Research Article

A New Solution for the Compression of a Two-Layer Strip and Its Application to Analysis of Bonding by Rolling

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Received 28 December 2013; Revised 24 February 2014; Accepted 24 February 2014; Published 25 March 2014

Academic Editor: Roohollah Jamaati

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The paper presents a theoretical study on the compression of a two-layer strip of strain-hardening rigid-plastic materials between rigid platens. Semianalytical solutions are obtained for stress and velocity fields in each layer. Special attention is devoted to the conditions corresponding to the beginning of cold bond formation between the layers. Depending on input parameters various general deformation patterns are possible. In particular, there exists such a range of process parameters that the soft metal layer yields while the hard metal layer is rigid at the beginning of the process. As the deformation proceeds, yielding also starts in the hard metal layer and the entire strip becomes plastic. This is a typical deformation pattern adopted in describing the process of joining by rolling. However, at a certain range of input parameters plastic deformation of the entire strip begins at the initial instant. Moreover, it is possible that only the hard metal layer yields while the soft metal layer does not. This deformation pattern takes place when the thickness of the soft metal layer is much smaller than that of the hard metal layer.

1. Introduction

For many years rolling has been recognized as being an important process for joining by forming ([1–7] among many others). Approximate theoretical engineering solutions for this process have been mainly found by the slab method [8–10] or the upper bound method [11–14]. However, none of these methods is really suitable for describing stationary processes with unknown boundaries between layers of different materials. In particular, the upper bound theorem is applicable if and only if the surface configuration including possible interfaces between different materials and the distribution of the local yield stress are known [15]. It is evident that these conditions are not satisfied in multilayer rolling. A possible method to use the upper bound theorem to determine the surface configuration in stationary processes has been proposed in [16]. In order to find a solution by this method, two nested minimization loops are necessary. In the inside loop, the geometry is fixed and the rate of work is minimized with respect to velocities. In the external loop, minimization is carried out with respect to geometry.

This is consistent with the upper bound theorem and is not the same as lumping two loops together. The latter has been however used in solutions for rolling of multilayer sheets. To the best of author's knowledge, no correct upper bound solution is available for such a process. The slab method is also not suitable to describe the process of rolling of multilayer sheets since the shape of bimaterial interfaces should be known in advance to apply this method. Another difficulty with the aforementioned theoretical methods is that their area of applicability, strictly speaking, is restricted to perfectly plastic materials. A typical way to overcome this difficulty is to assume that the yield stress depends on an average value of the equivalent strain (see, e.g., [10, 12]). However, the through thickness distribution of the yield stress is highly nonuniform due to intensive shear deformation near bimaterial interfaces. It is confirmed by highly nonuniform through thickness distributions of hardness (see, e.g., [6]). This effect is revealed in various metal forming process, not only in multilayer rolling (see, e.g., [17–19]). The presence of such a layer of intensive plastic deformation in the vicinity of bimaterial interfaces has a significant effect on the beginning of cold bond formation

between the layers. For, the position of the bonding point is usually controlled by the increase in the yield stress of the softer layer in the vicinity of the bimaterial interface (see, e.g., [10]). It is however evident from the actual distribution of material properties [6, 17–19] that the replacement of the local yield stress in the vicinity of the bimaterial interface with an average yield stress leads to a significant error. It is worthy of note that since the thickness of the layer of intensive plastic deformation is very small this replacement cannot have a large effect on global parameters such as the rolling power and torque which are typically used to compare theoretical and experimental results. In summary, it is worthwhile to develop an approximate theoretical method for describing multilayer rolling free of the aforementioned drawbacks of the upper bound and slab methods. Actually, a general method satisfying the requirements specified has been proposed in [20]. The method is widely used and shows excellent agreement with experiment for different materials (see, e.g., [21–23]), including rolling of very thin sheets (nano- and mesoscopic scale) as reported in [24]. The method proposed in [20] is used to verify other theories of rolling (see, e.g., [25, 26]). In particular, the solution [26] based on a series expansion shows a better agreement with the Orowan solution as the number of terms in the series expansion increases. Yet, there are very fast methods for solving the final equation obtained in [20] numerically (see, e.g., [27–30]), which is of special importance for design purpose. In order to apply one of the versions of the method [20], it is necessary to have an analytical or semianalytical solution for compression of a multilayer strip between parallel plates. Such solutions are available for strips made of rigid perfectly plastic layers [31] and several combinations of viscoplastic and rigid perfectly plastic layers [32]. However, these material models are not appropriate to describe the change of yield stress due to intensive plastic deformation in the vicinity of the bimaterial interface. Therefore, in the present paper a solution for compression of a two-layer strip of rigid plastic, strain-hardening materials is given. This solution is an extension of the solution proposed in [33] and it constitutes a theoretical basis for using the method [20] to simulate the process of bonding by rolling.

