as shown in Figure 4(d). Result in Figure 5(a) represents a constant variation of couple stress parameter with skin friction while the rate of heat transfer along the moving belt increases with increasing value of the couple stress parameter as shown in Figure 5(b).

4. Concluding Remarks

The inclusion of couple stresses into the vertically moving thin film is the major contribution of this study. The exact solutions of the velocity and temperature fields are obtained and used to derive the entropy generation rate and heat irreversibility ratio. The result of the computation shows the following:

(i) Drainage and lifting problems are opposing phenomena.

(ii) Couple stress parameter contributes to entropy generation only at the adiabatic surface

(iii) Entropy generation decreases with couple stress parameter at the isothermal cold surface

(iv) Viscous heating of the fluid enhances the entropy generation rate

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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