Research Article

Solitons and Other Exact Solutions for Two Nonlinear PDEs in Mathematical Physics Using the Generalized Projective Riccati Equations Method

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We apply the generalized projective Riccati equations method with the aid of Maple software to construct many new soliton and periodic solutions with parameters for two higher-order nonlinear partial differential equations (PDEs), namely, the nonlinear Schrödinger (NLS) equation with fourth-order dispersion and dual power law nonlinearity and the nonlinear quantum Zakharov-Kuznetsov (QZK) equation. The obtained exact solutions include kink and antikink solitons, bell (bright) and antibell (dark) solitary wave solutions, and periodic solutions. The given nonlinear PDEs have been derived and can be reduced to nonlinear ordinary differential equations (ODEs) using a simple transformation. A comparison of our new results with the well-known results is made. Also, we drew some graphs of the exact solutions using Maple. The given method in this article is straightforward and concise, and it can also be applied to other nonlinear PDEs in mathematical physics.

1. Introduction

The investigation of exact traveling wave solutions to nonlinear PDEs plays an important role in the study of nonlinear physical phenomena. Nonlinear waves appear in various scientific fields, especially in physics such as fluid mechanics, plasma physics, optical fibers, and solid state physics. In recent years, many powerful tools have been established to determine soliton and periodic wave solutions of nonlinear PDEs, such as the (G'/G) -expansion method [1–6], the extended auxiliary equation method [7, 8], the new mapping method [9–11], the generalized projective Riccati equations method [12–17], and the $(G'/G, 1/G)$ -expansion method [18]. Conte and Musette [12] presented an indirect method to find solitary wave solutions of some nonlinear PDEs that can be expressed as polynomials in two elementary functions which satisfy a projective Riccati equation [19]. This method has been applied to many nonlinear PDEs and the solitary wave solutions of these equations can be found in [13–17].

Recently, Yan [16] has been given a generalization of Conte and Musette's method.

The objective of this article is to use the generalized projective Riccati equations method [13–17] to construct the soliton and periodic solutions of the following two higher-order nonlinear PDEs.

(1) The nonlinear Schrödinger (NLS) equation with fourth-order dispersion and dual power law nonlinearity [5] is

$$iq_t + aq_{xx} - bq_{xxxx} + c(|q|^{2m} + k_1|q|^{4m})q = 0, \quad (1)$$

which describes the propagation of optical pulse in a medium, and $q(x, t)$ is the slowly varying envelope of the electromagnetic field, where a, b, c are real numbers. If $b = 0$, (1) reduces to the NLS. In addition if $m = 1$, (1) reduces to parabolic law nonlinearity, which has been discussed in [20] using two direct algebraic methods. The coefficient of a represents the group velocity dispersion (GVD), while the

coefficient of c represents the self-phase modulation (SPM) with dual power law nonlinearity. The constant k_1 binds the two nonlinear terms and the exponent m governs the power law. Also, the coefficients of b are the fourth-order dispersion terms. Equation (1) has been studied in [5] using five different techniques, namely, the (G'/G) -expansion method, the improved Sub-ODE method, the extended auxiliary equation method, the new mapping method, and the Jacobi elliptic function method.

(2) The nonlinear quantum Zakharov-Kuznetsov (QZK) equation [6, 21–23] is

$$\frac{\partial U}{\partial T} + AU \frac{\partial U}{\partial Z} + B \frac{\partial^3 U}{\partial Z^3} + C \frac{\partial}{\partial Z} \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) U = 0, \quad (2)$$

which arises in quantum magnetoplasma, where A, B, C are constants. Equation (2) has been derived in [21] using the reductive perturbation technique and in [22] using a series of transformations method. Here U is the electrostatic potential, which X, Y, Z, T are the stretched space-time coordinates which are defined in [21]. Moslem et al. [21] have derived (2) for electron-ion quantum plasma and solitary explosive and periodic solutions are presented. In [23], the authors applied the auxiliary equation method and Hirota bilinear method to study (2) and some types of exact solutions are obtained. Recently Zayed and Alurfi have discussed (2) in [6] using the extended generalized (G'/G) -expansion method with the Jacobi elliptic equation and determined its exact traveling wave solutions.

This paper is organized as follows: in Section 2, the description of the well-known generalized projective Riccati equations method is given. In Section 3, we use the given method described in Section 2, to find new soliton and periodic solutions of the NLS equation (1) and the QZK equation (2). In Section 4, we draw some figures for some solutions of (1). In Section 5, some conclusions are obtained.

2. Description of the Generalized Projective Riccati Equations Method

Consider a nonlinear PDE in the following form:

$$P(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \quad (3)$$

where $u = u(x, t)$ is an unknown function, P is a polynomial in $u(x, t)$, and its partial derivatives in which the highest-order derivatives and nonlinear terms are involved. Let us now give the main steps of the generalized projective Riccati equations method [13–17].

Step 1. We use the following transformation:

$$u(x, t) = u(\xi), \quad \xi = x - vt, \quad (4)$$

to reduce (3) to the following nonlinear ODE:

$$H(u, u', u'', \dots) = 0, \quad (5)$$

where v is velocity of the propagation, H is a polynomial of $u(\xi)$ and its total derivatives $u'(\xi), u''(\xi), \dots$ and $' = d/d\xi$.

