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Research Article

A One-Dimensional Thermoelastic Problem due to a Moving Heat Source under Fractional Order Theory of Thermoelasticity

Tianhu He and Ying Guo

School of Science, Lanzhou University of Technology, Lanzhou 730050, China

Correspondence should be addressed to Tianhu He; heth@lut.cn

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The dynamic response of a one-dimensional problem for a thermoelastic rod with finite length is investigated in the context of the fractional order theory of thermoelasticity in the present work. The rod is fixed at both ends and subjected to a moving heat source. The fractional order thermoelastic coupled governing equations for the rod are formulated. Laplace transform as well as its numerical inversion is applied to solving the governing equations. The variations of the considered temperature, displacement, and stress in the rod are obtained and demonstrated graphically. The effects of time, velocity of the moving heat source, and fractional order parameter on the distributions of the considered variables are of concern and discussed in detail.

1. Introduction

The classical coupled thermoelasticity proposed by Biot [1] predicts an infinite speed for heat propagating in elastic media, which is physically impossible. To eliminate such an inherent paradox and predict finite speed propagation for heat propagation, the generalized thermoelastic theories have been developed. Lord and Shulman (L-S) [2] developed the first generalized thermoelasticity by postulating a wave-type heat conduction law to replace the classical Fourier law. This law is the same as that suggested by Cattaneo [3] and Vernotte [4], which contains the heat flux vector as well as its time derivative and also contains a new constant that acts as a relaxation time. Later on, Green and Lindsay (G-L) [5] proposed another theory called the temperature rate dependent thermoelasticity by modifying both the energy equation and the Duhamel-Neumann relation, in which two relaxation times were introduced. Subsequently, Green and Naghdi [6–8] advocated the theory of thermoelasticity without energy dissipation and the theory of thermoelasticity with energy dissipation. There also exist other generalized thermoelastic theories such as the two-temperature generalized thermoelasticity by Youssef [9] and the dual-phase-lag thermoelasticity by Tzou [10].

Fractional calculus has been used successfully to modify many existing models of physical processes. One can state that the whole theory of fractional derivatives and integrals was established in the second half of the 19th century. The first application of fractional derivatives was given by Abel who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. The generalization of the concept of derivative and integral to a noninteger order has been subjected to several approaches, and some various alternative definitions of fractional derivatives appeared [11–13]. In the last few years, fractional calculus was applied successfully in various areas to modify many existing models of physical processes especially in the field of heat conduction, diffusion, viscoelasticity, mechanics of solids, control theory, and electricity [14–17]. A survey of applications of the fractional calculus in area of science and engineering can be found in [18].

There exist many materials and physical situations such as low-temperature regimes, amorphous media, colloids, glassy and porous materials, man-made and biological materials/polymers, and transient loading, where the classical coupled thermoelasticity and the generalized thermoelastic theories fail. In such cases, it may be necessary to introduce time-fractional derivatives into thermoelasticity. Povstenko [19] proposed a quasistatic uncoupled theory of thermoelasticity based on the heat conduction equation with a time derivative of fractional order. Later on, he [20] investigated thermal stresses of central-symmetric Cauchy and source

problems for time-fractional heat conduction equation with the fundamental solutions. Youssef [21] and Youssef and Al-Lehaibi [22] formulated the theory of fractional order generalized thermoelasticity by introducing the Riemann-Liouville fractional integral operator into the generalized heat conduction. Based on this theory, Youssef [23] investigated two-dimensional thermal shock problems by Laplace and Fourier transforms; Youssef and Al-Lehaibi [24] solved half-space problems subjected to ramp-type thermal loading by employing Laplace transform and state-space method; Sarkar and Lahiri [25] were concerned with a two-dimensional generalized thermoelastic problem with a rotating elastic medium under the theory of fractional order; Youssef [26] dealt with a two-temperature generalized thermoelastic medium subjected to a moving heat source. Very recently, a completely new theory on fractional order generalized thermoelasticity has been introduced by Sherief et al. [27]. By employing this theory, Kothari and Mukhopadhyay [28] solved an elastic half-space problem with Laplace transform and state-space method. Sherief and Abd El-Latief [29] investigated a half-space problem with different thermal conductivity under the theory of fractional order. In the theory of Sherief et al. [27], the heat conduction equation has a new form as

$$q_i + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha} q_i = -\kappa_{ij}\theta_{,j}, \qquad (1^*)$$

where q_i are the components of the heat flux vector, θ is the temperature increment, τ_0 is the thermal relaxation time, κ_{ij} is the thermal conductivity tensor, and α is a constant parameter such that $0 < \alpha \leq 1$. To derive this theory, the authors use the definition of fractional derivatives of order $\alpha \in (0, 1]$ of the absolutely continuous function $f(t)$ defined by Caputo [30] as

$$\frac{d^\alpha}{dt^\alpha} f(t) = I^{1-\alpha} f'(t), \qquad (1)$$

where I^β is the fractional integral of the function $f(t)$ of order β defined by Miller and Ross [11] as

$$I^\beta f(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} f(s)\, ds. \qquad (2)$$

Here $f(t)$ is a Lebesgue integrable function and $\beta > 0$. The uniqueness theorem, reciprocity theorem, and a variational principle on this theory are also established in the article by Sherief et al. [27]. The heat conduction equation (1^*) reduces to the Cattaneo-Vernotte [3, 4] law in the case $\alpha = 1$. It should be mentioned here that the Cattaneo-Vernotte law

$$q_i + \tau_0 \frac{\partial}{\partial t} q_i = -\kappa_{ij}\theta_{,j} \qquad (3)$$

has been employed by Lord and Shulman [2] to develop the first generalized theory of thermoelasticity.

