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Research Article

One-Dimensional Consolidation of Double-Layered Foundation with Depth-Dependent Initial Excess Pore Pressure and Additional Stress

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A model for one-dimensional consolidation of a double-layered foundation considering the depth-dependent initial excess pore pressure and additional stress and time-dependent loading under different drainage conditions was presented in this study and its general analytical solution was deduced. The consolidation solutions of several special cases of single-drained and double-drained conditions under an instantaneous loading and a single-level uniform loading were derived. Then, the average degree of consolidation of the double-layered foundation defined by settlement was gained and verified. Finally, the effects of the initial excess pore pressure distributions, depth-dependent additional stress, and loading modes on the consolidation of the soft foundation with an upper crust with different drainage conditions were revealed. The results show that the distributions of initial excess pore pressure and additional stress with depth and loading rates have a significant influence on the consolidation process of the soft foundation with an upper crust. This influence is larger with the single-drained condition than that with the double-drained condition. Comparing the consolidation rate with a uniform initial pore pressure and additional stress, their decreasing distribution with depth quickens the consolidation at the former and middle stages. Moreover, the larger the loading rate is, the quicker the consolidation of the soft foundation with an upper crust is.

1. Introduction

Soft soil foundations are widely distributed in coastal and inland lake areas. Because of some natural actions or long-term engineering practices, a kind of natural or artificial hard crust with a thickness of a few meters on the soft substratum forms. The consolidation behavior of this kind of double-layered foundation with an upper crust is quite different from that of a homogeneous foundation.

Since Gray [1] first discussed the consolidation of two contiguous layers, many research methods have been used by researchers to evaluate the effects of different factors on the consolidation behavior of a double-layered soil. In the analytical method aspect, Schiffmann and Stein [2] developed a solution of a layered foundation according to a series of complex boundary conditions and stress history. Garlanger [3] and Mesri and Rokhsar [4] analyzed the one-dimensional nonlinear consolidation for a double-layered overconsolidated soil foundation. Subsequently, Zhu and Yin [5] presented a model of the double-layered foundation to account for the depth-dependent vertical stress and time-dependent loading. Xie et al. [6] gained a fully explicit analytical solution for one-dimensional consolidation of two-layered soils with partially drained boundaries. Tang and Onitsuka [7] gave an analytical solution for consolidation of double-layered foundation with vertical drains under quasi-equal strain condition. Xie et al. [8] derived an analytical solution for one-dimensional nonlinear consolidation of double-layered soil considering a decreasing permeability. Miao et al. [9] proposed an analytical solution for consolidation

of a double-layered foundation partially penetrated by deep mixed columns considering one-side or two-side vertical drainage. And, then, Miao et al. [10] studied the analytical solution for one-dimensional consolidation of the double-layered foundation with a multilevel loading condition. Zhu and Yin [11] derived several analytical solutions to analyze the consolidation of a soil layer with fairly general laws of variation of permeability and compressibility for different drainage conditions. In the finite element method aspect, Desai and Saxena [12] studied the consolidation behaviour of layered anisotropic foundations. Pyrah [13] evaluated the effects of the permeability and coefficient of volume compressibility of soil on the consolidation of layered foundations. Huang and Griffiths [14] solved the coupled (settlement and excess pore pressure), uncoupled (excess pore pressure only), and classical Terzaghi equation. In addition, Chen et al. [15] introduced a differential quadrature method to analyze the one-dimensional nonlinear consolidation of multilayered soil under partially drained boundaries and arbitrary loading condition. Fox et al. [16] built a numerical model for one-dimensional large strain consolidation of layered soils with different material properties, which is able to account for time-dependent loading and boundary conditions. Kim and Mission [17] and Sadiku [18] presented solutions to the consolidation of multilayered soil based on the finite differences method.

For a double-layered foundation, if the loading, which is always time-dependent, applied on the foundation surface is large enough, such as a high embankment or a high building, its influence region inside the foundation is deep and the vertical additional stress may vary with depth. At the same time, there may exist different distributions of initial excess pore pressure for a practical project. Therefore, a consolidation model of the double-layered foundation which takes account of the depth-dependent initial excess pore pressure and additional stress, time-dependent loading needs to be built. Based on the preceding discussions, the current researchers are not able to consider the above three factors in one consolidation model. Though Zhu and Yin [5] considered the depth-dependent vertical stress and time-dependent loading, their model assumed the initial excess pore pressure to be zero. They also did not analyze the effect of the depth-dependent vertical stress on the consolidation behavior of the double-layered foundation. At the same time, the numerical model built by Fox et al. [16] is able to account for time-dependent loading. However, the solution of this model should be gained using the numeric method and is an approximate solution. Therefore, this study aims at building a one-dimensional (1D) consolidation model for a double-layered foundation to consider the depth-dependent initial pore pressure and additional stress and time-dependent loading and analyzing the effect of them on the consolidation behavior of the soft foundation with an upper crust.

The paper is organized as follows. The forthcoming section introduces the mathematical model used in this study. Subsequently, the general analytical consolidation solutions of the double-layered foundation considering the depth-dependent initial excess pore pressure and additional stress, time-dependent loading under different drainage conditions are derived. The next section deduces the consolidation solutions of some several cases of a bilinear initial excess pore pressure and additional stress with depth under an instantaneous loading and a single-level uniform loading. After that, the average degree of consolidation defined by settlement of the special cases is verified and the effects of the depth-dependent initial excess pore pressure and additional stress as well as loading rate on the consolidation of the soft foundation with an upper crust under different drainage conditions are discussed. The final section summarizes the major findings of this study.