Step 2. We suppose that the solution of (5) has the following form:

$$u(\xi) = A_0 + \sum_{i=1}^N \sigma^{i-1}(\xi) [A_i \sigma(\xi) + B_i \tau(\xi)], \quad (6)$$

where A_0, A_i , and B_i are constants to be determined. The functions $\sigma(\xi)$ and $\tau(\xi)$ satisfy the ODEs:

$$\sigma'(\xi) = \varepsilon \sigma(\xi) \tau(\xi), \quad (7)$$

$$\tau'(\xi) = R + \varepsilon \tau^2(\xi) - \mu \sigma(\xi), \quad \varepsilon = \pm 1,$$

where

$$\tau^2(\xi) = -\varepsilon \left(R - 2\mu \sigma(\xi) + \frac{\mu^2 + r}{R} \sigma^2(\xi) \right), \quad (8)$$

and here $r = \pm 1$ and R, μ are nonzero constants.

If $R = \mu = 0$, (5) has the formal solution:

$$u(\xi) = \sum_{i=0}^N A_i \tau^i(\xi), \quad (9)$$

where $\tau(\xi)$ satisfies the nonlinear ODE:

$$\tau'(\xi) = \tau^2(\xi). \quad (10)$$

Step 3. The positive integer number N in (6) must be determined by using the homogeneous balance between the highest-order derivatives and the highest nonlinear terms in (5).

Step 4. Substitute (6) along with (7) and (8) into (5). Collecting all terms of the same order of $\sigma^j(\xi) \tau^i(\xi)$ ($j = 0, 1, \dots; i = 0, 1$). Setting each coefficient to zero yields a set of algebraic equations which can be solved to find the values of A_0, A_i, B_i, v, μ , and R .

Step 5. It is well-known [13–17] that (7) admits the following solutions.

Case 1. When $\varepsilon = -1, r = -1, R > 0$,

$$\begin{aligned} \sigma_1(\xi) &= \frac{R \operatorname{sech}(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1}, \\ \tau_1(\xi) &= \frac{\sqrt{R} \tanh(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1}. \end{aligned} \quad (11)$$

Case 2. When $\varepsilon = -1, r = 1, R > 0$,

$$\begin{aligned} \sigma_2(\xi) &= \frac{R \operatorname{csch}(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1}, \\ \tau_2(\xi) &= \frac{\sqrt{R} \coth(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1}. \end{aligned} \quad (12)$$

Case 3. When $\varepsilon = 1$, $r = -1$, $R > 0$,

$$\begin{aligned}\sigma_3(\xi) &= \frac{R \sec(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1}, \\ \tau_3(\xi) &= \frac{\sqrt{R} \tan(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1}, \\ \sigma_4(\xi) &= \frac{R \csc(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1}, \\ \tau_4(\xi) &= -\frac{\sqrt{R} \cot(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1}.\end{aligned}\quad (13)$$

Case 4. When $R = \mu = 0$,

$$\begin{aligned}\sigma_5(\xi) &= \frac{C}{\xi}, \\ \tau_5(\xi) &= \frac{1}{\varepsilon \xi},\end{aligned}\quad (14)$$

where C is a nonzero constant.

Step 6. Substituting the values of A_0 , A_i , B_i , v , μ , and R as well as the solutions (11)–(14) into (6), we obtain the exact solutions of (3).

3. Applications

In this section, we apply the generalized projective Riccati equations method described in Section 2 to find many new soliton and periodic solutions of (1) and (2) in the following subsections.

3.1. On Solving (1) Using the Method of Section 2. In order to solve (1), we assume that the solution of (1) has the following form:

$$q(x, t) = \phi(\xi) e^{iQ(x, t)}, \quad i = \sqrt{-1}, \quad (15)$$

where $\phi(\xi)$ is the amplitude portion which is a real function of ξ , while $Q(x, t)$ is the phase portion of the soliton. It is assumed that ξ and $Q(x, t)$ are given by

$$\begin{aligned}\xi &= x - vt, \\ Q(x, t) &= -kx + \omega t + \theta,\end{aligned}\quad (16)$$

where v , k , ω , and θ are constants, such that v is the velocity of the soliton, ω is the frequency of the soliton, k is the wave number, and θ is a phase constant.

Substituting (15) into (1) and separating the real and imaginary parts, we get

$$\begin{aligned}\text{Re: } & -(\omega + ak^2 + bk^4)\phi + (a + 6bk^2)\phi'' - b\phi'''' \\ & + c(\phi^{2m+1} + k_1\phi^{4m+1}) = 0,\end{aligned}\quad (17)$$

$$\text{Im: } -(\nu + 2ak + 4bk^3)\phi' + 4bk\phi''' = 0, \quad (18)$$

and, differentiating (18) and substituting the resulting equation in (17), we have the following nonlinear ODE:

$$a_1\phi'' - b_1\phi + c_1(\phi^{2m+1} + k_1\phi^{4m+1}) = 0, \quad (19)$$

where a_1 , b_1 , and c_1 are given by

$$\begin{aligned}a_1 &= 2ak + 20bk^3 - \nu, \\ b_1 &= -4k(\omega + ak^2 + bk^4), \\ c_1 &= 4kc.\end{aligned}\quad (20)$$

