Research Article

Crack Extension Resistance of Normal-Strength Concrete Subjected to Elevated Temperatures

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Determination of the residual crack extension resistance curves (K_R -curves) associated with cohesive force distribution on fictitious crack zone of complete fracture process is implemented in present research. The cohesive force distributes according to bilinear softening traction-separation law proposed by Petersson. Totally ten temperatures varying from 20°C to 600°C and the specimen size of 230 × 200 × 200 mm with initial-notch depth ratios 0.4 are considered. The load-crack mouth opening displacement curves (P -CMOD) of postfire specimens are obtained by wedge-splitting method from which the stress intensity factor curves (K -curves) are calculated. In each temperature, with the distribution of cohesive force along the fracture process zone, the residual fracture toughness K_R (Δa) increases with increasing crack length Δa , whereas the K_R -curves decrease with increasing temperatures T_m for the thermal damage induced. The stability analysis on crack propagation demonstrates that when the residual K_R -curve is higher than K -curve, the crack propagates steadily; otherwise, the crack propagates unsteadily.

1. Introduction

To describe the crack propagation in concrete structures, the crack extension resistance in terms of K_R has been measured and the related characteristics were investigated by Hilsdorf and Brameshuber in 1984 [1], Mai in 1984 [2], and Karihaloo and Shah in 1987 [3] as well as Xu and Reinhardt in 1999 [4] which was calculated using the conventional approach proposed by Irwin et al. in the early 1960s. Xu and Reinhardt in 1998 [5] proposed an analytical method to determine the crack extension resistance (K_R -curve) according to the cohesive force on the fictitious crack zone that is described directly by the softening traction-separation law. The basic principle of the approach is that the crack extension resistance is composed of two parts. One part is the inherent toughness K_{Ic}^{ini} , which resists the initial propagation of an initial crack under loading. This means that a crack does not propagate when the stress intensity factor at the initial crack tip is less than the inherent toughness K_{Ic}^{ini} . Another part is induced by the cohesive force distributed on the fictitious crack during crack propagation. Therefore, it is

a function of the cohesive force distribution $f(\sigma)$, the tensile strength f_t of the material, and the length of the propagating crack.

The main requirement for determining crack extension resistance curve based on cohesive force distribution during crack propagation is to know the load-crack mouth opening displacement (P -CMOD) curve a priori. Some characteristics of K_R -curve were investigated numerically on standard TPBT specimen for different concrete strength and specimen sizes [6]. It was observed that K_R -curve was dependent on compressive strength of concrete and had almost the same S-shape. It increased with increasing crack length and increasing strength of concrete. The obtained K_R -curves were almost independent of the specimen sizes. However, it was found that some difference could be noticed on the gained K_R -curve by using bilinear [7] and nonlinear [8] softening functions of concrete. The influence of specimen geometry (TPB and CT specimens) on the K_R -curves is considered by Kumar and Barai [9], and the influence of the specimen geometry on the K_R -curves was not observed for the specified specimen size and initial-crack length/depth ratio.

FIGURE 1: Four stages of crack propagation during the fracture process.

The influence of temperature on the fracture properties was considered by several researchers, mainly on the fracture energy and material brittleness [10–14], relative fewer discussions on the fracture toughness [15, 16], not mention the crack extension resistance of the complete fracture process. For the stability analyses of crack propagation, the comparison between K_R -curves and the corresponding stress intensity factor curves (K -curves) can be taken as a crack propagation criterion to judge the stability of the crack in the loaded structures. This criterion could also be used in the analysis of the stability of structures that suffered high temperature or fire in a real situation.

The main objective of this research is to determine the residual crack extension resistance (K_R -curve) of postfire concrete based on the cohesive force distributed on the fictitious crack zone, and the influence of temperatures on the K_R -curve is discussed. The wedge-splitting experiments of a total of ten temperatures varying from 20°C to 600°C and

the specimens size 230 × 200 × 200 mm with initial-notch depth ratios of 0.4 are implemented.

