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Research Article

Kaluza-Klein State in Bekenstein-Hawking Entropy Cycles

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A Kaluza-Klein state configuration in black-hole qubit correspondence (BHQC) is considered in cyclic cycles of its Bekenstein-Hawking entropy. After a sequence of Peccei-Quinn transformations on the Kaluza-Klein state in cyclic cycles alternating between large and small extremal black hole (EBH) configurations, we obtain the corresponding amount of variation in the initial Bekenstein-Hawking entropy in cyclic cycles. We consider different cases where the EBH state alternates between small and large states. We then demonstrate that the total Bekenstein-Hawking entropy increases in these processes.

1. Introduction

An important class of extremal black holes (EBHs) is the special cases of black holes that cease evaporation due the presence of supersymmetric bounds [1–3]. The consequence of the bounds is to create a relation between the mass of the EBHs and their electric and magnetic charges [4, 5]. In the final state, the EBH can be described completely by their magnetic and electric charges and consequently is independent of the scalar fields that describe the nonstationary states [2]. A consequence is that a mapping with a Hilbert space can be done with the symmetries mapped on classes of entanglement, as a black hole qubit correspondence (BHQC) [6–9].

In recent years the BHQC of EBHs have been studied relating the entanglement classes with groups associated with string theories that lead to a description of the stationary states of EBHs, when the Hawking radiation is stopped and the state of the EBH is described completely in terms of their electric and magnetic charges [1].

Different configurations of EBH have been classified and the BHQC allows the classification of different configurations according to the entanglement class the EBH belongs. In particular, the association with qubits and the realization of quantum information protocols. An important advance in this field is the BHQC connection with quantum circuit protocols [10].

Recently these class of operations on a EBH state where investigated by means of Peccei-Quinn (PQ) transformations [11]. These classes of operations on an EBH state can be associated on a BHQC to operators that does not change the magnetic charges of an EBH state. These transformations produce changes in the Bekenstein-Hawking entropy [12, 13].

Here we investigate the class of Kaluza-Klein EBHs under the action of PQ operators in cycles alternating between large and small EBH configurations associated with their Bekenstein-Hawking entropy. The cases analyzed are transitions at fixed entropy and transitions at increasing or decreasing entropy. We consider different possible combinations where we finally compute the total amount of entropy exchange in the cycles. In the BHQC scenario this means it evolves as a set of gate operations applied in such a way to achieve cycles in EBH state configuration.

The paper is organized as follows: in Sections 2 and 3, some aspects of BHQC that will be applied in the next sections are revised; in Section 4, the relation between Bekenstein-Hawking entropy and the G_4-invariant is discussed; in Section 5 the Kaluza-Klein (KK) state configurations in small and large configurations of EBH are discussed; in Section 6, operations on EBH charge states are discussed; in Section 7, the protocols of cyclic cycles of Bekenstein-Hawking entropy in KK EBH configurations are implemented; in Section 8, conclusions remarks are addressed.

2. BHQC

BHQC has been established a bridge between certain solutions of black holes and quantum information theory (QIT).

This correspondence allows the problem of classification of EBHs be associated with the classification of entanglement classes in QIT systems. The Bekenstein-Hawking entropy is in a class of black hole solutions in string theory and classes of multipartite entanglement measures associated in BHQC. The correspondence can also shed new light on the way EBHs evolve under certain QIT conditions [6, 10].

3. BHQC States

In the BHQC realization, a general dyonic charge vector state is given by the superposition of type

$$|\mathcal{Q}\rangle := p^0 |0\rangle + q_0 |1\rangle + \sum_{i=1}^{n} p^i |2i\rangle + \sum_{i=1}^{n} q_i |2i+1\rangle, \quad (1)$$

where a Fock basis is used $\{|i\rangle,\ i = 0, \ldots, n\}$ as the symplectic basis of the Hilbert space associated with the electric and magnetic charges of the EBH. For convenience, the even states are associated with the magnetic charge configurations and the odd states to the electric charge configurations of the EBH. The dimension n is associated with the number of vector multiplets $n = n_V$.