Balancing ϕ'' with ϕ^{4m+1} in (19), then the following relation is attained:

$$N + 2 = N(4m + 1) \implies N = \frac{1}{2m}. \quad (21)$$

Since the balance number N is not integer, then we use the following new transformation:

$$\phi(\xi) = [u(\xi)]^{1/2m}, \quad (22)$$

where u is a new function of ξ . Substituting (22) into (19), we have the new equation

$$\begin{aligned}2a_1muu'' + a_1(1 - 2m)(u')^2 + 4b_1m^2u^2 + 4c_1m^2u^3 \\ + 4k_1c_1m^2u^4 = 0.\end{aligned}\quad (23)$$

Balancing uu'' and u^4 in (23), then the following relation is obtained:

$$N + N + 2 = 4N \implies N = 1. \quad (24)$$

From (6) the formal solution of (23) has the following form:

$$u(\xi) = A_0 + A_1\sigma(\xi) + B_1\tau(\xi), \quad (25)$$

where A_0 , A_1 , and B_1 are constants to be determined such that $A_1 \neq 0$ or $B_1 \neq 0$.

Substituting (25) into (23) and using (7) and (8), the left-hand side of (23) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$. Setting the coefficients of this polynomial to be zero yields the following system of algebraic equations:

$$\begin{aligned}\sigma^4: & 4m^2A_1^4c_1k_1 - \frac{1}{R}\varepsilon(\mu^2 + r)(24c_1k_1m^2A_1^2B_1^2 \\ & + 4a_1m\varepsilon^2A_1^2 + (1 - 2m)a_1\varepsilon^2A_1^2) + \frac{1}{R^2}\varepsilon^2(\mu^2 \\ & + r)^2(4c_1k_1m^2B_1^4 + 4a_1m\varepsilon^2B_1^2 + (1 - 2m)a_1\varepsilon^2B_1^2) \\ & = 0,\end{aligned}$$

$$\begin{aligned}\sigma^3 : & -\frac{1}{R}\varepsilon(\mu^2 + r)(12m^2 A_1 B_1^2 c_1 - 2(1-2m)\mu\varepsilon B_1^2 a_1 \\ & - 6m\mu\varepsilon B_1^2 a_1 + 4m\varepsilon^2 A_0 A_1 a_1 \\ & + 48m^2 A_0 A_1 B_1^2 c_1 k_1) + 4m^2 A_1^3 c_1 \\ & + 16m^2 A_0 A_1^3 c_1 k_1 - 2m\mu\varepsilon A_1^2 a_1 - \frac{4}{R}\mu\varepsilon^2(\mu^2 + r) \\ & \cdot (4c_1 k_1 m^2 B_1^4 + 4a_1 m\varepsilon^2 B_1^2 + (1-2m)a_1 \varepsilon^2 B_1^2) \\ & + 2\mu\varepsilon(24c_1 k_1 m^2 A_1^2 B_1^2 + 4a_1 m\varepsilon^2 A_1^2 \\ & + (1-2m)a_1 \varepsilon^2 A_1^2) = 0,\end{aligned}$$

$$\begin{aligned}\sigma^3 \tau : & \frac{1}{R}\varepsilon(\mu^2 + r)(16A_1 c_1 k_1 m^2 B_1^3 + 8A_1 a_1 m\varepsilon^2 B_1 \\ & + 2(1-2m)A_1 a_1 \varepsilon^2 B_1) - 16m^2 A_1^3 B_1 c_1 k_1 = 0,\end{aligned}$$

$$\begin{aligned}\sigma^2 : & 4m^2 A_1^2 b_1 + \varepsilon^2(6\mu^2 + 2r)(4c_1 k_1 m^2 B_1^4 \\ & + 4a_1 m\varepsilon^2 B_1^2 + (1-2m)a_1 \varepsilon^2 B_1^2) + (1-2m) \\ & \cdot \mu^2 B_1^2 a_1 + 2\mu\varepsilon(12m^2 A_1 B_1^2 c_1 - 2(1-2m)\mu\varepsilon B_1^2 a_1 \\ & - 6m\mu\varepsilon B_1^2 a_1 + 4m\varepsilon^2 A_0 A_1 a_1 \\ & + 48m^2 A_0 A_1 B_1^2 c_1 k_1) - \frac{1}{R}\varepsilon(\mu^2 + r) \\ & \cdot (24c_1 k_1 m^2 A_0^2 B_1^2 + 12c_1 m^2 A_0 B_1^2 + 4b_1 m^2 B_1^2 \\ & + 4R\varepsilon a_1 m B_1^2 + 2(1-2m)R\varepsilon a_1 B_1^2) \\ & + 24m^2 A_0^2 A_1^2 c_1 k_1 - 2m\mu\varepsilon A_0 A_1 a_1 \\ & - \varepsilon R(24c_1 k_1 m^2 A_1^2 B_1^2 + 4a_1 m\varepsilon^2 A_1^2 \\ & + (1-2m)a_1 \varepsilon^2 A_1^2) + 12m^2 A_0 A_1^2 c_1 + 2Rm\varepsilon A_1^2 a_1 \\ & = 0,\end{aligned}$$

$$\begin{aligned}\sigma^2 \tau : & 2\varepsilon\mu(16A_1 c_1 k_1 m^2 B_1^3 + 8A_1 a_1 m\varepsilon^2 B_1 \\ & + 2(1-2m)A_1 a_1 \varepsilon^2 B_1) + 12m^2 A_1^2 B_1 c_1 - 2(1 \\ & - 2m)\mu\varepsilon A_1 B_1 a_1 - \frac{1}{R}\varepsilon(\mu^2 + r)(4m^2 B_1^3 c_1 \\ & + 16m^2 A_0 B_1^3 c_1 k_1 + 4m\varepsilon^2 A_0 B_1 a_1) \\ & + 48m^2 A_0 A_1^2 B_1 c_1 k_1 - 8m\mu\varepsilon A_1 B_1 a_1 = 0,\end{aligned}$$

