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Research Article

Effect of Cylinder Size on the Modulus of Elasticity and Compressive Strength of Concrete from Static and Dynamic Tests

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The primary objective of this study is to investigate the effects of cylinder size (150 by 300 mm and 100 by 200 mm) on empirical equations that relate static elastic moduli and compressive strength and static and dynamic elastic moduli of concrete. For the purposes, two sets of one hundred and twenty concrete cylinders, 150 by 300 mm and 100 by 200 mm, were prepared from three different mixtures with target compressive strengths of 30, 35, and 40 MPa. Static and dynamic tests were performed at 4, 7, 14, and 28 days to evaluate compressive strength and static and dynamic moduli of cylinders. The effects of the two different cylinder sizes were investigated through experiments in this study and database collected from the literature. For normal strength concrete (≤40 MPa), the two different cylinder sizes do not result in significant differences in test results including experimental variability, compressive strength, and static and dynamic elastic moduli. However, it was observed that the size effect became substantial in high strength concrete greater than 40 MPa. Therefore, special care is still needed to compare the static and dynamic properties of high strength concrete from the two different cylinder sizes.

1. Introduction

The elastic modulus of concrete E_c is of great interest for design of new structures and condition assessment of existing structures. In structure design, there are requirements for serviceability of concrete structures such as maximum permissible deflections and allowable story drift for high-rise buildings in general building codes [1, 2]. E_c is a fundamental parameter for calculating the static and dynamic behavior of structural elements (e.g., deflection, side sway of tall buildings, and vibration of concrete elements). Furthermore, E_c is a good indicator of degree of concrete deterioration: more degradation results in lower E_c. Therefore, E_c is popularly used for condition assessment of concrete structures such as building components, pavements, and bridge decks [3].

Elastic modulus of concrete E_c is directly measured by the static uniaxial compressive test in accordance with ASTM C469 [4], which is called static elastic modulus. In practice, E_c is generally determined from compressive strength based on design codes rather than on the direct measurement. ACI 318 committee [1] proposes an empirical equation that relates E_c and F_c:

$$E_{\text{ACI 318}} = 0.043 w_c^{1.5} \sqrt{F_c}\,(\text{MPa}), \tag{1}$$

where F_c is compressive strength of concrete in MPa and w_c is a unit weight of concrete in kg/m^3 (for $1440 \le w_c \le 2560$ kg/m^3) for a value of F_c less than 38 MPa [5]. Furthermore, ACI 363 committee [6] proposes a different equation for linking E_c and F_c for a value of F_c between 21 MPa and 83 Mpa:

$$E_{\text{ACI 363}} = \left(\frac{w_c}{2300}\right)^{1.5} \left(3320\sqrt{F_c} + 6900\right)(\text{MPa}). \tag{2}$$

For both normal strength and high strength concrete, the Comité-Euro-International du Béton and the Fédération Internationale de la Précontrainte (CEB-FIP) Model code and Eurocode 2 suggests an empirical equation relating E_c and F_c as follows:

$$E_{\text{CEB-FIP}} = 22000\sqrt[3]{\frac{F_c}{10}}\ (\text{MPa})\,. \tag{3}$$

However, it has been reported that the simple code equations may not always produce accurate E_c compared to the value based on direct measurements [7, 8]. In fact, there was no standard test method for determining E_c when the equation adopted by ACI 318 [1] was developed: consequently, there was a substantial variation according to the definition of elastic modulus of concrete (i.e., initial, tangent, or secant modulus) [5]. Furthermore, the code equations (see (1)–(4)) do not take into account the critical parameters such as the type of coarse aggregates, mineral admixtures, and size of test specimens for compressive strength of concrete.

Elastic modulus of concrete E_c can be determined by dynamic test methods such as ultrasonic pulse velocity and resonance frequency tests [9]. The resulting elastic modulus is commonly referred to as dynamic elastic modulus E_d, which is larger than static elastic modulus E_c [10]. There are several empirical equations that relate E_d and E_c. Lydon and Balendran [11] proposed the following empirical relationship between E_d and E_c:

$$E_c = 0.83E_d\ (\text{GPa})\,. \tag{4}$$

The British testing standard BS8100 Part 2 [12] provides another empirical equation for E_c as follows:

$$E_c = 1.25E_d - 19\ (\text{GPa})\,. \tag{5}$$

It is noteworthy that this equation does not apply to concrete containing more than 500 kg/m^3 or to lightweight aggregate concrete [10]. A more general relationship was proposed by Popovics [13] for both lightweight and normal concrete, taking into account the effect of a unit weight of concrete:

$$E_c = \frac{446.09{E_d}^{1.4}}{w_c}\ (\text{GPa})\,. \tag{6}$$

However, E_c values for a given concrete predicted by different empirical equations do not agree with each other. In fact, it is known that the value of E_d may vary significantly according to test methods and size and type of test specimens [7]. Therefore, it is difficult to select a correct equation that produces the least error for different dynamic tests and test specimens.

Requirements for the cylinder size of concrete are described in ASTM C192 [14] and ASTM C31 [15], which are adopted by general building code [1] and other ASTM standards for measuring compressive strength, static elastic modulus, and dynamic elastic moduli of concrete. Even though dimensions are not stipulated in a specification, test method, or practice, the concrete cylinder should have a diameter at least three times nominal maximum aggregate size and the height-to-diameter ratio of 2. In practice, maximum aggregate size ranges between 12.5 mm and 25 mm; therefore, a size of 100 by 200 mm cylinder is accepted by the standard tests method. The use of 100 by 200 mm cylinders has many advantages against using larger specimens (e.g., 150 by 300 mm cylinders) because it facilitates handling in practice and needs smaller space and reduces construction wastes. For dynamic elastic modulus, the frequency equations in ASTM C215 [16] are supposed to produce the same value of E_d when test cylinder has the same diameter-to-height ratio without regard to cylinder sizes: however, different size of cylinder may produce inelastic effect and dispersion and consequently affect a value of E_d. For static tests, a number of researchers [17–24] have observed that the size of cylinders affects compressive strength and elastic modulus of concrete. In summary, the cylinder size may affect the accuracy of the empirical equations that relate compressive strength and static elastic modulus (see (1)–(3)) and dynamic and static elastic moduli (see (4)–(6)). However, it is difficult to quantitatively say the size effect due to scarcity of experimental data comparing various empirical equations for estimating static elastic modulus of concrete E_c from compressive strength F_c and dynamic elastic modulus of concrete E_d.

The primary objective of this study is to investigate the effect of cylinder size (100 by 200 mm and 150 by 300 mm) on empirical equations that relate static and dynamic elastic moduli of concrete and static elastic modulus and compressive strength of concrete. For the purposes, a series of experimental studies was performed in the laboratory, which is described in Section 2. Main experimental variables in this study include the cylinder size (100 by 200 mm and 150 by 300 mm cylinders), concrete ages at the test (4, 7, 14, and 28 days), and compressive strength (20, 30, and 40 MPa). The results (i.e., experimental variability, compressive strength, and static and dynamic elastic moduli) from various static and dynamic test methods are compared in Section 3. For a comparison, the experimental results in this study were compared with database collected from the literature.

