Research Article

Spontaneous Emission of an Excited Atom in a Dusty Unmagnetized Plasma Medium

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Investigation of spontaneous decay of an excited atom in dusty unmagnetized plasma is presented in this paper. The transverse contribution to the decay rate is normally associated with spontaneous emission. The rate of spontaneous emission can be obtained by Fermi's golden rule. In this calculation, the transverse contribution to dielectric permittivity and Green function technique are used. Calculation of the decay rate of atoms is applicable to understand the particular structure of the vacuum state of the electromagnetic field.

1. Introduction

Spontaneous decay of an excited state arises from the interaction between an excited atom or molecule and the ground state of the quantized electromagnetic field [1]. The rate of spontaneous emission depends partly on the environment of a light source. This means that, by placing the light source in a special environment, the rate of spontaneous emission can be modified [2]. Spontaneous emission is a quantum effect, which in a semiclassical picture can be described as an emission which is stimulated by vacuum noise, that is, by the zero point fluctuations of the electromagnetic field [3]. In a well-known paper published in 1948, Welton [4] wrote that spontaneous emission “can be thought of as forced emission taking place under the action of the fluctuating field.”

Spontaneous emission of light or luminescence is a fundamental process that plays an essential role in many phenomena in nature and forms the basis of many applications, such as fluorescent tubes, older television screens (cathode ray tubes), plasma display panels, lasers (for startup normal continues operation works by stimulated emission instead), and light emitting diodes.

Spontaneous emission is ultimately responsible for most of the light around us. We would not be here without it. The first person to derive the rate of spontaneous emission

directly from the first principles was Dirac [5], who used the newly formulated quantum theory of radiation.

The rate of spontaneous emission (i.e., the radiative rate) can be described by Fermi's golden rule [6]; the rate of emission depends on two factors: an “atomic part,” which describes the internal structure of light source, and a “field part,” which describes the density of electromagnetic modes of the environment. The atomic part describes the strength of a transition between two states in terms of transition moments. In a homogeneous medium, such as free space, the rate of spontaneous emission in the dipole approximation is given by [7]

$$\Gamma_0 = \frac{\omega^3 d^2}{3\pi\hbar\epsilon_0 c^3}. \quad (1)$$

Clearly, the rate of spontaneous emission in free space increases with ω^3 . In contrast with atom, which has a discrete emission spectrum, quantum dots can be tuned continuously by changing their size. This property has been used to check the ω^3 -frequency dependence of the spontaneous emission rate as described by Fermi's golden rule.

The purpose of present work is to investigate the decay rate of an excited atom in dusty unmagnetized plasma, using Green function technique. Dusty plasma is loosely defined as

normal electron-ion plasma with an additional charged component which increases the complexity of the system even further. Dusty plasma is low temperature fully or partially ionized electrically conducting gases whose constituents are electron, ions, charged dust grains, and neutral atoms. Dust grains are massive and their sizes range from nanometers to millimeters. Dust grains may be metallic, conducting, or made of ice particulates [8].

The plasma frequency ω_p in uniform, cold, dusty unmagnetized plasma can be described as follows [8]:

$$\omega_p^2 = \sum_s \frac{4\pi n_{s0} q_s^2}{m_s} = \sum_s \omega_{ps}^2, \quad (2)$$

where n_{s0} , q_s , and m_s are, respectively, the unperturbed number density, the charge, and the mass of the plasma species s (s equals e for electrons, i for ions, and d for dust grains). And $\omega_{ps} = (4\pi n_{s0} q_s^2 / m_s)^{1/2}$ represents the plasma frequency associated with the plasma species s .

Considering the usual expression for the decay rate of an excited atom in dipole approximation [9] and since we are interested in the decay rate of an excited atom in dusty unmagnetized plasma, at the first stage, we calculate Green's function in this medium.

