Hindawi
Advances in Mathematical Physics
Volume 2020, Article ID 5602373, 6 pages
https://doi.org/10.1155/2020/5602373

*Research Article*

# The Solitary Wave Solution for Quantum Plasma Nonlinear Dynamic Model

**Yihu Feng [1,2] and Lei Hou [1]**

[1] *Department of Mathematics, Shanghai University, Shanghai 200444, China*
[2] *Department of Electronics and Information Engineering, Bozhou University, Bozhou, Anhui 236800, China*

Correspondence should be addressed to Yihu Feng; fengyihubzxy@163.com

Received 2 October 2019; Accepted 19 November 2019; Published 6 January 2020

Academic Editor: P. Areias



In this paper, we discussed the quantum plasma system. A nonlinear dynamic disturbed model is studied. We used the undetermined coefficients method, dimensionless transformation and traveling wave transformation for the hyperbolic functions, and perturbation theory and method; then, the solitary wave solution for the quantum plasma nonlinear dynamic model is solved. Finally, the characteristics of the corresponding physical quantity are described.

## 1. Introduction

In recent years, there have been many discussions on the study of various kinds of plasma, such as laser plasma, dense celestial plasma, and dust magnetic plasma. Jung [1] studied the quantum-mechanical effects on electron-electron scattering cross-sections which are investigated in dense high-temperature plasmas; an effective pseudopotential model taking into account both quantum-mechanical effects and plasma screening effects is applied to describe electron-electron interactions in dense high-temperature plasmas. Kremp et al. [2] discussed a kinetic theory for quantum many-particle systems in time-dependent electromagnetic fields which is developed based on a gauge-invariant formulation. The resulting kinetic equation generalizes previous results to quantum systems and includes many-body effects. It is, in particular, applicable to the interaction of strong laser fields with dense correlated plasmas. Shukla et al. [3] studied the decay of a magnetic-field-aligned Alfven wave into inertial and kinetic Alfven waves in plasmas and so on [4–9]. Some quantum plasma acoustic wave and quantum-related properties have also been discussed in depth [10, 11].

Nowadays, many scholars [12] have made a lot of discussions on the solutions of nonlinear problems. Ramos [13] studied the existence, multiplicity, and shape of positive solutions of the system $-\varepsilon^2\Delta u + V(x)u = K(x)g(v), -\varepsilon^2\Delta v + V(x)v = H(x)f(u)$ in $R^N$, as $\varepsilon \to 0$. The functions $f$ and $g$ are power-like nonlinearities with superlinear and subcritical growth at infinity, and $V, H,$ and $K$ are positive and locally Hölder continuous.

Teresa and Angela [14] studied the existence of some new positive interior spike solutions to a semilinear Neumann problem.

Faye et al. [15] built models for short-term, mean-term, and long-term dynamics of dune and megariple morphodynamics. They are models that degenerated parabolic equations which are, moreover, singularly perturbed. Then, they give an existence and uniqueness result for the short-term and mean-term models. This result is based on a time-space periodic solution existence result for degenerated parabolic equation that they set out. Finally, the short-term model is homogenized.

Sirendaoerji and Tangetusang [16] constructed some new exact solitary solutions to the generalized mKdV equation and generalized Zakharov-Kuznetsov equation based on hyperbolic tanh-function method and homogeneous balance method, and auxiliary equation method, by the method of auxiliary equation with function transformation with aid of symbolic computation system Mathematica. The method is of important significance in seeking new exact solutions to the evolution equation with arbitrary nonlinear term.

Mao et al. [17] discussed nonlinear waves in an inhomogeneous quantum plasma; for an inhomogeneous quantum magnetoplasma system with density and temperature gradients, a two-dimensional nonlinear fluid dynamic equation is derived in the case where the collision frequency between ions and neutrals is minor. They also discussed how the shock, explosion, and vortex solutions of the potential for this system are obtained as well as the changes on the potential in the dense astrophysical environment.