A recent finite element solution for the roll-bonding process has been given in [34]. The solution is based on the numerical procedure developed in [35]. A simplified statement of the boundary value problem has been adopted. In particular, the regime of sticking and the existence of a rigid (or elastic) zone near the neutral point have been ignored. On the other hand, the present exact solution shows that the regime of sticking must occur under certain conditions. Moreover, it follows from the general asymptotic analysis of solution behavior near some friction surfaces for plastic hardening materials carried out in [36] that the sticking zone may occupy the entire friction surface. According to the material model adopted in [34], the yield stress becomes constant if the equivalent strain is high enough. In this case the velocity field becomes singular in the vicinity of some friction surfaces [37]. Presumably, these qualitative features of actual solutions cause difficulties with application of standard finite element methods to boundary value problems in

FIGURE 1: Compression of a two-layer strip between rough rigid plates.

metal forming involving surfaces with high friction [38–41]. Therefore, the exact solution given in the present paper can also be used as a benchmark problem for verifying numerical codes developed for simulation of the roll-bonding process. The verification of numerical codes by closed form solutions is a necessary step towards their successful applications to engineering problems [42, 43].

2. Statement of the Problem

The progressive plane-strain compression of a two-layer strip between parallel, rough plates is considered. The Cartesian coordinate system (x, y) is chosen such that the x -axis is parallel to the strip and y measures distance through the strip thickness, as shown in Figure 1.

It will be shown later that the bimaterial interface is a straight line parallel to the x -axis. Then the lower surface of the strip is determined by the equation $y = 0$, its upper surface by the equation $y = 2h^{(1)} + 2h^{(2)}$, the bimaterial interface by the equation $y = 2h^{(1)}$, and its ends by the equation $x = \pm L$. Here $2h^{(1)}$ is the current thickness of layer 1 and $2h^{(2)}$ is the current thickness of layer 2. The initial values of $h^{(1)}$ and $h^{(2)}$ are denoted by $h_0^{(1)}$ and $h_0^{(2)}$, respectively. The ends $x = \pm L$ are traction-free. The standard stress and velocity boundary conditions are prescribed at the axis of symmetry $x = 0$. The other velocity boundary conditions are

$$u_y = -U, \quad (1)$$

for $y = 2(h^{(1)} + h^{(2)})$, and

$$u_y = 0, \quad (2)$$

for $y = 0$. The bimaterial interface moves with the velocity U_i whose value should be found from the solution. Thus

$$u_y = -U_i, \quad (3)$$

for $y = 2h^{(1)}$. The friction law adopted is associated with the material model and, therefore, the corresponding boundary conditions at $y = 0$, $y = 2h^{(1)}$, and $y = 2(h^{(1)} + h^{(2)})$ will be formulated later. The boundary value problem is symmetric relative to the y -axis. Therefore, it is sufficient to find the solution in the range $x \geq 0$. The standard constitutive equations of

rigid/plastic, strain-hardening material are adopted for each layer. In particular, the yield criteria are

$$(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 = 4[k_0^{(1)}\Phi_1(\varepsilon_{eq})]^2, \quad (4a)$$

$$(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 = 4[k_0^{(2)}\Phi_2(\varepsilon_{eq})]^2, \quad (4b)$$

for layers 1 and 2, respectively. Here σ_{xx} , σ_{yy} , and σ_{xy} are the stress components in the Cartesian coordinates, $k_0^{(1)}$ is the initial shear yield stress in layer 1, $k_0^{(2)}$ is the initial shear yield stress in layer 2, and $\Phi_1(\varepsilon_{eq})$ and $\Phi_2(\varepsilon_{eq})$ are arbitrary monotonically increasing functions of the equivalent strain, ε_{eq} , subject to $\Phi_1(0) = \Phi_2(0) = 1$. The equivalent strain is defined as

