Research Article

Autonomous Jerk Oscillator with Cosine Hyperbolic Nonlinearity: Analysis, FPGA Implementation, and Synchronization

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A two-parameter autonomous jerk oscillator with a cosine hyperbolic nonlinearity is proposed in this paper. Firstly, the stability of equilibrium points of proposed autonomous jerk oscillator is investigated by analyzing the characteristic equation and the existence of Hopf bifurcation is verified using one of the two parameters as a bifurcation parameter. By tuning its two parameters, various dynamical behaviors are found in the proposed autonomous jerk oscillator including periodic attractor, one-scroll chaotic attractor, and coexistence between chaotic and periodic attractors. The proposed autonomous jerk oscillator has period-doubling route to chaos with the variation of one of its parameters and reverse period-doubling route to chaos with the variation of its other parameter. The proposed autonomous jerk oscillator is modelled on Field Programmable Gate Array (FPGA) and the FPGA chip statistics and phase portraits are derived. The chaotic and coexistence of attractors generated in the proposed autonomous jerk oscillator are confirmed by FPGA implementation of the proposed autonomous jerk oscillator. A good qualitative agreement is illustrated between the numerical and FPGA results. Finally synchronization of unidirectional coupled identical proposed autonomous jerk oscillators is achieved using adaptive sliding mode control method.

1. Introduction

There has been a significant increase in studies of chaotic oscillators over the past decades because of their complex behaviors and their promising applications [1–9]. There are increasing works on chaos in jerk oscillators [10–12]. It is worth noting that if the scalar $x(t)$ is denoted as a physical variable at time $t$, the third derivative $\dot{x}(t)$ presents the jerk [13]. Different jerk oscillators with chaos were summarized by Sprott [14]. By using Josephson junctions, Yalcin constructed a general jerk circuit with multiscroll and hypercube attractors [15]. Multiscroll attractors in jerk circuits were presented by Ma et al. [16]. An elegant chaotic oscillators based on jerk equation was implemented in a circuit where the nonlinearity was provided by a single diode [17]. A simple chaotic jerk circuit was used in a sound encryption scheme [18]. Especially, a three-dimensional novel jerk chaotic oscillator with two hyperbolic sinusoidal nonlinearities was reported in [19].

Researchers have shown an increased interest in multistability [20–26]. Multistability is associated with the presence
of coexisting attractors with different initial conditions [20]. Jerk oscillators can exhibit multistability despite their simplicity [27–31]. Recently, Njiteck et al. discovered the coexistence of multiple attractors and crisis route to chaos in a chaotic jerk circuit [27] and a simple autonomous jerk oscillator with multiple attractors was introduced in [28]. Kengne et al. performed an analysis of a jerk circuit including a pair of semiconductor diodes connected in antiparallel and investigated its multiple attractors [29]. It is interesting that, by applying the diode bridge memristor into the original jerk circuit, authors proposed a novel hybrid diode based jerk circuit [30]. This work revealed that the hybrid diode based jerk circuit exhibits rich dynamical behaviors including multiple coexisting self-excited attractors. Derived from the autonomous jerk circuit, an autonomous memristor-based jerk circuit was constructed [31]. Interestingly, for the same values of system parameters, the coexistence of four different attractors was obtained in such a memristor-based jerk circuit.

Motivated by published results related to jerk oscillators, some questions arose as to know, e.g., if a jerk oscillator with cosine hyperbolic nonlinearity can exhibit multistability or how such oscillator can synchronize. The aim of our work is to explore aspects of the unanswered questions. Our paper is structured as follows. Section 2 is devoted to the theoretical analysis of proposed autonomous jerk oscillator with a cosine hyperbolic term. FPGA implementation of proposed autonomous jerk circuit exhibits rich dynamical behaviors including multiple coexisting self-excited attractors. Derived from the autonomous jerk circuit, authors proposed a novel hybrid diode based jerk circuit as function of parameter $a$ [three.fitted/zero.fitted].

\[ \frac{dx}{dt} = z, \quad \frac{dy}{dt} = ax - y - bz - \cosh(x), \quad \frac{dz}{dt} = y, \]