$$\begin{aligned}\sigma : & -\varepsilon R(12m^2 A_1 B_1^2 c_1 - 2(1-2m)\mu\varepsilon B_1^2 a_1 \\ & - 6m\mu\varepsilon B_1^2 a_1 + 4m\varepsilon^2 A_0 A_1 a_1 \\ & + 48m^2 A_0 A_1 B_1^2 c_1 k_1) + 2\varepsilon\mu(24c_1 k_1 m^2 A_0^2 B_1^2 \\ & + 12c_1 m^2 A_0 B_1^2 + 4b_1 m^2 B_1^2 + 4R\varepsilon a_1 m B_1^2\end{aligned}$$

$$\begin{aligned}& + 2(1-2m)R\varepsilon a_1 B_1^2) + 8m^2 A_0 A_1 b_1 \\ & + 12m^2 A_0^2 A_1 c_1 - 4R\mu\varepsilon^2(4c_1 k_1 m^2 B_1^4 + 4a_1 m\varepsilon^2 B_1^2 \\ & + (1-2m)a_1 \varepsilon^2 B_1^2) + 16m^2 A_0^3 A_1 c_1 k_1 - 2(1 \\ & - 2m)R\mu B_1^2 a_1 + 2Rm\varepsilon A_0 A_1 a_1 = 0,\end{aligned}$$

$$\begin{aligned}\sigma\tau : & 8m^2 A_1 B_1 b_1 - \varepsilon R(16A_1 c_1 k_1 m^2 B_1^3 \\ & + 8A_1 a_1 m\varepsilon^2 B_1 + 2(1-2m)A_1 a_1 \varepsilon^2 B_1) \\ & + 2\varepsilon\mu(4m^2 B_1^3 c_1 + 16m^2 A_0 B_1^3 c_1 k_1 \\ & + 4m\varepsilon^2 A_0 B_1 a_1) + 24m^2 A_0 A_1 B_1 c_1 + 2(1-2m) \\ & \cdot R\varepsilon A_1 B_1 a_1 + 6Rm\varepsilon A_1 B_1 a_1 - 6m\mu\varepsilon A_0 B_1 a_1 \\ & + 48m^2 A_0^2 A_1 B_1 c_1 k_1 = 0,\end{aligned}$$

$$\begin{aligned}\tau : & 8m^2 A_0 B_1 b_1 - R\varepsilon(4m^2 B_1^3 c_1 + 16m^2 A_0 B_1^3 c_1 k_1 \\ & + 4m\varepsilon^2 A_0 B_1 a_1) + 12m^2 A_0^2 B_1 c_1 + 16m^2 A_0^3 B_1 c_1 k_1 \\ & + 4Rm\varepsilon A_0 B_1 a_1 = 0,\end{aligned}$$

$$\begin{aligned}\sigma^0 : & R^2 \varepsilon^2(4c_1 k_1 m^2 B_1^4 + 4a_1 m\varepsilon^2 B_1^2 + (1-2m)a_1 \varepsilon^2 B_1^2) \\ & + 4m^2 A_0^2 b_1 + 4m^2 A_0^3 c_1 + (1-2m)R^2 B_1^2 a_1 \\ & - R\varepsilon(24c_1 k_1 m^2 A_0^2 B_1^2 + 12c_1 m^2 A_0 B_1^2 + 4b_1 m^2 B_1^2 \\ & + 4R\varepsilon a_1 m B_1^2 + 2(1-2m)R\varepsilon a_1 B_1^2) + 4m^2 A_0^4 c_1 k_1 \\ & = 0.\end{aligned}$$

(26)

According to Step 5, of Section 2, there are three cases of solutions of the algebraic equations (26) to be discussed as follows:

Case 1 ($\varepsilon = -1, r = -1$). It leads to the following results.

Result 1. We have

$$\begin{aligned}A_0 &= -\frac{b_1(m+1)}{c_1}, \\ A_1 &= 0, \\ B_1 &= \pm \frac{(m+1)\sqrt{-b_1 a_1}}{c_1 m}, \\ \mu &= 0, \\ R &= -\frac{b_1 m^2}{a_1}, \\ k_1 &= \frac{c_1(2m+1)}{4b_1(m+1)^2}.\end{aligned}\tag{27}$$

From (11), (25), and (27), we deduce the kink-shaped soliton solutions of (1) as follows:

$$q_{11}(x, t) = \left\{ -\frac{b_1(m+1)}{c_1} \left(1 \pm \tanh \left(\sqrt{-\frac{b_1 m^2}{a_1}} \xi \right) \right) \right\}^{1/2m} \cdot \exp[i(-kx + \omega t + \theta)], \quad (28)$$

where $a_1 b_1 < 0$ and $c_1 b_1 < 0$.