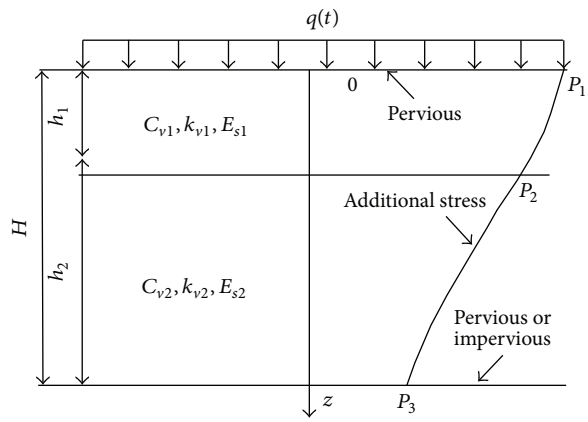


Figure 1: Consolidation problem considered.

2. Mathematical Modelling

A double-layered soil profile model is presented in Figure 1. The upper layer may be a natural or an artificial hard crust. The lower layer is a native soft soil layer. There are 2 kinds of drainage conditions in this model: (1) a single-drained condition, that is, only the top of upper layer is drained; (2) a double-drained condition, that is, both top and bottom of layers are freely drained. The soil properties of the ith layer are the coefficient of consolidation C_{vi}, the coefficient of permeability k_{vi}, and the compression modulus E_{si}. The compressible stratum has a total thickness of H. A time-dependent loading, $q(t)$, is applied on the foundation surface, as shown in Figure 2.

When the time-dependent loading, $q(t)$, as shown in Figure 2 is applied on the foundation surface, the resulting additional stress along the depth inside the foundation, $\sigma(z,t)$, can be expressed as follows:

$$\sigma(z,t) = [q(t) - q_0] K(z) + p(z),$$
$$\frac{\partial \sigma}{\partial t} = K(z) R(t), \tag{1}$$

where q_0 is the initial loading; t is duration and z is the vertical coordinate; $K(z)$ and $p(z)$ are the additional stress coefficient and initial excess pore pressure, respectively; $R(t)$ is the loading rate and $R(t) = dq/dt$.

For a 1D consolidation problem, the assumptions in Terzaghi's [19] consolidation theory are retained except for

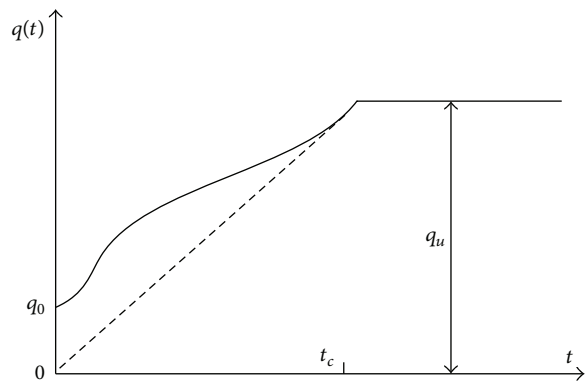


Figure 2: Time-dependent loading.

the depth-dependent initial pore pressure and additional stress and time-dependent loading. Then, let $L(z,t) = \partial\sigma/\partial t$; the partial differential equation for 1D consolidation of soils by vertical drainage is given as follows:

$$\frac{\partial u}{\partial t} = \begin{cases} C_{v1}\dfrac{\partial^2 u}{\partial z^2} + L(z,t) & 0 \le z \le h_1 \\ C_{v2}\dfrac{\partial^2 u}{\partial z^2} + L(z,t) & h_1 \le z \le h_2, \end{cases} \tag{2}$$

where u is the excess pore pressure.

Two commonly encountered drainage conditions are studied in this study:

$$\begin{cases} u(0,t) = 0 \\ u(H,t) = 0, \end{cases}$$
$$\begin{cases} u(0,t) = 0 \\ \left.\dfrac{\partial u}{\partial z}\right|_{z=H} = 0. \end{cases} \tag{3}$$

The former and the latter represent the drainage conditions (2) and (1) above, respectively.

The initial and continuity boundary conditions are given in the following forms:

$$u_i(z,0) = p_i(z) \tag{4}$$

$$u_1(h_1,t) = u_2(h_1,t),$$
$$k_{v1}\frac{\partial u_1(h_1,t)}{\partial z} = k_{v2}\frac{\partial u_2(h_1,t)}{\partial z}, \tag{5}$$

where u_i and $p_i(z)$ are the excess pore pressure and initial excess pore pressure in soil layer i at depth z, respectively ($i = 1,2$). Equation (4) is the initial condition. Equation (5) is the continuity condition of the two layers.

3. General Analytical Consolidation Solutions

The general consolidation solutions to consider depth-dependent initial excess pore pressure and additional stress, time-dependent loading, and different drainage conditions are deduced as follows.

3.1. Single-Drained Condition. The following dimensionless parameters were defined to simplify the expression for consolidation:

$$a = \frac{k_{v2}}{k_{v1}},$$
$$b = \frac{m_{v2}}{m_{v1}} = \frac{E_{s1}}{E_{s2}}, \tag{6}$$
$$c = \frac{h_2}{h_1},$$

where m_{vi} is the compression coefficient of the layers ($i = 1,2$).

The excess pore pressure, u_i, was described according to Terzaghi's 1D consolidation solution [20]:

$$u_i = \sum_{m=1}^{\infty} g_{mi}(z)\, e^{-\beta_m t}\left[B_m + C_m T_m(t)\right] \quad (i = 1,2), \tag{7}$$

where $g_{m1}(z) = \sin(\lambda_m z/h_1)$ and $g_{m2}(z) = A_m \cos(\mu\lambda_m(H - z)/h_1)$; β_m, μ, λ_m, A_m, B_m, and C_m are undetermined coefficients, which can be derived according to (2)~(4). According to (5),

$$\sum_{m=1}^{\infty} \sin(\lambda_m)\, e^{-\beta_m t}\left[B_m + C_m T_m(t)\right] = \sum_{m=1}^{\infty} A_m \cos(\mu c\lambda_m)\, e^{-\beta_m t}\left[B_m + C_m T_m(t)\right], \tag{8}$$

$$k_{v1}\sum_{m=1}^{\infty}\frac{\lambda_m}{h_1}\cos(\lambda_m)\, e^{-\beta_m t}\left[B_m + C_m T_m(t)\right] = k_{v2}\sum_{m=1}^{\infty} A_m \mu\frac{\lambda_m}{h_1}\sin(\mu c\lambda_m)\, e^{-\beta_m t}\left[B_m + C_m T_m(t)\right]. \tag{9}$$