2. Green Function

The associated electromagnetic fields are determined by Maxwell's equations

$$\begin{aligned} \nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \\ \nabla \times \mathbf{H} &= \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \end{aligned} \quad (3)$$

where $\mathbf{E}$, $\mathbf{H}$, and the electric displacement $\mathbf{D}$ have positive and negative frequency decomposition as follows:

$$\hat{\mathbf{E}}(\mathbf{r}, t) = \int_0^\infty d\omega \left\{ \hat{\mathbf{E}}^+(\mathbf{r}, \omega) e^{-i\omega t} + \hat{\mathbf{E}}^-(\mathbf{r}, \omega) e^{+i\omega t} \right\}. \quad (4)$$

The positive and negative frequency parts in the integrand involve only annihilation and creation operators, respectively.

Taking the Fourier transform of these equations with respect to t , we obtain for positive frequency part of the fields

$$\begin{aligned} \nabla \times \mathbf{E}^+(\mathbf{r}, \omega) &= i\omega \mathbf{B}^+(\mathbf{r}, \omega), \\ \nabla \times \mathbf{H}^+(\mathbf{r}, \omega) &= -i\omega \mathbf{D}^+(\mathbf{r}, \omega) + \mathbf{J}^+(\mathbf{r}, \omega). \end{aligned} \quad (5)$$

Using the relationship between the displacement and the electric field

$$\begin{aligned} D_i^+(\mathbf{r}, t) &= \int_{-\infty}^t dt' \int d\mathbf{r}' \varepsilon_{ij}(\mathbf{r} - \mathbf{r}', t - t') E_j(\mathbf{r}', t'), \\ D_i(\mathbf{r}, \omega) &= \varepsilon_{ij}(\mathbf{r}, \omega) E_j(\mathbf{r}, \omega). \end{aligned} \quad (6)$$

And utilize the gauge in which the scalar potential vanishes:

$$\mathbf{E}^+(\mathbf{r}, \omega) = i\omega \mathbf{A}^+(\mathbf{r}, \omega). \quad (7)$$

We find the inhomogeneous wave equation for $\mathbf{A}^+(\mathbf{r}, \omega)$ in the frequency domain

$$\nabla \times \nabla \times \mathbf{A}^+(\mathbf{r}, \omega) - q^2 n^2 \mathbf{A}^+(\mathbf{r}, \omega) = \frac{\mu(\omega)}{\varepsilon_0 c^2} \mathbf{J}^+(\mathbf{r}, \omega), \quad (8)$$

where $n = \sqrt{\varepsilon(\mathbf{r}, \omega) \mu(\omega)}$, $c^2 = 1/\varepsilon_0 \mu_0$, and $q = \omega/c$. Since the medium is unmagnetized, then $\mu(\omega) = 1$.

This equation can be solved by using Fourier time transform Green function as

$$\begin{aligned} \mathbf{A}^+(\mathbf{r}, \omega) &= \int d^3 r' G(\mathbf{r} - \mathbf{r}', \omega) \cdot \mathbf{J}^+(\mathbf{r}', \omega) \\ J^+(r, \omega) &= \int d^3 r' \delta(\mathbf{r} - \mathbf{r}') J^+(\mathbf{r}', \omega). \end{aligned} \quad (9)$$

Substitution of (9) into (8) shows that the classical Green function $G(\mathbf{r}, \mathbf{r}', \omega)$ satisfies the differential equation:

$$\begin{aligned} \nabla \times \nabla \times G(\mathbf{r} - \mathbf{r}', \omega) - q^2 \varepsilon(\mathbf{k}, \omega) G(\mathbf{r} - \mathbf{r}', \omega) \\ = \frac{\mathbf{I}}{\varepsilon_0 c^2} \delta(\mathbf{r} - \mathbf{r}'), \end{aligned} \quad (10)$$

where $\mathbf{I}$ is the unit tensor.