Mo and others [18–26] have also discussed a class of nonlinear physical problems by means of some asymptotic analysis methods.

In this paper, a quantum plasma system with temperature gradient and density is discussed in the case of electrons and ions. The solitary wave solutions of its nonlinear system are discussed by using the relevant mathematical physical methods and theories.

The rest of this paper is organized as follows. In Section 2, we used dimensionless transformation and traveling wave transformation to discuss a nonlinear partial differential mathematical physical equation. In Section 3, we obtained zero solitary wave of plasma electrostatic potential. In Section 4, we obtained each solitary wave of the electrostatic potential of plasma. In Section 5, we obtained the force function of plasma solitary wave. Finally, a brief conclusion is given in Section 6, in which we used the approximate method to solve it; the hyperbolic function method of undetermined coefficients and perturbation theory is an effective way. The method of solution is parse operation. So it continues to study solitary wave solutions related to physical quantities of other physical state.

## 2. Quantum Plasma Dynamics Model

According to the theory of quantum plasma dynamics, when the collision frequency of quantum is relatively low, the nonlinear dynamic model of a class of unhomogeneous quantum plasma is as follows [17].

$$\frac{3}{2}\frac{\partial^2 u}{\partial t^2} + a\frac{\partial^2 u^2}{\partial t^2} + \left(H^2 - \lambda^2 - \rho^2\right)\frac{\partial^4 u}{\partial t^2 \partial y^2} - \omega\rho^2\frac{\partial^3 u}{\partial t \partial y^2} + \frac{3u}{2}\frac{\partial^2 u}{\partial t \partial y} - D_0\frac{\partial^2 u^2}{\partial t \partial y} - c^2\frac{\partial^2 u}{\partial z^2} = f(\varepsilon, u). \tag{1}$$

In the above expression, $u$ is the electrostatic potential, $a = -3e/4lT$, $l$ is the Boltzmann constant, $e$ is the electric quantity of electrons, $T$ is the Fermi temperature of electrons, $\lambda$ is the Fermi wavelength of the electron, $\rho = \sqrt{lT/m\Omega^2}$ is the Fermi radius of the electron, $H$ is the quantum parameter, $\omega$ is the ion cyclotron frequency, $m$ is the mass of ions, and $v$ is the drift velocity of quantum ions. $D_0 = 3(l_n - l_t)/4B_0$, where $l_n$ is the density coefficient, $l_t$ is the temperature gradient, $B_0$ is the measure of uniform external magnetic field, $\Omega$ is the ion cyclotron frequency, $c$ is the sound velocity of quantum ions, and $f$ is the perturbation term caused by the related factors of quantum plasma. Let it be a sufficiently smooth function and $f(0, u) = 0$.

Equation (1) is a nonlinear partial differential mathematical physical equation. We use a special method to discuss the perturbed solitary wave solutions of Eq. (1).

Dimensionless transformation for Eq. (1): $v = eu/lT$, $y_1 = y/\rho$, $z_1 = z/\rho$, $t_1 = \Omega t$, form normalization and Eq. (1) becomes

$$\frac{\partial^2 v}{\partial t_1^2} - b_1\frac{\partial^2 v^2}{\partial t_1^2} - b_2\frac{\partial^4 v}{\partial t_1^2 \partial y_1^2} - b_3\frac{\partial^3 v}{\partial t_1 \partial y_1^2} - b_4\frac{\partial^2 v}{\partial z_1^2} - b_5\frac{\partial^2 v}{\partial t_1 \partial y_1} - b_6\frac{\partial^2 v^2}{\partial t_1 \partial y_1} = g(\varepsilon, v), \tag{2}$$

where $g(\varepsilon, v) = f(\varepsilon, lT/ev)$ and the expressions of dimensionless coefficient $b_i (i = 1, 2, \cdots, 6)$ are omitted.