According to (8), A_m can be gained:

$$A_m = \frac{\sin(\lambda_m)}{\cos(\mu c\lambda_m)}. \tag{10}$$

Substituting (10) into (9), (11) is derived:

$$\mu a \tan(\lambda_m)\tan(\mu c\lambda_m) = 1. \tag{11}$$

Substituting (7) into the consolidation equation (2), (12) and (13) can be gained:

$$\sum_{m=1}^{\infty}\sin\left(\lambda_m\frac{z}{h_1}\right)\left\{-\beta_m e^{-\beta_m t}\left[B_m + C_m T_m(t)\right] + e^{-\beta_m t} C_m T'_m(t)\right\} = c_{v1}\sum_{m=1}^{\infty}(-) \cdot \frac{\lambda_m^2}{h_1^2}\sin\left(\lambda_m\frac{z}{h_1}\right) e^{-\beta_m t}\left[B_m + C_m T_m(t)\right] + L(z,t), \tag{12}$$

$$\sum_{m=1}^{\infty} A_m \cos\left(\mu\lambda_m \frac{H-z}{h_1}\right)\left\{-\beta_m e^{-\beta_m t}\left[B_m + C_m T_m(t)\right]\right.$$
$$\left. + e^{-\beta_m t} C_m T'_m(t)\right\} = c_{v2}\sum_{m=1}^{\infty} A_m(-)$$
$$\cdot \frac{\mu^2\lambda_m^2}{h_1^2}\cos\left(\mu\lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t}\left[B_m + C_m T_m(t)\right]$$
$$+ L(z,t). \tag{13}$$

$T_m(t)$ is a time coefficient caused by $L(z,t)$. If $L(z,t) = 0$, $T_m(t) = 0$. So (12) is simplified:

$$\sin\left(\lambda_m \frac{z}{h_1}\right)(-)\beta_m e^{-\beta_m t} B_m$$
$$= c_{v1}(-)\frac{\lambda_m^2}{h_1^2}\sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\beta_m t} B_m. \tag{14}$$

Then, the coefficient β_m can be determined as follows:

$$\beta_m = \frac{C_{v1}\lambda_m^2}{h_1^2}. \tag{15}$$

According to similar principle, another β_m is determined based on (13):

$$\beta_m = \frac{C_{v2}\mu^2\lambda_m^2}{h_1^2}. \tag{16}$$

So the parameter μ is derived:

$$\mu = \sqrt{\frac{C_{v1}}{C_{v2}}} = \sqrt{\frac{b}{a}}. \tag{17}$$

Substituting (15) into (12) and (16) into (13), two equations are gained as follows:

$$\sum_{m=1}^{\infty} C_m \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\beta_m t} T'_m(t) = L(z,t);$$
$$\sum_{m=1}^{\infty} A_m C_m \cos\left(\mu\lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t} T'_m(t) = L(z,t). \tag{18}$$

The above two equations should satisfy

$$T_m = \int_0^t e^{\beta_m \tau} L_m(\tau)\, d\tau,$$
$$\sum_{m=1}^{\infty} C_m g_{m1}(z) = 1, \tag{19}$$
$$\sum_{m=1}^{\infty} A_m C_m g_{m2}(z) = 1.$$

Using the following orthogonality relationship:

$$\int_0^{h_1} m_{v1} g_{m1}(z)\cdot g_{n1}(z)\, dz$$
$$+ \int_{h_1}^{H} m_{v2} g_{m2}(z)\cdot g_{n2}(z)\, dz$$
$$= \begin{cases} 0 & (m \neq n) \\ \frac{1}{2} h_1 m_{v1}\left(1 + bcA_m^2\right) & (m = n), \end{cases} \tag{20}$$

the coefficient C_m is

$$C_m = \frac{2\left[\int_0^{h_1} K_1(z)\, g_{m1}(z)\, dz + b\int_{h_1}^{H} K_2(z)\, g_{m2}(z)\, dz\right]}{h_1\left(1 + bcA_m^2\right)}$$
$$= \frac{2\left[\int_0^{h_1} K_1(z)\sin\left(\lambda_m (z/h_1)\right) dz + b\int_{h_1}^{H} K_2(z)\left(\sin(\lambda_m)/\cos(\mu c\lambda_m)\right)\cos\left(\mu\lambda_m\left((H-z)/h_1\right)\right) dz\right]}{h_1\left(1 + bcA_m^2\right)}, \tag{21}$$

where $K_i(z)$ is the additional stress coefficient in the soil layers at depth z $(i = 1, 2)$.

According to the derivation processes of C_m and (13), the coefficient B_m can be given as follows:

$$B_m = \frac{2\left[\int_0^{h_1} p_1(z)\, g_{m1}(z)\, dz + b\int_{h_1}^{H} p_2(z)\, g_{m2}(z)\, dz\right]}{h_1\left(1 + bcA_m^2\right)}$$
$$= \frac{2\left[\int_0^{h_1} p_1(z)\sin\left(\lambda_m (z/h_1)\right) dz + b\int_{h_1}^{H} p_2(z)\left(\sin(\lambda_m)/\cos(\mu c\lambda_m)\right)\cos\left(\mu\lambda_m\left((H-z)/h_1\right)\right) dz\right]}{h_1\left(1 + bcA_m^2\right)}, \tag{22}$$

where λ_m is determined by eigen-equation $\mu a \tan(\lambda_m)\tan(\mu c\lambda_m) = 1$.