Result 2. We have

$$\begin{aligned} A_0 &= 0, \\ A_1 &= \frac{a_1 \mu (m+1)}{2c_1 m^2}, \\ B_1 &= 0, \\ R &= -\frac{4b_1 m^2}{a_1}, \\ k_1 &= \frac{c_1 (\mu^2 - 1) (2m+1)}{4b_1 \mu^2 (m+1)^2}. \end{aligned} \quad (29)$$

In this result, we deduce the bell-shaped soliton solutions of (1) as follows:

$$q_{12}(x, t) = \left\{ -\frac{2b_1 \mu (m+1)}{c_1} \left(\frac{\operatorname{sech} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right)}{\mu \operatorname{sech} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right) + 1} \right) \right\}^{1/2m} \cdot \exp[i(-kx + \omega t + \theta)], \quad (30)$$

where $a_1 b_1 < 0$ and $c_1 b_1 \mu < 0$.

Result 3. We have

$$\begin{aligned} A_0 &= -\frac{b_1(m+1)}{c_1}, \\ A_1 &= \pm \frac{a_1(m+1) \sqrt{\mu^2 - 1}}{4c_1 m^2}, \\ B_1 &= \pm \frac{(m+1) \sqrt{-a_1 b_1}}{2c_1 m}, \\ R &= -\frac{4b_1 m^2}{a_1}, \\ k_1 &= \frac{c_1 (2m+1)}{4b_1 (m+1)^2}. \end{aligned} \quad (31)$$

In this result, we deduce the bell-kink-shaped soliton solutions of (1) as follows:

$$q_{13}(x, t) = \left\{ -\frac{b_1(m+1)}{c_1} \left(1 \pm \frac{\sqrt{\mu^2 - 1} \operatorname{sech} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right)}{\mu \operatorname{sech} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right) + 1} \pm \frac{\tanh \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right)}{\mu \operatorname{sech} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right) + 1} \right) \right\}^{1/2m} \cdot \exp[i(-kx + \omega t + \theta)], \quad (32)$$

where $a_1 b_1 < 0$, $c_1 b_1 < 0$ and $\mu^2 > 1$.

Case 2 ($\varepsilon = -1$, $r = 1$). It leads to the following results.

Result 1. We have the same result (27).

From (12), (25), and (27), we deduce the singular kink-shaped soliton solutions of (1) as follows:

$$q_{21}(x, t) = \left\{ -\frac{b_1(m+1)}{c_1} \left(1 \pm \coth \left(\sqrt{-\frac{b_1 m^2}{a_1}} \xi \right) \right) \right\}^{1/2m} \cdot \exp[i(-kx + \omega t + \theta)], \quad (33)$$

where $a_1 b_1 < 0$ and $c_1 b_1 < 0$.

Result 2. We have

$$\begin{aligned} A_0 &= 0, \\ A_1 &= \frac{a_1 \mu (m+1)}{2c_1 m^2}, \\ B_1 &= 0, \\ R &= -\frac{4b_1 m^2}{a_1}, \\ k_1 &= \frac{c_1 (\mu^2 + 1) (2m+1)}{4b_1 \mu^2 (m+1)^2}. \end{aligned} \quad (34)$$

In this result, we deduce the singular bell-shaped soliton solutions of (1) as follows:

$$q_{22}(x, t) = \left\{ -\frac{2b_1 \mu (m+1)}{c_1} \left(\frac{\operatorname{csch} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right)}{\mu \operatorname{csch} \left(\sqrt{-(4b_1 m^2/a_1)} \xi \right) + 1} \right) \right\}^{1/2m} \cdot \exp[i(-kx + \omega t + \theta)], \quad (35)$$

where $a_1 b_1 < 0$ and $c_1 b_1 \mu < 0$.

Result 3. We have

$$\begin{aligned}
 A_0 &= -\frac{b_1(m+1)}{c_1}, \\
 A_1 &= \pm \frac{a_1(m+1)\sqrt{\mu^2+1}}{4c_1m^2}, \\
 B_1 &= \pm \frac{(m+1)\sqrt{-a_1b_1}}{2c_1m}, \\
 R &= -\frac{4b_1m^2}{a_1}, \\
 k_1 &= \frac{c_1(2m+1)}{4b_1(m+1)^2}.
 \end{aligned} \tag{36}$$

In this result, we deduce the singular bell-kink-shaped soliton solutions of (1) as follows:

$$\begin{aligned}
 q_{23}(x, t) &= \left\{ -\frac{b_1(m+1)}{c_1} \left(1 \right. \right. \\
 &\quad \pm \frac{\sqrt{\mu^2+1} \operatorname{csch}\left(\sqrt{-(4b_1m^2/a_1)}\xi\right)}{\mu \operatorname{csch}\left(\sqrt{-(4b_1m^2/a_1)}\xi\right)+1} \\
 &\quad \left. \left. \pm \frac{\coth\left(\sqrt{-(4b_1m^2/a_1)}\xi\right)}{\mu \operatorname{csch}\left(\sqrt{-(4b_1m^2/a_1)}\xi\right)+1} \right) \right\}^{1/2m} \\
 &\quad \cdot \exp[i(-kx + \omega t + \theta)],
 \end{aligned} \tag{37}$$

where $a_1b_1 < 0$ and $c_1b_1 < 0$.

Case 3 ($\varepsilon = 1, r = -1$). It leads to the following results.