2. Experimental Program

2.1. Preparation of Specimens. For experimental studies, two sets of one hundred and twenty concrete cylinders with different sizes (100 by 200 mm and 150 by 300 mm cylinders) were prepared in the laboratory. Concrete used in this study has the same mix proportions of Type I Portland cement, river sand, crushed granite with maximum size of 25 mm, and mineral admixtures (fly ash and granulate-furnace slag), except for three different water-to-binder ratios (W/B), 0.3, 0.35, and 0.45. Specific concrete mix proportions are summarized in Table 1. Concrete was cast in two standard plastic molds with dimensions of 100 by 200 mm and 150 by 300 mm and placed in a curing chamber within 30 minutes. Plastic molds were removed after 24 h and specimens were cured in a water pond. A series of static and dynamic tests were conducted at different ages: 4, 7, 14, and 28 days. One day before testing, ten 100 by 200 mm and 150 by 300 mm cylinders for each test series were taken out of a water pond and air-cured in a constant temperature-and-humidity room.

Table 1: Mix proportions of concrete design.

ID	Cement type	W/B	S/A	Unit quantity (kg/m^3)							
				W	C	S	G	Mineral admixture		Chemical admixture	
								FA	GBFS	AE (binder %)	SP (binder %)
Mix 1	Type I	0.45	0.46	259	121	777	934	58	69	0.9	—
Mix 2		0.35	0.47	308	166	761	886	81	85	—	1
Mix 3		0.3	0.46	357	165	714	868	94	99	—	1

Note. W: water; C: cement; S: sand; G: gravel; FA: fly ash; GBFS: granulated blast furnace slag; AE: air-entraining agent; SP: superplasticizer.

Figure 1: Testing setup and instrumentation for the uniaxial compressive test of a concrete cylinder.

2.2. Static Tests for Compressive Strength and Elastic Modulus. The cylinders were ground at both ends before testing to remove any surface irregularity as well as ensure the ends to be perpendicular to the sides of the specimen. Elastic modulus and compressive strength of the cylinders were measured using a Universal Testing Machine (UTM) with a capacity of 1000 kN according to ASTM C469 [4] and ASTM C39 [25], respectively. Tests were performed at a loading rate of approximately 0.28 MPa/s. Deformations were measured using three sets of linear voltage differential transducers attached to two fixed rings (see Figure 1). The apparatus consisted of two aluminum rings with screws for attachment to the specimen. The spacing between screws on the top and bottom rings was 70 mm and 150 mm for 100 by 200 mm and 150 by 300 mm cylinders, respectively, which served as a gauge length for calculating axial strain from the measured deformations. The static elastic modulus of concrete is defined as a chord modulus from the stress-strain curve with a first point at strain level of 0.00005 (ε_1) and second point at 40% of the maximum stress as follows:

$$E_c = \frac{0.4 f_c - \sigma(\varepsilon_1)}{\varepsilon(0.4 f_c) - \varepsilon_1}. \tag{7}$$

2.3. Dynamic Tests for Estimating Dynamic Elastic Modulus. Dynamic elastic modulus of concrete was estimated by measuring fundamental longitudinal and transverse resonance frequencies of cylinders in accordance with ASTM C215 [16]. Shown in Figure 2(a) is the test setup and data acquisition system for the transverse resonance frequency test. For the longitudinal resonance frequency test, an accelerometer was placed on the center of one concrete surface, and an impact source was hit on the center of the other concrete surface. A steel ball having a diameter of 10 mm was used as an impact source for generating incident stress waves in a concrete specimen: the steel ball was effective for generating wideband frequency signals from very low to 20 kHz, which covers a frequency range of the resonance tests for the 100 by 200 mm and 150 by 300 mm cylinders in this study. Dynamic response of concrete cylinder was measured by an accelerometer attached to concrete specimen according to ASTM C215 [16]. Resulting time signals were converted to the frequency domain using the FFT (Fast Fourier Transformation) algorithm. The resonance frequencies of concrete are manifested as dominant amplitude in the frequency domain. The most dominant frequency was regarded as fundamental resonance frequencies of longitudinal (or transverse) mode. Dynamic elastic moduli based on the fundamental longitudinal frequency ($E_{d,LR}$) were estimated using the following equation:

$$E_{d,LR} = \beta_L M {f_L}^2 \text{ (Pa)}, \tag{8}$$

where β_L is constant dependent on dimensions and Poisson's ratio of concrete specimen (= $5.093(L/D^2)$) for a cylinder in N·s^2 (kg·m^2), M is mass of specimen in kg, and f_L is fundamental longitudinal resonance frequency in Hz. In addition, dynamic elastic moduli of concrete based on the fundamental transverse frequency ($E_{d,TR}$) were estimated using the following equation:

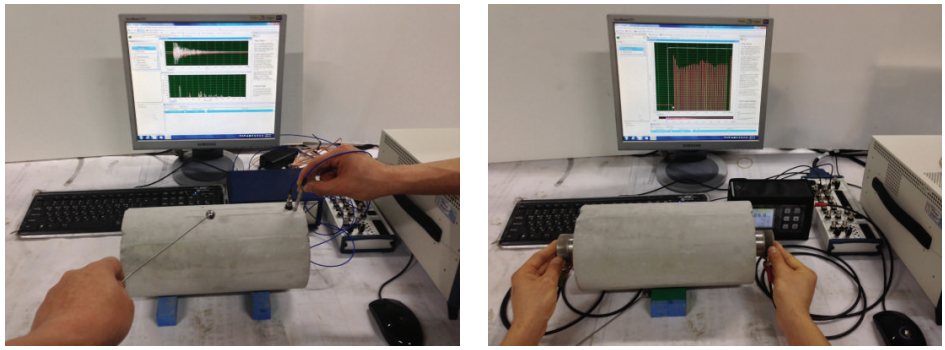
$$E_{d,TR} = \beta_T M {f_T}^2 \text{ (Pa)}, \tag{9}$$

where β_T is constant dependent on dimensions of concrete cylinder (= $1.6067[L^3T/D^4]$ for a cylinder, and T is a correction factor dependent on ratio of the radius of gyration K to the height of specimen H [for concrete cylinder $K/H = D/4H$] and Poisson's ratio) in N·s^2 (kg·m^2) and f_T is fundamental transverse resonance frequency in Hz.

For comparison purposes, the P-wave velocity of concrete, C_P, was measured according to ASTM C597 [26] using a pair of P-wave transducers (see Figure 2(b)), each of which generates and receives a longitudinal ultrasonic pulse of about 52 kHz through a concrete cylinder. Dynamic elastic modulus based on C_P ($E_{d,P}$) is determined using the following equation:

$$E_{d,P} = \alpha_P \rho {C_P}^2, \tag{10}$$

where α_P is constant dependent on Poisson's ratio υ, that is, $(1+\upsilon)(1-2\upsilon)/(1-\upsilon)$.



