Research Article

A One-Dimensional Thermoelastic Problem due to a Moving Heat Source under Fractional Order Theory of Thermoelasticity

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The dynamic response of a one-dimensional problem for a thermoelastic rod with finite length is investigated in the context of the fractional order theory of thermoelasticity in the present work. The rod is fixed at both ends and subjected to a moving heat source. The fractional order thermoelastic coupled governing equations for the rod are formulated. Laplace transform as well as its numerical inversion is applied to solving the governing equations. The variations of the considered temperature, displacement, and stress in the rod are obtained and demonstrated graphically. The effects of time, velocity of the moving heat source, and fractional order parameter on the distributions of the considered variables are of concern and discussed in detail.

1. Introduction

The classical coupled thermoelasticity proposed by Biot [1] predicts an infinite speed for heat propagating in elastic media, which is physically impossible. To eliminate such an inherent paradox and predict finite speed propagation for heat propagation, the generalized thermoelastic theories have been developed. Lord and Shulman (L-S) [2] developed the first generalized thermoelasticity by postulating a wave-type heat conduction law to replace the classical Fourier law. This law is the same as that suggested by Cattaneo [3] and Vernotte [4], which contains the heat flux vector as well as its time derivative and also contains a new constant that acts as a relaxation time. Later on, Green and Lindsay (G-L) [5] proposed another theory called the temperature rate dependent thermoelasticity by modifying both the energy equation and the Duhamel-Neumann relation, in which two relaxation times were introduced. Subsequently, Green and Naghdi [6–8] advocated the theory of thermoelasticity without energy dissipation and the theory of thermoelasticity with energy dissipation. There also exist other generalized thermoelastic theories such as the two-temperature generalized thermoelasticity by Youssef [9] and the dual-phase-lag thermoelasticity by Tzou [10].

Fractional calculus has been used successfully to modify many existing models of physical processes. One can state that the whole theory of fractional derivatives and integrals was established in the second half of the 19th century. The first application of fractional derivatives was given by Abel who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. The generalization of the concept of derivative and integral to a noninteger order has been subjected to several approaches, and some various alternative definitions of fractional derivatives appeared [11–13]. In the last few years, fractional calculus was applied successfully in various areas to modify many existing models of physical processes especially in the field of heat conduction, diffusion, viscoelasticity, mechanics of solids, control theory, and electricity [14–17]. A survey of applications of the fractional calculus in area of science and engineering can be found in [18].

There exist many materials and physical situations such as low-temperature regimes, amorphous media, colloids, glassy and porous materials, man-made and biological materials/polymers, and transient loading, where the classical coupled thermoelasticity and the generalized thermoelastic theories fail. In such cases, it may be necessary to introduce time-fractional derivatives into thermoelasticity. Povstenko [19] proposed a quasistatic uncoupled theory of thermoelasticity based on the heat conduction equation with a time derivative of fractional order. Later on, he [20] investigated thermal stresses of central-symmetric Cauchy and source
problems for time-fractional heat conduction equation with the fundamental solutions. Youssef [21] and Youssef and Al-Lehaibi [22] formulated the theory of fractional order generalized thermoelasticity by introducing the Riemann–Liouville fractional integral operator into the generalized heat conduction. Based on this theory, Youssef [23] investigated two-dimensional thermal shock problems by Laplace and Fourier transforms; Youssef and Al-Lehaibi [24] solved half-space problems subjected to ramp-type thermal loading by employing Laplace transform and state-space method; Sarkar and Lahiri [25] were concerned with a two-dimensional generalized thermoelastic problem with a rotating elastic medium under the theory of fractional order; Youssef [26] dealt with a two-temperature generalized thermoelastic medium subjected to a moving heat source. Very recently, a completely new theory on fractional order generalized thermoelasticity has been introduced by Sherief et al. [27]. By employing this theory, Kothari and Mukhopadhyay [28] solved an elastic half-space problem with Laplace transform and state-space method. Sherief and Abd El-Latif [29] investigated a half-space problem with different thermal conductivity under the theory of fractional order. In the theory of Sherief et al. [27], the heat conduction equation has a new form as