So far, there are few works on the investigation of problems involving heat source in the context of the fractional order theory of thermoelasticity. In the present work, we consider a one-dimensional problem for a rod subjected to a moving heat source under fractional order theory of generalized thermoelasticity proposed by Sherief et al. [27]. The problem is solved by means of Laplace transform and its numerical inversion. The variations of the considered variables are obtained and illustrated graphically.

2. Basic Equations

We investigate the dynamic problem of a thermoelastic rod subjected to a moving heat source in the context of fractional order theory of thermoelasticity. The rod is fixed at both ends and the applied heat source propagates along x direction with a constant velocity v. For the rod, it can be assumed that the geometrical dimension along x-axis is much greater than those along the other two directions orthogonal to x-axis; thus, the dynamic problem of the rod can be treated as a one-dimensional problem. So, all the considered variables are only functions of x and time t.

In the absence of body force, the governing equations for homogeneous and isotropic elastic media in the context of fractional order of thermoelasticity advocated by Sherief et al. [27] are

$$\sigma_{ij,j} = \rho \ddot{u}_i, \qquad (4)$$

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma\theta)\delta_{ij}, \qquad (5)$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \qquad (6)$$

$$q_{i,i} = -\rho T_0 \dot{\eta} + Q, \qquad (7)$$

$$\rho\eta = \gamma e_{kk} + \frac{\rho C_E}{T_0}\theta, \qquad (8)$$

$$\kappa\theta_{,ii} = \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho C_E \dot{\theta} + T_0 \gamma \dot{e}_{kk} - Q\right). \qquad (9)$$

For this one-dimensional problem, the only remaining displacement component is $u_x = u(x, t)$; therefore, (4), (5), and (9) are reduced to

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial\theta}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2},$$

$$\sigma = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma\theta, \qquad (10)$$

$$\kappa\frac{\partial^2\theta}{\partial x^2} = \left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho C_E\frac{\partial\theta}{\partial t} + \gamma T_0\frac{\partial^2 u}{\partial x \partial t} - Q\right).$$

For convenience, the following nondimensional quantities are introduced:

$$x^* = c_0\eta_0 x, \qquad u^* = c_0\eta_0 u, \qquad t^* = c_0^2\eta_0 t,$$
$$\tau_0^* = c_0^2\eta_0\tau_0, \qquad \theta^* = \frac{\theta}{T_0}, \qquad \sigma^* = \frac{\sigma}{\mu}, \tag{11}$$
$$Q^* = \frac{Q}{kT_0c_0^2\eta_0^2}, \qquad c_0^2 = \frac{\lambda+2\mu}{\rho}, \qquad \eta_0 = \frac{\rho C_E}{k}.$$

In terms of these nondimensional quantities, (10) take the following forms (dropping the asterisks for convenience):

$$\frac{\partial^2 u}{\partial x^2} - \frac{b}{\beta^2}\frac{\partial\theta}{\partial x} = \frac{\partial^2 u}{\partial t^2},$$
$$\sigma = \beta^2\frac{\partial u}{\partial x} - b\theta, \tag{12}$$
$$\frac{\partial^2\theta}{\partial x^2} = \left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial\theta}{\partial t} + g\frac{\partial^2 u}{\partial x\partial t} - Q\right),$$

where

$$\beta^2 = \frac{\lambda+2\mu}{\mu}, \qquad b = \frac{\gamma T_0}{\mu}, \qquad g = \frac{\gamma}{\rho C_E}. \tag{13}$$

The rod is assumed to have a reference temperature T_0 and homogeneous initial conditions as

$$u(x,0) = \dot{u}(x,0) = 0, \qquad \theta(x,0) = \dot{\theta}(x,0) = 0. \tag{14}$$

Assume the rod has a nondimensional length l and both fixed ends are heat insulation; the boundary conditions can be given as

$$u(0,t) = u(l,t) = 0, \qquad \frac{\partial\theta(0,t)}{\partial x} = \frac{\partial\theta(l,t)}{\partial x} = 0. \tag{15}$$

The moving heat source with constant strength, releasing its energy continuously in the process of moving along x-axis with a constant velocity v, is assumed to have the following nondimensional form:

$$Q = Q_0\delta(x - vt), \tag{16}$$

where Q_0 is constant and δ is the delta function.

Applying the Laplace transform defined by

$$L[f(t)] = \overline{f}(p) = \int_0^\infty e^{-pt} f(t)\,dt, \quad \mathrm{Re}(p) > 0 \tag{17}$$

to (12) with (14), we obtain

$$\overline{\sigma} = \beta^2\frac{d\overline{u}}{dx} - b\overline{\theta}, \tag{18}$$

$$\frac{d^2\overline{u}}{dx^2} - \frac{b}{\beta^2}\frac{d\overline{\theta}}{dx} = p^2\overline{u}, \tag{19}$$

$$\frac{d^2\overline{\theta}}{dx^2} = (1+\tau_0 p^\alpha)\left(p\overline{\theta} + gp\frac{d\overline{u}}{dx} - \omega e^{-(p/v)x}\right), \tag{20}$$

where $\omega = Q_0/v$.