(a) (b)

Figure 2: Test setups and data acquisition system for evaluating dynamic elastic moduli using two different nondestructive evaluation tests: (a) resonance frequency test (transverse mode) and (b) ultrasonic pulse velocity test (longitudinal mode).

Table 2: Summary of test results measured from 100 by 200 mm and 150 by 300 mm cylinders.

Cylinder size	Mix	Day	F_c		E_c		C_p		f_L		f_T	
			μ [MPa]	COV [%]	μ [GPa]	COV [%]	μ [m/s]	COV [%]	μ [Hz]	COV [%]	μ [Hz]	COV [%]
100 × 200	Mix 1	D4	7.76	0.96	9.6	3.50	3327	2.05	6845	1.32	4165	1.86
		D7	10.3	3.64	11.6	7.70	3511	1.18	7220	1.28	4455	2.38
		D14	14.4	2.44	15.4	10.62	3855	0.82	7795	1.58	4810	1.29
		D28	18.9	5.05	15.7	5.41	4032	1.24	8420	0.66	5165	1.15
	Mix 2	D4	25.6	3.50	16.6	2.54	3803	1.10	8360	0.95	5160	1.55
		D7	29.3	3.40	17.0	2.10	4108	0.83	8570	0.53	5215	0.74
		D14	36.7	2.25	19.1	2.44	4224	1.42	8995	1.32	5455	1.19
		D28	39.3	3.63	23.1	3.18	4375	1.25	9280	0.91	5690	1.41
	Mix 3	D4	22.9	3.79	1654	4.60	3978	1.90	8365	1.17	5150	1.37
		D7	28.2	4.59	19.8	8.21	4122	0.92	8730	0.52	5335	1.32
		D14	34.0	5.75	20.4	3.23	4261	1.21	8995	1.01	5445	1.33
		D28	40.8	5.76	22.3	5.82	4361	1.04	9185	1.96	5555	3.05
150 × 300	Mix 1	D4	7.7	2.75	10.4	7.33	3223	2.16	4585	0.98	2800	0.80
		D7	10.1	3.63	12.2	6.56	3487	1.53	4870	1.31	3015	1.66
		D14	13.8	3.55	14.7	7.08	3723	1.39	5220	1.15	3215	1.21
		D28	19.3	4.34	15.7	5.23	3937	1.42	5600	1.38	3430	1.16
	Mix 2	D4	24.9	3.02	16.6	5.67	3934	1.47	5630	1.32	3450	1.74
		D7	29.9	2.60	17.5	3.32	4067	1.59	5730	0.89	3485	1.58
		D14	36.9	4.37	20.9	2.26	4219	0.72	6010	0.73	3645	1.29
		D28	43.2	4.92	23.7	3.37	4296	0.99	6200	0.81	3800	1.02
	Mix 3	D4	23.1	2.33	16.9	3.44	3922	1.23	5605	0.84	3435	0.93
		D7	29.0	1.50	18.7	2.22	4056	0.84	5825	1.10	3535	1.10
		D14	39.7	2.84	21.5	7.09	4192	0.97	6060	0.81	3675	0.91
		D28	44.4	4.47	21.3	7.21	4295	0.63	6260	1.17	3820	1.33

3. Result and Discussion

3.1. Experimental Variability. In this study, the coefficient of variation (COV, the standard deviation, σ, divided by the mean value, μ, of a set of samples) was used as a means of evaluating the experimental variability of compressive strength and static and dynamic properties of concrete. Table 2 compares the statistical parameters (μ and COV) of test results (F_c, E_c, C_p, f_L, and f_T) obtained from 100 by 200 mm and 150 by 300 mm cylinders.

The average COVs of the compressive strength of concrete F_c from different mix proportions and testing ages are 4.41% and 3.65% for 100 by 200 mm and 150 by 300 mm cylinders, respectively. The 100 by 200 mm cylinders have about 10% higher within-test variability than 150 by 300 mm cylinders. The result is consistent with the observation by

previous researchers that 100 by 200 mm cylinder tends to have about 20% higher within-test variability than 150 by 300 mm cylinder [27]. The COVs from the 150 by 300 mm cylinder are between good (4.0 to 5.0) and excellent (<2.0) categories according to ACI 214R [28], whereas for 100 by 200 mm cylinder the COVs are between fair (5.0 to 6.0) and excellent categories.

The average COVs of the static elastic modulus E_c from 100 by 200 mm and 150 by 300 mm cylinders are 6.32% and 5.83%, respectively. A slightly higher COV of E_c is mainly due to imperfect concrete specimens and testing procedure: the opposite faces of the specimens were slightly skew and their deformations under compression were not uniform. This may also affect compressive strength of concrete but appears to be more influential to determination of elastic modulus. Table 2 shows that the COVs of both F_c and E_c from the three different mixes are reasonably consistent at different ages in 4, 7, 14, and 28 days.

For the dynamic tests, the 100 by 200 mm cylinders produce equivalent or slightly higher variability than the 150 by 300 mm cylinders: however, the differences appear to be insignificant. The average COVs of the fundamental longitudinal f_L and transverse f_T frequencies measured from 100 by 200 mm and 150 by 300 mm cylinders are 1.64% and 1.20%, and 1.62% and 1.17%, respectively. Furthermore, the average COVs of the P-wave velocity (C_L) for 100 by 200 mm and 150 by 300 mm cylinders are 1.40% and 1.23%, respectively, both of which are consistent with observations by ACI committee 228 [29].

3.2. Compressive Strength. Figure 3 compares compressive strengths of concrete cylinders measured from 100 by 200 mm and 150 by 300 mm cylinders (F_{c100} and F_{c150}, resp.). The experimental data, represented as open symbols, show that there is no significant difference between F_{c100} and F_{c150} in the lower strength range of 8 MPa to 30 MPa, with a mean absolute error (MAE) less than 1 MPa. In this study, MAE was defined as follows:

$$\mathrm{MAE} = \frac{\sum \left|F_{c150} - F_{c100}\right|}{N}, \tag{11}$$

where N is the number of the experimental data (i.e., in this study $N = 120$). The result for low compressive strength is consistent with observations by prior researchers [22, 24, 30].