Result 1. We have

$$\begin{aligned}
 A_0 &= 0, \\
 A_1 &= -\frac{a_1\mu(m+1)}{2c_1m^2}, \\
 B_1 &= 0, \\
 R &= \frac{4b_1m^2}{a_1}, \\
 k_1 &= \frac{c_1(\mu^2-1)(2m+1)}{4b_1\mu^2(m+1)^2}.
 \end{aligned} \tag{38}$$

From (13), (25), and (38), we deduce the periodic solutions of (1) as follows:

$$\begin{aligned}
 q_{31}(x, t) &= \left\{ -\frac{2b_1\mu(m+1)}{c_1} \left(\frac{\sec\left(\sqrt{(4b_1m^2/a_1)}\xi\right)}{\mu \sec\left(\sqrt{(4b_1m^2/a_1)}\xi\right)+1} \right) \right\}^{1/2m} \\
 &\quad \cdot \exp[i(-kx + \omega t + \theta)],
 \end{aligned} \tag{39}$$

$$\begin{aligned}
 q_{32}(x, t) &= \left\{ -\frac{2b_1\mu(m+1)}{c_1} \left(\frac{\csc\left(\sqrt{(4b_1m^2/a_1)}\xi\right)}{\mu \csc\left(\sqrt{(4b_1m^2/a_1)}\xi\right)+1} \right) \right\}^{1/2m} \\
 &\quad \cdot \exp[i(-kx + \omega t + \theta)],
 \end{aligned} \tag{40}$$

where $a_1b_1 > 0$ and $c_1b_1\mu < 0$.

Remark 1. Note that our results (28) and (33) are in agreement with the results obtained in [5], while the other results are new, which are not found elsewhere.

Remark 2. Note that, if $R = \mu = 0$, then we have the trivial solution.

3.2. On Solving (2) Using the Method of Section 2. In this subsection, we apply the generalized projective Riccati equations method of Section 2 to find new soliton and periodic solutions of (2). To this aim, we use the following wave transformation:

$$U(X, Y, Z, T) = U(\xi), \quad \xi = \alpha X + \beta Y + \gamma Z + \nu T, \tag{41}$$

to reduce (2) to the following nonlinear ODE:

$$VU' - AUU' - (B\gamma^2 + C(\alpha^2 + \beta^2))U''' = 0, \tag{42}$$

where α, β, γ are the direction cosines, and ν is the quantum ion-acoustic wave speed and $V = \nu/\gamma$. Integrating (42) with respect to ξ once and vanishing the constant of integration, we find the following nonlinear ODE:

$$VU - \frac{1}{2}AU^2 - (B\gamma^2 + C(\alpha^2 + \beta^2))U'' = 0. \tag{43}$$

Balancing U'' with U^2 gives $N = 2$. Therefore, (6) reduces to

$$\begin{aligned}
 U(\xi) &= A_0 + A_1\sigma(\xi) + A_2\sigma^2(\xi) + B_1\tau(\xi) \\
 &\quad + B_2\sigma(\xi)\tau(\xi),
 \end{aligned} \tag{44}$$

where A_i ($i = 0, 1, 2$), B_1 and B_2 are constants to be determined such that $A_2 \neq 0$ or $B_2 \neq 0$.

Substituting (44) and using (7) and (8) into (43), the left-hand side of (43) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$.

Setting the coefficients of this polynomial to be zero yields the following system of algebraic equations:

$$\begin{aligned}
 \sigma^4 : & \frac{1}{2} \frac{A}{R} \varepsilon B_2^2 (\mu^2 + r) - \frac{1}{2} A A_2^2 = 0, \\
 \sigma^3 : & 2 (B\gamma^2 + C(\alpha^2 + \beta^2)) \mu \varepsilon A_2 - A A_1 A_2 \\
 & - \frac{1}{2} A \varepsilon \left(2\mu B_2^2 - \frac{2}{R} B_1 B_2 (\mu^2 + r) \right) = 0, \\
 \sigma^3 \tau : & A A_2 B_2 \\
 & - \frac{4}{R} \varepsilon^3 B_2 (\mu^2 + r) (B\gamma^2 + C(\alpha^2 + \beta^2)) = 0, \\
 \sigma^2 : & V A_2 - \frac{1}{2} A (A_1^2 + 2A_0 A_2) \\
 & + \frac{1}{2} A \varepsilon \left(R B_2^2 + \frac{1}{R} B_1^2 (\mu^2 + r) - 4\mu B_1 B_2 \right) = 0, \\
 \sigma^2 \tau : & A (A_1 B_2 + A_2 B_1) \\
 & + (B\gamma^2 + C(\alpha^2 + \beta^2)) (\varepsilon^2 A_1 - 6\mu \varepsilon B_2) \\
 & + 8 (B\gamma^2 + C(\alpha^2 + \beta^2)) \mu \varepsilon^3 B_2 = 0, \\
 \sigma : & V A_1 - \frac{1}{2} A \varepsilon (2\mu B_1^2 - 2R B_1 B_2) - A A_0 A_1 = 0, \\
 \sigma \tau : & V B_2 - (B\gamma^2 + C(\alpha^2 + \beta^2)) (5R \varepsilon B_2 - 3\mu \varepsilon B_1) \\
 & - A (A_0 B_2 + A_1 B_1) \\
 & + 4 (B\gamma^2 + C(\alpha^2 + \beta^2)) R \varepsilon^3 B_2 = 0, \\
 \tau : & A A_0 B_1 - V B_1 + 2 (B\gamma^2 + C(\alpha^2 + \beta^2)) R \varepsilon B_1 = 0, \\
 \sigma^0 : & V A_0 - \frac{1}{2} A A_0^2 - (B\gamma^2 + C(\alpha^2 + \beta^2)) \varepsilon B_2 \\
 & + \frac{1}{2} A R \varepsilon B_1^2 = 0.
 \end{aligned} \tag{45}$$

According to Step 5 of Section 2, there are three cases of solutions of the algebraic equations (45) to be discussed as follows.