A BHQC of Peccei-Quinn (PQ) symplectic transformation on the charge state is given by the action of the corresponding operator $\widehat{\mathcal{O}}_{PQ}$ on the charge state being given by $|\mathcal{Q}_{PQ}\rangle = \widehat{\mathcal{O}}_{PQ}|\mathcal{Q}\rangle$ and has the following explicit expression:

$$\begin{aligned} |\mathcal{Q}_{PQ}\rangle = {} & p^0 |0\rangle + \left(q_0 + \rho p^0 + \sum_{i=1}^{n} c_i p^i\right) |1\rangle + \sum_{i=1}^{n} p^i |2i\rangle \\ & + \sum_{i=1}^{n} \left(q_i + c_i p^0 + \sum_{j=1}^{n} p^j \Theta_{ji}\right) |2i+1\rangle, \end{aligned} \quad (2)$$

where the coefficients allows to a Peccei-Quinn group, $PQ(n+1)$ [11], whose invariants are of the form

$$W = \begin{pmatrix} \mathbb{I}_{n+1} & 0_{n+1} \\ \begin{pmatrix} \varrho & c_j \\ c_i & \Theta_{ij} \end{pmatrix} & \mathbb{I}_{n+1} \end{pmatrix} \in PQ(n+1), \quad (3)$$

with $\mathbb{I}_{n+1}$ the identity and 0_{n+1} the null $(n+1)\times(n+1)$ matrices, where the submatrices with $\varrho, c_i, \Theta_{ij}$ are defined as a $(n+1)\times(n+1)$ real matrix $(i, j = 1, \ldots, n, \Theta_{ij} = \Theta_{(ij)})$.

As a consequence, PQ operators leave the EBH magnetic charges invariant,

$$p^0 \longrightarrow p^0, \quad (4)$$

$$p^i \longrightarrow p^i, \quad (5)$$

while changing the electric charges

$$q_0 \longrightarrow q_0 + \rho p^0 + \sum_{i=1}^{n} c_i p^i, \quad (6)$$

$$q_i \longrightarrow q_i + c_i p^0 + \sum_{j=1}^{n} p^j \Theta_{ji}. \quad (7)$$

4. Bekenstein-Hawking Entropy and the G_4-Invariant

A general EBH charge state configuration (1) is associated with a unique G_4-invariant:

$$\begin{aligned} \mathcal{I}_4(\mathcal{Q}) = {} & -\left(p^0 q_0 + \sum_{i=1}^{n} p^i q_i\right)^2 + 4 q_0 \mathcal{I}_3(p) \\ & - 4 p^0 \mathcal{I}_3(q) + 4\{\mathcal{I}_3(p), \mathcal{I}_3(q)\}, \end{aligned} \quad (8)$$

where the cubic invariants are given by

$$\mathcal{I}_3(p) = \frac{1}{3!} \sum_{i,j,k=1}^{n} d_{ijk} p^i p^j p^k, \quad (9)$$

$$\mathcal{I}_3(q) = \frac{1}{3!} \sum_{i,j,k=1}^{n} d^{ijk} q_i q_j q_k, \quad (10)$$

and the Poisson brackets are defined as

$$\{\mathcal{I}_3(p), \mathcal{I}_3(q)\} = \sum_{i=1}^{n} \frac{\partial \mathcal{I}_3(p)}{\partial p^i} \frac{\partial \mathcal{I}_3(q)}{\partial q_i} \quad (11)$$

The Bekenstein-Hawking entropy formula for a general EBH state is given in terms of this invariant as [6]

$$S(\mathcal{Q}) = \pi \sqrt{|\mathcal{I}_4(\mathcal{Q})|}. \quad (12)$$

5. Large and Small Kaluza-Klein EBH Configurations

We say that the EBH configuration is large if

$$S(\mathcal{Q}) \neq 0 \quad (13)$$

and the EBH is small if instead

$$S(\mathcal{Q}) = 0. \quad (14)$$

An important type of EBH configuration particularly to BHQC due to the form correspondence with a qubit is a Kaluza-Klein EBH configuration.

