Research Article

Toughness Calculation of Postfire Normal Concrete

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Fracture tests of postfire normal concrete with ten temperatures up to 600°C are implemented. Residual fracture toughness using analytical method is determined. Two situations are divided at critical load when calculating the cohesive fracture toughness. The initial and critical fracture toughness could be calculated from the complete load-crack opening displacement curves. Finally, the validation of double- K fracture model to the postfire concrete specimens is proved.

1. Introduction

The fracture process of concrete structures underwent three main stages: (i) crack initiation, (ii) stable crack propagation, and (iii) unstable fracture. Accordingly, the double- K fracture criterion initially introduced by Xu and Reinhardt [1] showed the crack initiation, crack propagation, and failure during a fracture process until the maximum load is reached. And the two size-independent parameters, initial cracking toughness, K_I^{ini} , and unstable fracture toughness, K_I^{un} , can be used to study the crack propagation of concrete.

In order to determine the double- K fracture parameters analytically [2, 3] the value of cohesion toughness K_I^c due to cohesive stress distribution in the fictitious fracture zone was computed using method proposed by Jenq and Shah [4]. The influences of geometrical parameter [5–7] and size effect [2, 3, 8] on fracture toughness were studied by various researchers. It was found that the influence of a_0/D ratio and shape of test specimen on the values of fracture parameters were relatively less than the one of size effect.

The influence of temperature on the fracture parameters was also considered by several researchers, but mainly on the fracture energy and material brittleness [9–13] and relatively fewer discussions on the fracture toughness [14, 15]. In the present paper, the calculation of residual fracture toughness of concrete is carried out. Wedge-splitting experiments of totally ten temperatures varying from 20°C to 600°C are

implemented. The specimen sizes are of $230 \times 200 \times 200$ mm with initial-notch depth ratios of 0.4. The validation of double- K fracture model to the postfire normal concrete is proved.

2. Analytical Determination of Cohesive Fracture Toughness

2.1. Effective Crack Extension Length and Residual Young's Modulus. The linear asymptotic superposition assumption was considered in the analytical method presented by Reinhardt et al. [2, 3] to introduce the concept of linear elastic fracture mechanics for calculating the double- K fracture parameters. Detailed explanation of the above assumption can be found elsewhere [2].

Based on this assumption, the value of the equivalent-elastic crack length for WS specimen is expressed as

$$a = (h + h_0) \left\{ 1 - \left(\frac{13.18}{E \cdot b \cdot c + 9.16} \right)^{1/2} \right\} - h_0, \quad (1)$$

where $c = \text{CMOD}/P$ is the compliance of specimens and b is specimens thickness; h is specimens height and h_0 is the thickness of the clip gauge holder. For calculation of critical value of equivalent-elastic crack length a_c , the value of crack mouth opening displacement (CMOD) and P is taken as CMOD_c and P_u , respectively.

FIGURE 1: The bilinear softening traction-separation law.

The residual Young's modulus E is calculated using the P -CMOD curve as

$$E = \frac{1}{bc_i} [13.18 \times (1 - \alpha)^2 - 9.16], \quad (2)$$

where $c_i = \text{CMOD}_{\text{ini}}/P_{\text{ini}}$ is the initial compliance before cracking, $\alpha = (a_0 + h_0)/(h + h_0)$. The values of critical equivalent-elastic crack length a_c and residual Young's modulus E are listed in Table 1.

2.2. Crack Opening Displacement along the Fracture Process Zone. Since the cohesive stress distribution along the fracture process zone depends on the crack opening displacement and the specified softening law, it is important to know the value of crack opening displacement along the fracture line. It is difficult to measure directly the value of COD along the fracture process zone; for practical purposes the value of COD (x) at the crack length x is computed using the following expression [3]:

COD (x)

$$= \text{CMOD} \left\{ \left(1 - \frac{x}{a} \right)^2 + \left(1.018 - 1.149 \frac{a}{h} \right) \left[\frac{x}{a} - \left(\frac{x}{a} \right)^2 \right] \right\}^{1/2}. \quad (3)$$

For calculation of critical value of crack tip opening displacement CTOD_c , the value of x and a (see Figure 3) in (3) is taken to be a_0 and a_c , respectively. The value of cohesive stress along the fictitious fracture zone to the corresponding crack opening displacement is evaluated using bilinear stress-displacement softening law as given in (6).