Case 1 ($\varepsilon = -1, r = -1$). It leads to the following results.

Result 1. We have

$$\begin{aligned}
 A_0 &= 0, \\
 A_1 &= \frac{6\mu (B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\
 A_2 &= -\frac{6(\mu^2 - 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\
 B_1 &= 0,
 \end{aligned}$$

$$\begin{aligned}
 B_2 &= \pm \frac{6\sqrt{\mu^2 - 1} (B\gamma^2 + C(\alpha^2 + \beta^2))}{A\sqrt{R}}, \\
 V &= R (B\gamma^2 + C(\alpha^2 + \beta^2)).
 \end{aligned} \tag{46}$$

From (11), (44), and (46), we deduce the bell-kink-shaped soliton solution of (2) as follows:

$$\begin{aligned}
 U_{11}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\
 &\cdot \frac{R \operatorname{sech}(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} \left(\frac{(1 - \mu^2) \operatorname{sech}(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} \right. \\
 &\left. \left. \pm \frac{\sqrt{\mu^2 - 1} \tanh(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} + \mu \right) \right\},
 \end{aligned} \tag{47}$$

where $R > 0$, $\mu^2 > 1$ and $\xi = \alpha X + \beta Y + \gamma Z + [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

Result 2. We have

$$\begin{aligned}
 A_0 &= -\frac{2R (B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\
 A_1 &= \frac{6\mu (B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\
 A_2 &= -\frac{6(\mu^2 - 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\
 B_1 &= 0, \\
 B_2 &= \pm \frac{6\sqrt{\mu^2 - 1} (B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\
 V &= -R (B\gamma^2 + C(\alpha^2 + \beta^2)).
 \end{aligned} \tag{48}$$

In this result, we deduce the bell-kink-shaped soliton solution of (2) as follows:

$$\begin{aligned}
 U_{12}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\
 &\cdot \frac{R \operatorname{sech}(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} \left(\frac{(1 - \mu^2) \operatorname{sech}(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} \right. \\
 &\left. \left. \pm \frac{\sqrt{\mu^2 - 1} \tanh(\sqrt{R}\xi)}{\mu \operatorname{sech}(\sqrt{R}\xi) + 1} + \mu \right) \right. \\
 &\left. - \frac{2R (B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right\},
 \end{aligned} \tag{49}$$

where $R > 0$, $\mu^2 > 1$ and $\xi = \alpha X + \beta Y + \gamma Z - [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

Case 2 ($\varepsilon = -1$, $r = 1$). It leads to the following results:

Result 1. We have

$$\begin{aligned} A_0 &= 0, \\ A_1 &= \frac{6\mu(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_2 &= -\frac{6(\mu^2 + 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\ B_1 &= 0, \\ B_2 &= \pm \frac{6\sqrt{\mu^2 + 1}(B\gamma^2 + C(\alpha^2 + \beta^2))}{A\sqrt{R}}, \\ V &= R(B\gamma^2 + C(\alpha^2 + \beta^2)). \end{aligned} \quad (50)$$

From (12), (44), and (50), we deduce the singular bell-kink-shaped soliton solution of (2) as follows:

$$\begin{aligned} U_{21}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\ &\cdot \frac{R \operatorname{csch}(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} \left(-\frac{(\mu^2 + 1) \operatorname{csch}(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} \right. \\ &\left. \left. \pm \frac{\sqrt{\mu^2 + 1} \coth(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} + \mu \right) \right\}, \end{aligned} \quad (51)$$

where $R > 0$ and $\xi = \alpha X + \beta Y + \gamma Z + [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

Result 2. We have

$$\begin{aligned} A_0 &= -\frac{2R(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_1 &= \frac{6\mu(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_2 &= -\frac{6(\mu^2 + 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\ B_1 &= 0, \\ B_2 &= \pm \frac{6\sqrt{\mu^2 + 1}(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\ V &= -R(B\gamma^2 + C(\alpha^2 + \beta^2)). \end{aligned} \quad (52)$$

In this result, we deduce the singular bell-kink-shaped soliton solution of (2) as follows:

$$\begin{aligned} U_{22}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\ &\cdot \frac{R \operatorname{csch}(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} \left(-\frac{(\mu^2 + 1) \operatorname{csch}(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} \right. \\ &\left. \left. \pm \frac{\sqrt{\mu^2 + 1} \coth(\sqrt{R}\xi)}{\mu \operatorname{csch}(\sqrt{R}\xi) + 1} + \mu \right) \right. \\ &\left. - \frac{2R(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right\}, \end{aligned} \quad (53)$$

where $R > 0$ and $\xi = \alpha X + \beta Y + \gamma Z - [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

Case 3 ($\varepsilon = 1$, $r = -1$). It leads to the following results.