A Kaluza-Klein (KK) EBH state configuration is given by the following state in BHQC:

$$|\mathcal{Q}_{KK}\rangle = p^0 |0\rangle + q_0 |1\rangle \quad (15)$$

The KK invariant is given by

$$\mathcal{I}_4(\mathcal{Q}_{KK}) = -\left(p^0\right)^2 q_0^2 < 0, \quad (16)$$

where

$$\begin{aligned} \frac{\partial \mathcal{I}_3(q)}{\partial q_i} &= 0, \\ \frac{\partial \mathcal{I}_3(p)}{\partial p^i} &= 0, \\ \forall i. \end{aligned} \quad (17)$$

It follows that the Bekenstein-Hawking formula for a general EBH configuration of a Kaluza-Klein type is

$$S(\mathcal{Q}_{KK}) = \pi \left| p^0 q_0 \right|. \tag{18}$$

In the case with both p^0 and q_0 nonvanishing, one obtains a dyonic large EBH configuration, corresponding to the maximal rank= 4 element in the related Freudenthal Triple System [11].

If some of p^0 or q_0 vanish, one has a small EBH configuration, since in this case

$$S(\mathcal{Q}_{KK}) = 0. \tag{19}$$

These cases are given by corresponding pure states $p^0|0\rangle$ and $q_0|1\rangle$.

6. Operations on EBH Charge States

After a PQ transformation, the Kaluza-Klein state dyonic EBH state acquires a superposition of change states

$$p^0 \longrightarrow p'^0 = p^0 \neq 0, \tag{20}$$

$$p^i = 0 \longrightarrow p'^i = 0 \tag{21}$$

$$q_0 \longrightarrow q_0' = q_0 + \rho p^0 + \sum_{i=1}^{n} c_i p^i \neq 0, \tag{22}$$

$$q_i = 0 \longrightarrow q_i' = c_i p^0 \neq 0. \tag{23}$$

This will alter the Bekenstein-Hawking EBH entropy. The state is written

$$\begin{aligned} \left|\mathcal{Q}_{PQ(KK)}\right\rangle &= p^0 |0\rangle + \left(q_0 + \rho p^0\right)|1\rangle \\ &\quad + \sum_{i=1}^{n} p^0 c_i |2i+1\rangle. \end{aligned} \tag{24}$$

And the invariants are given by

$$\mathcal{I}_3(p) = 0, \tag{25}$$

$$\mathcal{I}_3(q) = \frac{\left(p^0\right)^3}{3!} \sum_{i,j,k=1}^{n} d^{ijk} c_i c_j c_k, \tag{26}$$

$$\left\{\mathcal{I}_3(p), \mathcal{I}_3(q)\right\} = 0. \tag{27}$$

$$\begin{aligned} \mathcal{I}_4(\mathcal{Q}) &= -\left(p^0 q_0 + \rho \left(p^0\right)^2\right)^2 \\ &\quad - \frac{2\left(p^0\right)^4}{3} \sum_{i,j,k=1}^{n} d^{ijk} c_i c_j c_k, \end{aligned} \tag{28}$$

We can define the constant of the PQ transformation

$$\lambda = \sum_{i,j,k=1}^{n} d^{ijk} c_i c_j c_k, \tag{29}$$

such that the quartic invariant is given by

$$\begin{aligned} \mathcal{I}_4(\mathcal{Q}) &= -\left[q_0^2\right]\left(p^0\right)^2 - \left[2\rho q_0\right]\left(p^0\right)^3 \\ &\quad - \left[\rho^2 - \frac{2\lambda}{3}\right]\left(p^0\right)^4, \end{aligned} \tag{30}$$

Consequently, the Bekestein-Hawking entropy is written in the form

$$S(\mathcal{Q}) = \left|p^0\right| \sqrt{q_0^2 + (2\rho q_0)\left(p^0\right) + \left(\rho^2 - \frac{2\lambda}{3}\right)\left(p^0\right)^2} \tag{31}$$