Solitary wave solutions for the nonlinear dimensionless quantum plasma (Eq. (2)) are obtained by introducing traveling wave transformation.

$$\zeta = \bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t_1, \tag{3}$$

where $\bar{l}_1$ and $\bar{l}_2$ are wave numbers and $\bar{l}_3$ is the wave frequency, substituting Eq. (3) into Eq. (2).

$$\frac{d^4 v}{d\zeta^4} + \alpha\frac{d^3 v}{d\zeta^3} + \beta\frac{d^2 v}{d\zeta^2} + \gamma v\frac{d^2 v}{d\zeta^2} + \kappa\frac{d^2 v^2}{d\zeta^2} = G(\varepsilon, v), \tag{4}$$

where

$$\begin{aligned}
\alpha &= -\frac{b_3}{b_2\bar{l}_3}, \\
\beta &= -\frac{\bar{l}_3^2 - b_4\bar{l}_2^2 + b_5\bar{l}_1\bar{l}_3}{b_2\bar{l}_1^2\bar{l}_3^2}, \\
\gamma &= \frac{-l_3^2 + b_4\bar{l}_2^2 - b_5\bar{l}_1\bar{l}_3}{b_2\bar{l}_1^2\bar{l}_3^2}, \\
\kappa &= \frac{b_1\bar{l}_3 - b_6\bar{l}_1}{b_2\bar{l}_1^2\bar{l}_3}, \\
G(\varepsilon, v) &= -\frac{g(\varepsilon, v)}{b_2\bar{l}_1^2\bar{l}_3^2}.
\end{aligned} \tag{5}$$

## 3. Zeroth Solitary Wave of the Plasma Electrostatic Potential

Let the perturbation solution for the nonlinear dimensionless traveling wave (Eq. (4)) be [12]

$$v(\zeta, \varepsilon) = \sum_{i=0}^{\infty} v_i(\zeta)\varepsilon^i. \tag{6}$$

Substituting Eq. (6) into the traveling wave (Eq. (4)), the nonlinear term is expanded according to perturbation parameter $\varepsilon$, combine the same power terms of $\varepsilon^i (i = 0, 1, \cdots)$, and let their coefficients equal to zero.

From the coefficients of $\varepsilon^0$ equal zero, the nonlinear equation is obtained.

$$\frac{d^4 v_0}{d\zeta^4} + \alpha \frac{d^3 v_0}{d\zeta^3} + \beta \frac{d^2 v_0}{d\zeta^2} + \gamma v_0 \frac{d^2 v_0}{d\zeta^2} + \kappa \frac{d^2 v_0^2}{d\zeta^2} = 0, \quad (7)$$

where $\alpha$, $\beta$, $\gamma$, and $\kappa$ are represented by (5).

The solution of Eq. (7) is obtained by the undetermined coefficient method of hyperbolic function [12]. Let Eq. (7) have a solitary wave solution.

$$v_0(\zeta) = P_0 + P_1 \tanh(\zeta) + P_2 \tanh^2(\zeta) + \frac{Q_1}{\tanh(\zeta)} + \frac{Q_2}{\tanh^2(\zeta)}, \quad (8)$$

where $P_i, Q_j (i = 0, 1, 2, j = 1, 2)$ are undetermined constants. By substituting Eq. (8) into the nonlinear Eq. (7), combining the positive and negative powers coefficients of $\tanh(\zeta)$, and setting them to zero, we can determine the coefficients of $P_i$ and $Q_j$. Then, we substituted $P_i$ and $Q_j$ into Eq. (8), which have the following two solitary wave solutions.

$$v_{10}(\zeta) = T_0 + T_1 \frac{(2\tanh(\zeta) + 1)}{\tanh^2(\zeta)}, \quad (9)$$

$$v_{20}(\zeta) = T_0 + T_1 (2 - \tanh(\zeta)) \tanh(\zeta), \quad (10)$$

where $T_0$, $T_1$ are constants,

$$T_0 = -\frac{12 l_1^2 b_2 b_3^2 + 10 l_1 b_1 b_3 b_5 - 100 l_2^2 b_2^2 b_4 + b_3^2}{2 b_3 (b_1 b_3 - 10 l_1 b_2 b_6)}, \quad T_1 = \frac{6 l_1^2 b_2 b_3}{b_1 b_3 - 10 l_1 b_2 b_6}. \quad (11)$$

Let $T_0 = 0, T_1 = 10$. The solitary wave solutions (9) are described in the normal solitary wave solution of the singular soliton $v_{10}(\zeta)$, which is shown in Figure 1.