\[ q_i + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha} q_i = -\kappa_{ij} \theta_{ij}, \quad (1^*) \]

where \( q_i \) are the components of the heat flux vector, \( \theta \) is the temperature increment, \( \tau_0 \) is the thermal relaxation time, \( \kappa_{ij} \) is the thermal conductivity tensor, and \( \alpha \) is a constant parameter such that \( 0 < \alpha \leq 1 \). To derive this theory, the authors use the definition of fractional derivatives of order \( \alpha \in (0,1) \) of the absolutely continuous function \( f(t) \) defined by Caputo [30] as

\[ \frac{d^\alpha}{dt^\alpha} f(t) = 1^{1-\alpha} \frac{d}{dt} f(t), \]

where \( f^\beta \) is the fractional integral of the function \( f(t) \) of order \( \beta \) defined by Miller and Ross [11] as

\[ f^\beta = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} f(s) \, ds. \]

Here \( f(t) \) is a Lebesgue integrable function and \( \beta > 0 \). The uniqueness theorem, reciprocity theorem, and a variational principle on this theory are also established in the article by Sherief et al. [27]. The heat conduction equation (1*) reduces to the Cattaneo-Vernotte [3, 4] law in the case \( \alpha = 1 \). It should be mentioned here that the Cattaneo-Vernotte law

\[ q_i + \tau_0 \frac{\partial}{\partial t} q_i = -\kappa_{ij} \theta_{ij}, \quad (3) \]

has been employed by Lord and Shulman [2] to develop the first generalized theory of thermoelasticity.

So far, there are few works on the investigation of problems involving heat source in the context of the fractional order theory of thermoelasticity. In the present work, we consider a one-dimensional problem for a rod subjected to a moving heat source under fractional order theory of generalized thermoelasticity proposed by Sherief et al. [27]. The problem is solved by means of Laplace transform and its numerical inversion. The variations of the considered variables are obtained and illustrated graphically.

2. Basic Equations

We investigate the dynamic problem of a thermoelastic rod subjected to a moving heat source in the context of fractional order theory of thermoelasticity. The rod is fixed at both ends and the applied heat source propagates along \( x \)-direction with a constant velocity \( v \). For the rod, it can be assumed that the geometrical dimension along \( x \)-axis is much greater than those along the other two directions orthogonal to \( x \)-axis; thus, the dynamic problem of the rod can be treated as a one-dimensional problem. So, all the considered variables are only functions of \( x \) and time \( t \).

In the absence of body force, the governing equations for homogeneous and isotropic elastic media in the context of fractional order theory of thermoelasticity advocated by Sherief et al. [27] are

\[ \begin{align*}
\sigma_{ij,j} &= \rho \ddot{u}_i, \\
\sigma_{ij} &= 2\mu e_{ij} + (\lambda e_{kk} - \gamma \theta) \delta_{ij}, \\
e_{ij} &= \frac{1}{2} (u_{ij,j} + u_{jj,i}), \\
q_{ij} &= -\rho T_0 \dot{\eta} + Q, \\
\rho \dot{\eta} &= \gamma e_{kk} + \frac{\rho C_E}{T_0} \theta, \\
\kappa \theta_{jj} &= \left( 1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha} \right) \left( \rho C_B \dot{\theta} + T_0 \psi_{kk} - Q \right). 
\end{align*} \]

For this one-dimensional problem, the only remaining displacement component is \( u_x = u(x, t) \); therefore, (4), (5), and (8) are reduced to

\[ \begin{align*}
(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \theta}{\partial x} &= \rho \frac{\partial^2 u}{\partial t^2}, \\
\sigma &= (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \theta, \\
\kappa \frac{\partial^2 \theta}{\partial x^2} &= \left( 1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha} \right) \left( \rho C_B \frac{\partial \theta}{\partial t} + \gamma T_0 \frac{\partial^2 u}{\partial x \partial t} - Q \right). 
\end{align*} \]
For convenience, the following nondimensional quantities are introduced:

\[ x^* = \xi_0 x, \quad u^* = \xi_0 u, \quad t^* = \xi_0 t, \]
\[ \tau^*_0 = \xi_0 \tau_0, \quad \theta^* = \frac{\theta}{\theta_0}, \quad \sigma^* = \frac{\sigma}{\mu}, \]
\[ Q^* = \frac{Q}{k T_0 \xi_0^2 \theta_0}. \]

