Research Article

Anti-Synchronization of Fractional-Order Chaotic Circuit with Memristor via Periodic Intermittent Control

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In this paper, the anti-synchronization of fractional-order chaotic circuit with memristor (FCCM) is investigated via a periodic intermittent control scheme. Based on the principle of periodic intermittent control and the Lyapunov stability theory, a novel criterion is adopted to realize the anti-synchronization of FCCM. Finally, some examples of numerical simulations are exploited to verify the feasibility of theoretical analysis.

1. Introduction

Fractional calculus has a history of more than 300 years. It is worth pointing out that fractional-order system has provided infinite memory and more accurately describes natural phenomena than other integer order systems [1]. In recent decades, fractional differential equations due to its potential applications in many fields such as fluid mechanics [2], physics [3], encryption [4], and control processing [5]. Especially, the dynamical behavior of fractional order systems exhibits chaos, such as fractional-order Chen system, fractional-order Chua system, fractional-order Lorenz system, and fractional-order Lü system, etc., [6].

The memristor was firstly raised by Chua [7], but it had not aroused any attention until 2008 when the invention of the memristor had been published by the researchers in Hewlett–Packard lab [8, 9]. The memristor could “remember” its state when the voltage is turned off. Because of the characteristics of memristor, the potential applications of the chaotic system with memristor have been discovered in quite a few fields such as cryptography, filter, image encryption, etc., [10–15]. Therefore, the behaviors and properties of memristor have attracted the attention of many researchers attention.

Meanwhile, many scholars investigate the synchronization problems [16–18], especially the synchronization of fractional-order chaotic systems such as lag-synchronization [19], projective synchronization [20], impulsive synchronization [21], and anti-synchronization [22]. Examples of synchronization occur in different fields of engineering and science like coupled cardiac, circuits in electronics and respiratory systems in physiology, and coupled laser systems in nonlinear optics. Many synchronization methods have been put forward for chaotic systems, such as sliding mode control method [23, 24], impulsive control method [25, 26], active control method [27], periodic intermittent control method [28, 29], etc.

Intermittent control, which was first introduced to control linear econometric models in [30], has been widely used in engineering fields such as manufacturing, transportation, and communication for its practical and easy implementation in engineering control. Intermittent control is a discontinuous control method, its control input is activated during certain nonzero time intervals and closed during other time intervals [31]. Therefore, compared with the continuous control methods, intermittent control is more economical and efficient [32]. Recently, much effort has been devoted to study the issue of stabilization and synchronization of chaotic systems and dynamical networks by using intermittent control, and many important and interesting results have been obtained [33–38]. In [33], pinning synchronization for directed networks with node balance via adaptive intermittent control was researched. Zhang et al. [34] studied the lag synchronization for fractional-order memristive neural networks via periodic intermittent control. References [35, 36] proposed the finite-time synchronization via periodic intermittent control. Liu et al.
Motivated by the above discussions, we propose a periodic intermittent control method for the anti-synchronization of FCCM in this paper. Based on the Lyapunov stability theory, a novel and useful criterion of periodic intermittent control is developed by using the differential inequality method. Finally, we have illustrated the effectiveness and feasibility of the proposed approaches by numerical simulations.

This paper is arranged as follows: Section 2 describes some fundamental definitions, the lemmas and the model formulation. The anti-synchronization of FCCM via periodic intermittent control is discussed in Section 3. In Section 4, some numerical examples are provided to illustrate the effectiveness of the theoretical approach. The conclusions are put forward in the last section.

2. Preliminaries

In this paper, let $R^n$ denote the n-dimensional Euclidean space, $x = (x_1, x_2, x_3, x_4) \in R^4$, $y = (y_1, y_2, y_3, y_4) \in R^4$. In this section, some fundamental definitions and lemmas are recalled. In addition, we introduce fractional-order generalization form of the chaotic circuit with memristor.