2. Determination of Residual K_R -Curve Based on Cohesive Stress Distribution

2.1. Background. According to K_R -curve criterion [5], the crack extension resistance of a cracked solid consists of the inherent toughness K_{Ic}^{ini} and the cohesive toughness $K_c(\Delta a)$ which increases with the increasing amount of crack extension. The cohesive toughness depends upon cohesive stress distribution $f(\sigma)$ which is a function of crack opening displacement w and tensile strength of concrete f_t and the propagating crack length a . At the onset of unstable crack propagation, the stress intensity factor K at the tip of the propagating crack is expressed as

$$K = K_R(\Delta a), \quad (1)$$

FIGURE 2: The bilinear softening traction-separation law.

where $K_R(\Delta a)$ is the crack extension resistance at crack extension length $\Delta a = a - a_0$. Also $K_R(\Delta a)$ is expressed in the following relation:

$$K_R(\Delta a) = K_{lc}^{ini} + K_c(\Delta a), \quad (2)$$

$$K_c(\Delta a) = F_1(f_t, f(\sigma), \Delta a). \quad (3)$$

In order to develop the K_R -curve for complete fracture process considering the cohesive stress in fictitious fracture zone, the value of cohesive toughness K_c at every stage of loading is important to determine. During crack propagation, four different stages are considered with the help of three characteristic crack lengths (a_0 , a_c , and a_{wc}) as represented in Figure 1 in which r_u is the undamaged portion of the ligament, a_0 is initial crack length, a_c is crack length at critical condition of unstable crack propagation, and a_{wc} is the length of fully developed fictitious fracture zone after which the stress-free crack propagation will begin.

2.2. Softening Traction-Separation Law of Postfire Concrete. The softening traction-separation law is a priori to determine the K_R -curve, at room temperature, and many expressions have been proposed based on direct tensile test [8, 17–20]. Based on numerical studies, simplified bilinear expressions for the softening traction-separation law (Figure 2) were suggested by Petersson in 1981 [17], Hilsdorf and Bramehuber in 1991 [19], and Phillips and Zhang in 1993. The area under the softening curve was defined as the fracture energy G_F by Hillerborg et al. in 1976 [21]. Therefore, one can get the following equation:

$$G_F = \frac{1}{2} (f_t w_s + \sigma_s w_0). \quad (4)$$

As a consequence, a general form of the simplified bilinear expression of the softening traction-separation law is given as follows:

$$\sigma = \begin{cases} \frac{f_t - (f_t - \sigma_s)w}{w_0} & 0 \leq w \leq w_s, \\ \frac{\sigma_s(w_0 - w)}{(w_0 - w_s)} & w_s \leq w \leq w_0, \\ 0 & w \geq w_0. \end{cases} \quad (5)$$

Different values of the break point (σ_s , w_s) and the crack width w_0 at stress-free point were used for the expression proposed by different researchers. In present work, the bilinear softening function of concrete proposed by Petersson is used for postfire specimens as follows:

$$\begin{aligned} \sigma_s &= \frac{f_t}{3}, \\ w_s &= \frac{0.8G_F}{f_t}, \\ w_0 &= \frac{3.6G_F}{f_t}. \end{aligned} \quad (6)$$

2.3. Analytical Method. The standard Green function [22] for the edge cracks with finite width of plate subjected to a pair of normal forces is used to evaluate the value of cohesive toughness. The general expression for the crack extension resistance for complete fracture associated with cohesive stress distribution in the fictitious fracture zone for Mode I fracture is given as follows:

$$K_R(\Delta a) = K_{lc}^{ini} + \int_{a_0}^a \frac{2\sigma(x)F(x/a, a/h)}{\sqrt{\pi a}} dx, \quad (7)$$

where

$$\begin{aligned} F\left(\frac{x}{a}, \frac{a}{h}\right) &= \frac{3.52(1-x/a)}{(1-a/h)^{3/2}} - \frac{4.35-5.28x/a}{(1-a/h)} \\ &+ \left\{ \frac{1.30-0.30(x/a)^{3/2}}{\sqrt{1-(x/a)^2}} + 0.83 - 1.76\frac{x}{a} \right\} \\ &\times \left\{ 1 - \left(1 - \frac{x}{a}\right)\frac{a}{h} \right\}. \end{aligned} \quad (8)$$

Equation (7) is employed according to the conditions of four stages of crack propagation as mentioned below.