A large EBH is achieved for $S(\mathcal{Q}) \neq 0$, while the set of small EBHs after the transformation satisfy $S(\mathcal{Q}) = 0$; consequently

$$q_0^2 + (2\rho q_0)\left(p^0\right) + \left(\rho^2 - \frac{2\lambda}{3}\right)\left(p^0\right)^2 = 0. \tag{32}$$

In this case, two possible solutions are found

$$p_+^0 = \frac{-q_0}{\rho - \sqrt{(2\lambda/3)}}, \tag{33}$$

$$p_-^0 = \frac{-q_0}{\rho + \sqrt{(2\lambda/3)}}. \tag{34}$$

7. Cyclic Cycles of Bekenstein-Hawking Entropy in Kaluza-Klein EBH Configurations

In order to consider a cyclic cycle of Kaluza-Klein EBH states, we consider a set of PQ operations as a BHQC quantum circuit in different protocols alternating small and large EBH configurations.

7.1. Alternating Small and Large Kaluza-Klein EBH Configurations. We consider the interchange between large and small Kaluza-Klein EBH configurations obeying the following protocol (see Figure 1):

$$S(\mathcal{Q}_1) \neq 0 \longrightarrow S(\mathcal{Q}_2) = 0 \tag{35}$$

$$S(\mathcal{Q}_2) = 0 \longrightarrow S(\mathcal{Q}_3) \neq 0 \tag{36}$$

$$S(\mathcal{Q}_3) \neq 0 \longrightarrow S(\mathcal{Q}_4) = 0 \tag{37}$$

$$S(\mathcal{Q}_4) = 0 \longrightarrow S(\mathcal{Q}_5) = S(\mathcal{Q}_1) \tag{38}$$

We start with a Kaluza-Klein state

$$\left|\mathcal{Q}_1\right\rangle = \left|\mathcal{Q}_{KK}\right\rangle = p_0 |0\rangle + q_0 |1\rangle, \tag{39}$$

where $p_0 > 0$ and $q_0 > 0$, whose Bekenstein-Hawking entropy is given by

$$S(\mathcal{Q}_1) = \pi \left|p_0 q_0\right|. \tag{40}$$

The PQ symplectic transformation on this EBH configuration will lead to

$$\begin{aligned} \left|\mathcal{Q}_{PQ(KK)}\right\rangle &= p^0 |0\rangle + \left(q_0 + \rho_1 p^0\right)|1\rangle \\ &\quad + \sum_{i=1}^{n} \left(c_{i,1} p^0\right)|2i+1\rangle. \end{aligned} \tag{41}$$

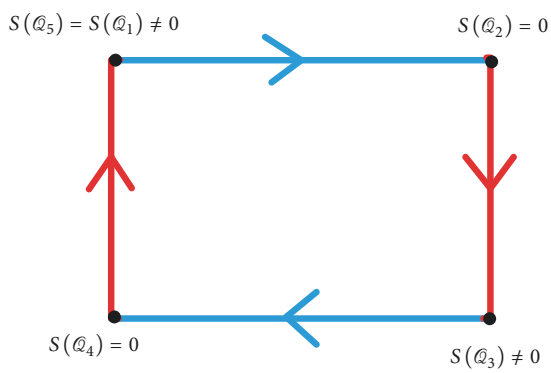

FIGURE 1: Cycle with alternating small and large EBH configurations.