For solving this equation, we used the formal approach that relies on Fourier transform, where the differential equations are transformed into reciprocal space. At the end, after algebraic calculation and separating Green function to transverse and longitudinal component in coordinate of wave vector [7], we obtain

$$G_{ij}^T(\mathbf{k}, \omega) = \frac{(\delta_{ij} - k_i k_j / k^2)}{\varepsilon_0 \omega^2 [(kc/\omega)^2 - \varepsilon^t]}, \quad (11)$$

$$G_{ij}^L(\mathbf{k}, \omega) = \frac{-k_i k_j}{k^2 \varepsilon_0 \varepsilon^l \omega^2}. \quad (12)$$

The dielectric tensor for dusty unmagnetized plasma is obtained by Poisson-Maxwell equation [10]. Consider

$$\begin{aligned} \varepsilon_{ij}(\mathbf{k}, \omega) &= \frac{k_i k_j}{k^2} \varepsilon^l(\mathbf{k}, \omega) + \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \varepsilon^t(\mathbf{k}, \omega) \\ &+ \left(\delta_{ij} - \frac{\Omega_i \Omega_j}{\Omega^2} \right) \varepsilon^d(\mathbf{k}, \omega), \end{aligned} \quad (13)$$

where Ω_i is the angular velocity of the i th grain. The influence of the dust grain rotation is described by $\varepsilon^d(\mathbf{k}, \omega)$. Since in this paper the unmagnetized plasma ($\mathbf{B}_0 = 0$) is investigated, the influence of the dust grain rotation is neglected. Therefore $\varepsilon^d(\omega, \mathbf{k}) = 0$.

Also $\varepsilon^l(\mathbf{k}, \omega)$ and $\varepsilon^t(\mathbf{k}, \omega)$ are the longitudinal and the transverse dielectric permittivity, respectively. They are given by

$$\varepsilon^l(\mathbf{k}, \omega) = 1 + \sum_s \frac{\omega_{ps}^2}{k^2 V_{Ts}^2} \left[1 - I_+ \left(\frac{\omega}{k V_{Ts}} \right) \right], \quad (14)$$

$$\varepsilon^t(\mathbf{k}, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} I_+ \left(\frac{\omega}{k V_{Ts}} \right),$$

where $V_{Ts} = (k_B T_s / m_s)^{1/2}$ is the thermal speed of the species s and s includes ion, electron, and dust grain.

The function $I_+(x)$ is

$$I_+(x) = \frac{x}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \frac{\exp(-z^2/2)}{x-z}. \quad (15)$$

The asymptotic forms of (15) are as follows.

For $|x| \gg 1$, $|\operatorname{Re} x| \gg |\operatorname{Im} x|$ and $\operatorname{Im} x < 0$,

$$I_+(x) \approx 1 + \frac{1}{x^2} + \cdots - i\sqrt{\frac{\pi}{2}} x \exp\left(-\frac{x^2}{2}\right). \quad (16)$$

And for $|x| \ll 1$,

$$I_+(x) \approx -i\sqrt{\frac{\pi}{2}} x. \quad (17)$$

Substituting the transverse dielectric permittivity in (11), we find that the transverse Green function has the following expansion:

$$G_{ij}^T(\mathbf{k}, \omega) = \frac{(\delta_{ij} - k_i k_j / k^2)}{\epsilon_0 \omega^2 [(kc/\omega)^2 - (1 - \sum_s (\omega_{ps}^2 / \omega^2) I_+(\omega / kV_{Ts}))]}. \quad (18)$$

Here, we study the range of high frequencies; then

$$\left| \frac{\omega}{kV_{Ts}} \right| \gg 1, \quad \text{and then } I_+\left(\frac{\omega}{kV_{Ts}}\right) \approx 1. \quad (19)$$

