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Research Article

Expanded (G/G^2) Expansion Method to Solve Separated Variables for the 2+1-Dimensional NNV Equation

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The traditional (G/G^2) expansion method is modified to extend the symmetric extension to the negative power term in the solution to the positive power term. The general traveling wave solution is extended to a generalized solution that can separate variables. By using this method, the solution to the detached variables of the symmetric extended form of the 2+1-dimensional NNV equation can be solved, also the soliton structure and fractal structure of Dromion can be studied well.

1. Introduction

The nature of this world is nonlinear, and linearity is only an approximation under certain conditions. And the reason why this large world is so colorful and varied is that there is a nonlinear relationship between things. In nonlinear science we start from the actual nonlinear phenomena; according to the laws of physics, the corresponding mathematical models are established under reasonable approximations, and then the corresponding laws are obtained by studying these mathematical models. A large part of these equations is nonlinear partial differential equations. It is not easy to solve these partial differential equations. For many years people have done a lot of work for this and have achieved a series of important results, such as the Inverse Scattering Transformation(IST) [1], Darboux transform [2, 3], Lie group and nonclassical Lie group method [4, 5], Hirota bilinear method [6–8], and other important methods. Among them, the function expansion [9–19] has the advantages of simplicity, directness, and ease of operation. So there is a lot of use in solving nonlinear partial differential equations.

Li proposed an expansion method in [20] to solve the new traveling wave solution for the Vakhnenko equation. However, for a large number of nonlinear systems, it is difficult to fully describe only the traveling wave solution, and it will inevitably lose other solutions with a rich structure. This paper reconstructs this method and extends the expansion of the solution symmetrically to negative power. Moreover, the variable is separated by the solution, and more forms of solutions are obtained. The symmetric extension of the 2+1-dimensional NNV equation is solved by using this method. A solution of this equation is studied to get soliton solution and fractal structure solution.

2. Separation Variable Solutions to Symmetric Extensions of 2+1-Dimensional NNV Equations

The 2+1-dimensional NNV equation

$$u_t + au_{xxx} + bu_{yyy} + cu_x + du_y - 3a(uv)_x - 3b(uw)_y = 0 \tag{1}$$

$$u_x = v_y \tag{2}$$

$$u_y = w_x \tag{3}$$

is a Lax integrable generalization of all isotropic properties of the 1+1-dimensional KdV equation. We first determine the maximum power of the expansion term in u, v, w expansion formula by the principle of homogeneous equilibrium [21–23] to be 2 and extend the symmetry extension of the solution to the negative power term (-2). Then set the expression for u, v, w to

$$u = a_{-2}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-2} + a_{-1}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-1} + a_0 + a_1\frac{G'(\phi)}{G(\phi)^2} + a_2\left(\frac{G'(\phi)}{G(\phi)^2}\right)^2 \quad (4)$$

$$v = b_{-2}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-2} + b_{-1}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-1} + b_0 + b_1\frac{G'(\phi)}{G(\phi)^2} + b_2\left(\frac{G'(\phi)}{G(\phi)^2}\right)^2 \quad (5)$$

$$w = c_{-2}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-2} + c_{-1}\left(\frac{G'(\phi)}{G(\phi)^2}\right)^{-1} + c_0 + c_1\frac{G'(\phi)}{G(\phi)^2} + c_2\left(\frac{G'(\phi)}{G(\phi)^2}\right)^2 \quad (6)$$

where $\phi, a_i, b_j, c_k, (i, j, k = -2..2)$ are functions of x, y, t.

At the same time $G(\xi)$ achieves an ordinary differential equation

$$\left(\frac{G'(\xi)}{G(\xi)^2}\right)' = \mu + \lambda\left(\frac{G'(\xi)}{G(\xi)^2}\right)^2 \quad (7)$$

It can be obtained from (7)

$$\mu\lambda < 0 : \frac{G'}{G^2} = -\frac{\sqrt{|\mu\lambda|}}{\lambda} + \frac{\sqrt{|\mu\lambda|}}{2}\frac{C_1\sinh\left(\sqrt{|\mu\lambda|}\xi\right) + C_2\cosh\left(\sqrt{|\mu\lambda|}\xi\right)}{C_1\cosh\left(\sqrt{|\mu\lambda|}\xi\right) + C_2\sinh\left(\sqrt{|\mu\lambda|}\xi\right)} \quad (8)$$

$$\mu\lambda > 0 : \frac{G'}{G^2} = \frac{C_1\cos\left(\sqrt{\mu\lambda}\xi\right) + C_2\sin\left(\sqrt{\mu\lambda}\xi\right)}{C_1\sin\left(\sqrt{\mu\lambda}\xi\right) - C_2\sin\left(\sqrt{\mu\lambda}\xi\right)} \quad (9)$$

$$\mu = 0, \lambda \neq 0 : \frac{G'}{G^2} = \frac{C_1}{\lambda(C_1\xi + C_2)} \quad (10)$$