We can consider the case where the transformation leaves the configuration in a Kaluza-Klein state but converts the EBH in a small configuration. This will occur under the condition

$$c_{i,1} = 0, \quad i = 1, \ldots, n. \tag{42}$$

$$\rho_1 = -\frac{q_0}{p_0} \tag{43}$$

Consequently, we will have

$$|\mathcal{Q}_2\rangle = |\mathcal{Q}_{PQ(1)}\rangle = p^0 |0\rangle, \tag{44}$$

which is small

$$S(\mathcal{Q}_2) = 0. \tag{45}$$

The second transformation will leave the state in the following charge configuration:

$$|\mathcal{Q}_{PQ(2)}\rangle = p^0 |0\rangle + (\rho_2 p^0) |1\rangle + \sum_{i=1}^{n} (c_{i,2} p^0) |2i+1\rangle \tag{46}$$

where the simplest case is taken

$$\rho_2 > 0 \tag{47}$$

$$c_{i,2} = 0 \tag{48}$$

and again we put the state in a Kaluza-Klein configuration

$$|\mathcal{Q}_3\rangle = p^0 |0\rangle + (\rho_2 p^0) |1\rangle \tag{49}$$

This is then a large EBH with a Bekenstein-Hawking entropy given by

$$S(\mathcal{Q}_3) = \pi p_0^2 |\rho_2|. \tag{50}$$

Again, we apply a PQ symplectic transformation on this EBH configuration, and we are left in the following state:

$$|\mathcal{Q}_{PQ(3)}\rangle = p^0 |0\rangle + (\rho_2 + \rho_3) p^0 |1\rangle + \sum_{i=1}^{n} (c_{i,3} p^0) |2i+1\rangle, \tag{51}$$

We now have that the small configuration implies

$$\rho_3 = -\rho_2, \tag{52}$$

$$c_{i,3} = 0. \tag{53}$$

And the state

$$|\mathcal{Q}_4\rangle = |\mathcal{Q}_{PQ(3)}\rangle = p^0 |0\rangle, \tag{54}$$

that is, a small EBH configuration $|\mathcal{Q}_4\rangle = |\mathcal{Q}_2\rangle$

$$S(\mathcal{Q}_4) = 0. \tag{55}$$

Finally, the cyclic transformation is achieved by the transformation

$$|\mathcal{Q}_{PQ(4)}\rangle = p^0 |0\rangle + (\rho_4 p^0) |1\rangle + \sum_{i=1}^{n} (c_{i,4} p^0) |2i+1\rangle \tag{56}$$

where we keep the Kaluza-Klein configuration with

$$c_{i,4} = 0, \tag{57}$$

and

$$|\mathcal{Q}_5\rangle = |\mathcal{Q}_{PQ(4)}\rangle = p^0 |0\rangle + (\rho_4 p^0) |1\rangle \tag{58}$$

$$S(\mathcal{Q}_5) = \pi |\rho_4| p_0^2. \tag{59}$$

The condition

$$|\mathcal{Q}_5\rangle = |\mathcal{Q}_1\rangle \tag{60}$$

will lead to

$$\rho_4 = \frac{q_0}{p^0} = -\rho_1. \tag{61}$$

The whole cyclic transformation is then realized under the conditions

$$c_{i,k} = 0, \quad k = 1, \ldots, 4, \; i = 1, \ldots, n \tag{62}$$

$$\rho_2 + \rho_3 = 0, \tag{63}$$

$$\rho_1 + \rho_4 = 0. \tag{64}$$

In this case the Kaluza-Klein EBH states are alternating between large and small configurations.

The set of variations of Bekenstein-Hawking entropies in each step are given by

$$\Delta S_{21} = \pi |p_0 q_0| \tag{65}$$

$$\Delta S_{32} = \pi p_0^2 |\rho_2| \tag{66}$$

$$\Delta S_{43} = -\pi p_0^2 |\rho_2| \tag{67}$$

$$\Delta S_{54} = \pi p_0^2 |\rho_4| \tag{68}$$

It follows that the whole process has a total increasing entropy

$$\Delta S = \pi |p_0 q_0| + \pi p_0^2 |\rho_4|. \tag{69}$$

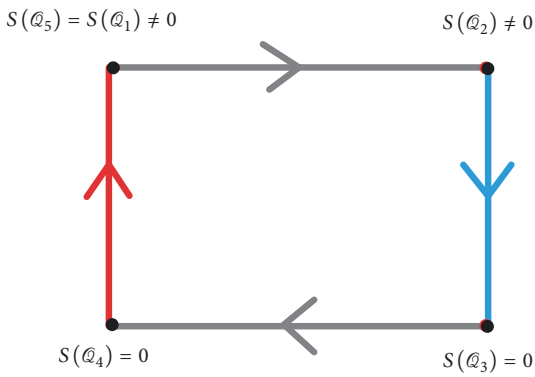

FIGURE 2: Cycle with sections of fixed large and small EBH configurations.