Comparing (21) with (22), it is found that the expressions of B_m and C_m are similar. However, $K_i(z)$ of (21) and $p_i(z)$ of (22) represent the additional stresses coefficient and initial excess pore pressure in soil layer i at depth z $(i = 1, 2)$.

So, for the consolidation problem of a double-layered foundation with the pervious top surface of the upper layer, the 1D consolidation solution is gained as follows:

$$\begin{aligned} u_1 &= \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) \\ &\quad \cdot e^{-\beta_m t}\left(B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau)\, d\tau\right), \\ u_2 &= \sum_{m=1}^{\infty} \frac{\sin(\lambda_m)}{\cos(\mu c\lambda_m)} \cos\left(\mu\lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t}\left(B_m \right. \\ &\quad \left. + C_m \int_0^t e^{\beta_m \tau} R(\tau)\, d\tau\right). \end{aligned} \tag{23}$$

3.2. Double-Drained Condition. According to the similar principle, for the consolidation problem of a double-layered foundation with a double-drained condition, the consolidation solution is

$$\begin{aligned} u_1 &= \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) \\ &\quad \cdot e^{-\beta_m t}\left(B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau)\, d\tau\right), \\ u_2 &= \sum_{m=1}^{\infty} \frac{\sin(\lambda_m)}{\cos(\mu c\lambda_m)} \sin\left(\mu\lambda_m \frac{H-z}{h_1}\right) \\ &\quad \cdot e^{-\beta_m t}\left(B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau)\, d\tau\right), \end{aligned} \tag{24}$$

where β_m and μ are similar to (15)~(17); λ_m is determined by $\sqrt{ab}\tan(\lambda_m)\cdot\cot(\mu c\lambda_m) = -1$;

$$\begin{aligned} A_m &= \frac{\sin(\lambda_m)}{\sin(\mu c\lambda_m)}; \\ B_m &= \frac{2\left[\int_0^{h_1} p_1(z)\sin(\lambda_m(z/h_1))\, dz + b\int_{h_1}^{H} p_2(z) A_m \sin(\mu\lambda_m((H-z)/h_1))\, dz\right]}{h_1(1+bcA_m^2)}; \\ C_m &= \frac{2\left[\int_0^{h_1} K_1(z)\sin(\lambda_m(z/h_1))\, dz + b\int_{h_1}^{H} K_2(z) A_m \sin(\mu\lambda_m((H-z)/h_1))\, dz\right]}{h_1(1+bcA_m^2)}. \end{aligned} \tag{25}$$

4. Special Cases

Based on the above solutions, results for two kinds of loading conditions, (1) instantaneous loading and (2) single-level uniform loading, are given. The bilinear distributions with depth of initial excess pore pressure and additional stress are considered.

4.1. Solution with an Instantaneous Loading. For the instantaneous loading, $R_t = 0$. So, (23)~(24) can be simplified as follows:

$$\begin{aligned} u_1 &= \sum_{m=1}^{\infty} B_m \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\beta_m t} \quad \text{(Single or double-drained condition)}, \\ u_2 &= \begin{cases} \displaystyle\sum_{m=1}^{\infty} B_m \frac{\sin(\lambda_m)}{\cos(\mu c\lambda_m)} \sin\left(\mu\lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t} & \text{(Single-drained condition)} \\ \displaystyle\sum_{m=1}^{\infty} B_m \frac{\sin(\lambda_m)}{\sin(\mu c\lambda_m)} \sin\left(\mu\lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t} & \text{(Double-drained condition)}. \end{cases} \end{aligned} \tag{26}$$

Assume the initial excess pore pressure has a bilinear distribution with depth; that is,

$$\begin{aligned} p_1(z) &= \frac{p_1}{\varsigma}\left[1 + (\varsigma - 1)\frac{h_1 - z}{h_1}\right], \quad \varsigma = \frac{p_1}{p_2}; \\ p_2(z) &= \frac{p_1}{\varsigma\xi h_2}\left[\xi H + (1-\xi)z - h_1\right], \quad \xi = \frac{p_2}{p_3}, \end{aligned} \tag{27}$$

where p_1, p_2, and p_3 are the initial excess pore pressure at the top of the upper layer, interface between the upper and lower layers, and bottom of the lower layer, respectively.

Substituting (27) into (22) and (25), the expression of B_m is determined.

For the single-drained condition,

$$B_m = \frac{2\left[\int_0^{h_1} p_1(z)\, g_{m1}(z)\, dz + b\int_{h_1}^{H} p_2(z)\, g_{m2}(z)\, dz\right]}{h_1\left(1+bcA_m^2\right)} = \frac{2p_1\left\{\int_0^{h_1}\left[1/\varsigma + (1-1/\varsigma)\left((h_1-z)/h_1\right)\right]\sin\left(\lambda_m\left(z/h_1\right)\right)dz + b\int_{h_1}^{H}\left(\left[\xi H + (1-\xi)z - h_1\right]/\varsigma\xi h_2\right)\left(\sin\left(\lambda_m\right)/\cos\left(\mu c\lambda_m\right)\right)\cos\left(\mu\lambda_m\left((H-z)/h_1\right)\right)dz\right\}}{h_1\left[1+bc\left(\sin\left(\lambda_m\right)/\cos\left(\mu c\lambda_m\right)\right)^2\right]}. \tag{28}$$