Take $T_0 = 0, T_1 = 1$; at this time, the solitary wave solutions (9) is described in the solitary wave solution $\bar{v}_{10}(\zeta)$, which is shown in Figure 2.

Comparing Figures 1 and 2, it can be seen that the two corresponding solitary waves ($v_{10}$ and $\bar{v}_{10}$) are quite different in the strength (dimensionless scale) of the corresponding physical quantities by choosing different physical parameters.

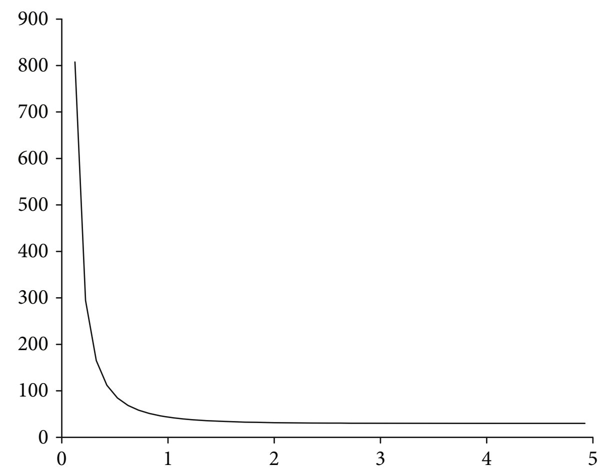


Figure 1: Illustration for the zeroth solitary wave $v_{10}(\zeta)$ curve in quantum plasma electrostatic potential ($T_0 = 0, T_1 = 10$) ($\zeta$ for the abscissa, $v_{10}(\zeta)$ for the ordinate).

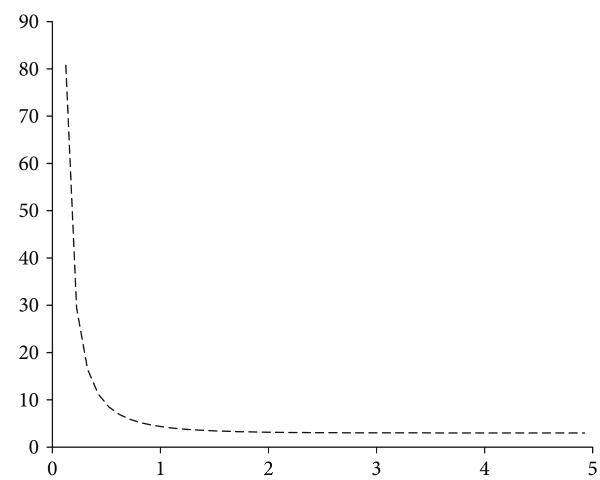


Figure 2: Illustration for the zeroth solitary wave $\bar{v}_{10}(\zeta)$ curve in quantum plasma electrostatic potential ($T_0 = 0, T_1 = 1$) ($\zeta$ for the abscissa, $\bar{v}_{10}(\zeta)$ for the ordinate).