2.3.1. Case 1: When $a = a_0$. There is no advancement in the initial notch length at this stage of loading and the body remains in elastic condition, subjected to small load (up to P_{ini}) without any slow crack growth. Hence, cohesive stress $\sigma(x) = 0$; the crack growth resistance remains equal to initiation toughness of the material. From (7), it is expressed as

$$K_R = K_{lc}^{ini}. \quad (9)$$

FIGURE 3: Two different situations for $CTOD_c$ and w_s .

FIGURE 4: The cohesive distribution of crack propagation stage II.

2.3.2. *Case 2: When $a_0 \leq a \leq a_c$.* The stable slow crack growth will take place until the effective crack extension a_c corresponding to the maximum load P_u is achieved. The cohesive force will start acting across the fictitious fracture zone resulting in the increase of crack extension resistance.

(a) For specimens subjected to temperatures less than 120°C , the critical $CTOD_c$ corresponding to maximum load P_u is less than w_s , as shown in Figure 3(a). The distribution of cohesive stress along the fictitious fracture zone is approximated to be linear, as shown in Figure 4(a). The variation of cohesive stress along the fictitious fracture zone for this loading condition, that is, $a_0 \leq a \leq a_c$ or $0 \leq CTOD \leq CTOD_c$, is written as

$$\sigma(x) = \frac{\sigma(w) + (f_t - \sigma(w))(x - a_0)}{(a - a_0)}, \quad (10)$$

where $\sigma(w)$ and w are the values of cohesive stress and crack opening displacement, being at the tip of initial notch,

respectively. The value of $\sigma(w)$ is determined by using bilinear softening function as follows:

$$\sigma(w) = \sigma_s(w_s) + \frac{w_s - w}{w_s} (f_t - \sigma_s(w_s)). \quad (11)$$

The crack extension resistance in this case is evaluated using (7) and (8).

(b) For specimens subjected to temperatures higher than 120°C , the critical $CTOD_c$ corresponding to maximum load P_u is wider than w_s , as shown in Figure 3(b). The distribution of cohesive stress along the fictitious fracture zone is approximated to be bilinear, as shown in Figure 4(b). The variation of cohesive stress along the fictitious fracture zone for this loading condition, also, $a_0 \leq a \leq a_c$ or $0 \leq CTOD \leq CTOD_c$, is written as

$$\begin{aligned} \sigma_1(x) &= \sigma(w) + (\sigma_s(w_s) - \sigma(w)) \frac{(x - a_0)}{(a_s - a_0)} \quad a_s \leq x \leq a_0, \\ \sigma_2(x) &= \sigma_s(w_s) + (f_t - \sigma_s(w_s)) \frac{(x - a_s)}{(a - a_s)} \quad a_s \leq x \leq a. \end{aligned} \quad (12)$$

The value of $\sigma(w)$ is determined by using bilinear softening function as follows:

$$\sigma(w) = \frac{w_0 - w}{w_0 - w_s} \sigma_s(w_s). \quad (13)$$

The limits of integration of (7) should be taken in two steps: $a_0 \leq x \leq a_s$ for cohesive stress $\sigma_1(x)$ and $a_s \leq x \leq a$ for cohesive stress $\sigma_2(x)$, respectively. The same Green's function $F(x/a, a/h)$ for a given effective crack extension will be determined using (8). The calculated formula is listed as follows:

$$K_R = K_{Ic}^{ini} + \int_{a_0}^{a_s} \frac{2\sigma_2(x) F(x/a, a/h)}{\sqrt{\pi a}} dx + \int_{a_s}^a \frac{2\sigma_3(x) F(x/a, a/h)}{\sqrt{\pi a}} dx. \quad (14)$$

The effective crack length at break point a_s (as shown in Figure 4(b)) is computed from the following nonlinear expression [23] by substituting COD(a_s), CMOD, a , and h :

$$\begin{aligned} \text{COD}(a_s) = \text{CMOD} & \left\{ \left(1 - \frac{a_s}{a} \right)^2 \right. \\ & + \left(1.018 - 1.149 \frac{a}{h} \right) \\ & \times \left[\frac{a_s}{a} - \left(\frac{a_s}{a} \right)^2 \right] \left. \right\}^{1/2}, \end{aligned} \quad (15)$$

where COD(a_s) is the crack opening displacement at a_s , a is the effective crack length (according to (18)), and h is the specimen height.