In terms of these nondimensional quantities, (10) take the following forms (dropping the asterisks for convenience):

\[ \frac{\partial^4 u}{\partial x^4} - \frac{m_1}{\partial^2 u}{\partial x^2} + \frac{m_2}{\partial^4 u}{\partial x^4} = \frac{m_4}{\partial^4 u}{\partial x^4} - m_1 \frac{\partial^4 u}{\partial x^4} + m_2 \frac{\partial^4 u}{\partial x^4} = m_4 e^{-\frac{p}{\mu} x}, \]

where \( m_4 = \omega \frac{p}{\mu} \left( 1 + \tau_0 \frac{\alpha}{\beta} \right) \left( 1 - \frac{1}{\mu^2} \right) \).

The boundary conditions in (14) can be transformed to

\[ \pi(0, p) = \pi(l, p) = 0, \quad \frac{\partial \theta}{\partial x}(0, p) = \frac{\partial \theta}{\partial x}(l, p) = 0. \]

### 3. Solutions in the Laplace Domain

Eliminating \( \tilde{\theta} \) between (19) and (20), we obtain the following equation satisfied by \( \tilde{u} \):

\[ \frac{d^4 \tilde{u}}{dx^4} - m_1 \frac{d^2 \tilde{u}}{dx^2} + m_2 \tilde{u} = m_4 e^{-\frac{p}{\mu} x}, \]

where

\[ m_1 = \left( 1 + \frac{gb}{\beta^2} \right) \left( 1 + \tau_0 \frac{p^2}{\beta^2} \right) \rho + \rho^2, \]
\[ m_2 = \left( 1 + \tau_0 \frac{p^2}{\beta^2} \right) \rho^3, \quad m_3 = \frac{b\omega p \left( 1 + \tau_0 \rho^2 \right)}{\beta^2 \rho}. \]

The general solution of (22) is

\[ \tilde{\Pi} = C_1 e^{-k_1 x} + C_2 e^{k_2 x} + C_3 e^{-k_3 x} + C_4 e^{k_4 x} + C_5 e^{-\frac{p}{\mu} x}, \]

where \( C_i \) (i = 1, 2, 3, 4) are parameters depending on \( p \) to be determined from the boundary conditions, and

\[ C_5 = \frac{m_3}{\left( k_1^2 - m_1 \left( \frac{p}{\mu} \right)^2 + m_2 \right)}; \]

\[ k_1 \] and \( k_2 \) are the roots of the characteristic equation

\[ k^4 - m_1 k^2 + m_2 = 0; \]

\[ k_1 \text{ and } k_2 \text{ are given by} \]

\[ k_1 = \sqrt{\frac{m_1 + \sqrt{m_1^2 - 4m_2}}{2}}, \quad k_2 = \sqrt{\frac{m_1 - \sqrt{m_1^2 - 4m_2}}{2}}. \]

In a similar manner, eliminating \( \tilde{u} \) between (19) and (20), we obtain the following equation satisfied by \( \tilde{\theta} \):

\[ \frac{d^4 \tilde{\theta}}{dx^4} - m_1 \frac{d^2 \tilde{\theta}}{dx^2} + m_2 \tilde{\theta} = m_4 e^{-\frac{p}{\mu} x}, \]

where \( m_4 = \omega p \left( 1 + \tau_0 \rho^2 \right) \left( p \left( 1 - \frac{1}{\rho^2} \right) \right) \).
The general solution of (28) is
\[
\bar{\theta} = C_{11}e^{-k_1x} + C_{22}e^{k_2x} + C_{33}e^{-k_3x} + C_{44}e^{k_4x} + C_{55}e^{-\left(p/\alpha\right)x},
\]
where \(C_{ii}\) \((i = 1, 2, 3, 4)\) are parameters depending on \(p\).