The boundary conditions in (14) can be transformed to

$$\overline{u}(0,p) = \overline{u}(l,p) = 0, \qquad \frac{d\overline{\theta}(0,p)}{dx} = \frac{d\overline{\theta}(l,p)}{dx} = 0. \tag{21}$$

3. Solutions in the Laplace Domain

Eliminating $\overline{\theta}$ between (19) and (20), we obtain the following equation satisfied by $\overline{u}$:

$$\frac{d^4\overline{u}}{dx^4} - m_1\frac{d^2\overline{u}}{dx^2} + m_2\overline{u} = m_3 e^{-(p/v)x}, \tag{22}$$

where

$$m_1 = \left(1+\frac{gb}{\beta^2}\right)(1+\tau_0 p^\alpha)\,p + p^2,$$
$$m_2 = (1+\tau_0 p^\alpha)\,p^3, \qquad m_3 = \frac{b\omega p(1+\tau_0 p^\alpha)}{\beta^2 v}. \tag{23}$$

The general solution of (22) is

$$\overline{u} = C_1 e^{-k_1 x} + C_2 e^{k_1 x} + C_3 e^{-k_2 x} + C_4 e^{k_2 x} + C_5 e^{-(p/v)x}, \tag{24}$$

where C_i $(i = 1,2,3,4)$ are parameters depending on p to be determined from the boundary conditions, and

$$C_5 = \frac{m_3}{\left[(p/v)^4 - m_1(p/v)^2 + m_2\right]}; \tag{25}$$

k_1 and k_2 are the roots of the characteristic equation

$$k^4 - m_1 k^2 + m_2 = 0; \tag{26}$$

k_1 and k_2 are given by

$$k_1 = \sqrt{\frac{m_1 + \sqrt{m_1^2 - 4m_2}}{2}}, \qquad k_2 = \sqrt{\frac{m_1 - \sqrt{m_1^2 - 4m_2}}{2}}. \tag{27}$$

In a similar manner, eliminating $\overline{u}$ between (19) and (20), we obtain the following equation satisfied by $\overline{\theta}$:

$$\frac{d^4\overline{\theta}}{dx^4} - m_1\frac{d^2\overline{\theta}}{dx^2} + m_2\overline{\theta} = m_4 e^{-(p/v)x}, \tag{28}$$

where $m_4 = \omega p(1+\tau_0 p^\alpha)[p(1-1/v^2)]$.

The general solution of (28) is

$$\overline{\theta} = C_{11}e^{-k_1x} + C_{22}e^{k_1x} + C_{33}e^{-k_2x} + C_{44}e^{k_2x} + C_{55}e^{-(p/v)x}, \tag{29}$$

where C_{ii} $(i = 1, 2, 3, 4)$ are parameters depending on p.

Substituting $\overline{u}$ from (24) and $\overline{\theta}$ from (29) into (19), we can get the following relationships:

$$\begin{aligned} C_{11} &= -\frac{\beta^2\left(k_1^2 - p^2\right)}{bk_1}C_1, \qquad C_{22} = \frac{\beta^2\left(k_1^2 - p^2\right)}{bk_1}C_2, \\ C_{33} &= -\frac{\beta^2\left(k_2^2 - p^2\right)}{bk_2}C_3, \qquad C_{44} = \frac{\beta^2\left(k_2^2 - p^2\right)}{bk_2}C_4, \\ C_{55} &= \frac{\beta^2\left(v^2p - p\right)}{bv}C_5. \end{aligned} \tag{30}$$

In order to determine the parameters C_i $(i = 1, 2, 3, 4)$ and C_{ii} $(i = 1, 2, 3, 4)$, from the boundary conditions in (15), we get

$$\begin{aligned} C_1 + C_2 + C_3 + C_4 &= -C_5, \\ C_1e^{-k_1l} + C_2e^{k_1l} + C_3e^{-k_2l} + C_4e^{k_2l} &= -C_5e^{-(p/v)l}, \\ -C_{11}k_1 + C_{22}k_1 - C_{33}k_2 + C_{44}k_2 &= \left(\frac{p}{v}\right)C_{55}, \\ -C_{11}k_1e^{-k_1l} + C_{22}k_1e^{k_1l} - C_{33}k_2e^{-k_2l} & \\ + C_{44}k_2e^{k_2l} &= \left(\frac{p}{v}\right)C_{55}e^{-(p/v)l}. \end{aligned} \tag{31}$$

Solving (31) with (30), we obtain

$$\begin{aligned} C_1 &= \frac{\left(k_2^2 - p^2/v^2\right)\left(e^{k_1l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5, \\ C_2 &= -\frac{\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5, \\ C_3 &= -\frac{\left(k_1^2 - p^2/v^2\right)\left(e^{k_2l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5, \\ C_4 &= \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5. \end{aligned} \tag{32}$$

Substituting C_i $(i = 1, 2, 3, 4)$ into (24), we obtain

$$\begin{aligned} \overline{u} = &\frac{\left(k_2^2 - p^2/v^2\right)\left(e^{k_1l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5e^{-k_1x} \\ &- \frac{\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5e^{k_1x} \\ &- \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{k_2l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5e^{-k_2x} \\ &+ \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5e^{k_2x} + C_5e^{-(p/v)x}. \end{aligned} \tag{33}$$

From the relationships between C_i and C_{ii} in (30), we get

$$\begin{aligned} C_{11} &= -\frac{\beta^2\left(k_1^2 - p^2\right)\left(k_2^2 - p^2/v^2\right)\left(e^{k_1l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5, \\ C_{22} &= -\frac{\beta^2\left(k_1^2 - p^2\right)\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5, \\ C_{33} &= \frac{\beta^2\left(k_2^2 - p^2\right)\left(k_1^2 - p^2/v^2\right)\left(e^{k_2l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5, \\ C_{44} &= \frac{\beta^2\left(k_2^2 - p^2\right)\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5, \\ C_{55} &= \frac{\beta^2\left(v^2p - p\right)}{bv}C_5. \end{aligned} \tag{34}$$