2.3.3. Case 3: When $a_c \leq a \leq a_{w0}$. During this stage of the applied load for all temperatures, corresponding CTOD and effective crack length have increased more than maximum load P_u , CTOD_c, and critical effective crack extension a_c , respectively. The cohesive stress distribution for this case will take a bilinear shape, as shown in Figure 4(b). It is the same situation as case 2 part (b), and the crack extension resistance K_R at this stage would be calculated according to (14). This case is shown in Figure 5.

2.3.4. Case 4: When $a \geq a_{w0}$. This situation of loading corresponds to the descending portion of P -CMOD curve. At the effective crack extension, a_{w0} , full shape of cohesive stress distribution is allowed to develop, and beyond further extension in crack during loading, a new stress-free crack in front of the initial notch tip will form. This case is shown

FIGURE 5: Crack propagation stage III.

FIGURE 6: Crack propagation stage IV.

in Figure 6 and the stress distribution is expressed with the following relation:

$$\begin{aligned} \sigma_1(x) &= 0 \quad a_0 \leq x \leq a_{w0}, \\ \sigma_2(x) &= \sigma_s(w_s) \frac{(x - a_{w0})}{(a_s - a_{w0})} \quad a_s \leq x \leq a_{w0}, \\ \sigma_3(x) &= \sigma_s(w_s) + (f_t - \sigma_s(w_s)) \frac{(x - a_s)}{(a - a_s)} \quad a_s \leq x \leq a. \end{aligned} \quad (16)$$

Similar to (15), the effective crack length a_{w0} corresponding to zero stress of new fictitious fracture zone as shown in Figure 6 is computed from the following nonlinear expression:

$$\begin{aligned} \text{COD}(a_{w0}) = \text{CMOD} & \left\{ \left(1 - \frac{a_{w0}}{a} \right)^2 \right. \\ & + \left(1.018 - 1.149 \frac{a}{h} \right) \\ & \times \left[\frac{a_{w0}}{a} - \left(\frac{a_{w0}}{a} \right)^2 \right] \left. \right\}^{1/2}, \end{aligned}$$

$$\begin{aligned} \text{COD}(a_s) = \text{CMOD} & \left\{ \left(1 - \frac{a_s}{a}\right)^2 \right. \\ & + \left(1.018 - 1.149 \frac{a}{h}\right) \\ & \times \left[\frac{a_s}{a} - \left(\frac{a_s}{a}\right)^2 \right] \left. \right\}^{1/2}. \end{aligned} \quad (17)$$

The evaluation of crack extension resistance in this case is also done using (7) and (8).

2.4. Determination of Equivalent Crack Extension

2.4.1. Assumption. Linear asymptotic superposition assumption [5, 6] is considered to introduce the nonlinearity effect in P -CMOD curves during loading of concrete test specimens. This assumption enables us to introduce linear elastic fracture mechanics (LEFM) for calculating fracture parameters during every stage of loading in concrete structures. The hypotheses of the assumption are given as follows.

- (1) The nonlinear characteristic of the P -CMOD curve is caused by fictitious crack extension in front of a stress-free crack.
- (2) An effective crack consists of an equivalent-elastic stress-free crack and equivalent-elastic fictitious crack extension.

2.4.2. Equivalent Crack Extension for WS Specimen. The equivalent-elastic crack length for WS specimen is expressed as [24]

$$a = (h + h_0) \left\{ 1 - \left(\frac{13.18}{\text{CMODEt}/P + 9.16} \right)^{1/2} \right\} - h_0. \quad (18)$$

The empirical expression (18) is valid within 2% accuracy for $0.2 \leq \alpha \leq 0.8$. The residual Young's modulus E is calculated using the P -CMOD curve as

$$E = \frac{1}{t c_i} \left[13.18 \times \left(1 - \frac{a_0 + h_0}{h + h_0} \right)^2 - 9.16 \right], \quad (19)$$

where $c_i = \text{CMOD}/P$, the segment compliance; t is the specimens thickness; h is the specimens height; and h_0 is the thickness of clip gauge holder. The value of equivalent-elastic crack length a and residual E is listed in Table 1.