## 4. Each Solitary Wave of the Plasma Electrostatic Potential

Substituting Eq. (7) into travelling wave Eq. (4), expand nonlinear terms according to perturbation parameter $\varepsilon$ and combine the terms of the same power of $\varepsilon^i (i = 0, 1, \cdots)$. From the coefficient of $\varepsilon^i$ which is equal to zero, the linear equation is obtained.

$$\frac{d^4 v_1}{d\zeta^4} + \alpha \frac{d^3 v_1}{d\zeta^3} + (\beta + (2\kappa + \gamma) v_0) \frac{d^2 v_1}{d\zeta^2} + 4\kappa \frac{d v_0}{d\zeta} \frac{d v_1}{d\zeta} + (2\kappa + \gamma v_0) \frac{d^2 v_0}{d\zeta^2} v_1 = G(0, v_0), \quad (12)$$

where $v_0$ is a known function determined by Eqs. (9) and (10). The solutions $v_{11}(\zeta)$ and $v_{21}(\zeta)$ under zero initial value can be obtained from linear equation (12).

From the perturbation theory ([12] and Eq. (7)), we can obtain $v_1(\zeta, \varepsilon), v_2(\zeta, \varepsilon)$ of the first approximate solitary wave solutions which are two nonuniform nonlinear

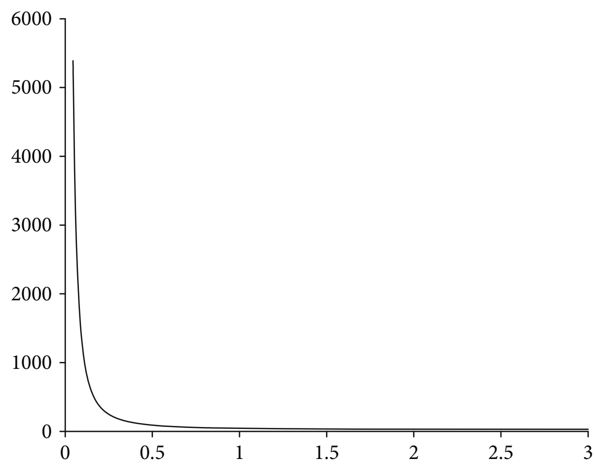


Figure 3: Illustration for the first solitary wave $v_{11}(\zeta)$ curve in quantum plasma electrostatic potential ($T_0 = 0, T_1 = 10$) ($\zeta$ for the abscissas, $v_{11}(\zeta)$ for the ordinate).

dimensionless quantum plasmas for the electrostatic potential (Eq. (4)).

$$v_1(\zeta, \varepsilon) = T_0 + T_1 \frac{(2 \tanh(\zeta) + 1)}{\tanh^2(\zeta)} + \varepsilon v_{11}(\zeta) + O(\varepsilon^2), \quad 0 < \varepsilon \ll 1, \tag{13}$$

$$v_2(\zeta, \varepsilon) = T_0 + T_1(2 - \tanh(\zeta)) \tanh(\zeta) + \varepsilon v_{21}(\zeta) + O(\varepsilon^2), \quad 0 < \varepsilon \ll 1. \tag{14}$$

Take $T_0 = 0$, $T_1 = 10$; at this time, Eq. (13) of the solitary wave solution describes the first perturbation function $v_{11}(\zeta)$ for the electrostatic potential of solitary wave, which is shown in Figure 3.

By the same way, the sequence of functions $\{v_{1n}\}$ and $\{v_{2n}\}$ can be obtained successively from the structure of the quantum plasma nonlinear dynamics (Eq. (4)).

According to the perturbation theory [12], the series can be determined by Eq. (7).

$$\bar{v}_1(\zeta, \varepsilon) = \sum_{i=0}^{\infty} v_{1i}(\zeta)\varepsilon^i, \bar{v}_2(\zeta, \varepsilon) = \sum_{i=0}^{\infty} v_{2i}(\zeta)\varepsilon^i, \tag{15}$$

where is uniformly valid for the finite interval of $\zeta$, and

$$\begin{aligned} \bar{v}_1(\zeta, \varepsilon) &= \sum_{i=0}^{n} v_{1i}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1, \\ \bar{v}_2(\zeta, \varepsilon) &= \sum_{i=0}^{n} v_{2i}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1. \end{aligned} \tag{16}$$