2.3. Softening Traction-Separation Law of Postfire Concrete. The softening traction-separation law is a prerequisite to determine the double- K fracture parameters, at room temperature; many expressions have been proposed based on direct tensile tests [16–20]. Based on numerical studies, simplified bilinear expressions for the softening traction-separation law (illustrated in Figure 1) were suggested by Petersson in 1981 [16], Hilsdorf and Bramehuber in 1991 [19], and Phillips and Zhang in 1993 [20]. The area under the softening curve was defined as the fracture energy G_F . Therefore, one could get the following equation:

$$G_F = \frac{1}{2} (f_t w_s + \sigma_s w_0), \quad (4)$$

where f_t is tensile strength of postfire specimens, w_s is the crack width at break point of softening curve, σ_s is the cohesive stress at the break point of softening curve, and w_0 is crack width at stress-free point.

As a consequence, a general form of the simplified bilinear expression of the softening traction-separation law is given as follows:

$$\begin{aligned} \sigma &= f_t - \frac{(f_t - \sigma_s)w}{w_0}, \quad 0 \leq w \leq w_s \\ \sigma &= \frac{\sigma_s(w_0 - w)}{(w_0 - w_s)}, \quad w_s \leq w \leq w_0 \\ \sigma &= 0, \quad w \geq w_0. \end{aligned} \quad (5)$$

Different values of the break point (σ_s, w_s) and the crack width w_0 at stress-free point were used for the expression proposed by different researchers. In the present work, the bilinear softening function of concrete proposed by Petersson is used for postfire specimens:

$$\begin{aligned} \sigma_s &= \frac{f_t}{3}, \\ w_s &= \frac{0.8G_F}{f_t}, \\ w_0 &= \frac{3.6G_F}{f_t}. \end{aligned} \quad (6)$$

2.4. Determination of Stress Intensity Factor Caused by Cohesive Force. The standard Green's function [21] for the edge cracks with finite width of plate subjected to a pair of normal forces is used to evaluate the value of cohesive toughness. The general expression for the crack extension resistance for complete fracture associated with cohesive stress distribution in the fictitious fracture zone for Mode I fracture is given as follows:

$$K_I^c = \int_{a_0}^a \frac{2\sigma(x) F(x/a, a/h)}{\sqrt{\pi a}} dx, \quad (7)$$

where

$$\begin{aligned} F\left(\frac{x}{a}, \frac{a}{h}\right) &= \frac{3.52(1 - x/a)}{(1 - a/h)^{3/2}} - \frac{4.35 - 5.28x/a}{(1 - a/h)} \\ &+ \left\{ \frac{1.30 - 0.30(x/a)^{3/2}}{\sqrt{1 - (x/a)^2}} + 0.83 - 1.76 \frac{x}{a} \right\} \\ &\times \left\{ 1 - \left(1 - \frac{x}{a} \right) \frac{a}{h} \right\} \end{aligned} \quad (8)$$

and $\sigma(x)$ is the cohesive force at crack length x , see Figure 3, and its expression is shown in (9) or (11).

At critical condition the value of a is taken to be a_c in (7) and (8). The integration of (8) is done by using Gauss-Chebyshev quadrature method because of existence of singularity at the integral boundary.

TABLE 1: The experimental results of fracture parameters.