Substituting \(\bar{\theta}\) from (24) and \(\bar{\theta}\) from (29) into (19), we can get the following relationships:
\[
C_{11} = -\frac{\beta^2 \left(k_1^2 - p^2\right)}{bk_1} C_1, \quad C_{22} = -\frac{\beta^2 \left(k_2^2 - p^2\right)}{bk_1} C_2, \quad C_{33} = -\frac{\beta^2 \left(k_3^2 - p^2\right)}{bk_2} C_3, \quad C_{44} = -\frac{\beta^2 \left(k_4^2 - p^2\right)}{bk_2} C_4, \quad C_{55} = -\frac{\beta^2 (v^2 p - p)}{bu} C_5.
\]

In order to determine the parameters \(C_i\) \((i = 1, 2, 3, 4)\) and \(C_{ii}\) \((i = 1, 2, 3, 4)\), from the boundary conditions in (15), we get
\[
C_1 + C_2 + C_3 + C_4 = -C_5, \quad -C_{11}k_1 + C_{22}k_1 - C_{33}k_3 + C_{44}k_3 = \left(\frac{p}{v}\right)C_{55}, \quad -C_{11}k_1 e^{-k_1x} + C_{22}k_1 e^{k_1x} - C_{33}k_3 e^{-k_3x} + C_{44}k_4 e^{k_4x} = \left(\frac{p}{v}\right)C_{55} e^{-\left(p/\alpha\right)x}.
\]

Solving (31) with (30), we obtain
\[
C_1 = -\frac{\beta^2 \left(k_1^2 - p^2\right) \left(e^{k_1x} - e^{-\left(p/\alpha\right)x}\right)}{(k_1^2 - k_2^2) \left(e^{k_1x} - e^{-k_1x}\right)} C_5, \quad C_2 = -\frac{\beta^2 \left(k_2^2 - p^2\right) \left(e^{k_2x} - e^{-\left(p/\alpha\right)x}\right)}{(k_2^2 - k_3^2) \left(e^{k_2x} - e^{-k_2x}\right)} C_5, \quad C_3 = -\frac{\beta^2 \left(k_3^2 - p^2\right) \left(e^{k_3x} - e^{-\left(p/\alpha\right)x}\right)}{(k_3^2 - k_4^2) \left(e^{k_3x} - e^{-k_3x}\right)} C_5, \quad C_4 = -\frac{\beta^2 \left(k_4^2 - p^2\right) \left(e^{k_4x} - e^{-\left(p/\alpha\right)x}\right)}{(k_4^2 - k_5^2) \left(e^{k_4x} - e^{-k_4x}\right)} C_5.
\]

Substituting \(C_1\) \((i = 1, 2, 3, 4)\) into (24), we obtain
\[
\bar{u} = \left(\frac{k_2^2 - p^2/v^2}{k_2^2 - k_3^2}\right) \left(e^{k_2x} - e^{-\left(p/\alpha\right)x}\right) C_5 e^{k_2x},
\]
\[
-\left(\frac{k_2^2 - p^2/v^2}{k_2^2 - k_3^2}\right) \left(e^{k_2x} - e^{-\left(p/\alpha\right)x}\right) C_5 e^{k_2x},
\]
\[
-\left(\frac{k_3^2 - p^2/v^2}{k_3^2 - k_4^2}\right) \left(e^{k_3x} - e^{-\left(p/\alpha\right)x}\right) C_5 e^{k_3x},
\]
\[
-\left(\frac{k_4^2 - p^2/v^2}{k_4^2 - k_5^2}\right) \left(e^{k_4x} - e^{-\left(p/\alpha\right)x}\right) C_5 e^{k_4x},
\]
\[
\frac{\beta^2 (v^2 p - p)}{bu} C_5 e^{-\left(p/\alpha\right)x}.
\]
Substituting \( \bar{u} \) from (35) and \( \bar{\sigma} \) from (37) into (20), we obtain
\[
\bar{\sigma} = -\beta^2 p^2 \left( k_1^2 - p^2 / v^2 \right) \left( e^{k_1 l} - e^{-(p/v)l} \right) \frac{C_5}{k_1 (k_1^2 - k_2^2)} e^{-k_1 x} 
- \beta^2 p^2 \left( k_2^2 - p^2 / v^2 \right) \left( e^{k_2 l} - e^{-(p/v)l} \right) \frac{C_5}{k_2 (k_1^2 - k_2^2)} e^{-k_2 x} 
+ \frac{\beta^2 p^2}{k_2} \left( k_2^2 - p^2 / v^2 \right) \left( e^{k_2 l} - e^{-(p/v)l} \right) \frac{C_5}{k_2 (k_1^2 - k_2^2)} e^{-k_2 x} 
+ \frac{\beta^2 p^2}{k_2} \left( k_2^2 - p^2 / v^2 \right) \left( e^{k_2 l} - e^{-(p/v)l} \right) \frac{C_5}{k_2 (k_1^2 - k_2^2)} e^{-k_2 x} 
- \beta^2 \nu \rho C_5 e^{-(p/v)l} \nu \times
\]