For the double-drained condition,

$$B_m = \frac{2\left[\int_0^{h_1} p_1(z)\, g_{m1}(z)\, dz + b\int_{h_1}^{H} p_2(z)\, g_{m2}(z)\, dz\right]}{h_1\left(1+bcA_m^2\right)} = \frac{2p_1\left\{\int_0^{h_1}\left[1/\varsigma + (1-1/\varsigma)\left((h_1-z)/h_1\right)\right]\sin\left(\lambda_m\left(z/h_1\right)\right)dz + b\int_{h_1}^{H}\left(\left[\xi H + (1-\xi)z - h_1\right]/\varsigma\xi h_2\right)\left(\sin\left(\lambda_m\right)/\sin\left(\mu c\lambda_m\right)\right)\sin\left(\mu\lambda_m\left((H-z)/h_1\right)\right)dz\right\}}{h_1\left[1+bc\left(\sin\left(\lambda_m\right)/\sin\left(\mu c\lambda_m\right)\right)^2\right]}. \tag{29}$$

Then, the average excess pore pressure inside the upper and lower soils is derived as follows:

$$\overline{u_1} = \frac{\int_0^{h_1} u_1(z)\, dz}{h_1} = \sum_{m=1}^{\infty}\frac{B_m}{\lambda_m}\left(1-\cos\left(\lambda_m\right)\right)\exp\left(-\beta_m t\right), \tag{30a}$$

$$\overline{u_2} = \frac{\int_{h_1}^{H} u_2(z)\, dz}{h_2} = \begin{cases} \displaystyle\sum_{m=1}^{\infty}\frac{B_m}{bc\lambda_m}\cos\left(\lambda_m\right)\exp\left(-\beta_m t\right) & \text{(Single-drained condition)} \\ \displaystyle\sum_{m=1}^{\infty}\frac{B_m}{\mu c\lambda_m}\left[\frac{\sin\left(\lambda_m\right)}{\sin\left(\mu c\lambda_m\right)} + \frac{\cos\left(\lambda_m\right)}{\sqrt{ab}}\right]\exp\left(-\beta_m t\right) & \text{(Double-drained condition)}. \end{cases} \tag{30b}$$

So, the average consolidation degree inside the foundation defined by settlement is

$$U_s = \frac{S_t}{S_\infty} = \frac{\left(h_1/E_{s1}\right)\left(\overline{p_1}-\overline{u_1}\right) + \left(h_2/E_{s2}\right)\left(\overline{p_2}-\overline{u_2}\right)}{h_1\overline{p_1}/E_{s1} + h_2\overline{p_2}/E_{s2}} = \frac{\overline{p_1}-\overline{u_1} + bc\left(\overline{p_2}-\overline{u_2}\right)}{\overline{p_1}+bc\overline{p_2}} = 1 - \frac{\overline{u_1}+bc\overline{u_2}}{\overline{p_1}+bc\overline{p_2}}$$
$$= \begin{cases} 1 - \dfrac{B_m}{\lambda_m\left(\overline{p_1}+bc\overline{p_2}\right)}\exp\left(-\beta_m t\right) & \text{(Single-drained condition)} \\ 1 - \dfrac{B_m\left(\sin\left(\mu c\lambda_m\right) + \sqrt{ab}\sin\left(\lambda_m\right)\right)}{\lambda_m\left(\overline{p_1}+bc\overline{p_2}\right)\sin\left(\mu c\lambda_m\right)}\exp\left(-\beta_m t\right) & \text{(Double-drained condition)}, \end{cases} \tag{31}$$

where S_t is the compression of the two soil layers at time t and S_∞ is the final compression when the excess pore pressure is zero; $\overline{p_i}$ is the average initial excess pore pressure in the layers ($i = 1, 2$). From (31), it is evident that U_s only relates to ζ and ξ when the parameters of a, b, and c are constant.

4.2. Solution with a Single-Level Uniform Loading. If a single-level uniform loading (dash line in Figure 2) is applied on the foundation surface, the following relationships can be gained:

$$q(t) = \begin{cases} \dfrac{q_u t}{t_c} & (0 < t \le t_c) \\ q_u & (t \ge t_c); \end{cases}$$

$$R(t) = \begin{cases} \dfrac{q_u}{t_c} & (0 < t \le t_c) \\ 0 & (t \ge t_c), \end{cases} \tag{32}$$

where t_c is the time when the loading becomes a constant value, q_u.

When the self-weight consolidation is not considered, the initial excess pore pressure inside the foundation is equal to zero; that is, $p_i(z) = 0$; then $B_m = 0$. So, the consolidation solutions of the double-layered foundation with the depth-dependent additional stress are determined as follows.

When $0 < t \le t_c$,

$$u_1 = q_u \sum_{m=1}^{\infty} \frac{C_m}{\lambda_m^2 T_c} \sin\left(\lambda_m \frac{z}{h_1}\right)\left(1 - e^{-\lambda_m^2 T_v}\right) \quad \text{(Single or double-drained condition)},$$

$$u_2 = \begin{cases} q_u \displaystyle\sum_{m=1}^{\infty} \frac{C_m \sin(\lambda_m)}{\lambda_m^2 T_c \cos(\mu c \lambda_m)} \cos\left(\mu \lambda_m \frac{H - z}{h_1}\right)\left(1 - e^{-\lambda_m^2 T_v}\right) & \text{(Single-drained condition)} \\ q_u \displaystyle\sum_{m=1}^{\infty} \frac{C_m \sin(\lambda_m)}{\lambda_m^2 T_c \sin(\mu c \lambda_m)} \sin\left(\mu \lambda_m \frac{H - z}{h_1}\right)\left(1 - e^{-\lambda_m^2 T_v}\right) & \text{(Double-drained condition)}. \end{cases} \tag{33}$$

When $t \ge t_c$,

$$u_1 = q_u \sum_{m=1}^{\infty} \frac{C_m}{\lambda_m^2 T_c} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_v}\left(e^{\lambda_m^2 T_c} - 1\right) \quad \text{(Single or double-drained condition)},$$

$$u_2 = \begin{cases} q_u \displaystyle\sum_{m=1}^{\infty} \frac{C_m \sin(\lambda_m)}{\lambda_m^2 T_c \cos(\mu c \lambda_m)} \cos\left(\mu \lambda_m \frac{H - z}{h_1}\right) e^{-\lambda_m^2 T_v}\left(e^{\lambda_m^2 T_c} - 1\right) & \text{(Single-drained condition)} \\ q_u \displaystyle\sum_{m=1}^{\infty} \frac{C_m \sin(\lambda_m)}{\lambda_m^2 T_c \sin(\mu c \lambda_m)} \sin\left(\mu \lambda_m \frac{H - z}{h_1}\right) e^{-\lambda_m^2 T_v}\left(e^{\lambda_m^2 T_c} - 1\right) & \text{(Double-drained condition)}, \end{cases} \tag{34}$$

where $T_c = C_{v1} t_c / h_1^2$; $T_v = C_{v1} t / h_1^2$.