Considering a more general scenario where the final configuration is given by

$$\rho_4 = \alpha \frac{q_0}{p_0} \tag{70}$$

we have the total entropy in the process given by

$$\Delta S = \pi \left| p_0 q_0 \right| (1 + |\alpha|). \tag{71}$$

It follows that, after N cycles, the entropy will suffer the total variation obeying the following relation:

$$\Delta S = \pi \left| p_0 q_0 \right| \left(1 + \sum_{k=1}^{N} |\alpha_k| \right), \tag{72}$$

where α_k is the proportionality of last cycle step in each cycle. The perfect case is $|\alpha_k| = 1$,

$$\Delta S = \pi \left| p_0 q_0 \right| (N + 1). \tag{73}$$

7.2. Cycle with Sections of Small and Large EBH Configurations. We can also consider the following protocol (see Figure 2):

$$S(\mathcal{Q}_1) \neq 0, \tag{74}$$

$$S(\mathcal{Q}_2) \neq 0, \tag{75}$$

$$S(\mathcal{Q}_3) = 0, \tag{76}$$

$$S(\mathcal{Q}_4) = 0, \tag{77}$$

$$S(\mathcal{Q}_5) = S(\mathcal{Q}_1), \tag{78}$$

where now the operation does not change the entropy. Starting again with the Kaluza-Klein state

$$\left| \mathcal{Q}_1 \right\rangle = \left| \mathcal{Q}_{KK} \right\rangle = p_0 \left| 0 \right\rangle + q_0 \left| 1 \right\rangle, \tag{79}$$

with $p_0 > 0$ and $q_0 > 0$, whose Bekenstein-Hawking entropy is given by

$$S(\mathcal{Q}_1) = \pi \left| p_0 q_0 \right|. \tag{80}$$

The PQ transformation will lead to

$$\begin{aligned} \left| \mathcal{Q}_{PQ(1)} \right\rangle = {} & p^0 \left| 0 \right\rangle + \left(q_0 + \rho_1 p^0 \right) \left| 1 \right\rangle \\ & + \sum_{i=1}^{n} \left(c_{i,1} p^0 \right) \left| 2i + 1 \right\rangle \end{aligned} \tag{81}$$

Staying in a Kaluza-Klein configuration implies that

$$c_{i,1} = 0, \tag{82}$$

which will lead to the following state

$$\left| \mathcal{Q}_2 \right\rangle = p_0 \left| 0 \right\rangle + \left(q_0 + \rho_1 p_0 \right) \left| 1 \right\rangle \tag{83}$$

In this case, the Bekenstein-Hawking entropy is given by

$$S(\mathcal{Q}_2) = \pi \left| p_0 \left(q_0 + \rho_1 p_0 \right) \right| \tag{84}$$

Taking the case $q_0 > 0, \rho_1 > 0, p_0 > 0$, we can write

$$S(\mathcal{Q}_2) = S(\mathcal{Q}_1) + \pi \rho_1 p_0^2 \tag{85}$$

The PQ transformation on this state will now lead to the following:

$$\begin{aligned} \left| \mathcal{Q}_{PQ(2)} \right\rangle = {} & p^0 \left| 0 \right\rangle + \left(q_0 + \rho_1 p_0 + \rho_2 p^0 \right) \left| 1 \right\rangle \\ & + \sum_{i=1}^{n} \left(c_{i,2} p^0 \right) \left| 2i + 1 \right\rangle \end{aligned} \tag{86}$$