2.4.3. Calculation of Crack Opening Displacement. The crack opening displacement at position x along the fictitious crack line $\text{COD}(x)$ is computed from the known value of CMOD using the following expression [23]:

$$\begin{aligned} \text{COD}(x) = \text{CMOD} & \left\{ \left(1 - \frac{x}{a}\right)^2 \right. \\ & + \left(1.018 - 1.149 \frac{a}{h}\right) \left[\frac{x}{a} - \left(\frac{x}{a}\right)^2 \right] \left. \right\}^{1/2}. \end{aligned} \quad (20)$$

FIGURE 7: Geometry of specimens.

Also, the value of cohesive stress along the fictitious fracture zone corresponding to crack opening displacement at all stages of loading is evaluated using bilinear stress-displacement softening law as given in (3).

2.5. Calculation of Double-K Fracture Parameters. The parameters like initiation toughness K_{Ic}^{ini} and stress intensity factors (SIF) are required to be calculated to judge the stability of a propagating crack in a loaded structure using $K_R(\Delta a)$ curve analysis. The LEFM formula for the corresponding test specimen geometries is used for this purpose.

The SIF for WS test specimens is written as [24]

$$K(P, a) = \frac{P \times 10^{-3}}{t h^{1/2}} f(\alpha), \quad (21)$$

$$f(\alpha) = \frac{3.675 \times [1 - 0.12(\alpha - 0.45)]}{(1 - \alpha)^{3/2}}, \quad \alpha = \frac{a}{h}. \quad (22)$$

The empirical expression (21) is valid within 2% accuracy for $0.2 \leq \alpha \leq 0.8$.

3. Experimental Research

3.1. Experimental Program and Experimental Phenomena. To obtain the complete P -CMOD curves, the wedge-splitting tests were implemented. A total of 50 concrete specimens with the same dimensions $230 \times 200 \times 200$ mm were prepared, the geometry of the specimens is shown in Figure 7 ($b = 200$ mm, $d = 65$ mm, $h = 200$ mm, $f = 30$ mm, $a_0 = 80$ mm, and $\theta = 15^\circ$). The concrete mix ratios (by weight) were Cement : Sand : Coarse aggregate : Water = 1.00 : 3.44 : 4.39 : 0.80, with common Portland cement-mixed medium sand and 16 mm graded coarse aggregate. All the specimens had a precast notch of 80 mm height and 3 mm thickness, achieved by placing a piece of steel plate into the molds prior to casting. Each wedge splitting specimen was embedded with a thermal couple in the center of the specimens for temperature control.

Nine heating temperatures, ranging from 65°C to 600°C ($T_m = 65^\circ\text{C}, 120^\circ\text{C}, 200^\circ\text{C}, 300^\circ\text{C}, 350^\circ\text{C}, 400^\circ\text{C}, 450^\circ\text{C}$,

TABLE 1: Experimental results of fracture parameters.