According to the research results by Zhang et al. [20], the distribution of the depth-dependent additional stress can be simplified to a bilinear line; that is,

$$K_1(z) = \frac{1}{\psi} + (\psi - 1)\frac{h_1 - z}{h_1 \psi}, \quad \psi = \frac{P_1}{P_2};$$
$$K_2(z) = \frac{[\varphi H + (1 - \varphi) z - h_1]}{\psi \varphi h_2}, \quad \varphi = \frac{P_2}{P_3}, \tag{35}$$

where P_1, P_2, and P_3 are the additional stress at the top of the upper layer, the interface between the upper and lower layers, and the bottom of the lower layer, respectively, as shown in Figure 1.

Substituting (35) into (21) and (25), the consolidation solutions with a single-level loading are determined as follows.

For the single-drained condition,

$$C_m = \frac{2\left\{\int_0^{h_1} [1/\psi + (1 - 1/\psi)((h_1 - z)/h_1)] \sin(\lambda_m (z/h_1))\, dz + b \int_{h_1}^{H} ([\varphi H + (1 - \varphi) z - h_1]/\psi \varphi h_2)(\sin(\lambda_m)/\cos(\mu c \lambda_m)) \cos(\mu \lambda_m ((H - z)/h_1))\, dz\right\}}{h_1 \left[1 + bc (\sin(\lambda_m)/\cos(\mu c \lambda_m))^2\right]}. \tag{36}$$

For the double-drained condition,

$$C_m = \frac{2\left\{\int_0^{h_1}\left[1/\psi+(1-1/\psi)\left((h_1-z)/h_1\right)\right]\sin\left(\lambda_m\left(z/h_1\right)\right)dz + b\int_{h_1}^{H}\left(\left[\varphi H+(1-\varphi)z-h_1\right]/\psi\varphi h_2\right)\left(\sin\left(\lambda_m\right)/\sin\left(\mu c\lambda_m\right)\right)\sin\left(\mu\lambda_m\left((H-z)/h_1\right)\right)dz\right\}}{h_1\left[1+bc\left(\sin\left(\lambda_m\right)/\sin\left(\mu c\lambda_m\right)\right)^2\right]}. \quad (37)$$

So, the average degree of consolidation inside double-layered foundation defined by settlement is

$$U_s = \frac{S_t}{S_\infty} = \frac{\left(h_1/E_{s1}\right)\left[q(t)\overline{K_1}-\overline{u_1}\right]+\left(h_2/E_{s2}\right)\left[q(t)\overline{K_2}-\overline{u_2}\right]}{h_1 q_u\overline{K_1}/E_{s1}+h_2 q_u\overline{K_2}/E_{s2}} = \frac{q(t)}{q_u} - \frac{\overline{u_1}+bc\overline{u_2}}{q_u\left(\overline{K_1}+bc\overline{K_2}\right)}$$

$$= \begin{cases} \begin{cases} \dfrac{T_v}{T_c} - \displaystyle\sum_{m=1}^{\infty}\dfrac{C_m}{\lambda_m^3 T_c\left(\overline{K_1}+bc\overline{K_2}\right)}\left(1-e^{-\lambda_m^2 T_v}\right) & t_c \geq t \geq 0 \\ 1-\displaystyle\sum_{m=1}^{\infty}\dfrac{C_m}{\lambda_m^3 T_c\left(\overline{K_1}+bc\overline{K_2}\right)}e^{-\lambda_m^2 T_v}\left(e^{\lambda_m^2 T_c}-1\right) & t \geq t_c \end{cases} & \text{(Single-drained condition)} \\ \begin{cases} \dfrac{T_v}{T_c} - \dfrac{C_m\left(\sin\left(\mu c\lambda_m\right)+\sqrt{ab}\sin\left(\lambda_m\right)\right)}{\lambda_m^3 T_c\left(\overline{K_1}+bc\overline{K_2}\right)\sin\left(\mu c\lambda_m\right)}\left(1-e^{-\lambda_m^2 T_v}\right) & t_c \geq t \geq 0 \\ 1-\displaystyle\sum_{m=1}^{\infty}\dfrac{C_m\left(\sin\left(\mu c\lambda_m\right)+\sqrt{ab}\sin\left(\lambda_m\right)\right)}{\lambda_m^3 T_c\left(\overline{K_1}+bc\overline{K_2}\right)\sin\left(\mu c\lambda_m\right)}e^{-\lambda_m^2 T_v}\left(e^{\lambda_m^2 T_c}-1\right) & t \geq t_c \end{cases} & \text{(Double-drained condition)}, \end{cases} \quad (38)$$

where $\overline{K_i}$ is the average coefficient of additional stress in the layers, respectively ($i = 1, 2$).