4. Numerical Inversion of the Transforms

To obtain the distributions of the nondimensional temperature, displacement, and stress in time domain, \( \bar{\sigma}, \bar{u} \) and \( \bar{T} \) need to be inverted from Laplace domain. Unfortunately, the obtained solutions in Laplace domain are too complicated to be inverted analytically; thus, a feasible numerical method, the Riemann-sum approximation method, is used to complete the inversion. In this method, any function \( f(x, \beta) \) in Laplace domain can be inverted to the time domain as [31]
\[
f(x, t) = \frac{C_5}{\nu} \left[ \frac{1}{2} f(x, \beta) + \text{Re} \sum_{n=1}^{N} f \left( x, \beta + \frac{i \nu t}{\nu} \right) (-1)^n \right],
\]
where \( \text{Re} \) is the real part and \( i \) is imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of \( \beta \) satisfies the relation \( \beta t \approx 4.7 \) [32].

5. Numerical Results and Discussions

By using the Riemann-sum approximation given in (37), numerical Laplace inversion is implemented to obtain the nondimensional temperature, displacement, and stress in the rod in time domain. In simulation, the thermoelastic material is taken as copper, and the parameters are
\[
\lambda = 7.76 \times 10^{10} \text{ Nm}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ Nm}^{-2},
\rho = 8954 \text{ kgm}^{-3}, \quad \alpha = 1.78 \times 10^{-5} \text{ K}^{-1},
\]
\[
C_E = 383.1 \text{ Jkg}^{-1} \text{ K}^{-1}, \quad \kappa = 386 \text{ Wm}^{-1} \text{ K}^{-1},
T_0 = 293 \text{ K}.
\]
The other constants are specified as
\[
Q_0 = 10, \quad \tau_0 = 0.05, \quad l = 10.
\]
Numerical calculation is carried out to obtain the variations of the considered variables for three cases: case I, alter time while keeping the moving heat source velocity and fractional order parameter constant; case II, alter the moving heat source velocity while keeping time and fractional order parameter constant; case III, alter fractional order parameter while keeping time and the moving heat source velocity constant. In the calculation, no heat wave reflection from both ends of the rod is involved in the foregoing cases at any time. The obtained results are presented graphically in Figures 1–3.

Figure 1 shows the distributions of the nondimensional temperature, displacement, and stress in the rod for case I in (a), (b), and (c), respectively. In this case, three time instants, \( t = 1.0, t = 1.5, \) and \( t = 2.0, \) are considered, while the heat source velocity and the fractional order parameter remain constant as \( \nu = 2.0 \) and \( \alpha = 0.25, \) respectively. As shown in Figure 1(a), the peak value of the nondimensional temperature in solid line, dash line, and dot-dash line appears at \( x = 2, x = 3, \) and \( x = 4, \) respectively, which is due to the fact that the heat source moves with a constant velocity \( \nu \) and the distance that heat source moves across is \( x = \nu t. \) At location \( x = \nu t, \) heat source releases its maximum energy, which leads to a peak value. In Figure 1(b), it can be observed that the nondimensional displacement increases with the passage of time. Due to the applied moving heat source, the rod undergoes thermal expansion deformation. With the passage of time, the heat disturbed region enlarges so that thermal expansion deformation evolves along the rod. As seen in Figure 1(c), the nondimensional stress in the rod is compressive. Due to the fixed ends, thermal expansion deformation is restrained in between both ends, which leads to the occurrence of compressive thermal stress in the rod.