Substituting C_{ii} into (29), we obtain

$$\begin{aligned} \overline{\theta} = &-\frac{\beta^2\left(k_1^2 - p^2\right)\left(k_2^2 - p^2/v^2\right)\left(e^{k_1l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5e^{-k_1x} \\ &- \frac{\beta^2\left(k_1^2 - p^2\right)\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1l} - e^{-k_1l}\right)}C_5e^{k_1x} \\ &+ \frac{\beta^2\left(k_2^2 - p^2\right)\left(k_1^2 - p^2/v^2\right)\left(e^{k_2l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5e^{-k_2x} \\ &+ \frac{\beta^2\left(k_2^2 - p^2\right)\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2l} - e^{-k_2l}\right)}C_5e^{k_2x} \\ &+ \frac{\beta^2\left(v^2p - p\right)}{bv}C_5e^{-(p/v)x}. \end{aligned} \tag{35}$$

Substituting $\overline{u}$ from (35) and $\overline{\theta}$ from (37) into (20), we obtain

$$\begin{aligned}\overline{\sigma} = &-\frac{\beta^2 p^2 \left(k_2^2 - p^2/v^2\right)\left(e^{k_1 l} - e^{-(p/v)l}\right)}{k_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)} C_5 e^{-k_1 x} \\ &- \frac{\beta^2 p^2 \left(k_2^2 - p^2/v^2\right)\left(e^{-k_1 l} - e^{-(p/v)l}\right)}{k_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)} C_5 e^{k_1 x} \\ &+ \frac{\beta^2 p^2 \left(k_1^2 - p^2/v^2\right)\left(e^{k_2 l} - e^{-(p/v)l}\right)}{k_2\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)} C_5 e^{-k_2 x} \\ &+ \frac{\beta^2 p^2 \left(k_1^2 - p^2/v^2\right)\left(e^{-k_2 l} - e^{-(p/v)l}\right)}{k_2\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)} C_5 e^{k_2 x} \\ &- \beta^2 v p C_5 e^{-(p/v)x}.\end{aligned} \tag{36}$$

4. Numerical Inversion of the Transforms

To obtain the distributions of the nondimensional temperature, displacement, and stress in time domain, $\overline{\theta}$, $\overline{u}$ and $\overline{\sigma}$ need to be inverted from Laplace domain. Unfortunately, the obtained solutions in Laplace domain are too complicated to be inverted analytically; thus, a feasible numerical method, that is, the Riemann-sum approximation method, is used to complete the inversion. In this method, any function $\overline{f}(x, p)$ in Laplace domain can be inverted to the time domain as [31]

$$f(x,t) = \frac{e^{\beta t}}{t}\left[\frac{1}{2}\overline{f}(x,\beta) + \mathrm{Re}\sum_{n=1}^{N}\overline{f}\left(x, \beta + \frac{in\pi}{t}\right)(-1)^n\right], \tag{37}$$

where Re is the real part and i is imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of β satisfies the relation $\beta t \approx 4.7$ [32].

5. Numerical Results and Discussions

By using the Riemann-sum approximation given in (37), numerical Laplace inversion is implemented to obtain the nondimensional temperature, displacement, and stress in the rod in time domain. In simulation, the thermoelastic material is taken as copper, and the parameters are

$$\begin{gathered}\lambda = 7.76 \times 10^{10}\,\mathrm{Nm}^{-2}, \qquad \mu = 3.86 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ \rho = 8954\,\mathrm{kgm}^{-3}, \qquad \alpha_t = 1.78 \times 10^{-5}\,\mathrm{K}^{-1}, \\ C_E = 383.1\,\mathrm{JKg}^{-1}\mathrm{K}^{-1}, \qquad \kappa = 386\,\mathrm{Wm}^{-1}\mathrm{K}^{-1}, \\ T_0 = 293\,\mathrm{K}.\end{gathered} \tag{38}$$

The other constants are specified as

$$Q_0 = 10, \qquad \tau_0 = 0.05, \qquad l = 10. \tag{39}$$

Numerical calculation is carried out to obtain the variations of the considered variables for three cases: case I, alter time while keeping the moving heat source velocity and fractional order parameter constant; case II, alter the moving heat source velocity while keeping time and fractional order parameter constant; case III, alter fractional order parameter while keeping time and the moving heat source velocity constant. In the calculation, no heat wave reflection from both ends of the rod is involved in the foregoing cases at any time. The obtained results are presented graphically in Figures 1–3.

Figure 1 shows the distributions of the nondimensional temperature, displacement, and stress in the rod for case I in (a), (b), and (c), respectively. In this case, three time instants, $t = 1.0$, $t = 1.5$, and $t = 2.0$, are considered, while the heat source velocity and the fractional order parameter remain constant as $v = 2.0$ and $\alpha = 0.25$, respectively. As shown in Figure 1(a), the peak value of the nondimensional temperature in solid line, dash line, and dot-dash line appears at $x = 2$, $x = 3$, and $x = 4$, respectively, which is due to the fact that the heat source moves with a constant velocity v and the distance that heat source moves across is $x = vt$. At location $x = vt$, heat source releases its maximum energy, which leads to a peak value. In Figure 1(b), it can be observed that the nondimensional displacement increases with the passage of time. Due to the applied moving heat source, the rod undergoes thermal expansion deformation. With the passage of time, the heat disturbed region enlarges so that thermal expansion deformation evolves along the rod. As seen in Figure 1(c), the nondimensional stress in the rod is compressive. Due to the fixed ends, thermal expansion deformation is restrained in between both ends, which leads to the occurrence of compressive thermal stress in the rod. The absolute value of stress increases with the passage of time.