Therefore, $v_{1n}(\zeta, \varepsilon)$ and $v_{2n}(\zeta, \varepsilon)$ are the $n$th asymptotic approximate solutions for the electrostatic potential solitary waves of the two quantum plasmas in Eq. (4).

$$\begin{aligned} v_{1n}(\zeta, \varepsilon) &= T_0 + T_1 \frac{(2 \tanh(\zeta) + 1)}{\tanh^2(\zeta)} \\ &\quad + \sum_{i=1}^{n} v_{1i}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1, \\ v_{2n}(\zeta, \varepsilon) &= T_0 + T_1(2 - \tanh(\zeta)) \tanh(\zeta) \\ &\quad + \sum_{i=1}^{n} v_{2i}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1. \end{aligned} \tag{17}$$

Considering the transformation (3), $u_n(l_1 y_1 + l_2 z_1 - l_3 t_1)$ is the $n$th traveling wave solution for the two electrostatic potential solitary waves of the quantum plasma nonlinear dynamics dimensionless (Eq. (2)).

$$\begin{aligned} u_{1n}(x_1, y_1, z_1, \varepsilon) &= T_0 + T_1 \frac{(2 \tanh(\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t) + 1)}{\tanh^2(\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t)} \\ &\quad + \sum_{i=1}^{n} v_{1i}(\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t)\varepsilon^i + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1, \\ u_{2n}(x_1, y_1, z_1, \varepsilon) &= T_0 + T_1(2 - \tanh(\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t)) \tanh \\ &\quad \cdot (\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t) + \sum_{i=1}^{n} v_{2i}(\bar{l}_1 y_1 + \bar{l}_2 z_1 - \bar{l}_3 t)\varepsilon^i \\ &\quad + O(\varepsilon^{n+1}), \quad 0 < \varepsilon \ll 1. \end{aligned} \tag{18}$$

## 5. Force Function of the Plasma Solitary Wave

By using the electrostatic potential solitary wave perturbation solution for the quantum plasma nonlinear dynamics dimensionless (Eq. (4)) [12], all kinds of relevant physical quantities can be obtained. For example, from Eq. (4) and relational Eqs. (13) and (14), it is not difficult to obtain the subsolitary wave functions $F_{1n}(\zeta, \varepsilon)$ and $F_{2n}(\zeta, \varepsilon)$ of the two dimensionless force functions for the quantum plasma nonlinear dynamics.

$$\begin{aligned} F_{1n}(\zeta, \varepsilon) &= 2T_1 \left(1 + \frac{1}{\tanh(\zeta)} - \frac{1}{\tanh^2(\zeta)} - \frac{1}{\tanh^3(\zeta)}\right) \\ &\quad + \sum_{i=1}^{n} \frac{dv_{1i}}{d\zeta}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad n = 1, 2, \cdots, 0 < \varepsilon \ll 1, \\ F_{2n}(\zeta, \varepsilon) &= 2T_1(1 - \tanh(\zeta) - \tanh^2(\zeta) + \tanh^3(\zeta)) \\ &\quad + \sum_{1=1}^{n} \frac{dv_{2i}}{d\zeta}(\zeta)\varepsilon^i + O(\varepsilon^{n+1}), \quad n = 1, 2, \cdots, 0 < \varepsilon \ll 1. \end{aligned} \tag{19}$$

Take $T_1 = 10$; the curve of the first solitary wave function $F_{11}(\zeta, \varepsilon)$ for the dimensionless force function to the quantum plasma nonlinear dynamics model is shown Figure 4.