5. Consolidation Behaviour of the Soft Foundation with an Upper Crust

5.1. Model Verification. A calculation program was developed to analyze the consolidation of the soft foundation with an upper curst in terms of (26)~(38). In order to verify the consolidation model above, the degree of consolidation of a double-layered foundation with a uniform additional stress with depth under a single-level uniform loading was calculated as shown in Figure 3; that is, $\psi = \varphi = 1$, $a = 1$, $b = 5$, $c = 1$, and $T_c = C_{v1}t_c/h_1^2 = 0.2, 1, 2, 5$, and 10. In Figure 3, SDC and DDC denote the single-drained condition and double-drained condition. It is obvious from Figure 3 that the consolidation rate relates to the loading rates (the values of T_c) and drainage conditions, which is larger with the increasing loading rate (i.e., with a smaller value of T_c) and under the double-drained condition. Moreover, the degree of consolidation of a double-layered foundation with a uniform initial excess pore pressure with depth under an instantaneous loading was also obtained, as shown in Figure 4. For this calculation, $\zeta = \xi = 1$, $T_c = 0$, $c = 1$, $b = 0.1, 2, 5$, and 10, $a = 10$ for the single drainage condition, and $a = 1$ for the double drainage condition. It can be seen from Figure 4 that the degree of consolidation relates to both the ratios of the modulus of the upper and lower layers (the values of b) and the drainage conditions. The consolidation rate becomes smaller with the increasing b, which is larger under the double-drained condition than that under the single-drained condition. In addition, compared with the analytical results of Xie et al. [21] and Xie et al. [22], it can be found that the consolidation curves under the single and double drainage conditions in Figures 3 and 4 are perfectly similar to Figures 13 and 3 from Xie et al. [21] and Figures 9 and 2 from Xie et al. [22], respectively.

In addition, Pyrah [13] discussed the influence of soil parameters on the consolidation behaviour of a double-layered system through four idealized soil profiles using the finite element method. Zhu and Yin [5] gained the analytical solutions of the first three soil profiles and found that they are similar to the results of Pyrah [13]. In order to further verify the consolidation model in this study, the degree of consolidation of the first three soil profiles (namely, $a = b = 10, 0.1, 1$) was calculated with a uniform additional stress ($P_1 : P_2 : P_3 = 1:1:1$). The result with a depth-dependent additional stress ($P_1 : P_2 : P_3 = 1:0.4:0$), which decreases along the depth, also was gained to investigate the effects of the distribution of the additional stress. For all the calculations, $c = 1$ and $T_c = 0$. Figure 5 presents the calculation results with the corresponding curves of Zhu and Yin [5]. $C1$ and $C3$ denote the consolidation curves with the uniform and depth-dependent additional stress. $C2$ denotes

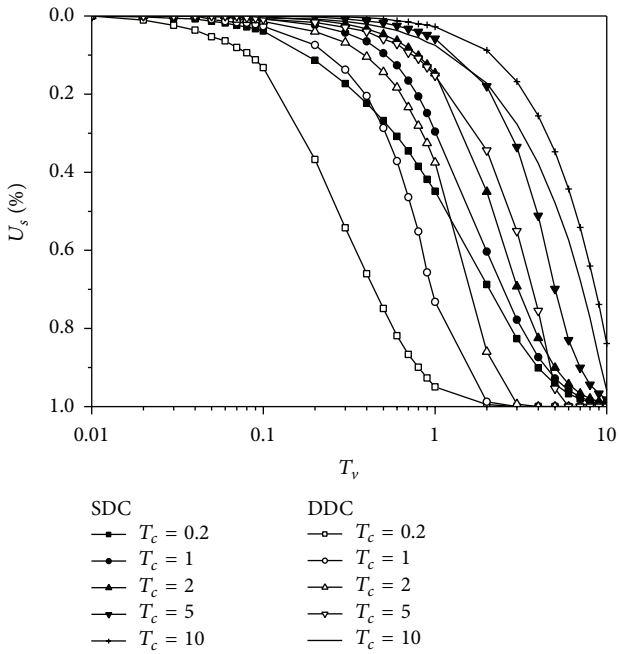


Figure 3: Consolidation curves with a uniform vertical additional stress.

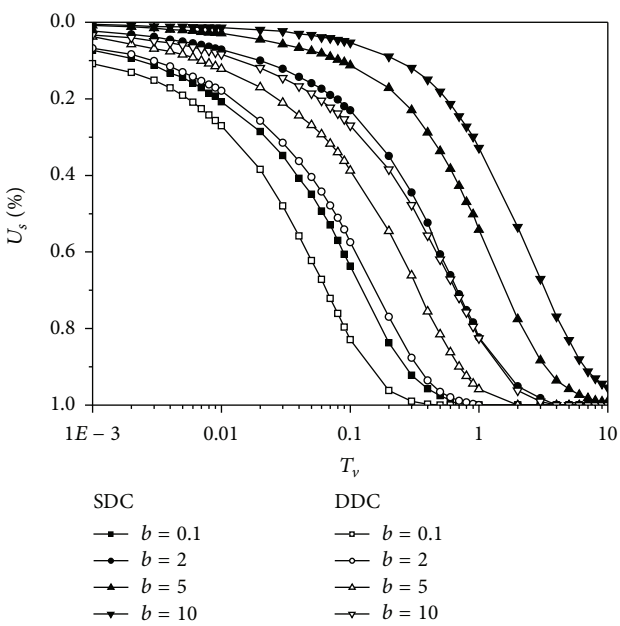


Figure 4: Consolidation curves with a uniform vertical additional stress initial excess pore pressure.

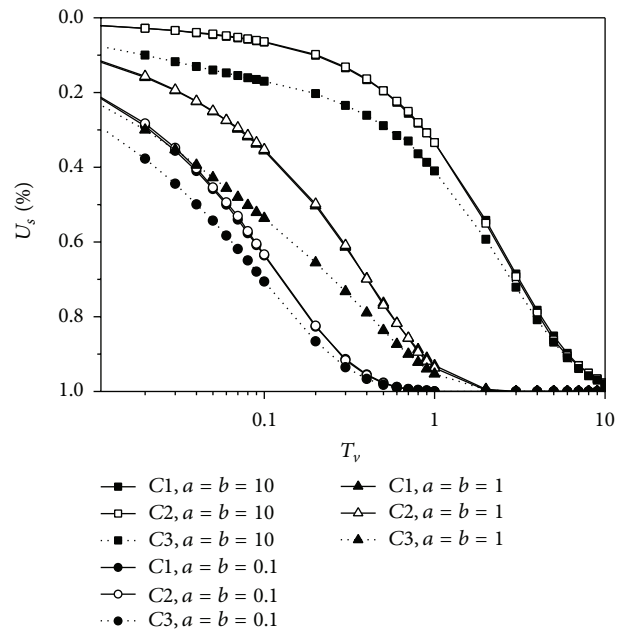


Figure 5: Degree of consolidation for different soil profiles.