Specimen	Temperature	P_{ini} /kN	P_{max} /kN	CMOD _c /mm	CTOD _c /mm	E GPa	G_F N/m	K_I^{ini} MPa·m ^{1/2}	K_I^{C-A} MPa·m ^{1/2}	$K_I^{\text{un-E}}$ MPa·m ^{1/2}	$K_I^{\text{un-A}}$ MPa·m ^{1/2}
WS6	65°C	6.98	11.31	0.195	0.078	21.73	425.91	0.550	1.007	1.594	1.557
WS7		3.88	8.23	0.163	0.100	24.79	482.62	0.303	1.63	1.664	1.933
WS8		6.88	10.41	0.212	0.087	19.43	487.75	0.557	1.006	1.518	1.563
WS9		7.94	10.71	0.164	0.087	23.25	480.51	0.511	1.174	1.685	1.685
WS10		6.32	11.67	0.229	0.086	16.60	522.36	0.562	0.903	1.507	1.465
Average		6.42	10.47	0.193	0.088	21.16	479.83	0.480	1.161	1.594	1.641
WS16	200°C	—	—	—	—	—	—	—	—	—	—
WS17		4.22	6.60	0.317	0.143	11.58	282.51	0.211	0.803	1.206	1.014
WS18		3.97	6.15	0.213	0.138	6.98	215.14	0.353	0.485	0.863	0.838
WS19		2.41	5.33	0.213	0.138	9.17	235.91	0.215	0.72	0.924	0.935
WS20		3.32	5.24	0.458	0.211	7.00	368.94	0.245	0.796	1.022	1.041
Average		3.50	5.72	0.352	0.157	8.68	376.95	0.256	0.688	1.044	0.957
WS31	400°C	1.55	3.61	1.032	0.492	2.56	532.09	0.138	0.572	0.795	0.710
WS32		1.78	3.19	1.218	0.548	1.42	396.93	0.158	0.333	0.574	0.491
WS33		2.37	3.27	0.768	0.299	2.12	434.89	0.174	0.236	0.621	0.410
WS34		2.36	4.53	0.736	0.302	1.71	598.93	0.125	0.356	0.468	0.494
WS35		—	—	—	—	—	—	—	—	—	—
Average		2.01	3.78	0.901	0.410	1.95	490.71	0.152	0.374	0.615	0.526
WS46	600°C	0.76	1.13	1.482	0.684	0.47	228.23	0.067	0.174	0.221	0.231
WS47		0.53	1.48	2.082	0.684	0.48	395.06	0.063	0.209	0.277	0.284
WS48		0.81	1.65	1.908	0.813	1.14	539.22	0.072	0.478	0.550	0.550
WS49		0.58	1.14	1.687	0.973	0.38	331.99	0.052	0.188	0.225	0.225
WS50		0.62	1.48	2.082	0.727	0.38	273.07	0.068	0.155	0.213	0.213
Average		0.62	1.38	1.848	0.799	0.57	353.51	0.064	0.241	0.297	0.301

FIGURE 2: Two different situations for $CTOD_c$ and w_s .

FIGURE 3: Cohesive force distribution along the crack length at critical load.

As shown in Figure 2, two conditions at critical load, that is, $CTOD_c \leq w_s$ and $w_s \leq CTOD_c \leq w_c$, may arise at the notch tip while using bilinear softening function. For specimens subjected to temperatures less than 120°C, the critical $CTOD_c$ is less than w_s , whereas, for temperatures higher than 120°C, the critical $CTOD_c$ is wider than w_s .

- (A) When the critical $CTOD_c$ corresponding to maximum load P_u is less than w_s as shown in Figure 2(a), the distribution of cohesive stress along the fictitious fracture zone is approximated to be linear as shown in Figure 3(a). The variation of cohesive stress along the fictitious fracture zone for this loading condition, that is, $a_0 \leq a \leq a_c$ or $0 \leq CTOD \leq CTOD_c$, is written as

$$\sigma(x) = \sigma(CTOD_c) + \frac{(f_t - \sigma(CTOD_c))(x - a_0)}{(a_c - a_0)}, \quad (9)$$

where, $\sigma(CTOD_c)$ is the critical values of cohesive stress being at the tip of initial notch. The value of $\sigma(CTOD_c)$ is determined by using bilinear softening function:

$$\sigma(CTOD_c) = \sigma_s(w_s) + \frac{w_s - CTOD_c}{w_s} (f_t - \sigma_s(w_s)). \quad (10)$$

- (B) When the critical $CTOD_c$ corresponding to maximum load P_u is wider than w_s as shown in Figure 2(b), the distribution of cohesive stress along the fictitious

fracture zone is approximated to be bilinear as shown in Figure 3(b). The variation of cohesive stress along the fictitious fracture zone for this loading condition, also, $a_0 \leq a \leq a_c$ or $0 \leq CTOD \leq CTOD_c$, is written as

$$\begin{aligned} \sigma_1(x) &= \sigma(CTOD_c) \\ &+ (\sigma_s(w_s) - \sigma(w)) \frac{(x - a_0)}{(a_s - a_0)}, \quad a_s \leq x \leq a_0, \\ \sigma_2(x) &= \sigma_s(w_s) \\ &+ (f_t - \sigma_s(w_s)) \frac{(x - a_s)}{(a_c - a_s)}, \quad a_s \leq x \leq a_c. \end{aligned} \quad (11)$$

The value of $\sigma(CTOD_c)$ is determined by using bilinear softening function:

$$\sigma(CTOD_c) = \frac{w_0 - CTOD_c}{w_0 - w_s} \sigma_s(w_s). \quad (12)$$

The limits of integration of (7) should be taken in two steps: $a_0 \leq x \leq a_s$ for cohesive stress $\sigma_1(x)$ and $a_s \leq x \leq a_c$ for cohesive stress $\sigma_2(x)$, respectively. The same Green's function $F(x/a, a/h)$ for a given effective crack extension will

(a) The tendency of K_I^{ini} with T_m (b) The tendency of K_I^{un} with T_m FIGURE 6: The tendency of residual fracture toughness with heating temperatures T_m . FIGURE 7: Comparison between $K_I^{\text{un-A}}$ and $K_I^{\text{un-E}}$.