By using this approximation and taking inverse Fourier transformation, the transverse Green function in coordinate space is obtained:

$$\begin{aligned} G_{ij}^T(\mathbf{R}, \omega) &= \frac{1}{4\pi\epsilon_0\omega^2 (1 - \omega_p^2/\omega^2)} \\ &\times \left\{ \left[\delta_{ij} - \frac{3R_i R_j}{R^2} \right] \right. \\ &\quad + q'^3 \left[\left(\frac{1}{q'R} + \frac{i}{(q'R)^2} - \frac{1}{(q'R)^3} \right) \delta_{ij} \right. \\ &\quad \left. - \left(\frac{1}{q'R} + \frac{3i}{(q'R)^2} - \frac{3}{(q'R)^3} \right) \frac{R_i R_j}{R^2} \right] \\ &\quad \left. \times e^{iq'R} \right\}, \end{aligned} \quad (20)$$

in which $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $q' = (\omega/c)\sqrt{1 - \omega_p^2/\omega^2}$.

Expanding the exponential term in (20), it is straightforward to show that, in the limit $R \rightarrow 0$, the transverse Green function takes the form

$$\lim_{R \rightarrow 0} G_{ij}^T(\mathbf{R}, \omega) = \frac{1}{4\pi\epsilon_0 c^2} \left\{ \frac{R_i R_j}{2R^3} + \frac{\delta_{ij}}{2R} + \frac{2i\omega}{3c} \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2}\right)} \delta_{ij} + o(R) \right\}. \quad (21)$$

The singular behavior of the real part arises from the transverse delta function associated with the canonical field commutation relation [1]. But the imaginary part of this Green function is well behaved in the limit as r tends to zero:

$$\operatorname{Im} G_{ij}^T(\mathbf{0}, \omega) = \frac{\operatorname{Real}\left(\sqrt{1 - \omega_p^2/\omega^2}\right) \omega}{6\pi\epsilon_0 c^3} \delta_{ij}. \quad (22)$$

Substituting the longitudinal dielectric permittivity in Green function, the longitudinal Green function is obtained. But because of considering only the transverse radiative modes of the electromagnetic field, the calculation of longitudinal component is omitted here.

3. Calculation of Decay Rate

According to the expression of decay rate [7],

$$\Gamma = \frac{2\omega^2}{\hbar} \operatorname{Im} \{d_i G_{ij}(\mathbf{0}, \omega) d_j\}. \quad (23)$$

The transverse contribution to the total transition rate can be written as follows.

For $|\omega/kV_{Ts}| \gg 1$,

$$\Gamma^T = \operatorname{Real}\left(\sqrt{1 - \frac{\omega_p^2}{\omega^2}}\right) \Gamma_0. \quad (24)$$

In this relation, if

$$\omega_p^2 > \omega^2, \quad (25)$$

then $\sqrt{1 - \omega_p^2/\omega^2}$ is imaginary and $\Gamma^T = 0$. This is linked to the nonexistence of propagating modes in the plasma at this range of frequency into which a photon can be emitted.

And if

$$\omega_p^2 < \omega^2, \quad (26)$$

then $\sqrt{1 - \omega_p^2/\omega^2}$ is real and

$$\Gamma^T = \left(\sqrt{1 - \frac{\omega_p^2}{\omega^2}}\right) \Gamma_0. \quad (27)$$

The result of calculation of transverse decay rate is applied in high energy astrophysical plasma, collisional radioactive model in plasma, fluorescence quenching, and other options.

In Figure 1 we plot the transverse decay rate as given in (24).

4. Conclusions

In this paper, we have calculated the transverse contribution of decay by using Green function technique and fluctuation dissipation theorem [11] in dusty unmagnetized plasma. Spontaneous emission is the process by which a light source

FIGURE 1: The transverse spontaneous emission rate (24). Transverse decay rate is zero for the range of frequency $\omega < \omega_p$ and for $\omega > \omega_p$; the diagram corresponds to (27). The plasma frequency in this model is considered 5.5×10^{10} Hz.

such as an atom, molecule, or nucleus in an excited state undergoes a transition to a state with a lower energy and emits a photon.

The transverse decay rate in dusty plasma is obtained by modifying the dielectric permittivity.

This behavior arises from the existence of the grain and radiating interaction of them with electrical field.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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