In order to achieve the separation of variables, we set

$$\phi(x, y, t) = p(x, t) + q(y, t) \quad (11)$$

Substituting (4)~(6) into (1)~(3), if there is a high-order derivative of $G(\phi)$, then use the ordinary differential equation (7) to replace the first derivative of G. That is, That is, (1)~(3) become three power polynomials about (G'/G^2). Let the coefficient of each power of (G'/G^2) of this polynomial be zero, resulting in a set containing a_i, b_j, c_k partial differential equations. Then, (11) is substituted into the partial differential equation group, and finally the partial differential equation group is solved to obtain three sets of solutions.

First group is

$$\begin{aligned}
&a_{-2} = b_{-2} = c_{-2} = a_{-1} = b_{-1} = c_{-1} = a_1 = 0,\\
&a_0 = 2\lambda\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\mu,\\
&b_0 = \frac{2}{3}\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu - \frac{F_1(t)b}{((\partial/\partial x)p)a} + \frac{1}{3}\left(\frac{c}{a} + \frac{(\partial^3/\partial x^3)p}{(\partial/\partial x)p} + \frac{(\partial/\partial t)p}{((\partial/\partial x)p)a}\right),\\
&c_0 = \frac{2}{3}\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu + \frac{1}{3}\left(\frac{d}{b} + \frac{(\partial^3/\partial y^3)q}{(\partial/\partial y)q} + \frac{(\partial/\partial t)q}{((\partial/\partial y)q)b}\right) + \frac{F_1(t)}{(\partial/\partial y)q},\\
&b_1 = 2\left(\frac{\partial^2}{\partial x^2}p\right)\lambda,\\
&c_1 = 2\lambda\frac{\partial^2}{\partial y^2}q,\\
&a_2 = 2\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\lambda^2,\\
&b_2 = 2\left(\frac{\partial}{\partial x}p\right)^2\lambda^2,\\
&c_2 = 2\lambda^2\left(\frac{\partial}{\partial y}q\right)^2
\end{aligned} \quad (12)$$

Second Group is

$$\begin{aligned}
&a_{-1} = a_1 = b_1 = c_1 = a_2 = b_2 = c_2 = 0,\\
&a_{-2} = 2\mu^2\left(\frac{\partial}{\partial x}p\right)\frac{\partial}{\partial y}q,\\
&b_{-2} = 2\left(\frac{\partial}{\partial x}p\right)^2\mu^2,\\
&c_{-2} = 2\left(\frac{\partial}{\partial y}q\right)^2\mu^2,\\
&b_{-1} = -2\mu\frac{\partial^2}{\partial x^2}p,\\
&c_{-1} = -2\mu\frac{\partial^2}{\partial y^2}q,
\end{aligned}$$

$$
\begin{aligned}
a_0 &= 2\lambda\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\mu,\\
b_0 &= \frac{2}{3}\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu\\
&\quad+\frac{1}{3}\left(\frac{c}{a}+\frac{\left(\partial^3/\partial x^3\right)p}{(\partial/\partial x)\,p}+\frac{(\partial/\partial t)\,p}{((\partial/\partial x)\,p)\,a}\right)\\
&\quad-\frac{F_2(t)\,b}{((\partial/\partial x)\,p)\,a},\\
c_0 &= \frac{2}{3}\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu\\
&\quad+\frac{1}{3}\left(\frac{d}{b}+\frac{\left(\partial^3/\partial y^3\right)q}{(\partial/\partial y)\,q}+\frac{(\partial/\partial t)\,q}{((\partial/\partial y)\,q)\,b}\right)\\
&\quad+\frac{F_2(t)}{(\partial/\partial y)\,q}
\end{aligned}
\tag{13}
$$