However, it was noticed that scattering of experimental data in this study becomes greater as compressive strength increases in the higher strength range greater than 30 MPa. F_{c150}-to-F_{c100} ratio in this study gradually increases as F_{c100} increases. An approximated equation that relates F_{c150} and F_{c100} was established by a linear regression analysis of the experimental data set in this study as follows:

$$F_{c150} = 1.11F_{c100} - 1.56\,[\mathrm{MPa}] \quad \left(R^2 = 0.96\right). \tag{12}$$

The best-fit line, presented as a red dash line in Figure 3, is compatible with the equations proposed by prior researchers [19, 20]. However, there is contradiction in the relationship between F_{c150} and F_{c100} reported by different researchers (see

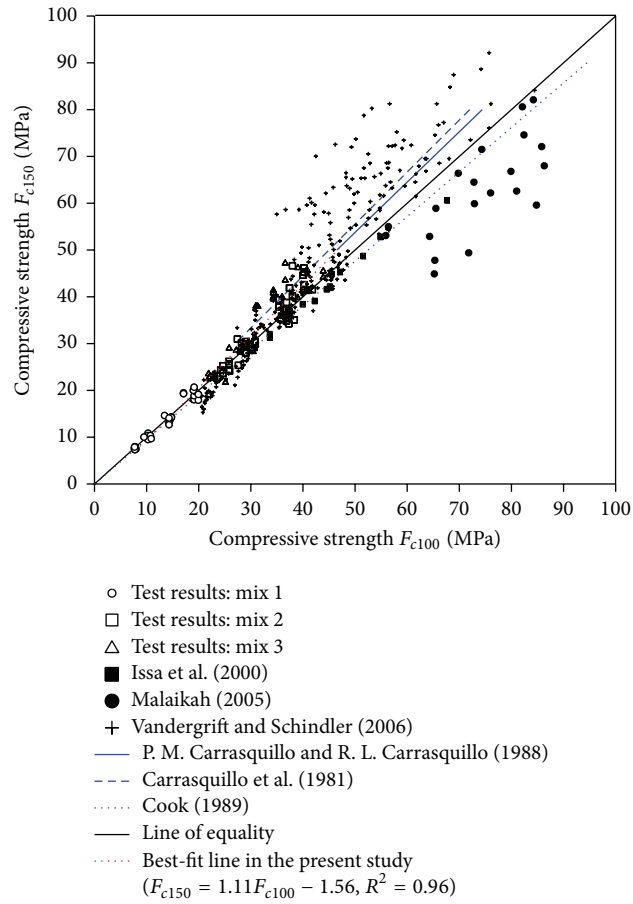


Figure 3: Comparison of compressive strength of concrete measured from 100 by 200 mm and 150 by 300 mm cylinders (F_{c100} and F_{c150}, resp.).

Figure 2) [18–22, 24, 30]. The inconsistency results in the higher strength range are attributed to complexity in the interfacial transition zone of concrete [10, 24]. It is known that considerable stresses are transferred at cement paste and aggregates' interface of high strength concrete due to lower porosity of the interfacial transition zone (ITZ). In fact, there are a number of factors affecting the ITZ, which include coarse aggregates, mineral admixtures, and curing methods, and various factors affect compressive strength in different ways for different cylinder sizes [14]. Therefore, special care is still needed for selecting the cylinder size for measuring compressive strength of relatively high strength concrete (>40 MPa).

3.3. Static Elastic Moduli. Figure 4 compares static elastic moduli of concrete measured from 100 by 200 mm and 150 by 300 mm cylinders (E_{c100} and E_{c150}, resp.) in this study. For a comparison, database collected from the literature [22, 30] was shown in the same figure. The experimental data set in this study, presented as open symbols, shows that E_{c100} is closely correlated with E_{c150} in the elastic modulus range of 10 GPa to 25 GPa. For the lower elastic modulus range of 10 to 15 GPa, E_{c100} values are comparable with E_{c150} values, with

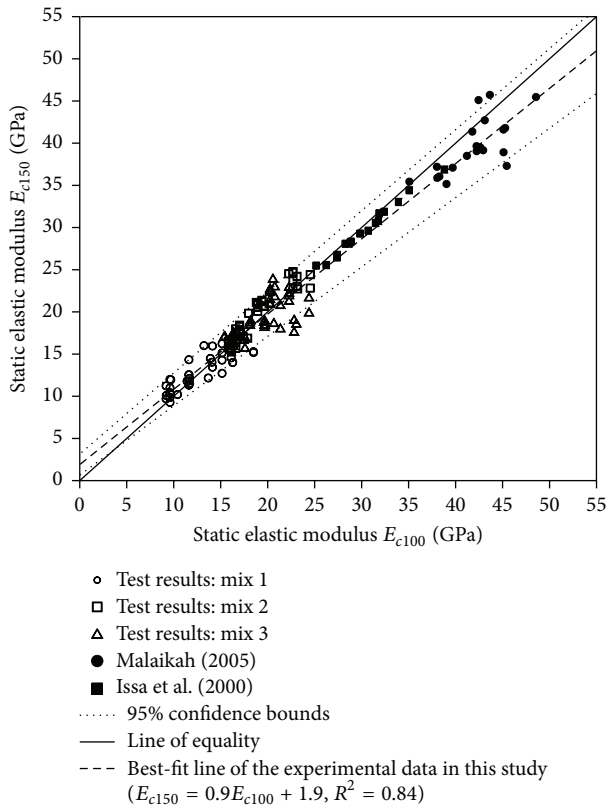


Figure 4: Comparison of static elastic moduli of concrete measured from 100 by 200 mm and 150 by 300 mm cylinders (E_{c100} and E_{c150}, resp.).

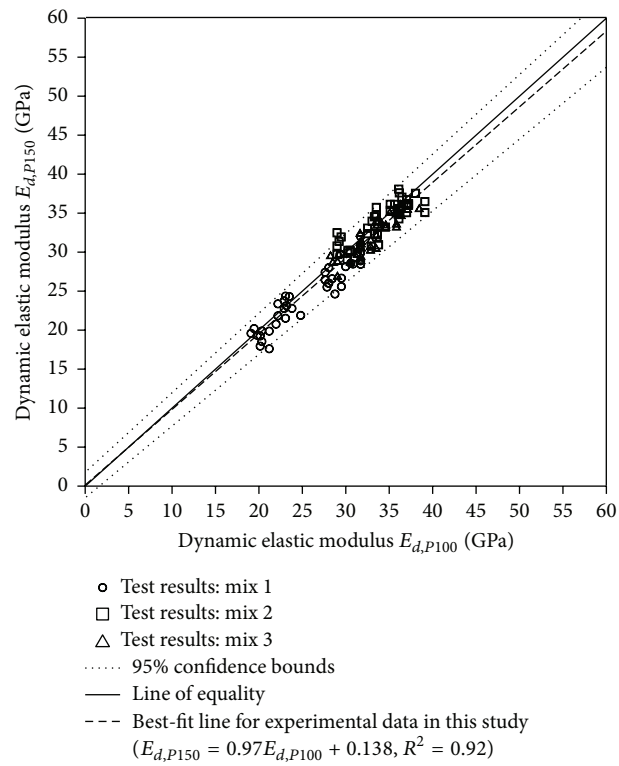


Figure 5: Comparison of velocity moduli from 100 by 200 mm and 150 by 300 mm cylinders.

MAE between E_{c100} and E_{c150} of about 0.45 GPa. Therefore, it may be acceptable to assume from a practical perspective that static elastic moduli using 100 by 200 mm and 150 by 300 mm cylinders are equivalent.