Figure 2 shows the distributions of the nondimensional temperature, displacement, and stress for case II in (a), (b), and (c), respectively. In this case, three heat source velocities, $v = 1.5$, $v = 2.0$, and $v = 2.5$, are considered, while the time instant and the fractional order parameter remain constant as $t = 1.5$ and $\alpha = 0.25$, respectively.

As observed in Figure 2(a), the nondimensional temperature decreases with the increasing of moving heat source velocity. At a given period, the energy that heat source can release is constant. The intensity of the released energy per unit length decreases as the heat source velocity increases, which leads to a reduction of the local temperature at each location. It can also be observed from Figures 2(b) and 2(c) that the magnitudes of the nondimensional displacement and stress decreases as the moving heat source velocity increases, which results from the reduction of the heat energy intensity per unit length at larger velocity.

Figure 3 shows the variations of the nondimensional temperature, displacement, and stress for case III in (a), (b), and (c), respectively, and aims to demonstrate the effect of the fractional order parameter on the variations of the considered variables. To this end, a series of values of the fractional order parameter α within the region $(0, 1]$ were tested in the numerical calculation. As representation of the effect of

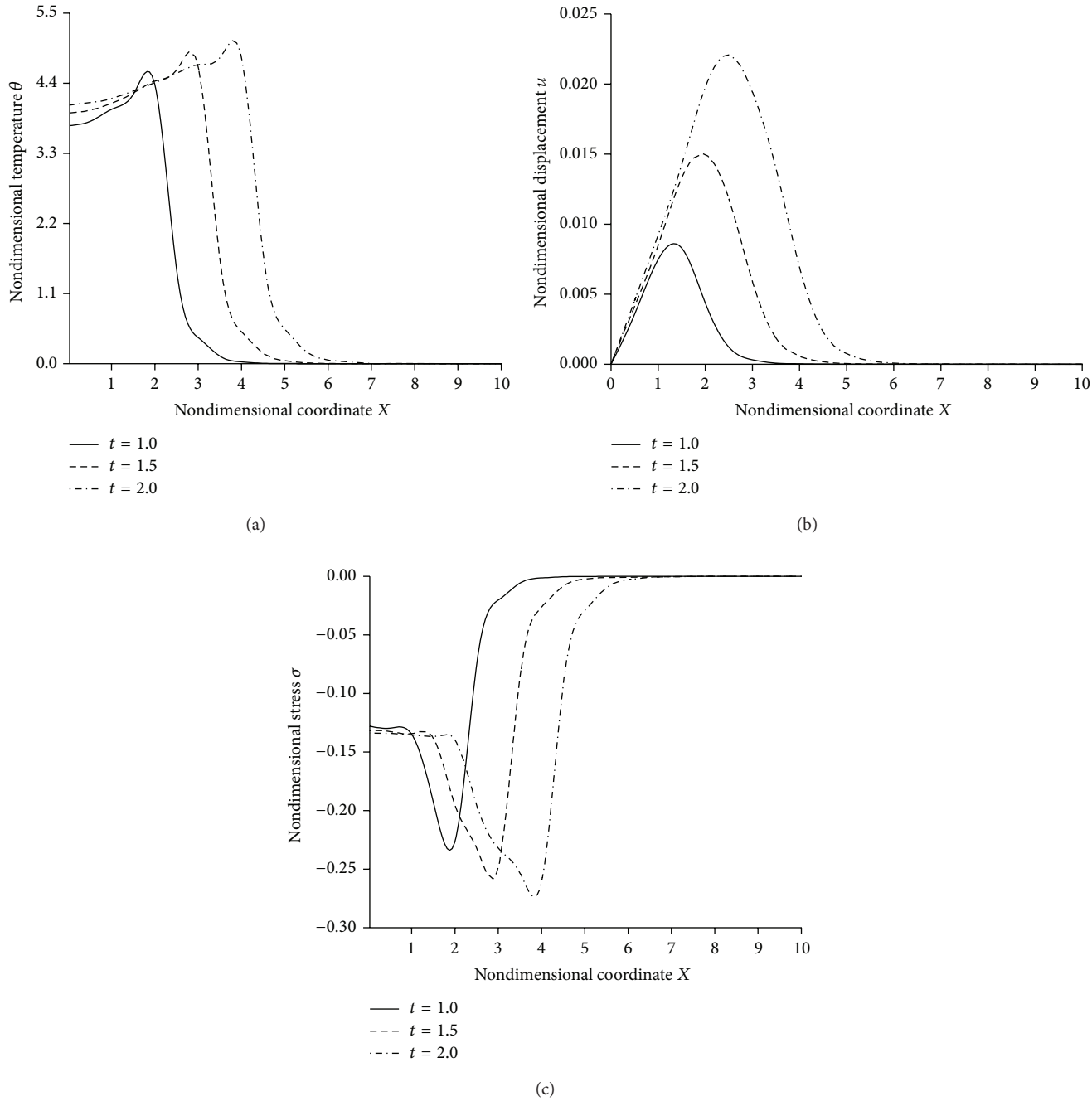

Figure 1: The variations of the nondimensional temperature, displacement, and stress in the rod when $\upsilon = 2.0$ and $\alpha = 0.25$.