(1a) (1b) (1c)

where $x, y, z$ are state variables of the oscillator and $a, b$ two positive parameters. System (1a), (1b), and (1c) can be converted to a jerk oscillator, as follows:

\[ \frac{d^3 x}{dt^3} = -\frac{d^2 x}{dt^2} - b \frac{dx}{dt} + ax - \cosh(x). \]

(2)

System (1a), (1b), and (1c) is dissipative because $\nabla V = \partial(dx/dt)/\partial x + \partial(dy/dt)/\partial y + \partial(dz/dt)/\partial z = -1$. The equilibrium points of system (1a), (1b), and (1c) are obtained by solving $dx/dt = 0, dy/dt = 0, dz/dt = 0$, which gives

\[ y^* = z^* = 0, \]

(3a)

\[ ax^* - \cosh(x^*) = 0. \]

(3b)

Equation (3b) cannot be solved analytically. We thus use the Newton-Raphson method to find the value of $x^*$. Depending on the value of parameter $a$, (3b) presents no roots for $0 < a < 1.509$ and one root ($x^*$) for $a \geq 1.509$. Therefore, for $0 < a < 1.509$, system (1a), (1b), and (1c) has no equilibrium points where, for $a \geq 1.509$, it has two equilibrium points $E_1 = (x_1^*, 0, 0)$ and $E_2 = (x_2^*, 0, 0)$ with $x_1^* > x_2^*$ (see Figure 1). The characteristic equation evaluated at the equilibrium point $E = (x^*, 0, 0)$ is

\[ \lambda^3 + \lambda^2 + b\lambda - a + \sinh(x^*) = 0. \]

(4)

According to the Routh-Hurwitz criterion, the real parts of all the roots $\lambda$ of (4) are negative if and only if

\[ -a + \sinh(x^*) > 0 \]

(4b)

\[ a + b - \sinh(x^*) > 0 \]

(4c)

The stability analysis of equilibrium point $E_1$ as function of the parameter $a$ of system (1a), (1b), and (1c) is depicted in Figure 1.

From Figure 1, one can notice that equilibrium point $E_2$ is always unstable for $1.65 \leq a \leq 4$, whereas the equilibrium point $E_1$ is stable for $1.65 \leq a \leq 2.269$ and unstable for $a > 2.269$. Since the equilibrium points $E_1$ is stable for $1.65 \leq a \leq 2.269$ but it changes stability properties at $a = 2.269$, system (1a), (1b), and (1c) can exhibit a Hopf bifurcation from equilibrium point $E_1$ when the parameter $a$ varies.

2. Theoretical Analysis of Proposed Autonomous Jerk Oscillator

In chapter three of “Elegant Chaos: Algebraically Simple flow” book published in 2010, Sprott proposed a list of sixteen autonomous chaotic jerk oscillators with different nonlinearities called memory oscillators [14]. The nonlinearities of memory oscillators include quadratic, cubic, quintic, absolute, maximum, sign, exponential, sine, and tangent hyperbolic functions. Inspired by [14], in this work we introduce an autonomous jerk oscillator with a cosine hyperbolic nonlinearity described by

\[ \frac{dx}{dt} = z, \]

\[ \frac{dy}{dt} = ax - y - bz - \cosh(x), \]

\[ \frac{dz}{dt} = y, \]

(1a) (1b) (1c)

Figure 1: (Color online) Stability diagram of equilibrium points $E_{1,2}$ versus the parameter $a$ for $b = 3$. Solid black lines indicate the stable branches and the dashed red lines the unstable branches.
Theorem 1. System (1a), (1b), and (1c) undergoes a Hopf bifurcation at the equilibrium point $E_1$ if $b > 0$ and the parameter $a$ passes through the critical value $a_H = -b + \sinh(x_1^*)$.