$$\frac{d\varepsilon_{eq}}{dt} = \xi_{eq}, \quad (5)$$

where d/dt stands for the convected derivative and ξ_{eq} is the equivalent strain rate defined by

$$\xi_{eq} = \sqrt{\frac{(\xi_{xx} - \xi_{yy})^2}{4} + \xi_{xy}^2}. \quad (6)$$

Here ξ_{xx} , ξ_{yy} , and ξ_{xy} are the components of the strain rate tensor in the Cartesian coordinate system. The associated flow rule gives

$$\begin{aligned} \xi_{xx} &= \frac{\xi_{eq}(\sigma_{xx} - \sigma_{yy})}{2k_0^{(1)}\Phi_1(\varepsilon_{eq})}, \\ \xi_{yy} &= \frac{\xi_{eq}(\sigma_{yy} - \sigma_{xx})}{2k_0^{(1)}\Phi_1(\varepsilon_{eq})}, \\ \xi_{xy} &= \frac{\xi_{eq}\sigma_{xy}}{k_0^{(1)}\Phi_1(\varepsilon_{eq})}, \end{aligned} \quad (7)$$

in layer 1, and

$$\begin{aligned} \xi_{xx} &= \frac{\xi_{eq}(\sigma_{xx} - \sigma_{yy})}{2k_0^{(2)}\Phi_2(\varepsilon_{eq})}, \\ \xi_{yy} &= \frac{\xi_{eq}(\sigma_{yy} - \sigma_{xx})}{2k_0^{(2)}\Phi_2(\varepsilon_{eq})}, \\ \xi_{xy} &= \frac{\xi_{eq}\sigma_{xy}}{k_0^{(2)}\Phi_2(\varepsilon_{eq})}, \end{aligned} \quad (8)$$

in layer 2. A conventional friction law in metal forming is to take the friction stress to be a constant fraction of the local yield shear stress. In this case the frictional boundary conditions are

$$T_2 \leq m_2 k_0^{(2)} \Phi_2(\varepsilon_{eq}), \quad (9)$$

FIGURE 2: Compression of a single layer between rough rigid plates.

for $y = 2(h^{(1)} + h^{(2)})$,

$$T_1 \leq m_1 k_0^{(1)} \Phi_1(\varepsilon_{eq}), \quad (10)$$

for $y = 0$, and

$$|T_i| \leq m_i \min \{k_0^{(1)}\Phi_1(\varepsilon_{eq}), k_0^{(2)}\Phi_2(\varepsilon_{eq})\}, \quad (11)$$

for $y = 2h^{(1)}$. With no loss of generality it is possible to assume that $k_0^{(1)} > k_0^{(2)}$. Then,

$$k_0^{(1)} = \alpha k_0^{(2)}, \quad \alpha > 1. \quad (12)$$

The system of (4a), (4b), (7), and (8) is supplemented with the equilibrium equations.

3. An Exact Instantaneous Solution for One Layer

An exact solution of (4a) and (7) (or (4b) and (8)) supplemented with the equilibrium equations has been derived in [33]. In order to use this solution, it is convenient to introduce new notations. A schematic diagram of the layer under consideration and Cartesian coordinates (x, z) are shown in Figure 2. The plates are now moving toward each other with speed V . The magnitude of the friction stress is equal to K_1 at $z = -w$ and K_2 at $z = w$. These stresses are directed as shown in Figure 2. The boundary conditions at $x = \pm L$ and $x = 0$ (Figure 1) are ignored. The constitutive equations (4a) and (7) (or (4b) and (8)) are replaced with

$$\begin{aligned} (\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 &= 4[k_0\Phi(\varepsilon_{eq})]^2, \\ \xi_{xx} &= \frac{\xi_{eq}(\sigma_{xx} - \sigma_{yy})}{2k_0\Phi(\varepsilon_{eq})}, \\ \xi_{yy} &= \frac{\xi_{eq}(\sigma_{yy} - \sigma_{xx})}{2k_0\Phi(\varepsilon_{eq})}, \\ \xi_{xy} &= \frac{\xi_{eq}\sigma_{xy}}{k_0\Phi(\varepsilon_{eq})}. \end{aligned} \quad (13)$$