Specimen	Temperature	P_{ini} (kN)	CMOD _{ini} (mm)	P_u (kN)	CMOD _c (mm)	CTOD _c (mm)	E (GPa)	a_c (h)	K_{Ic}^{ini} (MPa·m ^{1/2})	K_{Ic}^{un} (MPa·m ^{1/2})	G_F (N/m)
WS1	20°C	6.19	0.068	8.33	0.174	0.065	15.30	0.53	0.505	1.061	234.15
WS2		6.28	0.047	9.81	0.120	0.039	20.51	0.48	0.523	1.070	483.66
WS3		7.26	0.063	10.40	0.210	0.079	20.66	0.57	0.610	1.497	438.22
WS4		7.02	0.086	7.92	0.152	0.060	18.88	0.56	0.357	1.091	219.39
WS5		5.65	0.060	9.39	0.237	0.096	15.45	0.54	0.503	1.213	321.05
Average		6.55	0.065	9.17	0.178	0.068	18.16	0.54	0.498	1.186	339.30
WS6	65°C	6.98	0.055	11.31	0.195	0.078	21.73	0.56	0.550	1.594	425.91
WS7		3.88	0.050	8.23	0.163	0.100	24.79	0.66	0.303	1.664	482.62
WS8		6.88	0.078	10.41	0.212	0.087	19.43	0.57	0.557	1.518	487.75
WS9		7.94	0.052	10.71	0.164	0.087	23.25	0.60	0.511	1.685	480.51
WS10		6.32	0.056	11.67	0.229	0.086	16.60	0.54	0.562	1.507	522.36
Average		6.42	0.058	10.47	0.193	0.088	21.16	0.59	0.480	1.594	479.83
WS11	120°C	5.03	0.064	8.37	0.191	0.056	10.65	0.47	0.518	0.900	396.52
WS13		4.69	0.093	8.25	0.224	0.084	11.87	0.53	0.417	1.058	517.82
WS12		4.71	0.070	7.53	0.357	0.152	9.48	0.60	0.419	1.202	654.73
WS14		2.79	0.030	7.53	0.198	0.083	15.42	0.58	0.249	1.107	345.46
WS15		—	—	—	—	—	—	—	—	—	—
Average		4.31	0.064	7.92	0.243	0.094	11.86	0.55	0.401	1.067	478.63
WS21	300°C	1.89	0.182	3.40	0.653	0.283	2.45	0.61	0.168	0.556	437.92
WS22		3.48	0.185	5.53	0.667	0.280	3.49	0.59	0.309	0.841	611.47
WS23		1.82	0.121	3.38	0.672	0.271	1.91	0.57	0.162	0.480	341.77
WS24		2.61	0.194	4.97	0.577	0.262	1.99	0.52	0.232	0.589	564.12
WS25		2.03	0.096	4.17	0.651	0.361	4.03	0.68	0.175	0.913	549.99
Average		2.37	0.156	4.29	0.644	0.291	2.78	0.59	0.209	0.676	501.05
WS36	450°C	1.52	0.126	3.37	1.009	0.544	1.41	0.62	0.135	0.582	611.53
WS37		—	—	—	—	—	—	—	—	—	—
WS38		1.52	0.163	3.26	1.419	0.660	1.46	0.62	0.135	0.527	482.45
WS39		1.12	0.296	3.07	1.348	0.617	1.34	0.64	0.100	0.563	663.10
WS40		0.99	0.105	2.94	1.394	0.666	1.58	0.68	0.088	0.659	678.79
Average		1.29	0.172	3.16	1.293	0.622	1.48	0.64	0.115	0.583	608.97
WS46	600°C	0.76	0.443	1.13	1.482	0.684	0.47	0.65	0.067	0.221	228.23
WS47		0.53	0.139	1.48	2.082	0.684	0.48	0.64	0.063	0.277	395.06
WS48		0.81	0.324	1.65	1.908	0.813	1.14	0.76	0.072	0.550	539.22
WS49		0.58	0.436	1.14	1.687	0.973	0.38	0.65	0.052	0.225	331.99
WS50		0.62	0.279	1.48	2.082	0.727	0.38	0.62	0.068	0.213	273.07
Average		0.62	0.324	1.38	1.848	0.799	0.57	0.67	0.064	0.297	353.51

500°C, and 600°C), were adopted with the ambient temperature as a reference. Because it was recognized that the fracture behavior measurements were generally associated with significant scatter, five repetitions were performed for each temperature. An electric furnace with net dimensions 300 × 300 × 900 mm was used for heating. When the designated T_m was reached, the furnace was shut down, and the specimens were naturally cooled for 7 days prior to the test.

A closed-loop servocontrolled hydraulic jack with a maximum capacity of 1000 kN was employed to conduct the wedge splitting test. Two clip-on extensometers were suited at the mouth and the tip of the crack to measure the crack mouth opening displacement (CMOD) and crack tip opening displacement (CTOD). To obtain the complete P -CMOD curves (as shown in Figure 8), the test rate was fixed at 0.4 mm/min, such that it took approximately 20 minutes to complete a single test of specimens subjected to less than 300°C and 30 minutes for beyond 300°C.