Proof. Substituting $\lambda = i\omega (\omega > 0)$ into (4) and separating real and imaginary parts, we obtain

\[
\omega = \omega_0 = \sqrt{b}, \quad a = a_H = -b - \sinh(x_1^*). \tag{5a}
\]

Differentiating both sides of (4) with respect to $a$, we can obtain

\[
3\lambda^2 \frac{d\lambda}{da} + 2\lambda \frac{d\lambda}{da} + b \frac{d\lambda}{da} - 1 = 0 \tag{6a}
\]

and

\[
\frac{d\lambda}{da} = \frac{1}{3\lambda^2 + 2\lambda + b} \tag{6b}
\]

then

\[
\text{Re}\left(\frac{d\lambda}{da} \bigg|_{a=a_H, \lambda=\lambda_0}\right) = \frac{-1}{2(b+1)} \neq 0. \tag{7}
\]

All the conditions for Hopf bifurcation to occur are met; therefore system (1a), (1b), and (1c) undergoes a Hopf bifurcation at $E_1$ when $a_H = -b + \sinh(x_1^*)$ and periodic solutions will exist in a neighbourhood of the point $a_H$ (provided that $b > 0$ holds). If $b = 3$, the critical value is $a = a_H \approx 2.269$. For $a = 2.2 < a_H$, the trajectories of jerk system (1a), (1b), and (1c) converge to the equilibrium point $E_1$ whereas, for $a = 2.5 > a_H$, system (1a), (1b), and (1c) exhibits a limit cycle (not shown).

We plot in Figure 2 a two-parameter $(a, b)$ bifurcation diagram depicting the dynamical behaviors of system (1a), (1b), and (1c).

Figure 2 indicated that system (1a), (1b), and (1c) can display unbounded orbits and periodic and chaotic behaviors. In Figure 3, we firstly fix $b = 3$ and plot the bifurcation diagram with respect to $a$ and the related largest Lyapunov exponent.

The bifurcation diagram of the output $x(t)$ in Figure 2(a) shows that the trajectories of system (1a), (1b), and (1c) converge to the equilibrium point $E_1$ up to $a \approx 2.269$ where a Hopf bifurcation occurs followed by a limit cycle motion (i.e., period-1 oscillations) and period-doubling to chaos interspersed with periodic windows. The largest Lyapunov exponent found in Figure 3(b) confirms the chaotic behavior found in Figure 3(a). The chaotic behavior is illustrated in Figure 4 for a specific value of parameter $a$.

The trajectories of chaotic attractor are swirling around one of the two equilibrium points in Figure 4. This is a signature of one-scroll chaotic attractor.

Secondly, for $a = 3.3$, sample results showing the bifurcation diagram versus $b$ and the related plot of Largest Lyapunov exponent are provided in Figure 5.

By varying parameter $b$ from 2.87 to 3.8, the bifurcation diagram of the output $x(t)$ in Figure 5(a) displays chaotic behavior interspersed with periodic windows. For $b > 3.18$, period-12-oscillations, period-6-oscillations, and period-3-oscillations are found, respectively, up to $b < 3.2012$ followed by reverse period-doubling to chaos interspersed with periodic windows. By varying parameter $b$ from 3.8 to 2.87 [see red dots of Figure 5(a)], the output $x(t)$ displays the same dynamical behaviors as shown by black dots of Figure 5(a) in the ranges $2.87 \leq b \leq 3.182$ and $3.2012 \leq b \leq 3.8$, while, in the range $3.182 < b < 3.2012$, the output $x(t)$ presents chaotic behavior. By comparing black dots of Figure 5(a) and red dots of Figure 5(a), one can notice that system (1a), (1b), and (1c) displays coexistence of period-6-oscillations and chaotic attractors in the range $3.182 < b < 3.188$ and coexistence of period-3-oscillations and chaotic attractors in the range $3.188 < b < 3.2012$. The largest Lyapunov exponent shown in Figure 5(b) confirms the chaotic behavior found in Figure 5(a). Figure 6 depicts the phase portraits of coexisting attractors found in Figure 5(a) for specific values of parameter $b$.