α, three typical values of α, that is, $\alpha = 0.25$, $\alpha = 0.5$, and $\alpha = 1.0$, are considered, while the time and the heat source velocity remain constant as $t = 2.0$ and $\upsilon = 1.5$, respectively. As shown in Figures 3(a) and 3(c), the fractional order parameter significantly influences the peak values of the nondimensional temperature and stress, and the magnitude of the peak value of the nondimensional temperature as well as stress increases with the increasing of the fractional order parameter. However, the fractional order parameter barely influences the variation of the nondimensional displacement.

From Figures 1–3, it should be noted that the nonzero values of all the considered variables are only in a bounded region at a given time. This is governed by the nature that heat wave and thermoelastic wave propagate in the rod with finite speed, respectively.

6. Conclusions

The dynamic response of a thermoelastic rod with finite length subjected to a moving heat source is investigated in

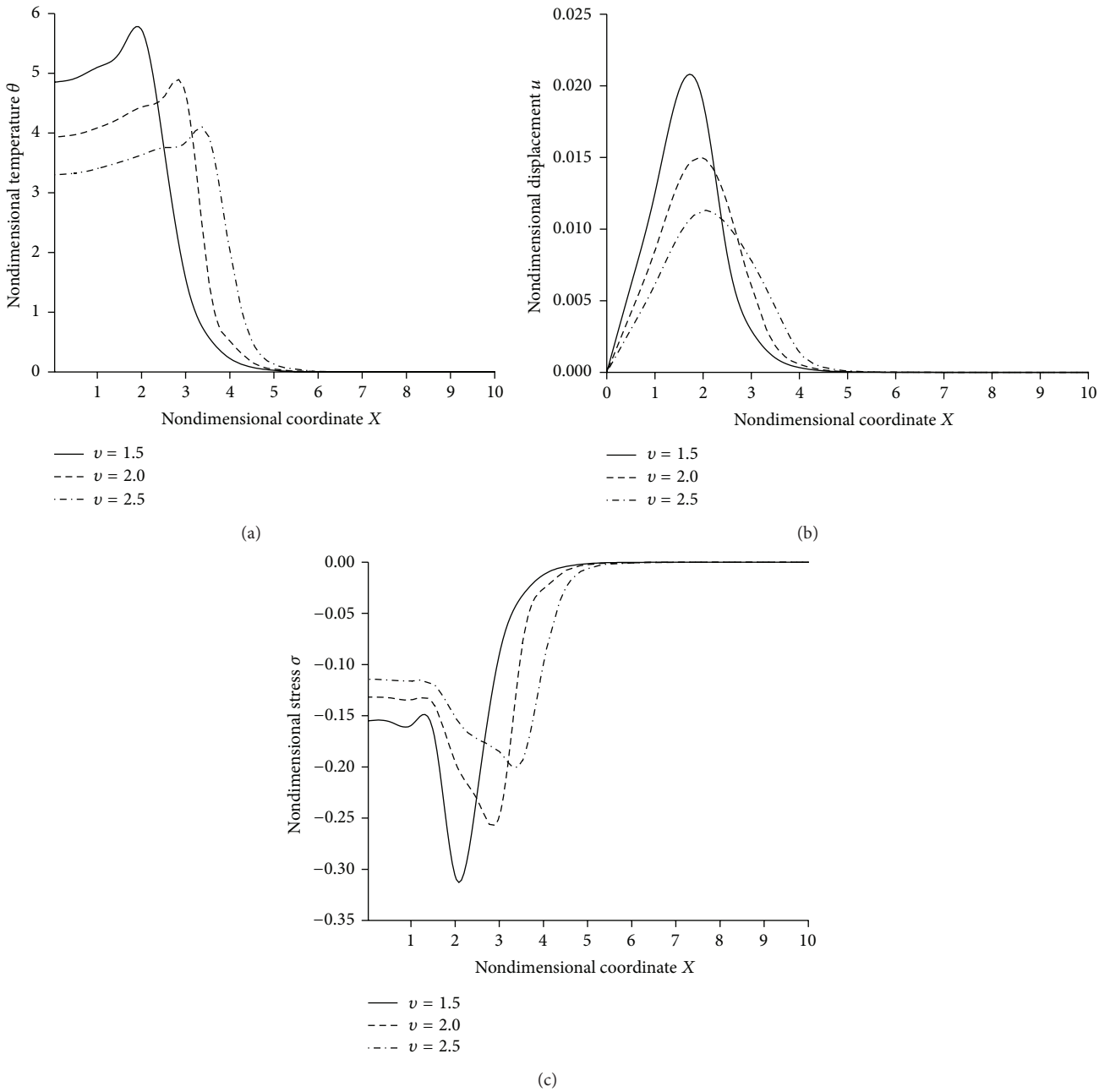

Figure 2: The variations of the nondimensional temperature, displacement, and stress when $t = 1.5$ and $\alpha = 0.25$.

the context of fractional order theory of thermoelasticity. From the above discussions, we can arrive at the following conclusions.

(1) Once the time is specified, the nondimensional temperature reaches its peak value at location $x = vt$.

(2) The magnitudes of the nondimensional temperature, displacement, and stress decrease with the increasing of the velocity of the moving heat source.

(3) The magnitude of the peak value of the nondimensional temperature as well as stress increases with the increasing of the fractional order parameter. However, the fractional order parameter barely influences the variation of the nondimensional displacement.