## 6. Conclusion

From nonuniform quantum dynamics equations of the plasma system, we can discuss the impact of the potential solutions of the system, explosion, and vortex solutions and

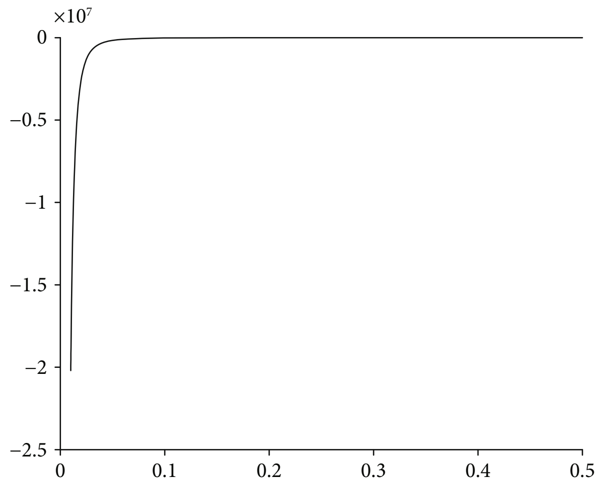


FIGURE 4: Illustration for the first solitary wave $F_{11}(\zeta)$ curve in quantum plasma force function $(T_1 = 10)$ ($\zeta$ for the abscissa, $F_{11}(\zeta)$ for the ordinate).

analyze the range of the electric potential of the shock wave and the blast wave width and density for the relationship between the changes to the drift velocity, to understand the stability of the static potential changing at any time in space.

Unhomogeneous quantum plasma disturbance system comes from a complex natural phenomena. In order to study the nonlinear solitary of more complex models, sometimes, we need to use the approximate method to solve it. In this paper, the hyperbolic function of undetermined coefficients method and perturbation theory is an effective way. The method of solution is parse operation. So it can continue to study the solitary wave solutions related to physical quantities of other physical state.

## Data Availability

The authors declare that the data in the article is available, shareable, and referenced. Readers can check to each article here: http://apps.webofknowledge.com/.

## Conflicts of Interest

No potential conflict of interest was reported by the authors.

## Acknowledgments

This work has been supported by the National Natural Science Foundation of China (11271247); the Project of Support Program for Excellent Youth Talent in Colleges and Universities of Anhui Province (gxyqZD2016520); the Key Project for Teaching Research in Anhui Province (2018jyxm0594, 2016jyxm0677); the Key Project of Teaching Research of Bozhou University (2017zdjy02); and the Key Project of the Natural Science Research of Bozhou University (BYZ2017B02).

## References


[1] Y. D. Jung, "Quantum-mechanical effects on electron-electron scattering in dense high-temperature plasmas," *Physics of Plasmas*, vol. 8, no. 8, pp. 3842–3844, 2001.

[2] D. Kremp, T. Bornath, and M. Bonitz, "Quantum kinetic theory of plasmas in strong laser fields," *Physical Review E*, vol. 60, no. 4, pp. 4725–4732, 1999.

[3] P. K. Shukla, M. Mond, I. Kourakis, and B. Eliasson, "Nonlinearly coupled whistlers and dust-acoustic perturbations in dusty plasmas," *Physics of Plasmas*, vol. 12, no. 12, article 124502, 2005.

[4] J. Yang, Y. Xu, Z. Meng, and T. Yang, *Physics of Plasmas*, vol. 15, no. 2, article 023503, 2008.

[5] T. Zhou, R. Yu, H. Li, and B. Wang, "Ocean forcing to changes in global monsoon precipitation over the recent half-century," *Journal of Climate*, vol. 21, no. 15, pp. 3833–3852, 2008.

[6] T. Zhou, B. Wu, and B. Wang, "How well do atmospheric general circulation models capture the leading modes of the interannual variability of the Asian–Australian monsoon?," *Journal of Climate*, vol. 22, no. 5, pp. 1159–1173, 2009.

[7] T. Zhou and J. Zhang, "The vertical structures of atmospheric temperature anomalies associated with two flavors of El Niño simulated by AMIP II models," *Journal of Climate*, vol. 24, no. 4, pp. 1053–1070, 2011.