For $b = 3.185$, system (1a), (1b), and (1c) can exhibit either one-scroll chaotic attractor or period-6-oscillations depending on initial conditions as shown in Figure 6(a), while, in Figure 6(b) for $b = 3.189$, system (1a), (1b), and (1c) can exhibit either one-scroll chaotic attractor or period-3-oscillations depending on initial conditions. Figure 7 presents the basin of attraction of system (1a), (1b), and (1c) in the plane $y = 0$ for $a = 3.3$ and $b = 3.189$.

In Figure 7, red, green, and black regions contain initial conditions that lead to unbounded orbits, periodic and chaotic attractors, respectively. One can see from the basin of attraction that the possibility of occurrence of unbounded orbits is greater than the ones of periodic and chaotic attractors.

\section*{3. FPGA Implementation of Proposed Autonomous Jerk Oscillator}

There have been many literatures about implementation of chaotic oscillators using FPGA like FPGA based multiscroll attractor discussed in [32–39], digital chaotic oscillators and
Figure 3: The bifurcation diagrams depicting the local maxima (black dots) and local minima (gray dots) of $x(t)$ (a) and the largest Lyapunov exponents (b) versus the parameter $a$ for $b = 3$.

Figure 4: 2D phase portrait of system (1a), (1b), and (1c) for $a = 3.3$ and $b = 3$. The initial conditions are $(x(0), y(0), z(0)) = (1, 0, 0.5)$.
the system (8a), (8b), and (8c) is implemented and the initial conditions for the next iteration can be obtained. By repeating this process of iteration we can derive the discrete chaotic system. The absolute value operation only needs to set the first bit of the 32-bit integer to 0. The discrete state equations are implemented using hardware and software cosimulations and the needed basic arithmetic operators are implemented using the Xilinx system generator toolkit. We used the Kintex 7 chipset (xc7k160tbg) for the cosimulations and Matlab Simulink is used to plot the phase plots. The latency of the arithmetic blocks is kept at 3 and the maximum clock frequency of the FPGA used is 437 MHz. The register-transfer level (RTL) schematic of hyperbolic cosine function given by (9) is not shown while the RTL schematic of proposed autonomous jerk oscillator is shown in Figure 8.

Table 1 shows the resources utilized by the proposed autonomous jerk oscillator for implementing in Kintex-7.

Figure 9 shows the 2D phase portraits of the proposed autonomous jerk oscillator.

Figure 10 shows the coexisting attractors exhibited by the FPGA implemented proposed autonomous jerk oscillator for two different initial conditions.

One-scroll chaotic attractor and coexistence of attractors are clearly seen from Figures 9 and 10, respectively. By
oscillators are using active control method.

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synchronization designs of this type of proposed jerk oscil-
proposed jerk oscillators in order to promote chaos-based
Here we present synchronization results of coupled chaotic
numerical simulations and FPGA results.

Table I: Resources consumed by the FPGA implemented of proposed autonomous jerk oscillator.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Utilization</th>
<th>Available</th>
<th>Utilization %</th>
</tr>
</thead>
<tbody>
<tr>
<td>LUT</td>
<td>880</td>
<td>101400</td>
<td>0.87</td>
</tr>
<tr>
<td>FF</td>
<td>522</td>
<td>202800</td>
<td>0.26</td>
</tr>
<tr>
<td>DSP</td>
<td>20</td>
<td>600</td>
<td>3.33</td>
</tr>
<tr>
<td>IO</td>
<td>97</td>
<td>285</td>
<td>34.04</td>
</tr>
<tr>
<td>BUFG</td>
<td>1</td>
<td>32</td>
<td>3.13</td>
</tr>
</tbody>
</table>

comparing Figures 9 and 10 with Figures 9 and 10, it can be noticed that there is a good qualitative agreement between the numerical simulations and FPGA results.