The solution to the equilibrium equations and (13) proposed in [33] is

$$\sigma_{xy} = -\frac{\bar{K}z}{w} + \Delta, \quad \sigma_{yy} = \frac{\bar{K}x}{w} + c, \quad (14)$$

$$\sigma_{xx} = \frac{\bar{K}x}{w} + c + 2\sqrt{k_0^2 \Phi^2(\varepsilon_{eq}) - \sigma_{xy}^2},$$

$$u_x = \frac{Vx}{w} - \frac{2V}{w} \int_{z_*}^z \cot 2\phi d\beta + C, \quad (15a)$$

$$u_z = -\frac{Vz}{w}. \quad (15b)$$

Here u_x and u_z are the velocity components in the Cartesian coordinates, c and C are constants of integration, z_* is an arbitrary constant introduced for further convenience, and β is a dummy variable of integration. ϕ depends on z and does not depend on x . Also,

$$\begin{aligned} \bar{K} &= \frac{1}{2}(K_1 + K_2), \quad \Delta = \frac{1}{2}(K_1 - K_2), \\ \cos 2\phi &= -\frac{\bar{K}z}{wk_0\Phi(\varepsilon_{eq})} + \frac{\Delta}{k_0\Phi(\varepsilon_{eq})}. \end{aligned} \quad (16)$$

The equation for ϕ is compatible with the requirement that ϕ is independent of x if ε_{eq} is independent of x . The latter is satisfied since it follows from (6) and (15a), (15b) that

$$\xi_{eq} = \frac{V}{w} |\operatorname{cosec} 2\phi|, \quad (17)$$

and, thus, the right hand side of (5) is independent of x . Equation (15b) demands that all lines parallel to the x -axis at the initial instant remain parallel to this axis in the progressive compression.

4. An Instantaneous Solution for the Strip of Two Plastic Layers

The solution described in the previous section can be adopted to find a solution for the boundary value problem formulated in Section 2. First it is assumed that both layers are plastic. Then the solution given in Section 3 is valid in each of these layers. The solution ((15a), (15b)) for u_x shows that the regime of sliding occurs at the frictional interfaces $y = 0$ and $y = 2(h^{(1)} + h^{(2)})$ (Figure 1). Therefore, the boundary conditions (9) and (10) become

$$T_2 = m_2 k_0^{(2)} \Phi_2(\varepsilon_{eq}), \quad (18)$$

for $y = 2(h^{(1)} + h^{(2)})$, and

$$T_1 = m_1 k_0^{(1)} \Phi_1(\varepsilon_{eq}), \quad (19)$$

for $y = 0$, respectively. The velocity boundary conditions (1) to (3) can be transformed to the velocity boundary conditions used in the solution given in Section 3 by superimposing

FIGURE 3: Boundary conditions and coordinate systems for each layer.

a rigid body translation on the velocity field in each layer. In particular, it is necessary to put

$$V = \frac{U_i}{2}, \quad (20a)$$

$$V = \frac{U - U_i}{2}, \quad (20b)$$

for layers 1 and 2, respectively. It is also convenient to introduce new coordinates (x, y_1) and (x, y_2) , as shown in Figure 3.

To apply the solution (14) and ((15a), (15b)) to layer 2, it is necessary to put (Figures 2 and 3) $k_0 = k_0^{(2)}$, $\Phi(\varepsilon_{eq}) = \Phi_2(\varepsilon_{eq})$, $c = c_2$, $C = C_2$, $K_2 = T_2$, $K_1 = T_i$, $z = y_2$, and $w = h^{(2)}$. Then, from (14) and (16),

$$\begin{aligned} \sigma_{xy} &= -\frac{(T_2 + T_i)y_2}{2h^{(2)}} + \frac{T_i - T_2}{2}, \\ \sigma_{yy} &= \frac{(T_2 + T_i)x}{2h^{(2)}} + c_2, \\ \sigma_{xx} &= \frac{(T_2 + T_i)x}{h^{(2)}} + c_2 + 2\sqrt{\left[k_0^{(2)}\Phi_2(\varepsilon_{eq})\right]^2 - \sigma_{xy}^2}. \end{aligned} \quad (21)$$