**Definition 1 [39].** The caputo’s fractional derivative for a function $z(t): [0, +\infty) \rightarrow R$ is defined by

$$\frac{\partial^\alpha}{\partial t^\alpha} z(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{z'(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (1)$$

where $t \geq 0$, $\alpha$ is fractional-order, $n \in Z^+$, $n-1 < \alpha < n$, and $\Gamma(\cdot)$ is the gamma function. Particularly, when $0 < \alpha < 1$,

$$\frac{\partial^\alpha}{\partial t^\alpha} z(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{z'(\tau)}{(t-\tau)^{\alpha}} d\tau. \quad (2)$$

**Definition 2 [39].** The Mittag–Leffler function of $\alpha$ is defined as

$$E_\alpha(x) = \sum_{m=0}^{\infty} \frac{x^m}{\Gamma(m\alpha+1)}, \quad (3)$$

where $\alpha > 0$ and $x \in C$.

**Lemma 1 [40].** Suppose $z(t) \in R^n$ is a differentiable function and continuous. For $\alpha \in (0, 1)$, the following inequality holds

**Lemma 2 [41].** For $\alpha \in (0, 1)$, $x \in R$, and $x > 0$, $E_\alpha(x)$ is a monotone increasing function.

**Lemma 3 [39].** Let $V(t)$ be a continuous function on $[t_0, +\infty)$ and satisfies

$$\int_{t_0}^t D^\alpha V(t) \leq \theta V(t), \quad (5)$$

where $0 < \alpha < 1$ and $\theta$ is a constant, then

$$V(t) \leq V(t_0)E_\alpha\left(\theta(t-t_0)^\alpha\right). \quad (6)$$

**Lemma 4 [42].** Let $0 < \alpha < 1$, $\lambda > 0$, and $t \geq 0$, the following inequality holds

$$0 \leq E_\alpha(-\lambda t^\alpha) \leq 1. \quad (7)$$

Moreover, $E_\alpha(0) = 1$.

According to the chaotic circuit with memristor [43] as shown in Figure 1, the flux-controlled memristor is defined by

$$q(\phi) = a\phi + b\phi^3, \quad (8)$$

$$W(\phi) = \frac{d q(\phi)}{d\phi} = a + 3b\phi^2,$$

where $\phi$ is the flux, $W(\phi)$ is the memductance, and $a$ and $b$ are constants.

Similar to [43], let $x_1 = u_x, x_2 = u_y, x_3 = \xi, x_4 = \phi, \beta = 1/C_p, \psi = 1/L, \xi = G, y = r/L, R = 1$, and $C_f = 1$, then the mathematical model of the chaotic circuit with memristor as follows

$$\begin{align}
\dot{x}_1 &= \beta(\xi x_1 - x_1 + x_2 - W(x_4)x_1), \\
\dot{x}_2 &= x_1 - x_2 + x_3, \\
\dot{x}_3 &= -\psi x_2 - \gamma x_3, \\
\dot{x}_4 &= x_1. 
\end{align} \quad (9)$$

Refer to the above model, the fractional-order generalization according to (9) is described as...
The response system with the controller can be rewritten as

\begin{equation}
\mathcal{O} D^\alpha_t x(t) = A x(t) + \phi(x(t)),
\end{equation}

where

\begin{equation}
A = \begin{bmatrix}
\alpha (\xi - 1) & \beta & 0 & 0 \\
1 & -1 & 1 & 0 \\
0 & -\psi & -\gamma & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}, \quad \phi(x) = \begin{bmatrix}
-\beta W(x_4) x_1 \\
0 \\
0 \\
0
\end{bmatrix},
\end{equation}

and \(a, b, \gamma, \xi, \beta, \psi\) are positive constants.