The third group is

$$
\begin{aligned}
a_{-2} &= 2\mu^2\left(\frac{\partial}{\partial x}p\right)\frac{\partial}{\partial y}q,\\
b_{-2} &= 2\left(\frac{\partial}{\partial x}p\right)^2\mu^2,\\
c_{-2} &= 2\left(\frac{\partial}{\partial y}q\right)^2\mu^2,\\
a_{-1} &= a_1 = 0,\\
b_{-1} &= -2\mu\frac{\partial^2}{\partial x^2}p,\\
c_{-1} &= -2\mu\frac{\partial^2}{\partial y^2}q,\\
a_0 &= 4\lambda\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\mu,\\
b_0 &= \frac{1}{3}\left(\frac{c}{a}+\frac{\left(\partial^3/\partial x^3\right)p}{(\partial/\partial x)\,p}+\frac{(\partial/\partial t)\,p}{((\partial/\partial x)\,p)\,a}\right)\\
&\quad-\frac{4}{3}\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu-\frac{F_3(t)\,b}{((\partial/\partial x)\,p)\,a},\\
c_0 &= \frac{1}{3}\left(\frac{d}{b}+\frac{\left(\partial^3/\partial y^3\right)q}{(\partial/\partial y)\,q}+\frac{(\partial/\partial t)\,q}{((\partial/\partial y)\,q)\,b}\right)\\
&\quad-\frac{4}{3}\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu+\frac{F_3(t)}{(\partial/\partial y)\,q},\\
b_1 &= 2\lambda\frac{\partial^2}{\partial x^2}p,\\
c_1 &= 2\lambda\frac{\partial^2}{\partial y^2}q,\\
a_2 &= 2\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\lambda^2,\\
b_2 &= 2\left(\frac{\partial}{\partial x}p\right)^2\lambda^2,\\
c_2 &= 2\lambda^2\left(\frac{\partial}{\partial y}q\right)^2
\end{aligned}
\tag{14}
$$

where $F_1(t), F_2(t), F_3(t)$ is any function about t.Then (12)~(14) are substituted into (4)~(5) and three solutions to the 2+1-dimensional NNV equation are obtained.

$$
u_1 = 2\lambda\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\left(\mu+\lambda\left(\frac{G'}{G^2}\right)^2\right) \tag{15}
$$

$$
\begin{aligned}
v_1 &= \frac{2}{3}\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu+\frac{1}{3}\left(\frac{c}{a}+\frac{\left(\partial^3/\partial x^3\right)p}{(\partial/\partial x)\,p}\right.\\
&\quad\left.+\frac{(\partial/\partial t)\,p}{((\partial/\partial x)\,p)\,a}\right)-\frac{F_1(t)\,b}{((\partial/\partial x)\,p)\,a}+2\left(\frac{\partial^2}{\partial x^2}p\right)\lambda\\
&\quad\cdot\frac{G'}{G^2}+2\left(\frac{\partial}{\partial x}p\right)^2\lambda^2\left(\frac{G'}{G^2}\right)^2
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
w_1 &= \frac{2}{3}\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu+\frac{1}{3}\left(\frac{d}{b}+\frac{\left(\partial^3/\partial y^3\right)q}{(\partial/\partial y)\,q}\right.\\
&\quad\left.+\frac{(\partial/\partial t)\,q}{((\partial/\partial y)\,q)\,b}\right)+\frac{F_1(t)}{(\partial/\partial y)\,q}+2\lambda\left(\frac{\partial^2}{\partial y^2}q\right)\frac{G'}{G^2}\\
&\quad+2\lambda^2\left(\frac{\partial}{\partial y}q\right)^2\left(\frac{G'}{G^2}\right)^2
\end{aligned}
\tag{17}
$$

$$
u_2 = 2\left(\frac{\partial}{\partial y}q\right)\left(\frac{\partial}{\partial x}p\right)\mu\left(\lambda+\mu\left(\frac{G'}{G^2}\right)^{-2}\right) \tag{18}
$$