Assuming $z_* = -h^{(2)}$ and eliminating V by means of (20b) the solution ((15a), (15b)) for the velocities is transformed to

$$\begin{aligned} u_x &= \frac{x(U - U_i)}{2h^{(2)}} - \frac{(U - U_i)}{h^{(2)}} \int_{-h^{(2)}}^{y_2} \cot 2\phi d\beta + C_2, \\ u_y &= -\frac{y_2(U - U_i)}{2h^{(2)}}. \end{aligned} \quad (22)$$

To apply the solution (14) and ((15a), (15b)) to layer 1, it is necessary to put $k_0 = k_0^{(1)}$, $\Phi(\varepsilon_{eq}) = \Phi_1(\varepsilon_{eq})$, $c = c_1$, $C = C_1$,

$K_2 = -T_i$, $K_1 = T_1$, $z = y_1$, and $w = h^{(1)}$. Then, from (14) and (16),

$$\begin{aligned}\sigma_{xy} &= -\frac{(T_1 - T_i) y_1}{2h^{(1)}} + \frac{T_1 + T_i}{2}, \\ \sigma_{yy} &= \frac{(T_1 - T_i) x}{2h^{(1)}} + c_1, \\ \sigma_{xx} &= \frac{(T_1 - T_i) x}{2h^{(1)}} + c_1 + 2\sqrt{\left[k_0^{(1)}\Phi_1(\varepsilon_{eq})\right]^2 - \sigma_{xy}^2}.\end{aligned}\quad (23)$$

Assuming $z_* = h^{(1)}$ and eliminating V by means of (20a) the solution ((15a), (15b)) for the velocities is transformed to

$$u_x = \frac{U_i x}{2h^{(1)}} - \frac{U_i}{h^{(1)}} \int_{h^{(1)}}^{y_1} \cot 2\phi d\beta + C_1, \quad u_y = -\frac{U_i y_1}{2h^{(1)}}. \quad (24)$$

The stress σ_{yy} must be continuous across the bimaterial interface. Therefore, it follows from (21) and (23) that

$$c_1 = c_2 \quad (25)$$

and $(T_2 + T_i)h^{(1)} = (T_1 - T_i)h^{(2)}$. Solving this equation for T_i yields

$$T_i = \frac{(sT_1 - T_2)}{(1 + s)}, \quad (26)$$

where $s = h^{(2)}/h^{(1)}$. The solution found exists if and only if (26) is compatible with (11). In general, there exist two critical values of s , s_{c1} , and s_{c2} such that (11) is not satisfied in the ranges $s > s_{c1}$ and $s < s_{c2}$, respectively. In these cases one of the layers is rigid and the regime of sliding occurs at the bimaterial interface. The values of s_{c1} and s_{c2} can be found from (26) assuming that

$$T_i = m_i \min \{k_0^{(1)}\Phi_1(\varepsilon_{eq}), k_0^{(2)}\Phi_2(\varepsilon_{eq})\}, \quad (27a)$$

$$T_i = -m_i \min \{k_0^{(1)}\Phi_1(\varepsilon_{eq}), k_0^{(2)}\Phi_2(\varepsilon_{eq})\}, \quad (27b)$$

respectively. In the range $s_{c2} < s < s_{c1}$ the regime of sticking occurs at the bimaterial interface. Therefore, the velocity component u_x must be continuous across this interface, where $y_2 = -h^{(2)}$ and $y_1 = h^{(1)}$. Using (22) and (24) it is possible to find that this condition is satisfied if

$$C_1 = C_2, \quad U_i = \frac{U}{s + 1}. \quad (28)$$

5. An Instantaneous Solution for the Strip of One Plastic Layer

Layer 1 is rigid when $s > s_{c1}$. The regime of sliding occurs at the bimaterial interface. Therefore, (11) is transformed to

$$T_i = m_i \min \{k_0^{(1)}\Phi_1(\varepsilon_{eq}), k_0^{(2)}\Phi_2(\varepsilon_{eq})\}. \quad (29)$$