3.2. Experimental Results and Discussion. The recorded maximum load P_u , the recorded crack mouth opening displacement CMOD_c at P_u , the calculated crack tip opening displacement CTOD_c based on (20), the initial cracking load P_{ini} determined by graphical method, the calculated residual Young's modulus E based on (18), the double- K fracture parameters, that is, K_{Ic}^{ini} and K_{Ic}^{un} , and the residual fracture energy G_F are summarized in Table 1.

Figure 8 shows typical complete load-displacement curves for different heating temperatures up to 600°C. The figure shows that the ultimate load P_u decreases significantly with increasing temperatures T_m , whereas the crack mouth opening displacement (CMOD) increases with T_m . The initial slope of ascending branches decreases with heating temperatures and the curves become gradually shorter and more extended.

From Table 1, it is found that the initial load P_{ini} , ultimate load P_u , the residual Young's modulus E , and the double- K fracture parameters decrease with the increasing temperatures, whereas the CMOD_{ini} , CMOD_c , CTOD_c , and a_c/h increase with T_m . The G_F sustains a hold-increase-decrease tendency with T_m ; the detailed explanation could be found in our previous work [25].

4. Residual Crack Extension Resistance Curves (K_R -Curves) and Stability Criterion

4.1. Crack Extension Resistance at Various Temperatures. Since the K_R -curve is considered as a criterion for complete description of crack propagation in structure, it is regarded as the material properties of the complete fracture process. The procedure to calculate the crack extension resistance curves (K_R -curves) is programmed using the analytical expressions given in Section 3. The P -CMOD curves shown in Figure 8 are used. The K_R -curves at different temperatures with crack extension from 65°C to 600°C are plotted in Figures 9(a)~9(c).

FIGURE 8: Load versus CMOD curves of specimens with temperatures.

When temperatures are less than 200°C, the K_R -curves have almost the same S-shape, which have a good coincidence with existed literature [5].

4.2. The Influence of Temperatures on Residual K_R -Curves. Figure 10 plots the K_R -curves of all temperatures; it is concluded that the K_R -curves decrease with increasing temperatures. Generally, temperatures under 120°C appear not to induce much thermal damage to concrete; the K_R -curves of 20°C, 65°C, and 120°C are close. Between 200°C and 600°C, higher temperatures cause more damage to the concrete, and the K_R -curves drop significantly. When temperature reaches 600°C, the K_R -curve almost increases linearly with a small margin.

4.3. The Stability Analysis of Crack Propagation. The K_R -curve presents the material properties of the complete fracture process. So, the K_R -curve can be taken as a criterion for describing the crack propagation in a structure or a structural component.

On the contrary, the stress intensity factor curve (K -curve) during the crack propagation in the structure or the structural component must be calculated already. At an arbitrary loading stage on a wedge-splitting specimen, the stress intensity factor at the tip of a propagating crack can be evaluated by inserting the load P and the length of the propagating crack a into formula (21). For the complete fracture process, the stress intensity factor at the tip of the propagating crack caused by the external load P can be plotted as a curve using formula (21).

Herein, the length of the propagating crack is taken as a horizontal axis, and the crack extension resistance $K_R(\Delta a)$, the stress intensity factor K , and the corresponding load P

FIGURE 9: K_R -curves with crack extension length of various temperatures.

are plotted on the vertical axis in Figures 11(a)~11(d). From those figures, the stability analysis of the crack propagation during the complete fracture process could be carried out.

From the initial point B to the critically unstable point C during the crack propagation, the toughness of the material increases with increasing crack extension from the inherent toughness K_{Ic}^{ini} to the value of the crack extension resistance

at the critically unstable point which is denoted by $K_R(\Delta a_c)$. The difference between $K_R(\Delta a_c)$ and K_{Ic}^{ini} is caused by the cohesive force on the fictitious crack zone during the crack propagation.

A common character of Figures 11(a)~11(d) is that when the maximum load P_{max} is reached, the stress intensity factor curve (K -curve) coincides with the crack extension resistance

FIGURE 10: K_R -curves with crack extension length of all temperatures.

curve (K_R -curve) at the point at which the value of the vertical coordinate is equal to the value of the unstable fracture toughness K_{Ic}^{un} . All of them correspond to the related critical crack length a_c .