$$
\begin{aligned}
v_2 &= \frac{2}{3}\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu+\frac{1}{3}\left(\frac{c}{a}+\frac{\left(\partial^3/\partial x^3\right)p}{(\partial/\partial x)\,p}\right.\\
&\quad\left.+\frac{(\partial/\partial t)\,p}{((\partial/\partial x)\,p)\,a}\right)-\frac{F_2(t)\,b}{((\partial/\partial x)\,p)\,a}-2\mu\left(\frac{\partial^2}{\partial x^2}p\right)\\
&\quad\cdot\left(\frac{G'}{G^2}\right)^{-1}+2\left(\frac{\partial}{\partial x}p\right)^2\mu^2\left(\frac{G'}{G^2}\right)^{-2}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
w_2 &= \frac{2}{3}\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu+\frac{1}{3}\left(\frac{d}{b}+\frac{\left(\partial^3/\partial y^3\right)q}{(\partial/\partial y)\,q}\right.\\
&\quad\left.+\frac{(\partial/\partial t)\,q}{((\partial/\partial y)\,q)\,b}\right)+\frac{F_2(t)}{(\mathrm{d}/\mathrm{d}y)\,q}-2\mu\frac{d^2}{\mathrm{d}y^2}q\left(\frac{G'}{G^2}\right)^{-1}\\
&\quad+2\left(\frac{\partial}{\partial y}q\right)^2\mu^2\left(\frac{G'}{G^2}\right)^{-2}
\end{aligned}
\tag{20}
$$

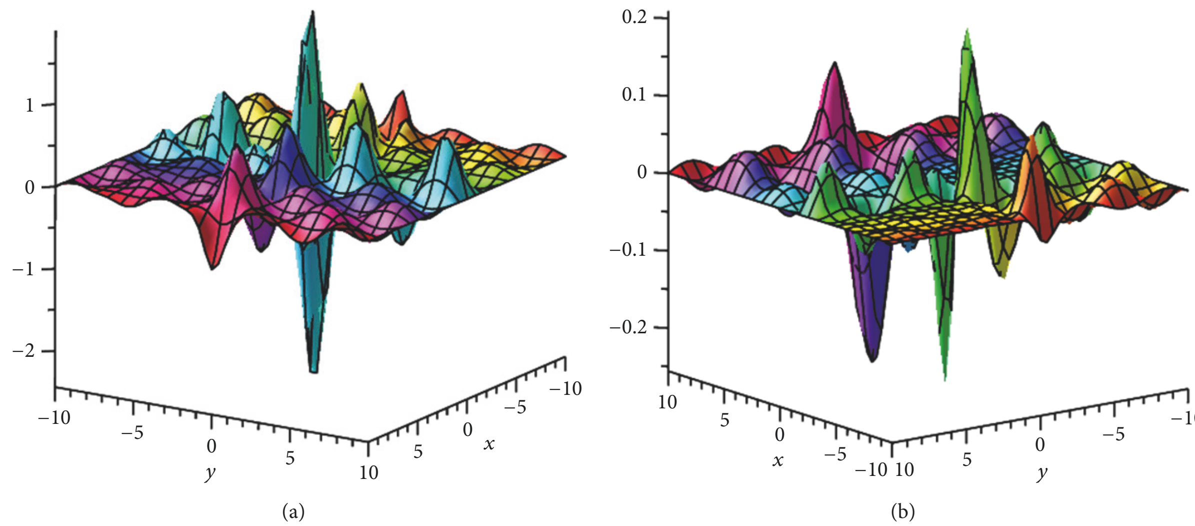

Figure 1: Figure (a) is a three-dimensional view of Dromion single soliton at t=-0.5 and figure (b) is a three-dimensional view of Dromion double soliton at t=2.9.

$$u_3 = 4\lambda\left(\frac{\partial}{\partial x}p\right)\left(\frac{\partial}{\partial y}q\right)\mu + 2\left(\frac{\partial}{\partial x}p\right)\left(\frac{\partial}{\partial y}q\right) \cdot \lambda^2\left(\frac{G'}{G^2}\right)^2 + 2\left(\frac{\partial}{\partial x}p\right)\left(\frac{\partial}{\partial y}q\right)\mu^2\left(\frac{G'}{G^2}\right)^{-2} \quad (21)$$

$$v_3 = \frac{1}{3}\left(\frac{c}{a} + \frac{\left(\partial^3/\partial x^3\right)p}{(\partial/\partial x)\,p} + \frac{(\partial/\partial t)\,p}{((\partial/\partial x)\,p)\,a} - 4\left(\frac{\partial}{\partial x}p\right)^2\lambda\mu\right) - \frac{F_3(t)\,b}{((\partial/\partial x)\,p)\,a} + 2\left(\frac{\partial^2}{\partial x^2}p\right)\lambda \cdot \frac{G'}{G^2} + 2\left(\frac{\partial}{\partial x}p\right)^2\lambda^2\left(\frac{G'}{G^2}\right)^2 - 2\mu\left(\frac{\partial^2}{\partial x^2}p\right) \cdot \left(\frac{G'}{G^2}\right)^{-1} + 2\left(\frac{\mathrm{d}}{\mathrm{d}x}p\right)^2\mu^2\left(\frac{G'}{G^2}\right)^{-2} \quad (22)$$