Result 1. We have

$$\begin{aligned} A_0 &= 0, \\ A_1 &= -\frac{6\mu(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_2 &= \frac{6(\mu^2 - 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\ B_1 &= 0, \\ B_2 &= \pm \frac{6\sqrt{1 - \mu^2}(B\gamma^2 + C(\alpha^2 + \beta^2))}{A\sqrt{R}}, \\ V &= -R(B\gamma^2 + C(\alpha^2 + \beta^2)). \end{aligned} \quad (54)$$

From (13), (44), and (54), we deduce the periodic solutions of (2) as follows:

$$\begin{aligned} U_{31}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\ &\cdot \frac{R \sec(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} \left(\frac{(\mu^2 - 1) \sec(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} \right. \\ &\left. \left. \pm \frac{\sqrt{1 - \mu^2} \tan(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} - \mu \right) \right\}, \\ U_{32}(X, Y, Z, T) &= \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \end{aligned}$$

FIGURE 1: Plot solution $|q_{11}(x, t)|$ of (28) with $a_1 = -1$, $b_1 = 11/5$, $c_1 = -2$, $m = \nu = 1$.

$$\left. \begin{aligned} & \cdot \frac{R \csc(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} \left(\frac{(\mu^2 - 1) \csc(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} \right. \\ & \left. \pm \frac{\sqrt{1 - \mu^2} \cot(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} - \mu \right) \Bigg\}, \end{aligned} \right\} \quad (55)$$

where $R > 0$, $\mu^2 < 1$ and $\xi = \alpha X + \beta Y + \gamma Z - [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

Result 2. We have

$$\begin{aligned} A_0 &= \frac{2R(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_1 &= -\frac{6\mu(B\gamma^2 + C(\alpha^2 + \beta^2))}{A}, \\ A_2 &= \frac{6(\mu^2 - 1)(B\gamma^2 + C(\alpha^2 + \beta^2))}{AR}, \\ B_1 &= 0, \\ B_2 &= \pm \frac{6\sqrt{1 - \mu^2}(B\gamma^2 + C(\alpha^2 + \beta^2))}{A\sqrt{R}}, \\ V &= R(B\gamma^2 + C(\alpha^2 + \beta^2)). \end{aligned} \quad (56)$$

In this result, we deduce the periodic solutions of (2) as follows:

$$U_{33}(X, Y, Z, T) = \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right. \\ \left. \cdot \frac{R \sec(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} \left(\frac{(\mu^2 - 1) \sec(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} \right. \right.$$

$$\left. \begin{aligned} & \pm \frac{\sqrt{1 - \mu^2} \tan(\sqrt{R}\xi)}{\mu \sec(\sqrt{R}\xi) + 1} - \mu \right) \\ & \left. + \frac{2R(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right\}, \end{aligned}$$

$$U_{34}(X, Y, Z, T) = \left\{ \frac{6(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right.$$

$$\left. \begin{aligned} & \cdot \frac{R \csc(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} \left(\frac{(\mu^2 - 1) \csc(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} \right. \\ & \pm \frac{\sqrt{1 - \mu^2} \cot(\sqrt{R}\xi)}{\mu \csc(\sqrt{R}\xi) + 1} - \mu \right) \\ & \left. + \frac{2R(B\gamma^2 + C(\alpha^2 + \beta^2))}{A} \right\}, \end{aligned} \right\} \quad (57)$$

where $R > 0$, $\mu^2 < 1$ and $\xi = \alpha X + \beta Y + \gamma Z + [R\gamma(B\gamma^2 + C(\alpha^2 + \beta^2))]T$.

4. Graphical Representations of Some Solutions

In this section, we draw graphs of some exact solutions. The obtained soliton and periodic solutions are kink and antikink solitons, bell (bright) and antibell (dark) solitary wave solutions, and trigonometric solutions.

Let us now examine Figures 1–3 as they illustrate some of our solutions obtained in this article. To this aim, we select

FIGURE 2: Plot solution $|q_{23}(x, t)|$ of (37) with $a_1 = 23$, $b_1 = -12$, $c_1 = 168$, $\mu = 1$, $\nu = -1$, $m = 2$.

FIGURE 3: Plot solution $|q_{31}(x, t)|$ of (39) with $a_1 = -19$, $b_1 = -4$, $c_1 = 20$, $\mu = 3$, $\nu = 1$, $m = 2$.

some special values of the parameters obtained, for example, in some of the solutions (28), (37), and (39) of the NLS equation with fourth-order dispersion and dual power law nonlinearity (1). For more convenience the graphical representations of these solutions are shown in Figures 1, 2, and 3.

From Figures 1, 2, and 3, one can see that the obtained solutions possess the solitary wave solutions and periodic wave solution. Also, these figures express the behavior of these solutions which give some perspective to readers on how the behavior solutions are produced.

5. Conclusions

In this article, based on the generalized projective Riccati equations method described in Section 2, we have obtained

many new types of soliton and periodic solutions for the two higher-order nonlinear PDEs, namely, the NLS equation (1) and the nonlinear QZK equation (2). The exact solutions of these equations include kink and antikink solitons, bell (bright) and antibell (dark) solitary wave solutions, and periodic solutions. Comparing our results in this article with the well-known results of [5, 6, 21–23], we conclude that all our results are new and not found elsewhere except for results (28) and (33) which are in agreement with results (19) and (20) obtained in [5]. The proposed method of this article is effective and can be applied to many other nonlinear PDEs. Finally, all solutions obtained in this article have been checked with the Maple 2015 by putting them back into the original equations (1) and (2).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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