Since layer 1 is rigid, then $U_i = 0$. The solution given by (14) and ((15a), (15b)) in which K_1 should be replaced with T_i , K_2 with T_2 , w with $h^{(2)}$, k_0 with $k_0^{(2)}$, $\Phi(\varepsilon_{eq})$ with $\Phi_2(\varepsilon_{eq})$, z with y_2 , and V with $U/2$ is valid. It is convenient to take $z_* = -h^{(2)}$. Then, using (16), (19), and (29),

$$\begin{aligned}\sigma_{xy} &= -\frac{(T_2 + T_i) y_2}{2h^{(2)}} + \frac{T_i - T_2}{2}, \quad \sigma_{yy} = \frac{(T_2 + T_i) x}{2h^{(2)}} + c, \\ \sigma_{xx} &= \frac{(T_2 + T_i) x}{2h^{(2)}} + c + 2\sqrt{\left[k_0^{(2)}\Phi_2(\varepsilon_{eq})\right]^2 - \sigma_{xy}^2}, \\ u_x &= \frac{Ux}{2h^{(2)}} - \frac{U}{h^{(2)}} \int_{-h^{(2)}}^{y_2} \cot 2\phi d\beta + C, \quad u_y = -\frac{Uy_2}{2h^{(2)}}.\end{aligned}\quad (30)$$

Layer 2 is rigid when $s < s_{c2}$. The regime of sliding occurs at the bimaterial interface. Therefore, (29) is valid. Since layer 2 is rigid, then $U_i = U$. The solution given by (14) and ((15a), (15b)) in which K_1 should be replaced with T_1 , K_2 with $-T_i$, w with $h^{(1)}$, k_0 with $k_0^{(1)}$, $\Phi(\varepsilon_{eq})$ with $\Phi_1(\varepsilon_{eq})$, z with y_1 , and V with $U/2$ is valid. It is convenient to take $z_* = h^{(1)}$. Then, using (16), (19), and (29),

$$\begin{aligned}\sigma_{xy} &= -\frac{(T_1 - T_i) y_1}{2h^{(1)}} + \frac{T_1 + T_i}{2}, \quad \sigma_{yy} = \frac{(T_1 - T_i) x}{2h^{(1)}} + c, \\ \sigma_{xx} &= \frac{(T_1 - T_i) x}{2h^{(1)}} + c + 2\sqrt{\left[k_0^{(1)}\Phi_1(\varepsilon_{eq})\right]^2 - \sigma_{xy}^2}, \\ u_x &= \frac{Ux}{2h^{(1)}} - \frac{U}{h^{(1)}} \int_{h^{(1)}}^{y_1} \cot 2\phi d\beta + C, \quad u_y = -\frac{Uy_1}{2h^{(1)}}.\end{aligned}\quad (31)$$

6. Cold Bond Formation in Continued Compression of Two-Layer Strips

Replacing T_i in (26) with (27a) and then with (27b) at $\varepsilon_{eq} = 0$ and using (12), (18), and (19) give the following equations for s_{c1} and s_{c2} at the initial instant:

$$s_{c1} = \frac{m_i + m_2}{\alpha m_1 - m_i}, \quad (32a)$$

$$s_{c2} = \frac{m_2 - m_i}{\alpha m_1 + m_i}. \quad (32b)$$

Each of these expressions involves four independent input parameters. Therefore, it is impossible to provide a clear geometric illustration of this solution in its generality. In order to give some insights into qualitative features of the solution, it is assumed that $m_1 = m_2 = 0.7$ and $\alpha = 3$. Then, both s_{c1} and s_{c2} solely depend on m_i . The corresponding curves are depicted in Figure 4, where $s_0 = h_0^{(2)}/h_0^{(1)}$. The domain between the curves corresponds to the regime of sticking at the bimaterial interface at the initial instant. The curves intersect at $m_i = 0$. It follows from ((32a), (32b)) that in this special case the regime of sticking at the initial instant can only occur if $h_0^{(2)}/h_0^{(1)} = m_2/(\alpha m_1)$. The lower curve intersects

FIGURE 4: Domain of sticking at the initial instant.