$$w_3 = \frac{1}{3}\left(\frac{d}{b} + \frac{\left(\partial^3/\partial y^3\right)q}{(\partial/\partial y)\,q}\frac{d}{b} + \frac{(\partial/\partial t)\,q}{((\partial/\partial y)\,q)\,b} - 4\left(\frac{\partial}{\partial y}q\right)^2\lambda\mu\right) + \frac{F_3(t)}{(\partial/\partial y)\,q} + 2\lambda\frac{\partial^2}{\partial y^2}q\frac{G'}{G^2} + 2\lambda^2\left(\frac{\partial}{\partial y}q\right)^2\left(\frac{G'}{G^2}\right)^2 - 2\mu\frac{\partial^2}{\partial y^2}q\left(\frac{G'}{G^2}\right)^{-1} + 2\left(\frac{\partial}{\partial y}q\right)^2\mu^2\left(\frac{G'}{G^2}\right)^{-2} \quad (23)$$

where $G = G(p+q)$ is the expression in (8)~(10).

3. Soliton Structure and Fractal Structure of the 2+1 Dimensional NNV Equation

Substituting (8) into (15) gives

$$u = 2\lambda\left(\frac{\partial}{\partial x}p\right)\left(\frac{\partial}{\partial y}q\right)\mu + 2\left(\frac{\partial}{\partial x}p\right)\left(\frac{\partial}{\partial y}q\right)\lambda^2\left(-\frac{\sqrt{|\mu\lambda|}}{\lambda} + \frac{\sqrt{|\mu\lambda|}}{2}B(p,q)\right)^2 \quad (24)$$

$$B(p,q) = \frac{C_1\sinh\left(\sqrt{|\mu\lambda|}\,(p+q)\right) + C_2\cosh\left(\sqrt{|\mu\lambda|}\,(p+q)\right)}{C_1\cosh\left(\sqrt{|\mu\lambda|}\,(p+q)\right) + C_2\sinh\left(\sqrt{|\mu\lambda|}\,(p+q)\right)} \quad (25)$$

Then select the parameters

$$C_1 = 4, \quad C_2 = 2, \quad \lambda = 1, \quad \mu = -1 \quad (26)$$

p, q selected as

$$p = \sin(x - 0.5t) + \tanh(x - 0.5t) \quad (27)$$

$$q = 0.4\sin(y) + \tanh(y) \quad (28)$$

as well as

$$p = 0.1\sin(x - 0.5t) + 0.5\tanh(x + t) + 0.5\tanh(0.5x - t) \quad (29)$$

$$q = 0.1\sin(0.9y) + 0.5\tanh(0.9y) \quad (30)$$

The periodic wave background Dromion soliton and double Dromion soliton can be obtained as shown in Figure 1.