The absolute value of stress increases with the passage of time.

Figure 2 shows the distributions of the nondimensional temperature, displacement, and stress for case II in (a), (b), and (c), respectively. In this case, three heat source velocities, \( \nu = 1.5, \nu = 2.0, \) and \( \nu = 2.5, \) are considered, while the time instant and the fractional order parameter remain constant as \( t = 1.5 \) and \( \alpha = 0.25, \) respectively.

As observed in Figure 2(a), the nondimensional temperature decreases with the increasing of moving heat source velocity. At a given period, the energy that heat source can release is constant. The intensity of the released energy per unit length decreases as the heat source velocity increases, which leads to a reduction of the local temperature at each location. It can also be observed from Figures 2(b) and 2(c) that the magnitudes of the nondimensional displacement and stress decreases as the moving heat source velocity increases, which results from the reduction of the heat energy intensity per unit length at larger velocity.

Figure 3 shows the variations of the nondimensional temperature, displacement, and stress for case III in (a), (b), and (c), respectively, and aims to demonstrate the effect of the fractional order parameter \( \alpha \) within the region \([0, 1]\) were tested in the numerical calculation. As representation of the effect of
\(\alpha\), three typical values of \(\alpha\), that is, \(\alpha = 0.25\), \(\alpha = 0.5\), and \(\alpha = 1.0\), are considered, while the time and the heat source velocity remain constant as \(t = 2.0\) and \(\nu = 1.5\), respectively. As shown in Figures 3(a) and 3(c), the fractional order parameter significantly influences the peak values of the nondimensional temperature and stress, and the magnitude of the peak value of the nondimensional temperature as well as stress increases with the increasing of the fractional order parameter. However, the fractional order parameter barely influences the variation of the nondimensional displacement.

From Figures 1–3, it should be noted that the nonzero values of all the considered variables are only in a bounded region at a given time. This is governed by the nature that heat wave and thermoelastic wave propagate in the rod with finite speed, respectively.

6. Conclusions

The dynamic response of a thermoelastic rod with finite length subjected to a moving heat source is investigated in
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Figure 2: The variations of the nondimensional temperature, displacement, and stress when $t = 1.5$ and $\alpha = 0.25$.

From the above discussions, we can arrive at the following conclusions.

(1) Once the time is specified, the nondimensional temperature reaches its peak value at location $x = vt$.

(2) The magnitudes of the nondimensional temperature, displacement, and stress decrease with the increasing of the velocity of the moving heat source.

(3) The magnitude of the peak value of the nondimensional temperature as well as stress increases with the increasing of the fractional order parameter.

However, the fractional order parameter barely influences the variation of the nondimensional displacement.

(4) At a given time, the nonzero values of the nondimensional temperature, displacement, and stress are only in a bounded region, which is governed by the nature that heat wave and thermoelastic wave propagate in the rod with finite speed, respectively.

Nomenclatures

$\sigma_{ij}$: The components of stress tensor

$\varepsilon_{ij}$: The components of strain tensor
Figure 3: The variations of the nondimensional temperature, displacement, and stress when $t = 2.0$ and $\nu = 1.5$.

- $\varepsilon_{ik}$: The cubic dilation
- $u_i$: The components of displacement vector
- $\theta$: $T - T_0$
- $T$: Absolute temperature of the medium
- $T_0$: Reference temperature
- $\kappa_{ij}$: The coefficient of thermal conductivity
- $\tau_0$: Thermal relaxation time
- $\rho$: Mass density
- $C_E$: Specific heat at constant strain
- $\lambda, \mu$: Lame's constants
- $\alpha$: Linear thermal expansion coefficient
- $Q$: The strength of the applied heat source per unit mass

$\gamma$: $(3\lambda + 2\mu)\alpha$
$\lambda$: The heat modulus
$\eta$: The entropy density
$q_i$: The components of heat flux vector
$v$: The velocity of the moving heat source
$\alpha$: Constant parameter such that $0 < \alpha \leq 1$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
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