To investigate the anti-synchronization of FCCM, the drive system can be rewritten as

\begin{equation}
\mathcal{O} D^\alpha_t x(t) = A x(t) + \phi(x(t)).
\end{equation}

Similarly, the response system with the controller can be described as

\begin{equation}
\mathcal{O} D^\alpha_t y(t) = A y(t) + \phi(y(t)) + u(t),
\end{equation}

where \(u(t)\) is the intermittent periodical controller which is proposed by

\begin{equation}
u(t) = \begin{cases}
-k(x(t) + y(t)), & mT \leq t < mT + \sigma, \\
0, & mT + \sigma \leq t < (m + 1)T,
\end{cases}
\end{equation}

where \(k > 0\) is a positive constant, \(T > 0\) is the control period, \(0 < \sigma < T\) is called the control width, and \(m = 0, 1, 2, \ldots\).

Let \(e(t) = y(t) - x(t)\) be the synchronization error between system (13) and system (14), the error system can be obtained by

\begin{equation}
\mathcal{O} D^\alpha_t e(t) = A e(t) + \phi(e(t)) - ke(t), \quad mT \leq t \leq mT + \sigma,
\end{equation}

\begin{equation}
\mathcal{O} D^\alpha_t e(t) = A e(t) + \phi(e(t)), \quad mT + \sigma < t < (m + 1)T,
\end{equation}

where

\begin{equation}
\phi(e(t)) = \phi(x(t)) + \phi(y(t)) = \begin{bmatrix}
-\beta W(x_4(t)) x_1(t) - \beta W(y_4(t)) y_1(t), 0, 0, 0
\end{bmatrix}^T.
\end{equation}

3. Main Result

In this section, the anti-synchronization problem of FCCM via periodic intermittent control is investigated. First of all, we propose the following assumption.

**Assumption 1.** It can be seen from Figure (2) that system (10) is a chaotic system with bounders, we assume \(M_1\) and \(M_2\) are positive constants, such that

\begin{equation}
|x_i(t)| \leq M_i, \quad |x_4(t)| \leq M_2.
\end{equation}
where \( \lambda_{\Delta} \) is the largest eigenvalue of \( \Lambda \), and \( \eta_1 \) and \( \eta_2 \) are nonnegative constants.

**Proof.** Construct the following Lyapunov function.

\[
\frac{d}{dt} V(t) = \frac{1}{2} e^T(t) e(t). 
\]

Taking the time derivative of \( V(t) \) along the solution \( e(t) \) of the system (22). From Lemma 1, when \( mT \leq t \leq mT + \sigma \), \( m = 0, 1, 2, \ldots \) we have

\[
E_\alpha(-\eta_1 \sigma^\alpha)E_\alpha((\eta_2 (T - \sigma)^\alpha) < 1, \quad (21)
\]

where \( \lambda_{\Delta} \) is the largest eigenvalue of \( A \), \( \eta_1 \), and \( \eta_2 \) are nonnegative constants.

**Theorem 1.** Suppose Assumption 1 holds. The systems (13) and (14) can be anti-synchronized under the periodic intermittent controller (15) if the following conditions are satisfied:

\[
2(\lambda_{\Delta} + 3\beta b M_1 M_2 - k) + \eta_1 \leq 0, \quad (19)
\]

\[
2(\lambda_{\Delta} + 3\beta b M_1 M_2) - \eta_2 \leq 0, \quad (20)
\]

Then, we derive the anti-synchronization criteria for the FCCM according to periodic intermittent control scheme and Assumption 1 in Theorem 1.
According to the condition (19), we have

\[ m^T e^T V(t) \leq e^T (t) m^T e(t) \]

\[ = e^T (t) (A e(t) + \phi(e(t)) - k e(t)) \]

\[ = e^T (t) A e(t) + e^T (t) \phi(e(t)) - k e^T (t) e(t) \]

\[ = e^T (t) A e(t) - \beta [a e^2 (t) + 3 b (y^2 (t) y_1 (t) + y^2 (t) x_1 (t)) \]

\[ x_1^2 (t) y_1 (t) - y_1^2 (t) x_1 (t)] e_1 (t) \right] - k e^T (t) e(t) \]

\[ \leq \lambda_\alpha e^T (t) e(t) + 3 \beta b M_1 M_2 e^T (t) e(t) - k e^T (t) e(t) \]

\[ = (\lambda_\alpha + 3 \beta b M_1 M_2 - k) e^T (t) e(t). \]  