the horizontal axis at $m_i = m_2$. It is evident from (32b). The physical meaning of this mathematical feature of the solution is that the soft layer is plastic at $m_i \geq m_2$ even if its thickness is vanishing. On the other hand, it follows from (32a) that $s_{c1} \rightarrow \infty$ as $m_i \rightarrow \alpha m_1$. It is evident that it is possible if and only if $\alpha m_1 < 1$. Therefore, this feature of the solution is not revealed in Figure 4. Its physical meaning is that the regime of sticking at the bimaterial interface occurs at any large value of the ratio $h_0^{(2)}/h_0^{(1)}$. In what follows, it is assumed that the set of input parameters is such that the hard layer is rigid and the soft layer is plastic at the initial instant. This is a typical situation in rolling of two-layer strips [6]. It will be also assumed that the materials are linear strain hardening. Thus

$$\Phi_1(\varepsilon_{eq}) = 1 + H_1 \varepsilon_{eq}, \quad (33a)$$

$$\Phi_2(\varepsilon_{eq}) = 1 + H_2 \varepsilon_{eq}, \quad (33b)$$

where H_1 and H_2 are constant hardening moduli.

Since the hard layer is rigid during an initial stage of the process of compression, $h^{(1)} = h_0^{(1)}$ and $\varepsilon_{eq} = 0$ in this layer. The solution (30) is valid in the soft layer. In order to use (26), it is necessary to find the dependence of T_i and T_2 on $\lambda^{(2)} = \ln(h^{(2)}/h_0^{(2)})$. Replacing K_1 with T_i , K_2 with T_2 , w with $h^{(2)}$, k_0 with $k_0^{(2)}$, z with y_2 , V with $U/2$, and $\Phi(\varepsilon_{eq})$ with (33b) in (16) and (17) yields

$$\begin{aligned} \cos 2\phi = & - \frac{[m_2(1 + H_2 \varepsilon_2) + m_i(1 + H_2 \varepsilon_i)] y_2}{2h^{(2)}(1 + H_2 \varepsilon_{eq})} \\ & + \frac{m_i(1 + H_2 \varepsilon_i) - m_2(1 + H_2 \varepsilon_2)}{2(1 + H_2 \varepsilon_{eq})}, \end{aligned} \quad (34a)$$

$$\xi_{eq} = \frac{U}{2h^{(2)}} |\operatorname{cosec} 2\phi|. \quad (34b)$$

Here ε_2 is the value of ε_{eq} at $y_2 = h^{(2)}$ and ε_i is the value of ε_{eq} at $y_2 = -h^{(2)}$. Equations (19) and (29) have been used to

eliminate T_1 and T_i . Note that $dh^{(2)}/dt = -U/2$. Therefore, it follows from (5) and (34b) that

$$\frac{d\varepsilon_{eq}}{d\lambda^{(2)}} = -|\operatorname{cosec} 2\phi|. \quad (35)$$

It follows from (34a) that

$$\cos 2\phi = -m_2, \quad \cos 2\phi = m_i, \quad (36)$$

at $y_2 = h^{(2)}$ and $y_2 = -h^{(2)}$, respectively. Substituting (36) into (35) and integrating with the use of the initial condition $\varepsilon_{eq} = 0$ at $\lambda^{(2)} = 0$ give

$$\varepsilon_2 = -\frac{\lambda^{(2)}}{\sqrt{1 - m_2^2}}, \quad \varepsilon_i = -\frac{\lambda^{(2)}}{\sqrt{1 - m_i^2}}. \quad (37)$$

Substituting (37) into (33a), (33b) and the resulting equations into (19) and (29) leads to

$$\begin{aligned} T_2 &= m_2 k_0^{(2)} \left(1 - H_2 \frac{\lambda^{(2)}}{\sqrt{1 - m_2^2}} \right), \\ T_i &= m_i k_0^{(2)} \left(1 - H_2 \frac{\lambda^{(2)}}{\sqrt{1 - m_i^2}} \right). \end{aligned} \quad (38)$$

Combining (26) and (38) gives the following equation for the value of $\lambda^{(2)} = \lambda_c$ corresponding to the beginning of cold bond formation between the layers:

$$\begin{aligned} & m_i \left(1 - H_2 \frac{\lambda_c}{\sqrt{1 - m_i^2}} \right) (1 + s_0 \exp \lambda_c) \\ &= \alpha m_1 s_0 \exp \lambda_c - m_2 \left(1 - H_2 \frac{\lambda_c}{\sqrt{1 - m_2^2}} \right). \end{aligned} \quad (39)$$