However, the ratio of E_{c150} to E_{c100} appears to gradually increase as E_{c150} increases to 25 GPa. The higher elasticity in the smaller size is attributed to the fact that the quantity of mortar required to fill the space between the particles of the coarse aggregate and the wall of the mold is greater than that necessary in the interior of the mass (i.e., wall effect) [10]. In general, the elasticity of cement mortar is greater than that of concrete, which consequently results in increasing the effective elasticity of concrete in smaller size. In this study, the best-fit line that approximates the relationship between E_{c150} and E_{c100} was established by a linear regression analysis as follows:

$$E_{c150} = 0.9E_{c100} + 1.9\,[\text{MPa}] \quad \left(R^2 = 0.84\right). \tag{13}$$

The best-fit line and 95% confidence bounds of the best-fit line are presented as solid and dash lines, respectively, in Figure 3. Interestingly, the best-fit line of the experimental data in this study (10 GPa $\leq$ E_{c100} $\leq$ 25 GPa) appears to be valid for predicting the data set in the higher elastic modulus range of 25 GPa to 55 GPa reported by prior researchers [22, 30]. However, it is still difficult to attain general conclusions on the relationship between E_{c150} and E_{c100}, especially for concrete in the higher strength range greater than 50 MPa, because of scarcity of available experimental data.

3.4. Dynamic Elastic Moduli. Figure 5 compares dynamic elastic moduli measured using UPV tests (i.e., velocity moduli) from 100 by 200 mm and 150 by 300 mm cylinders ($E_{d,P100}$ and $E_{d,P150}$, resp.) in accordance with ASTM C597 [26]. It was observed that 100 by 200 mm cylinders consistently result in slightly higher dynamic elastic moduli than 150 by 300 mm cylinders. Linear regression of $E_{d,P100}$ and $E_{d,P150}$ measured in this study shows that $E_{d,P100}$ is about 3% greater than $E_{d,P150}$ in a range of 20 GPa to 40 GPa (see Figure 5). The MAE from two different cylinders was about 0.5 GPa, which is only 25% of the MAE from static elastic moduli in this study (i.e., 2 GPa).

In addition, shown in Figures 6(a) and 6(b) is the comparison of dynamic elastic moduli measured from longitudinal and transverse resonance frequency tests (i.e., resonance moduli), $E_{d,LR}$ and $E_{d,TR}$, respectively, from 100 by 200 mm and 150 by 300 mm cylinders in accordance with ASTM C125 [16]. The use of 150 by 300 mm cylinders tends to result in slightly higher resonance moduli (both $E_{d,LR}$ and $E_{d,TR}$) than those from 100 by 200 mm cylinders. According to the linear regression analysis, it was found that the resonance moduli measured using the 150 by 300 mm cylinder ($E_{d,LR150}$ or $E_{d,RR150}$) are 1 to 2% greater than those from the 100 by 200 mm cylinder ($E_{d,LR100}$ or $E_{d,RR100}$). Approximated equations that relate the resonance moduli from the two different cylinder sizes are shown in Figures 6(a) and 6(b).

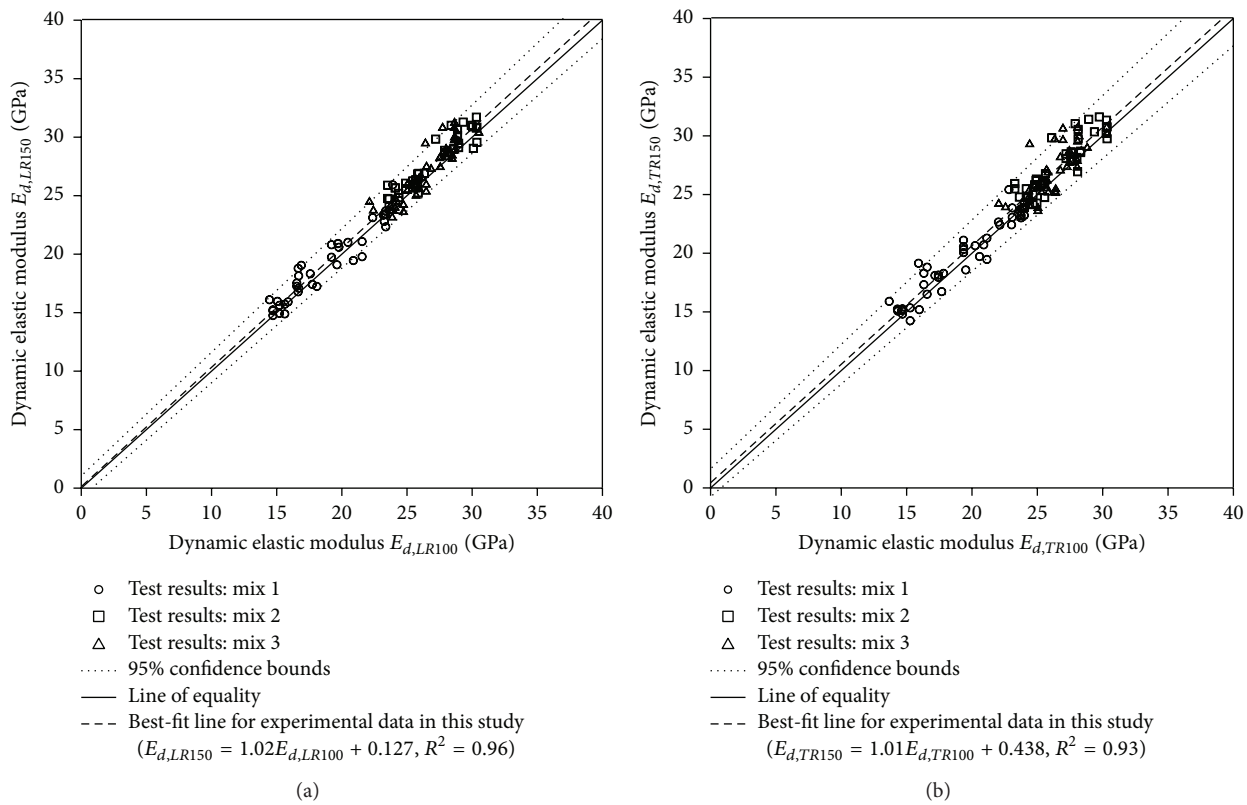


Figure 6: Comparison of resonance moduli from 100 by 200 mm and 150 by 300 mm cylinders: (a) $E_{d,LR150}$ versus $E_{d,LR100}$ and (b) $E_{d,TR150}$ versus $E_{d,TR100}$.

Furthermore, without regard to longitudinal or transverse modes, the MAE between the resonance moduli from the different cylinder is less than 0.3 GPa. Therefore, the findings in this study demonstrate that the dynamic elastic moduli (both velocity and resonance moduli) using the 100 by 200 mm and 150 by 300 mm cylinders are regarded as being equal from a practical standpoint.

However, it was observed that there is a precaution in measuring reliable and consistent resonance frequency using the 100 by 200 mm cylinder. The smaller cylinder has higher chance of mistakenly hitting an impact source (location and inclination to test surface), which may cause undesirable resonance modes such as torsional modes in the frequency domain. Consequently, it was often difficult to select a right frequency peak corresponding to the fundamental longitudinal or transverse modes. In this study, preliminary numerical simulations were conducted to calculate theoretical frequency peak values, which was helpful to select a right frequency peak. In addition, to improve accuracy and consistency, resonance tests were repeated until the COV of five successive testing procedures is less than 5%, and average of the five test results was finally accepted in this study.