120°C, 200°C, 300°C, 350°C, 400°C, 450°C, 500°C, 600°C), were adopted with the ambient temperature as a reference. Each wedge splitting specimen was embedded with a thermal couple in the center of specimen for temperature control. An electric furnace with net dimensions 300 × 300 × 900 mm was used for heating. When the designated T_m was reached, the furnace was shut down, and the specimens were naturally

cooled for 7 days prior to the test. It averagely took 50, 95, 135, 182, 218, 254, 294, 342, and 453 mins for the specimens to reach the final temperatures, respectively (from 65°C to 600°C). The detailed experimental information would be found elsewhere [22].

4.2. Experimental Results. Figure 5 shows typical complete load-displacement curves for different heating temperatures up to 600°C. The figure shows that the ultimate load P_u decreases significantly with increasing temperatures T_m , whereas the crack-mouth opening displacement (CMOD) increases with T_m . The initial slope of ascending branches decreases with heating temperatures, and the curves become gradually shorter and more extended.

The recorded maximum load P_u , the recorded crack mouth opening displacement CMOD_c at P_u , the calculated crack tip opening displacement CTOD_c based on (14), the initial cracking load P_{ini} determined by graphical method, the calculated residual Young's modulus E based on (2), the double-K fracture parameters, that is, K_I^{ini} and $K_I^{\text{un-E}}$, and the residual fracture energy G_F are summarized in Table 1. Here we only list part of the statistics.

4.3. Discussion. In order to express the influence on the residual fracture toughness in detail, Figure 6 plots the tendency of initial fracture toughness K_I^{ini} and the unstable fracture toughness K_I^{un} with heating temperatures T_m . It is concluded that the toughness of the two fractures decreases monotonously with T_m because of the thermal damage induced by the heating temperatures.

The initial fracture toughness continuously decreases from 0.498 MPa·m^{1/2} at room temperature to 0.269 MPa·m^{1/2}

at 200°C, 0.115 MPa·m^{1/2} at 450°C, and finally 0.064 MPa·m^{1/2} at 600°C, with a significant loss of 0.434 MPa·m^{1/2} or 96%. The unstable fracture toughness decreases from 1.186 MPa·m^{1/2} at room temperature to 0.297 MPa·m^{1/2} at 600°C, with a significant loss of 0.889 MPa·m^{1/2} or 75%.

Comparing the results shown in Table 1, it can be known that the value of $K_I^{\text{un-A}}$ evaluated by (17) has good coincidence with the one calculated by inserting the values of P_{max} and $a_c = h$ into (15), that is, the critical fracture toughness from analytical and experimental method. Figure 7 shows the relationship between the two parameters. In totally 45 effective specimens, the deviation between $K_I^{\text{un-A}}$ and $K_I^{\text{un-E}}$ of 22 specimens is below 5% and of 40 specimens is below 15%, which accounts for 89% of total specimens.

5. Conclusion

The determination of residual fracture parameter using analytical method is carried out in present research. In calculating the cohesive fracture toughness, two conditions are divided at critical load: for specimens subjected to temperatures less than 120°C, the critical CTOD_c is less than w_s , whereas, for temperatures higher than 120°C, the critical CTOD_c corresponding to maximum load P_u is wider than w_s . This part of work would be a useful supplement to the existed analysis.

Wedge-splitting tests with ten temperatures varying from 20°C to 600°C are implemented. The complete load-crack opening displacement curves are obtained and the initial and critical fracture toughness could be calculated experimentally.

The validation of double- K fracture model to the postfire concrete specimens is proved. In totally 45 effective specimens, the deviation between analytical value $K_I^{\text{un-A}}$ and experimental $K_I^{\text{un-E}}$ of 22 specimens is below 5% and of 40 specimens is below 15%, which accounts for 89% of total specimens.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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