In this case, we have the relations

$$\rho_1 = \rho_2 = -\frac{q_0}{p_0} \tag{87}$$

$$c_{i,2} = 0 \tag{88}$$

such that the Kaluza-Klein state is reduced to

$$\left| \mathcal{Q}_3 \right\rangle = p^0 \left| 0 \right\rangle, \tag{89}$$

and the EBH has a small configuration

$$S(\mathcal{Q}_3) = 0. \tag{90}$$

We then have the next transformation

$$\left| \mathcal{Q}_{PQ(3)} \right\rangle = p^0 \left| 0 \right\rangle + \left(\rho_3 p^0 \right) \left| 1 \right\rangle + \sum_{i=1}^{n} \left(c_{i,3} p^0 \right) \left| 2i + 1 \right\rangle. \tag{91}$$

Since in this case we also have a small EBH configuration,

$$\rho_3 = 0, \tag{92}$$

$$c_{i,3} = 0. \tag{93}$$

This will lead to the EBH state

$$\left| \mathcal{Q}_4 \right\rangle = p^0 \left| 0 \right\rangle. \tag{94}$$

This is finally after the transformation

$$\left|\mathcal{Q}_{PQ(4)}\right\rangle = p^0 \left|0\right\rangle + \left(\rho_4 p^0\right)\left|1\right\rangle + \sum_{i=1}^{n}\left(c_{i,4}p^0\right)\left|2i+1\right\rangle, \quad (95)$$

whocj will lead to

$$c_{i,4} = 0 \quad (96)$$

and we have

$$\left|\mathcal{Q}_5\right\rangle = p^0 \left|0\right\rangle + \left(\rho_4 p^0\right)\left|1\right\rangle, \quad (97)$$

The Bekenstein-Hawking entropy in this case is

$$S\left(\mathcal{Q}_5\right) = \pi\left|\rho_4\right| p_0^2 \quad (98)$$

The cyclic condition imposes the relation

$$\rho_4 = \frac{q_0}{p^0}. \quad (99)$$

The variations of Bekenstein-Hawking entropy in each step are given by

$$\Delta S_{21} = \pi\rho_1 p_0^2 \quad (100)$$

$$\Delta S_{32} = -\pi\rho_1 p_0^2 \quad (101)$$

$$\Delta S_{43} = 0 \quad (102)$$

$$\Delta S_{54} = \pi p_0^2\left|\rho_4\right| \quad (103)$$

The total entropy variation is given by

$$\Delta S = \pi p_0^2\left|\rho_4\right|, \quad (104)$$

and consequently it has increased.

In a more general case

$$\rho_4 = \alpha\frac{q_0}{p^0}. \quad (105)$$

This leads to

$$\Delta S = \alpha\pi\left|p_0 q_0\right| \quad (106)$$

In N cycles, the total entropy variation is given by

$$\Delta S = \sum_{k=1}^{N}\pi\left|\alpha_k\right|\left|p_0 q_0\right| \quad (107)$$

In particular, for $\alpha_k = 1$,

$$\Delta S = \pi N\left|p_0 q_0\right|. \quad (108)$$

The Bekenstein-Hawking entropy after N cycles is then an increasing quantity.

8. Concluding Remarks

Within the black-hole/qubit correspondence (BHQC), we considered the Bekenstein-Hawking entropy variation in cyclic cycles, where Peccei–Quinn (PQ) symplectic transformations are used on the transitions between EBH state configurations. We considered the cases where the entropy is alternating between small and large configurations and the situation where the transitions occur in fixed small or large configurations. We showed applying BHQC that the total Bekenstein-Hawking entropy in each state increases. This is in complete agreement with area theorem [12, 14]. As a consequence, the cyclic transport of a qubit like state in BHQC leads to an increasing variation of the corresponding total Bekenstein-Hawking entropy.

As a consequence, a sequence of gate operations in a KK state in a cyclic cycle with the initial state as a target state implies increasing of the total Bekenstein-Hawking entropy, in agreement with the expected in the thermodynamics of black holes, implying that such a scenario can be implemented physically under a cost of increasing entropy.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares there are no conflicts of interest in this paper.

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