4. Synchronization of Unidirectional Coupled Proposed Autonomous Jerk Oscillators Using Adaptive Sliding Mode Control Method

Here we present synchronization results of coupled chaotic proposed jerk oscillators in order to promote chaos-based synchronization designs of this type of proposed jerk oscillator. Synchronization of chaotic systems has applications in secure communication and cryptography. The highly sensitive nature of chaotic oscillators to initial conditions makes it difficult to synchronize the oscillators with uncertainties and disturbance. Some well-known ways to synchronize chaotic oscillators are using active control method [49, 50] adaptive control method [51, 52], extended back stepping control method [53, 54], sliding mode control method [55, 56], adaptive sliding mode control method [56–59], etc. Sliding mode control method is applied to provide robustness in the face of internal and external disturbances. In section, we use adaptive sliding mode control method for synchronization of unidirectional coupled identical proposed autonomous jerk oscillators using adaptive sliding mode control method.

4.1. Problem Statement. Let the generalized master system be defined as
\[
\dot{x} = f(x) + F(x)c
\]
and the slave system as
\[
\dot{y} = g(y) + G(y)d + u(t)
\]
where \(f(x), g(y)\) are \(n \times 1\) row vector and \(F(x), G(y)\) are \(m \times n\) matrix elements of the master and slave systems, respectively; \(c, d\) are the unknown parameters of the systems, and \(u(t)\) is the controller to synchronize the systems. The control objective is to synchronize the slave system with initial condition \(y(0)\) with master system of initial condition \(x(0)\) such that the synchronization errors (12) approaches zero.

\[
\lim_{t \to \infty} e_i = y_i - x_i.
\]

The sliding mode controller design for synchronizing the two systems involves selection of sliding surface for the desired dynamics and designing the reaching law such that any point on the phase space is brought to the sliding surface in the presence of uncertainties.

Let us define the proportional integral sliding surface [60] as
\[
s_i = e_i + K \int e_i (\tau) d\tau
\]
where \(K\) is the proportional constant vector \([k_1, k_2, k_3]\).

The first derivative of the sliding surface is derived as
\[
\dot{s}_i = \dot{e}_i + Ke_i.
\]

For the existence of the sliding mode, it is necessary and sufficient that the sliding surface and its first derivative should be equal to zero. The error dynamics can be derived as
\[
\dot{e} = g(x_i) + G(x_i) \tilde{d} + u(t) - f(x_m) - F(x_m)c.
\]

In order to avoid the chattering phenomenon caused by discontinuous control signals, the adaptive controller used to synchronize the master and slave system is chosen as
\[
u(t) = -g(x_i) - G(x_i) \tilde{d} + f(x_m) - F(x_m)c - \tilde{e} - k \eta \tanh(s) - \rho s.
\]

where \(k, \eta, \rho\) are positive gain values, \(\tilde{e}, \tilde{d}\) are parameter estimates of master and slave systems, and \(s\) is the sliding surface.

Using (16) in (15), the error dynamics simplifies to
\[
\dot{e} = G(x_i) [d - \tilde{d}] - F(x_m) [c - \tilde{c}] - \eta \tanh(s) - \rho s.
\]

The stability of the proposed controller can be analyzed using the Lyapunov candidate function:
\[
V = \frac{1}{2} s^2 + \frac{1}{2} (d - \tilde{d})^2 + \frac{1}{2} (c - \tilde{c})^2.
\]
Figure 8: (Color online) RTL schematic of proposed autonomous jerk oscillator implemented in Kintex 7.

Figure 9: (Color online) Phase portraits of the FPGA implemented of proposed autonomous jerk oscillator with hardware-software co-simulation for parameter values $a = 3.3, b = 3$ and initial conditions $[1,0,0.5]$ with $h = 0.001$. 
The dynamics of the Lyapunov candidate function can be derived as follows:

\[ \dot{V} = s\dot{s} + (d - \hat{d})(-\hat{c}) + (c - \hat{c})(-\dot{\hat{c}}) \]  

Using (17), (16), and (14) in (19),

\[ \dot{V} \leq ks \left[ G(x_s) (d - \hat{d}) - F(x_m) (c - \hat{c}) - \eta \tanh(s) - \rho s \right] - (d - \hat{d})(\dot{\hat{d}}) - (c - \hat{c})(\dot{\hat{c}}). \]  

Let us define the parameter estimate laws as

\[ \dot{\hat{c}} = K_c s G(x_s), \]  

\[ \dot{\hat{d}} = K_d \rho s F(x_m), \]  

where \( K_c, K_d \) are positive constants. Using (21a) and (21b) in (20), we can solve the Lyapunov function dynamics as follows:

\[ \dot{V} \leq -\eta|s_w| - \rho s^2, \]  

where \( \eta \) and \( \rho \) are all positive, and \( \dot{V} \) is negative definite.