Furthermore, it can be seen that when the curve of the stress intensity factor is lower than the crack extension resistance curve, the crack propagates steadily which can be observed in the region between point B and point C. Otherwise, when the K -curve coincides with or is higher than the K_R -curve, the crack propagates unsteadily. The stability analysis can be expressed mathematically as given below:

$$\begin{aligned} K_{Ic}^{ini} < K(P, a) < K_{Ic}^{un} & \quad \text{crack propagates steadily,} \\ K(P, a) > K_{Ic}^{un} & \quad \text{crack propagates unsteadily.} \end{aligned} \quad (23)$$

5. Conclusions

The residual crack extension resistance $K_R(\Delta a)$ associated with cohesive stress distribution in fictitious crack zone is evaluated for wedge-splitting specimens using analytical method. The analytical formulations of the residual $K_R(\Delta a)$ for the complete fracture process are presented which consist of two parts. One part is the initiation toughness of the material K_{Ic}^{ini} , and the other part is an overall value of stress intensity factor K_{Ic}^c caused by the cohesive stress along the fictitious crack zone that increases with the crack extension Δa . The distributions of the cohesive forces along the fictitious crack zone for varied loading stages are determined according to the softening traction-separation law of the concrete materials.

In the calculation of residual crack extension curves (K_R -curves) of wedge-splitting specimens subjected to different temperatures, it is found that in each temperature, the fracture toughness $K_R(\Delta a)$ increases with increasing crack length Δa , while the K_R -curves decrease with increasing temperatures. Generally, temperatures under 120°C appear not to induce much thermal damage to concrete; the K_R -curves of 20°C, 65°C, and 120°C are close. Between 200°C and 600°C, high temperatures cause more thermal damage to the concrete, and the K_R -curves drop significantly. When temperature reaches 600°C, the K_R -curve increases almost linearly with a small margin.

For the stability analyses of crack propagation, the comparison between K_R -curves and the corresponding stress intensity factor curves (K -curves) shows that K_R -curves can be taken as a crack propagation criterion to judge the stability of the crack in the loaded structures; that is, when the stress intensity factor $K(P, a)$ is smaller than the crack extension resistance $K_R(\Delta a)$, the crack propagates steadily. Contrarily, when $K(P, a)$ is larger than $K_R(\Delta a)$, the crack propagates unsteadily.

Nomenclature

a :	Equivalent-elastic crack length
a_c :	Critical notch depth of the specimen
a_s :	Effective crack length corresponding to w_s
a_{w0} :	Effective crack length corresponding to zero stress of new fictitious fracture zone
CMOD:	Crack mouth opening displacement
CTOD:	Crack tip opening displacement
E :	Residual Young's modulus
G_F :	Fracture energy: N/m
h :	Height of wedge-splitting specimens
K_R :	Crack extension resistance
$K_c(\Delta a)$:	Cohesive toughness
$\sigma(w)$:	Cohesive stress at the tip of initial notch
$\sigma(x)$:	Cohesive stress at equivalent-elastic crack length x
P_u :	Maximum load
w_0 :	Crack width at stress-free point
w :	Crack opening displacement at the tip of initial notch
a_0 :	Initial notch depth of the specimen
Δa :	Crack extension length
a_{wc} :	Fully developed fictitious fracture zone length
CMOD _c :	Critical crack mouth opening displacement
CTOD _c :	Critical crack tip opening displacement
$f(\sigma)$:	Cohesive force distribution
f_t :	Tensile strength
h_0 :	Thickness of the clip gauge holder
K_{Ic}^{ini} :	Initial fracture toughness
K :	Stress intensity factor
$\sigma_s(w_s)$:	Cohesive stress at the break point of softening curve

(a) Stability analysis of crack propagation at room temperature

(b) Stability analysis of crack propagation at 120°C

(c) Stability analysis of crack propagation at 300°C

(d) Stability analysis of crack propagation at 450°C

FIGURE 11: Stability analysis of crack propagation of various temperatures.

P_{ini} : The initial cracking load

T_m : Heating temperatures

w_s : Crack width at break point of softening curve.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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