(4) At a given time, the nonzero values of the nondimensional temperature, displacement, and stress are only in a bounded region, which is governed by the nature that heat wave and thermoelastic wave propagate in the rod with finite speed, respectively.

Nomenclatures

σ_{ij}: The components of stress tensor
e_{ij}: The components of strain tensor

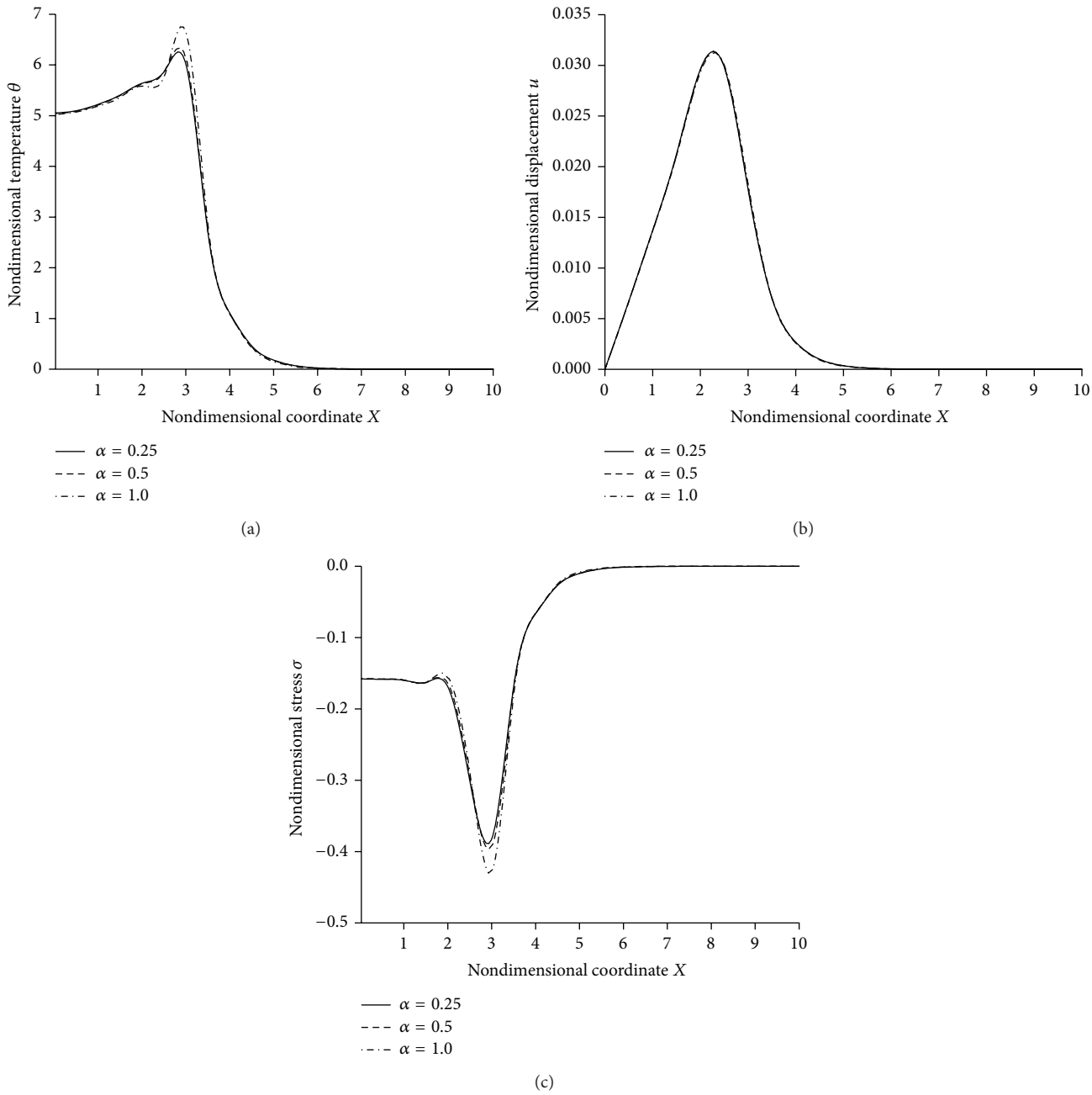

Figure 3: The variations of the nondimensional temperature, displacement, and stress when $t = 2.0$ and $v = 1.5$.

e_{kk}: The cubic dilation
u_i: The components of displacement vector
θ: $T - T_0$
T: Absolute temperature of the medium
T_0: Reference temperature
κ_{ij}: The coefficient of thermal conductivity
τ_0: Thermal relaxation time
ρ: Mass density
C_E: Specific heat at constant strain
λ, μ: Lame's constants
α_t: Linear thermal expansion coefficient
Q: The strength of the applied heat source per unit mass
γ: $(3\lambda + 2\mu)\alpha_t$
λ: The heat modulus
η: The entropy density
q_i: The components of heat flux vector
v: The velocity of the moving heat source
α: Constant parameter such that $0 < \alpha \leq 1$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] M. A. Biot, "Thermoelasticity and irreversible thermodynamics," *Journal of Applied Physics*, vol. 27, pp. 240–253, 1956.

[2] H. W. Lord and Y. Shulman, "A generalized dynamical theory of thermoelasticity," *Journal of the Mechanics and Physics of Solids*, vol. 15, no. 5, pp. 299–309, 1967.