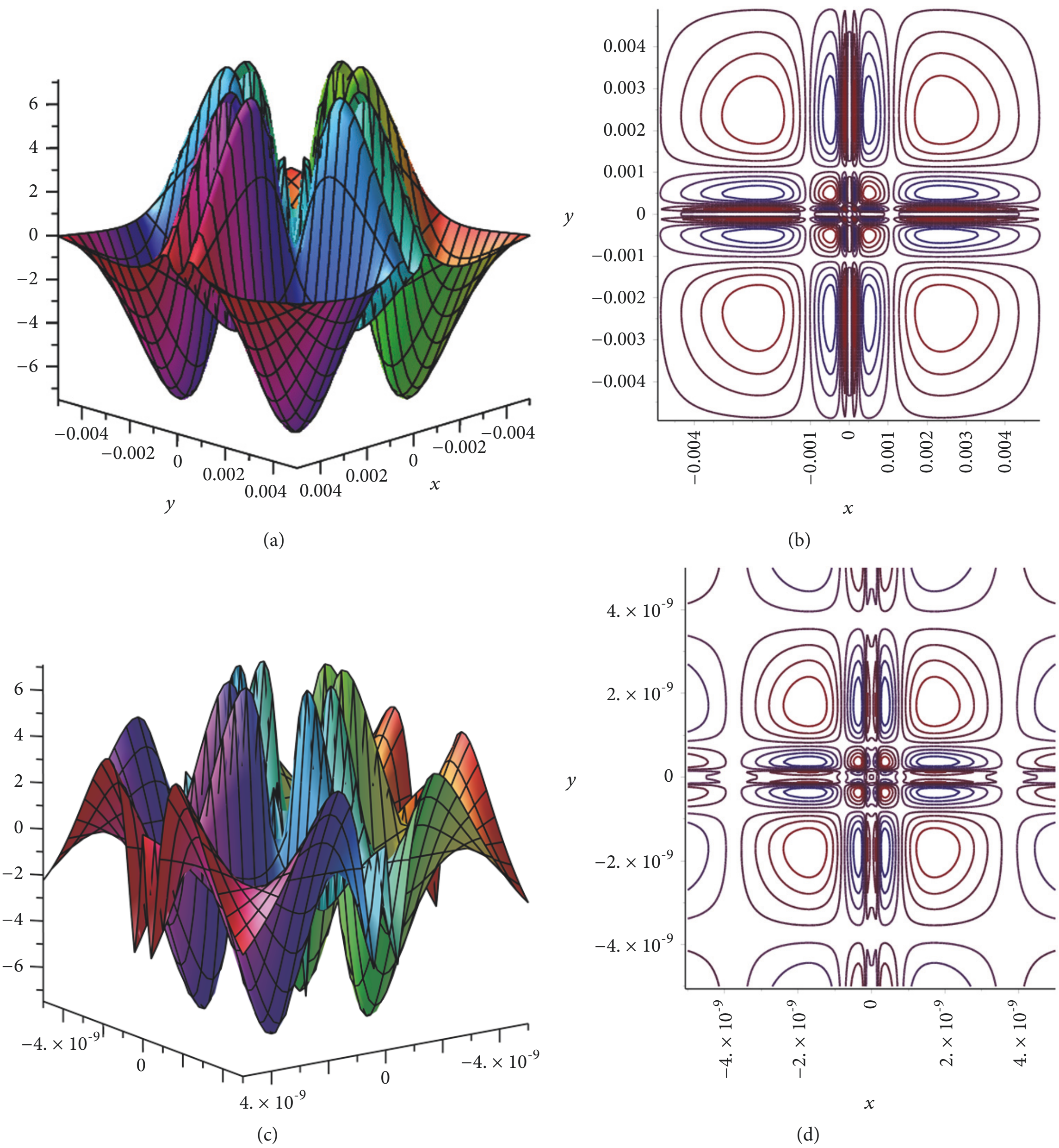

FIGURE 2: (a) Three-dimensional map, $x, y \in (-5\times10^{-3}, 5\times10^{-3})$. (b) Contour map $x, y \in (-5\times10^{-3}, 5\times10^{-3})$. (c) Three-dimensional map, $x, y \in (-5\times10^{-9}, 5\times10^{-9})$. (d) Contour map, $x, y \in (-5\times10^{-9}, 5\times10^{-9})$.

When we select the following parameters,

$$\begin{aligned} C_1 &= 3, \\ C_2 &= 2, \\ \lambda &= 1, \\ \mu &= -1 \end{aligned} \tag{31}$$

p, q selected as

$$p = 1 + e^{x(x+\sin(\ln(x^2)))} \tag{32}$$

$$q = 1 + e^{y(y+\sin(\ln(y^2)))} \tag{33}$$

we get Dromion's fractal structure.

It can be seen from Figure 2 that the Dromion structure remains basically unchanged at different scales and has self-similar properties, so it is a fractal structure.

4. Conclusion

In this paper, in order to obtain a richer solution of the nonlinear partial differential equations, improved the (G'/G^2) expansion method, starting from the solution to the positive exponential expansion symmetric extension to the negative power term and comparing the expressions (15)~(23) of the solution of the 2+1-dimensional NNV equation u, v, w, it can be found that symmetric extension to the negative power can indeed obtain more new formal solutions. At the same time, traveling wave solutions used for solving the equations are transformed into generalized solutions being capable of separating variables. Equation (11) plays an important role in achieving the separation of the equation variables, which also greatly expands the form of the nonlinear partial differential equation solution. The method is simple and direct. The advantages can be used to solve other nonlinear partial differential equations.

Data Availability

All data are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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