(23)

According to the condition (19), we have

\[ m^T e^T V(t) \leq -\eta_1 V(t). \]  

(24)

By Lemma 3, when \( mT \leq t \leq mT + \sigma \), we have

\[ V(t) \leq V(mT) E_a (-\eta_1 t - mT^\alpha). \]  

(25)

Similarly, when \( mT + \sigma < t < mT + T, m = 0, 1, 2, \ldots \) we have

\[ m^T e^T V(t) \leq e^T (t) m^T e(t) \]

\[ = e^T (t) (A e(t) + \phi(e(t))) \]

\[ = (\lambda_\alpha + 3 \beta b M_1 M_2) e^T (t) e(t). \]  

(26)

According to condition (20), we have

\[ m^T e^T V(t) \leq \eta_2 V(t). \]  

(27)

From Lemma 3, when \( mT + \sigma < t < (m + 1)T \), we have

\[ V(t) \leq V(mT + \sigma) E_a (\eta_2 (t - mT - \sigma)^\alpha). \]  

(28)

From inequality (25) and (28), we summarize that:

When \( 0 \leq t \leq \sigma \), we have

\[ V(t) \leq V(0) E_a (-\eta_1 t^\alpha). \]  

(29)

When \( \sigma < t < T \), we obtain
When \( (14) \) with periodic intermittent controller \((15)\), where \( \alpha = 0.985, \eta_1 = 30, \eta_2 = 34, k = 33, T = 1, \) and \( \sigma = 0.8 \).

\[
V(t) \leq V(\sigma)E_\alpha(\eta_2(t - \sigma)^\alpha) \\
\leq V(0)E_\alpha(-\eta_2 \sigma^\alpha)E_\alpha(\eta_2(t - \sigma)^\alpha). \tag{30}
\]

When \( T \leq t \leq T + \sigma \), we have

\[
V(t) \leq V(T)E_\alpha(-\eta_1(t - T)^\alpha) \\
\leq V(0)E_\alpha(-\eta_1 \sigma^\alpha)E_\alpha(\eta_1(T - \sigma)^\alpha)E_\alpha(-\eta_1(t - T)^\alpha). \tag{31}
\]

When \( T + \sigma < t < 2T \), we obtain

\[
V(t) \leq V(T + \sigma)E_\alpha(\eta_2(t - T - \sigma)^\alpha) \\
\leq V(0)E_\alpha(-\eta_2 \sigma^\alpha)E_\alpha(\eta_2(T - \sigma)^\alpha)E_\alpha(-\eta_1(t - T - \sigma)^\alpha) \\
\leq V(0)E_\alpha(-\eta_1 \sigma^\alpha)^2E_\alpha(\eta_2(T - \sigma)^\alpha)E_\alpha(\eta_2(t - T - \sigma)^\alpha). \tag{32}
\]

By induction, when \( mT \leq t \leq mT + \sigma \), we have

\[
V(t) \leq V(mT)E_\alpha(-\eta_1(t - mT)^\alpha) \\
\leq V(0)E_\alpha(-\eta_1 \sigma^\alpha)^mE_\alpha(\eta_2(T - \sigma)^\alpha)^mE_\alpha(-\eta_1(t - mT)^\alpha). \tag{33}
\]

From Lemma 4, we obtain

\[
V(t) \leq V(0)(E_\alpha(-\eta_1 \sigma^\alpha)E_\alpha(\eta_2(T - \sigma)^\alpha))^m. \tag{34}
\]

when \( mT + \sigma < t < (m + 1)T \), we have

\[
V(t) \leq V(mT + \sigma)E_\alpha(\eta_2(t - mT - \sigma)^\alpha) \\
\leq V(0)E_\alpha(-\eta_1 \sigma^\alpha)^mE_\alpha(\eta_2(T - \sigma)^\alpha)^mE_\alpha(-\eta_1 \sigma^\alpha)^mE_\alpha(\eta_2(t - mT - \sigma)^\alpha). \tag{35}
\]