[3] C. Catteneo, "A form of heat conduction equation which eliminates the paradox of instantaneous propagation," *Compute Rendus*, vol. 247, pp. 431–433, 1958.

[4] P. Vernotte, "Some possible complications in the phenomenon of thermal conduction," *Compute Rendus*, vol. 252, pp. 2190–2191, 1961.

[5] A. E. Green and K. A. Lindsay, "Thermoelasticity," *Journal of Elasticity*, vol. 2, no. 1, pp. 1–7, 1972.

[6] A. E. Green and P. M. Naghdi, "A re-examination of the basic postulates of thermomechanics," *Proceedings of the Royal Society*, vol. 432, pp. 171–194, 1991.

[7] A. E. Green and P. M. Naghdi, "On undamped heat waves in an elastic solid," *Journal of Thermal Stresses*, vol. 15, no. 2, pp. 253–264, 1992.

[8] A. E. Green and P. M. Naghdi, "Thermoelasticity without energy dissipation," *Journal of Elasticity*, vol. 31, no. 3, pp. 189–208, 1993.

[9] H. M. Youssef, "Theory of two-temperature-generalized thermoelasticity," *IMA Journal of Applied Mathematics*, vol. 71, no. 3, pp. 383–390, 2006.

[10] D. Y. Tzou, "Unified field approach for heat conduction from macro- to micro-scales," *Journal of Heat Transfer*, vol. 117, no. 1, pp. 8–16, 1995.

[11] K. S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley & Sons, New York, NY, USA, 1993.

[12] K. B. Oldham and J. Spanier, *The Fractional Calculus*, Academic Press, New York, NY, USA, 1974.

[13] R. Gorenflo and F. Mainardi, "Fractional calculus: integral and differential equations of fractional orders," in *Fractals and Fractional Calculus in Continuum Mechanics*, Springer, Wien, Austria, 1997.

[14] M. Caputo, "Vibrations on an infinite viscoelastic layer with a dissipative memory," *Journal of the Acoustical Society of America*, vol. 56, no. 3, pp. 897–904, 1974.

[15] R. L. Bagley and P. J. Torvik, "A theoretical basis for the application of fractional calculus to viscoelasticity," *Journal of Rheology*, vol. 27, no. 3, pp. 201–210, 1983.

[16] R. C. Koeller, "Applications of fractional calculus to the theory of viscoelasticity," *Journal of Applied Mechanics*, vol. 51, no. 2, pp. 299–307, 1984.

[17] Y. A. Rossikhin and M. V. Shitikova, "Applications of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids," *Applied Mechanics Reviews*, vol. 50, no. 1, pp. 15–67, 1997.

[18] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York, NY, USA, 1999.

[19] Y. Z. Povstenko, "Fractional heat conduction equation and associated thermal stress," *Journal of Thermal Stresses*, vol. 28, no. 1, pp. 83–102, 2005.

[20] Y. Z. Povstenko, "Fundamental solutions to central symmetric problems for fractional heat conduction equation and associated thermal stresses," *Journal of Thermal Stresses*, vol. 31, no. 2, pp. 127–148, 2008.

[21] H. M. Youssef, "Theory of fractional order generalized thermoelasticity," *Journal of Heat Transfer*, vol. 132, no. 6, pp. 1–7, 2010.

[22] H. M. Youssef and E. A. Al-Lehaibi, "Variational principle of fractional order generalized thermoelasticity," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1183–1187, 2010.

[23] H. M. Youssef, "Two-dimensional thermal shock problem of fractional order generalized thermoelasticity," *Acta Mechanica*, vol. 223, no. 6, pp. 1219–1231, 2012.

[24] H. M. Youssef and E. A. Al-Lehaibi, "Fractional order generalized thermoelastic half-space subjected to ramp-type heating," *Mechanics Research Communications*, vol. 37, no. 5, pp. 448–452, 2010.

[25] N. Sarkar and A. Lahiri, "Effect of fractional parameter on plane waves in a rotating elastic medium under fractional order generalized thermoelasticity," *International Journal of Applied Mechanics*, vol. 4, no. 3, Article ID 1250030, 2012.

[26] H. M. Youssef, "State-space approach to fractional order two-temperature generalized thermoelastic medium subjected to moving heat source," *Mechanics of Advanced Materials and Structures*, vol. 20, no. 1, pp. 47–60, 2013.

[27] H. H. Sherief, A. M. A. El-Sayed, and A. M. Abd El-Latief, "Fractional order theory of thermoelasticity," *International Journal of Solids and Structures*, vol. 47, no. 2, pp. 269–275, 2010.

[28] S. Kothari and S. Mukhopadhyay, "A problem on elastic half space under fractional order theory of thermoelasticity," *Journal of Thermal Stresses*, vol. 34, no. 7, pp. 724–739, 2011.

[29] H. Sherief and A. M. Abd El-Latief, "Effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity," *International Journal of Mechanical Sciences*, vol. 74, pp. 185–189, 2013.

[30] M. Caputo, "Linear model of dissipation whose Q is almost frequency independent-II," *Geophysical Journal of the Royal Astronomical Society*, vol. 13, pp. 529–539, 1967.

[31] F. Durbin, "Numerical inversion of Laplace transforms: an efficient improvement to Dubner and Abate's method," *Computer Journal*, vol. 17, pp. 371–376, 1974.

[32] G. Honig and U. Hirdes, "A method for the numerical inversion of Laplace transforms," *Journal of Computational and Applied Mathematics*, vol. 10, no. 1, pp. 113–132, 1984.