3.5. Relationship between Static Elastic Modulus and Compressive Strength. Figure 7 is a plot representing relationship between static elastic modulus and compressive strength measured in accordance with ASTM C469 [4] and ASTM C39 [25], respectively. The data points obtained from 100 by 200 mm and 150 by 300 mm cylinders are presented as open and solid symbols in Figures 7(a) and 7(b), respectively. For a comparison, Figure 7 presents three code equations adopted by ACI 318 and ACI 363 committees and CEB-FIP Model code and a practical equation proposed by Noguchi et al. [8] (see (14)), which was developed based on an extensive experimental database from normal to high strength concrete:

$$E_c = k_1 k_2 33500 \left(\frac{F_c}{60}\right)^{1/3} \left(\frac{w_c}{2400}\right)^2, \tag{14}$$

where k_1 and k_2 are correction factors for coarse aggregates and mineral admixtures.

It was found that the effect of cylinder size appears to be insignificant on the relationship between static elastic modulus and compressive strength of concrete for normal strength concrete (<40 MPa) in this study. Approximated equations that relate E_c and F_c from 100 by 200 mm and 150 by 300 mm cylinders were established by nonlinear regression analyses and shown in Figures 7(a) and 7(b), respectively. The MAE of the two best-fit curves is less than 0.1 GPa in the compressive strength range of 10 MPa to 40 MPa. In this study, MAE of the two continuous curves was defined as follows:

$$\mathrm{MAE}_{\mathrm{con}} = \frac{\left[\int g_1(F_c) - g_2(F_c)\, dF_c\right]}{\int dF_c}, \tag{15}$$

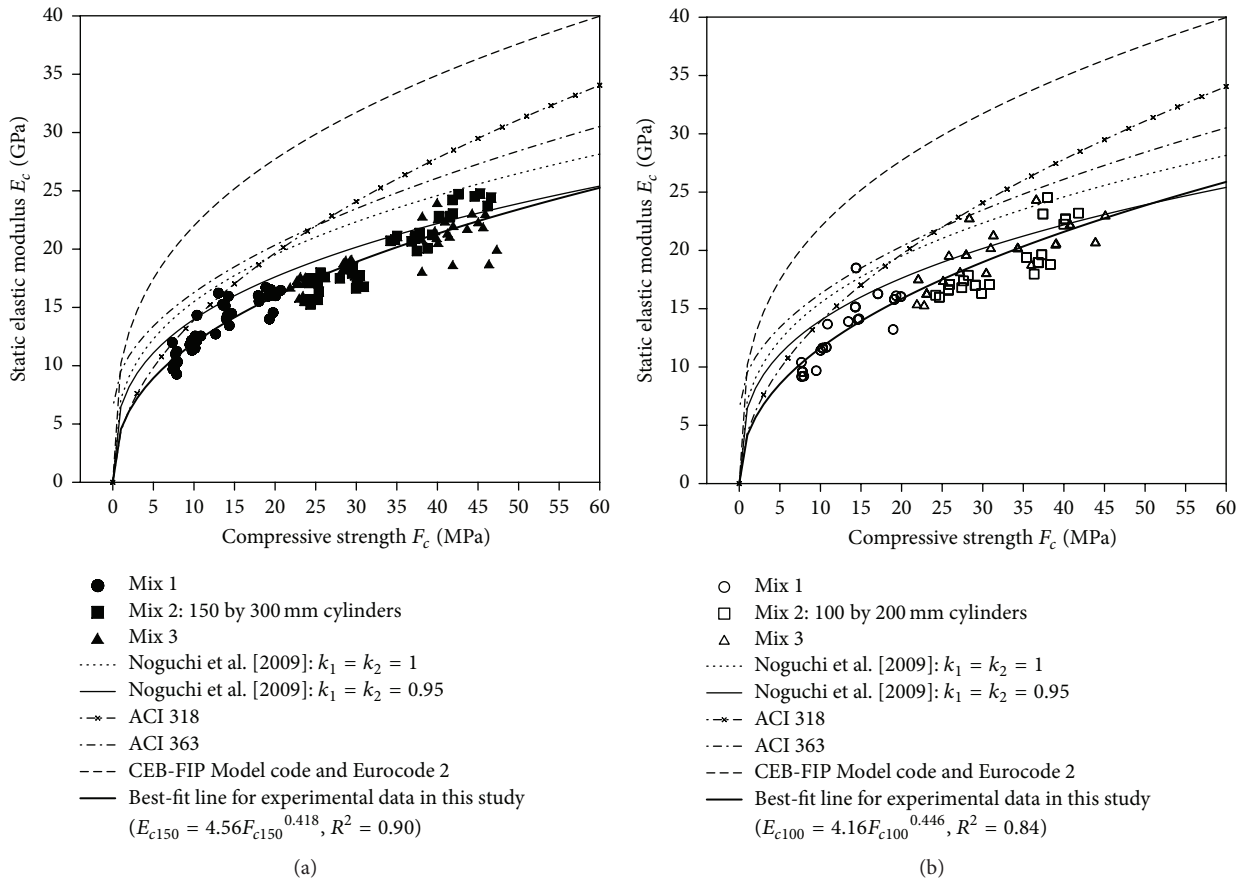


Figure 7: Comparison of compressive strength and static elastic modulus of concrete.

where $\mathrm{MAE}_{\mathrm{con}}$ is mean average error (MAE) of two continuous functions, g_i is a function presenting E_c expressed as F_c, and the subscript i indicates cylinder size (1 and 2 for the 100 by 200 mm and 150 by 300 mm cylinders, resp.). Furthermore, there is no significant difference in R^2 values of the two best-fit curves: R^2 values for the 100 by 200 mm and 150 by 300 mm cylinders are 0.88 and 0.90, respectively. Therefore, the 100 by 200 mm cylinders can be used instead of 150 by 300 mm cylinders to estimate static elastic modulus from compressive strength without reducing accuracy and consistency. However, the three code equations tend to overestimate static modulus of concrete compared to those from direct measurement according to ASTM C469 [4]. As summarized in Table 3, mean average error (MAE) between measured and predicted E_c from the three code equations is in a range of 4.4 to 12 GPa, corresponding to about 10% to 30% of the measured E_c. In contrast, the Noguchi equation [8] predicts experimental results with far more improved accuracy (i.e., MAE less than 2 GPa) by addressing correction factors for the effects of aggregates and mineral admixtures (i.e., $k_1 = k_2 = 0.95$).

3.6. Relationship between Static and Dynamic Elastic Moduli. Figure 8 presents the relationship between static and dynamic elastic moduli determined using different nondestructive testing methods, $E_{d,P}$, $E_{d,LR}$, and $E_{d,TR}$, respectively, with Poisson's ratio of 0.2. The use of Poisson's ratio of 0.2 is reasonable for common concrete in practice [10]. In Figure 8, $E_{d,P}$, $E_{d,LR}$, and $E_{d,TR}$ measured from 100 by 200 mm and 150 by 300 mm cylinders are shown as open and solid symbols in left and right columns. For comparison purposes, several well-known empirical equations (see (3), (4), and (5)) proposed by prior researchers are shown in the figures.