4.2. Numerical Verifications. For numerical validation of the proposed synchronization method, we use the proposed autonomous jerk oscillators as master and slave and apply the adaptive sliding mode control to achieve the synchronization. Let us define the master system as

\[ \dot{x}_m = z_m, \]  

\[ \dot{y}_m = a x_m - y_m - b z_m - \cosh(x_m), \]  

\[ \dot{z}_m = y_m, \]  

where \( a, b \) are the system parameters. The slave system with the adaptive sliding mode controllers \( (u_i) \) are defined as

\[ \dot{x}_s = z_s + u_x, \]  

\[ \dot{y}_s = a x_s - y_s - b z_s - \cosh(x_s) + u_y, \]  

\[ \dot{z}_s = y_s + u_z. \]  

The parameters of the slave system are assumed to be unknown with parameter estimates \( \hat{a}, \hat{b} \). Using the master
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Figure 11: Time history of synchronization error variables (a), synchronized states (b), and estimated parameters (c).

Using (17) and (18) with (13), the parameter update laws can be defined as follows:

\[
\dot{\hat{a}} = -s_y x_m, \quad (27a)
\]
\[
\dot{\hat{b}} = s_y z_m, \quad (27b)
\]
where \(\hat{a}, \hat{b}\) are the dynamics of the parameter estimates \(\hat{a}, \hat{b}\). For numerical simulations, we take the initial conditions of the master system (25a), (25b), and (25c) as \([-1, 1, 2]\), slave system as \([2, 2, 4]\), parameter estimates as \(\hat{a}(0) = 2, \hat{b}(0) = 5\), and the proportional constants \(K = [1, 1, 1]\). Figure 11 shows the dynamics of the synchronization errors and synchronized states of the master and slave systems and estimated parameters.

When \(t > 0.3\), the synchronization error variables converge to zero with exponentially asymptotical speed [see Figure 11(a)] and thereby guaranteeing the synchronization between the master system (23a), (23b), and (23c) and slave system (24a), (24b), and (24c) [see Figure 11(b)]. The unknown parameters of slave system (24a), (24b), and (24c) are simultaneously successfully estimated to their values [see Figure 11(c)].

Let the adaptive sliding mode controllers be chosen as follows:

\[
\begin{align*}
\dot{e}_x &= e_z + u_x, \quad (25a) \\
\dot{e}_y &= \hat{a} x_s - \hat{b} z_s - e_y - \cosh(x_s) - a x_m + b z_m \\
&\quad + \cosh(x_m) + u_y, \quad (25b) \\
\dot{e}_z &= e_y + u_z. \quad (25c)
\end{align*}
\]

where \(\rho_i > 0, \gamma_i > 0\) are the sliding surface gains and \(k_i\) is the controller gain for \(i = x, y, z\).
5. Conclusion
In this paper, an autonomous jerk oscillator with a cosine hyperbolic nonlinearity and two parameters was proposed and studied. It was demonstrated that the Hopf bifurcation occurs near the equilibrium point as one of jerk oscillator parameter crosses the critical value. The proposed autonomous jerk oscillator exhibits periodic attractors, one-scroll chaotic attractors, and coexistence between chaotic and periodic attractors. Period-doubling route to chaos and reverse period-doubling route to chaos was found in proposed autonomous jerk oscillator with the variation of each of its two parameters. Further, the proposed autonomous jerk oscillator was implemented using Field Programmable Gate Array in order to show that the proposed autonomous jerk oscillator is hardware realizable. Finally, synchronization of unidirectional coupled identical proposed autonomous jerk oscillators was achieved using adaptive sliding mode control method.

Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

References