Therefore, from inequality (34) and (35), we have

\[
V(0)(E_\alpha(-\eta_1 \sigma^\alpha)E_\alpha(\eta_2(T - \sigma)^\alpha))^m, \\
V(t) \leq V(0)(E_\alpha(-\eta_1 \sigma^\alpha)E_\alpha(\eta_2(T - \sigma)^\alpha))^mE_\alpha(\eta_2(t - mT - \sigma)^\alpha), \tag{36}
\]

According to condition (21), since \( T = 1, \sigma = 0.8 \). Hence, \( ||e(t)|| \to 0 \) as \( t \to \infty \). It follows that the error system (16) is globally stable. It means that system (14) is anti-synchronized with system (13). This completes the proof. \( \square \)

**Remark 1.** Because \( T, \sigma, \eta_1, \) and \( \eta_2 \) are nonnegative constants, we can get \( 0 \leq E_\alpha(-\eta_1 \sigma^\alpha) \leq 1, E_\alpha(0) = 1 \) based on Lemma 4. According to Lemma 2, when \( T > \sigma, E_\alpha(\eta_2(T - \sigma)^\alpha) \geq 1 \) is a monotone increasing function. Therefore, there are suitable constants \( T, \sigma, \eta_1, \) and \( \eta_2 \), which makes the condition (21) of Theorem 1 hold.

### 4. Numerical Simulations

In this section, some numerical simulations are given to illustrate the theoretical analysis.

Based on Figure 2 and Assumption 1, when \( M_1 = 1.5, M_2 = 1.5 \), \( \lambda = 425 \) are selected, \( \lambda, \beta, M, M_3 = 16.892 \) is obtained. According to condition (20) of Theorem 1, we can obtain \( \eta_1 \geq 33.784 \). Moreover we set \( \alpha = 0.985, \eta_2 = 34, T = 1, \) and \( \sigma = 0.8, \) by condition (21) of Theorem 1, we get
\[ E_a(-\eta_1 \alpha^\sigma)E_a(\eta_1(T - \sigma)\alpha^\sigma) = E_{0.985}(-\eta_1 \times 0.8^{0.985}) E_{0.985}(34 \times (1 - 0.8)^{0.985}) < 1, \quad (38) \]

by Matlab calculation program, we can get that \( \eta_1 > 27.57 \) satisfies condition (21) of Theorem 1. When we set \( \eta_1 = 30 \) and \( k \geq 31.892 \), the condition (19) of Theorem 1 holds.

Therefore, when we choose \( \alpha = 0.985 \), \( \eta_1 = 30 \), \( \eta_2 = 34 \), \( k = 33 \), \( T = 1 \), and \( \sigma = 0.8 \), all the conditions in Theorem 1 hold. The initial values of the systems of (13) and (14) are set to be \((0, 0.01, 0, 0)\) and \((1.0345, 0.2048, -1.9919, -0.5568)\), respectively. It follows from Theorem 1 that the system (14) is anti-synchronized with system (13) under the periodic intermittent control.

Figure 3 shows the time evolution curves of systems (13) and (14) without periodic intermittent controller (15), indicating that there are different trajectories over time. By several conditions, Figure 4 displays the state trajectories of systems (13) and (14) with the periodic intermittent controller (15). Figure 5 depicts the error dynamics of the two systems with a periodic intermittent controller (15), which indicates that anti-synchronization can be implemented infinite time. The time evolution of the intermittent feedback control gain \( k(t) \) is shown in Figure 6.

5. Conclusions

In this paper, the anti-synchronization of FCCM via periodic intermittent control has been achieved in finite time based on periodic intermittent control principle and Lyapunov stability theory. In addition, some numerical simulations have been provided to demonstrate the effectiveness of the proposed approach. The result will have potential applications for image encryption, cryptography, and chaotic radar. Our future research is to investigate the anti-synchronization of FCCM with time delay via nonperiodic intermittent control.

Data Availability

All data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interests.

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