Table 3: Mean absolute error (MAE) of expressions relating static elastic modulus and compressive strength measured using 100 by 200 mm and 150 by 300 mm cylinders.

Equation to convert E_d to E_c	MAE (GPa)	
	150 by 300 mm cylinder	100 by 200 mm cylinder
	$E_{d,LT}$	$E_{d,LT}$
Equation (1)	4.83	4.63
Equation (2)	4.54	4.43
Equation (3)	12.21	12.15
Equation (14) with $k_1 = k_2 = 1$	3.42	3.41
Equation (14) with $k_1 = k_2 = 0.95$	1.56	1.88
Best-fit line	1.32	1.57

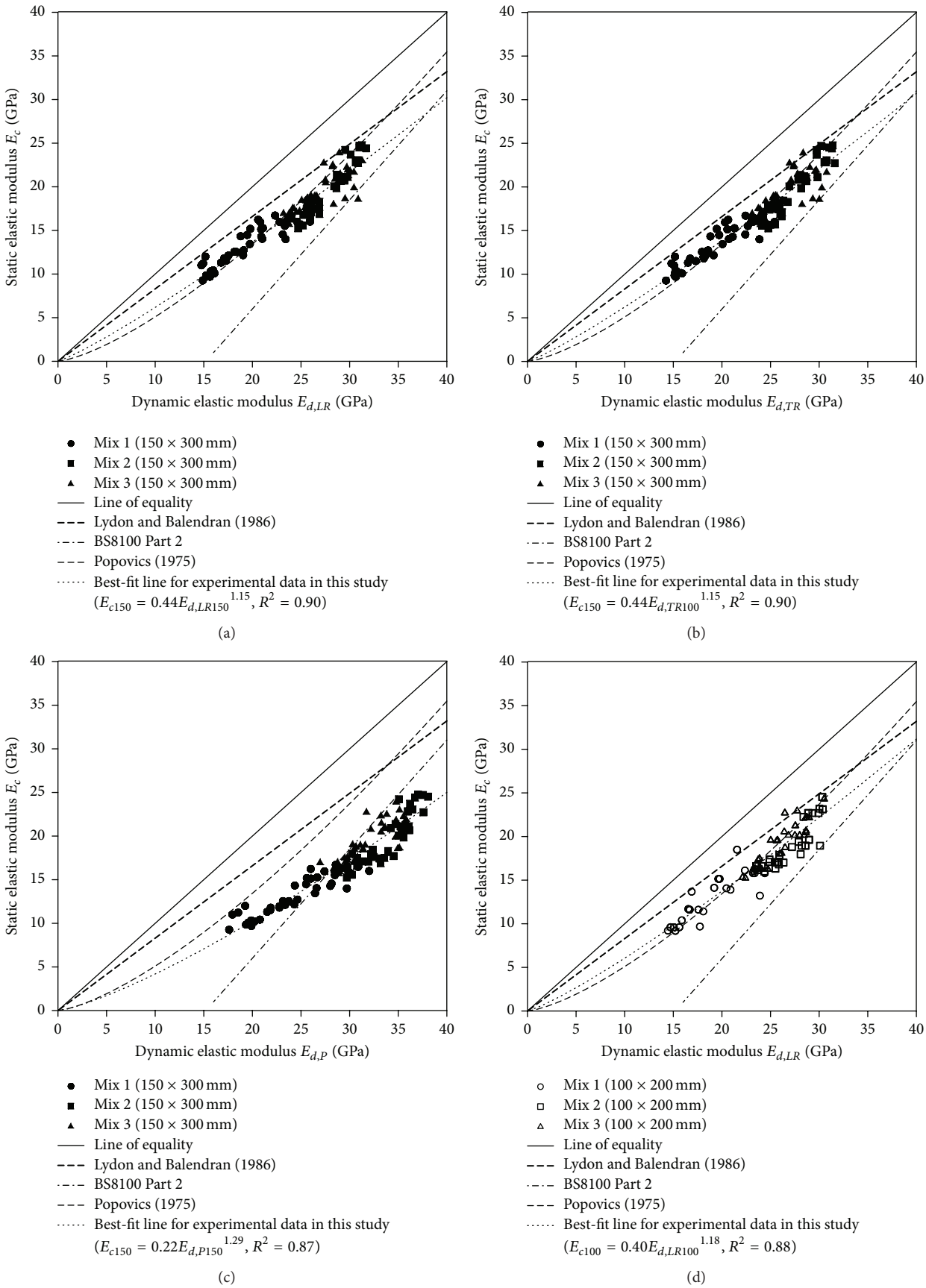


Figure 8: Continued.