It has been taken into account here that $T_1 = m_1 k_0^{(1)}$ at this instant. At $\lambda^{(2)} < \lambda_c$ the solution given in Section 4 should be used. In particular, it follows from (22) and (24) that

$$\frac{dh^{(2)}}{dt} = -\frac{U - U_i}{2}, \quad \frac{dh^{(1)}}{dt} = -\frac{U_i}{2}. \quad (40)$$

Combining these equations and (28) gives

$$\frac{dh^{(2)}}{dh^{(1)}} = \frac{U}{U_i} - 1 = s = \frac{h^{(2)}}{h^{(1)}}. \quad (41)$$

Integrating this equation with the use of the initial condition $h^{(1)} = h_0^{(1)}$ at $h^{(2)} = h_0^{(2)} \exp \lambda_c$ yields

$$\frac{h^{(2)}}{h^{(1)}} = \frac{h_0^{(2)}}{h_0^{(1)}} \exp \lambda_c. \quad (42)$$

FIGURE 5: Variation of λ_c with m_i at different values of H_2 (other input parameters are fixed).

FIGURE 6: Variation of λ_c with m_i at different values of $m_1 = m_2$ (other input parameters are fixed).

The total thickness of the strip at the end of the process is given by

$$h_f = 2(h^{(1)} + h^{(2)}). \quad (43)$$

Equations (42) and (43) can be used to find the thickness of the individual layers at the end of the process as

$$h_f^{(2)} = \frac{h_f \exp \lambda_c}{2(h_0^{(1)}/h_0^{(2)} + \exp \lambda_c)}, \quad (44)$$

$$h_f^{(1)} = \frac{h_f}{2[1 + (h_0^{(2)}/h_0^{(1)}) \exp \lambda_c]}.$$

It is seen from this equation that the parameter λ_c controls the final configuration of the product. In order to illustrate the dependence of λ_c on input parameters, (39) has been solved numerically. The variation of λ_c with m_i is depicted in Figures 5 to 7. In all cases the magnitude of λ_c decreases with m_i . Figure 5 illustrates the effect of the hardening modulus H_2 on λ_c . It is seen from this figure that the magnitude of λ_c decreases as H_2 increases. Figure 6 shows that the dependence of the magnitude of λ_c on m_1 and m_2 when $m_1 = m_2$ is not monotonic at small m_i . However, when m_i is large enough the magnitude of λ_c decreases with these friction factors. Finally, it is seen from Figure 7 that the magnitude of λ_c decreases with the initial ratio $s_0 = h_0^{(2)}/h_0^{(1)}$.

FIGURE 7: Variation of λ_c with m_i at different values of s_0 (other input parameters are fixed).

7. Conclusions

A semianalytical solution for the compression of a two-layer strip of strain-hardening materials has been given. Numerical treatment has been reduced to solving a transcendental equation (39) and evaluating ordinary integrals. The solution to the transcendental equation predicts the thickness of the soft layer corresponding to the beginning of cold bold formation between the layers. Then, the final thickness of each layer is determined from (44) if the final thickness of the strip is given. Evaluating ordinary integrals is required to obtain detailed information on the through thickness distribution of stresses, strains, and velocities. The solution found determines the conditions of the transition between the regimes of sticking and sliding. These conditions are of special importance for the process of bonding.

The process of bonding by rolling can be simulated using the approach proposed in [20]. In this case, s_0 is determined by the thickness of each layer at the entrance to the roll gap and h_f introduced in (43) by the thickness of the strip at the exit. It is worthy of note that powerful numerical methods have been already developed to simulate the rolling process using this approach (see, e.g., [28]) and the present solution can be directly incorporated in simulations of the roll-bonding process by means of these numerical methods.

The solution found reduces to the rigid perfectly plastic solution given in [31] if $H_1 = H_2 = 0$ in (33a), (33b). The latter is in qualitative agreement with the experimental data reported in [44]. The solution can also serve as a benchmark problem for verifying numerical codes since accurate closed form solutions are necessary for this purpose [42].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The research described in this paper was supported by Grants RFBR-13-08-00969 and NSH-1275.2014.1 (Russia).

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