[8] T. J. Zhou, B. Wu, A. A. Scaife et al., "The CLIVAR C20C project: which components of the Asian-Australian monsoon circulation variations are forced and reproducible?," *Climate Dynamics*, vol. 33, no. 7-8, pp. 1051–1068, 2009.

[9] T. Zhou, R. Yu, J. Zhang et al., "Why the Western Pacific subtropical high has extended westward since the late 1970s," *Journal of Climate*, vol. 22, no. 8, pp. 2199–2215, 2009.

[10] Q. Haque and S. Mahmood, "Drift solitons and shocks in inhomogeneous quantum magnetoplasmas," *Physics of Plasmas*, vol. 15, no. 3, article 034501, 2008.

[11] W. Masood, "Drift ion acoustic solitons in an inhomogeneous 2-D quantum magnetoplasma," *Physics Letters A*, vol. 373, no. 16, pp. 1455–1459, 2009.

[12] L. Barbu and G. Morosanu, *Singularly Perturbed Boundary-Value Problems*, Birkhauserm Verlag AG, Basel, 2007.

[13] M. Ramos, "On singular perturbations of superlinear elliptic systems," *Journal of Mathematical Analysis and Applications*, vol. 352, no. 1, pp. 246–258, 2009.

[14] T. D'Aprile and A. Pistoia, "On the existence of some new positive interior spike solutions to a semilinear Neumann problem," *Journal of Differential Equations*, vol. 248, no. 3, pp. 556–573, 2010.

[15] L. Faye, E. Frenod, and D. Seck, "Singularly perturbed degenerated parabolic equations and application to seabed morphodynamics in tided environment," *Discrete and Continuous Dynamical Systems*, vol. 29, no. 3, pp. 1001–1030, 2011.

[16] T. Sirendaoerji, "New exact solitary wave solutions to generalized mKdV equation and generalized Zakharov–Kuzentsov equation," *Chinese Physics*, vol. 15, no. 6, pp. 1143–1148, 2006.

[17] J. Mao, J. Yang, and C. Li, "Nonlinear waves in an inhomogeneous quantum plasma," *Acta Physica Sinica*, vol. 61, no. 2, article 020206, 2012.

[18] J. Mo, "Homotopic mapping solving method for gain fluency of a laser pulse amplifier," *Science in China Series G: Physics, Mechanics and Astronomy*, vol. 52, no. 7, pp. 1007–1010, 2009.

[19] J. Mo, "A variational iteration solving method for a class of generalized Boussinesq equations," *Chinese Physics Letters*, vol. 26, no. 6, article 060202, 2009.

[20] J. Mo, Y. Lin, and W. Lin, "Approximate solution of sea-air oscillator for El Nino-southern oscillation model," *Acta Physica Sinica*, vol. 59, no. 10, pp. 6707–6711, 2010.

[21] J. Mo, "Travelling wave solution of disturbed Vakhnenko equation for physical model," *Acta Physica Sinica*, vol. 60, no. 9, article 090203, 2011.

[22] J. Mo, "Generalized variational iteration solution of soliton for disturbed KdV equation," *Communications in Theoretical Physics*, vol. 53, no. 3, pp. 440–442, 2010.

[23] J. Mo, W. Lin, and Y. Lin, "Asymptotic solution for the El Niño time delay sea-air oscillator model," *Chinese Physics B*, vol. 20, no. 7, article 070205, 2011.

[24] L. Hou, H. Li, J. Zhang, D. Lin, and L. Qiu, "Boundary-layer eigen solutions for multi-field coupled equations in the contact interface," *Applied Mathematics and Mechanics*, vol. 31, no. 6, pp. 719–732, 2010.

[25] L. Hou and L. Qiu, "Computation and asymptotic analysis in the impact problem," *Acta Mathematicae Applicatae Sinica, English Series*, vol. 25, no. 1, pp. 117–126, 2009.

[26] E. M. De Jager and J. Furu, *The Theory of Singular Perturbation*, North-Holland Publishing Co, Amsterdam, 1996.