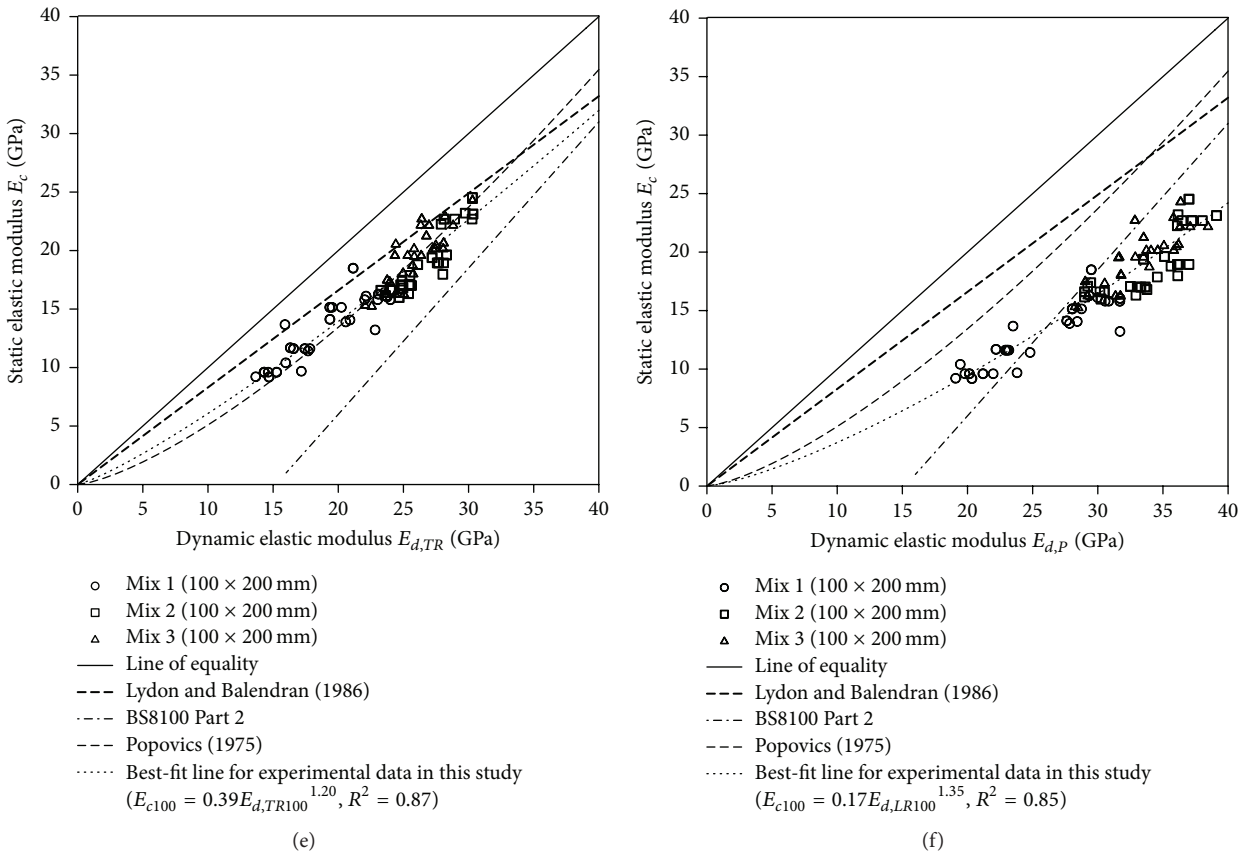


Figure 8: Comparison of static and resonance elastic moduli of concrete: (a) $E_{d,LR150}$ versus $E_{c,150}$, (b) $E_{d,TR150}$ versus $E_{c,150}$, (c) $E_{d,P150}$ versus $E_{c,150}$, (d) $E_{d,LR100}$ versus $E_{c,100}$, (e) $E_{d,TR100}$ versus $E_{c,100}$, and (f) $E_{d,P100}$ versus $E_{c,100}$.

Table 4: Mean absolute error (MAE) of expressions relating static and dynamic elastic modulus measured using 100 by 200 mm and 150 by 300 mm cylinders.

Equation to convert E_d to E_c	MAE (GPa)					
	150 by 300 mm cylinder			100 by 200 mm cylinder		
	$E_{d,LT}$	$E_{d,RT}$	$E_{d,P}$	$E_{d,LT}$	$E_{d,RT}$	$E_{d,P}$
Equation (4)	3.02	2.82	7.54	2.83	2.54	8.46
Equation (5)	5.59	5.91	2.77	6.07	6.49	3.35
Equation (6)	1.25	1.11	6.58	1.13	1.07	7.80
Best-fit line	0.90	0.95	1.26	1.09	1.10	1.17

It is observed that there is only slight difference in the relationship between E_c and E_d obtained from 100 by 200 mm and 150 by 300 mm cylinders. In this study, approximated equations that relate E_c and E_d ($E_{d,P}$, $E_{d,LR}$, or $E_{d,TR}$) from 100 by 200 mm and 150 by 300 mm cylinders were established by nonlinear regression analyses and shown in Figures 8(a)–8(f). The MAEs of the two best-fit curves are less than 0.1 MPa in the dynamic elastic modulus range of 10 GPa to 25 GPa. Furthermore, there is no significant difference in R^2 values of the two best-fit curves (see Figure 8). However, for both data sets from different cylinders, the three dynamic moduli ($E_{d,LR}$, $E_{d,TR}$, and $E_{d,P}$) obtained from resonance tests and UPV method are greater than the static elastic modulus E_c, with different static-to-dynamic elastic modulus ratio. $E_c/E_{d,P}$ ratio, with an average of 0.56 and COV of 9.2%, is too far away from the line of equality as well as from the three well-known equations relating static and dynamic elastic modulus. Therefore, the use of $E_{d,P}$ appears to be inappropriate to estimate static elastic modulus, which is consistent with observations from other researchers [7, 10]. In contrast, $E_c/E_{d,LR}$ (or $E_c/E_{d,TR}$) ratio was closer to the line of equality than $E_c/E_{d,P}$, with an average of 0.72 and COV of 7.15%. The equation proposed by Popovics [13] (see (6)) shows very good agreement with the experimental results regardless of cylinder size in this study (see Table 4). In addition, prior researcher [7] observed that the Popovics equation (see (6)) can be extended to high strength concrete up to 60 MPa without regard to cylinder size. However, it should be mentioned

that additional experimental data should be accumulated to better understand the size effect on the relationship between static and dynamic elastic moduli of high strength concrete (>60 MPa) due to scarcity of experimental studies in such a high strength range.

4. Conclusions

A series of experimental studies was conducted to explore the effect of cylinder size (100 by 200 mm and 150 by 300 mm cylinders) on the relationship between static elastic modulus and compressive strength and static and dynamic elastic moduli of concrete. Conclusions based on experiments in this study and database from the literature are drawn as follows:

(1) Experimental results in this study show that the COVs of test results from 100 by 200 mm cylinders are about 10% higher than those from 150 by 300 mm cylinders: however, the differences are statistically insignificant. The average COVs of static and dynamic elastic moduli using the UPV and longitudinal and transverse resonance frequency tests from 100 by 200 mm cylinders are 6.32%, 1.40%, 1.62%, and 1.17%, respectively, while those values from 150 by 300 mm cylinders are 5.83%, 1.23%, 1.62%, and 1.17%, respectively. For compressive strength, the average COVs from the 100 by 200 mm and 150 by 300 mm cylinders are 4.41% and 3.65%, respectively.

(2) According to the findings from similarity tests of best-fit curves and comparison of R^2 based on regression analyses, it has been demonstrated that the effect of cylinder size (100 by 200 mm and 150 by 300 mm cylinders) is insignificant on compressive strength and static elastic modulus and the relationship between the two parameters for normal strength concrete up to 30 MPa. However, in the higher strength range greater than 30 MPa, there is contradiction in the relationship between F_{c150} and F_{c100} based on experimental test in this study and database reported by different researchers. Therefore, it is difficult to establish a simple equation that relates F_{c100} and F_{c150}, and E_{c100} and E_{c150} because of limited experimental and theoretical studies.

(3) The three code equations tend to overestimate static modulus of concrete compared to those from direct measurement according to ASTM C469, with MAE in a range of 4.4 GPa to 12 GPa. In contrast, the Noguchi equation predicts experimental results with far more improved accuracy (i.e., MAE less than 2 GPa) by addressing correction factors for the effects of aggregates and mineral admixtures (i.e., $k_1 = k_2 = 0.95$).

(4) There is only a slight difference in the relationship between static and resonance moduli from 100 by 200 mm and 150 by 300 mm cylinders for normal strength concrete. Test results from this study and the literature show that the empirical equation suggested by Popovics [13] ((6) in this study) is effective for predicting static elastic moduli from resonance moduli of concrete with compressive strength up to 60 MPa regardless of the cylinder size, with MAE of less than 1.5 GPa.

(5) However, the velocity moduli are excessively greater than static elastic modulus. The empirical equations (see (4), (5), and (6)) produce considerable errors (with MAE in a range of 2.77 GPa to 8.46 GPa) between measured and predicted elastic moduli. Therefore, the use of velocity moduli appears to be inappropriate to estimate static elastic modulus.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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