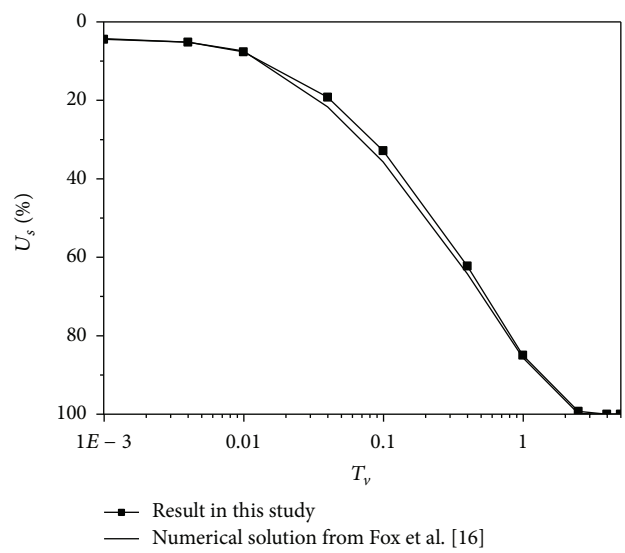
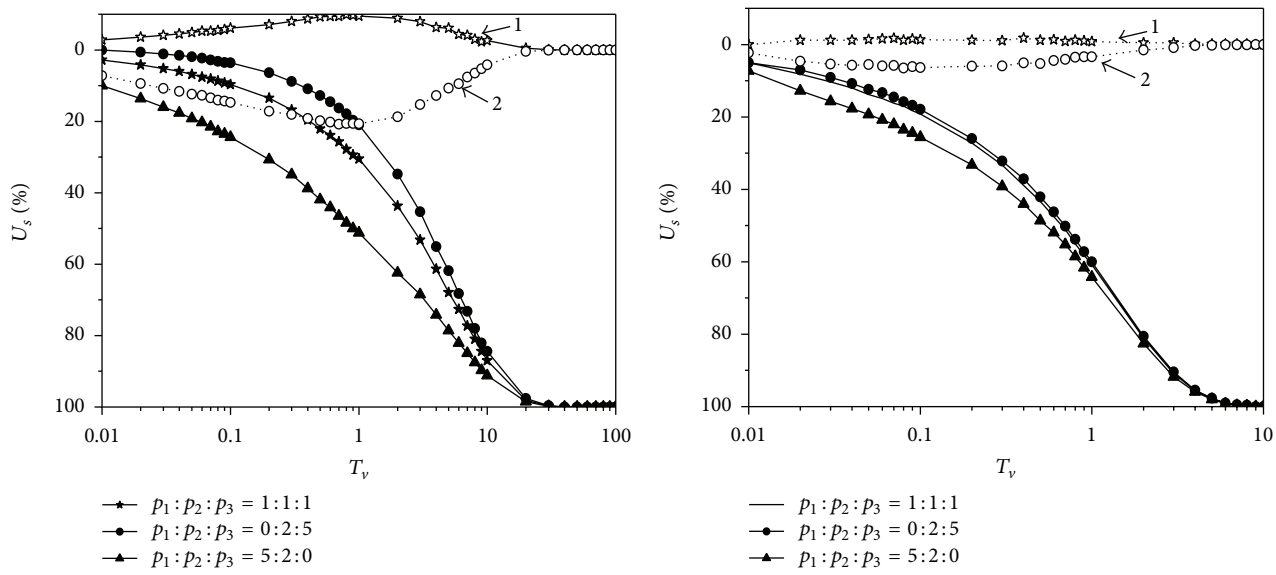


Figure 6: Degree of consolidation for a single layer foundation.

the curves of (1), (2), and (3) with T_c= 0 in Figure 10 from Zhu and Yin [5]. It is evident from Figure 5 that $C1$ curves are similar to $C2$ curves, which indicates that the results in this study agree well with the analytical solutions of Zhu and Yin [5] and the finite element results of Pyrah [13]. Meanwhile, the decreasing additional stress with depth ($P_1 : P_2 : P_3$ = 1 : 0.4 : 0) quickens the consolidation process of the double-layered foundation.

For the last comparison, the degree of consolidation was calculated for the single layer defined by Case 1 in Example 3 from literature [16], as shown in Figure 6. The analytical solution was gained by the model in this study and the numerical solution was calculated by the curve with the bilinear additional stress in Figure 12 and the parameters in Table 4 from literature [16]. It can be concluded from Figure 6 that the analytical solution matches the numerical results well except a minor difference at the middle stage.

Therefore, the consolidation model in this study is rational by several comparisons above with other analytical and numerical results, which can be further used to analyze the effects of distributions of initial excess pore pressure and



(a) Curves with a single-drained condition

(b) Curves with a double-drained condition

Figure 7: Consolidation curves with different drainage conditions.

depth-dependent additional stress on the consolidation of the double-layered foundation.

5.2. Effect of Distributions of Initial Excess Pore Pressure. For the soft foundation with an upper crust, the upper layer always forms due to the soil sedimentation or some long-term engineering practices; it has the same ingredient and smaller compressibility. Then, a will be smaller than 1 while b will be larger than 1. So, assume $a = 0.2$, $b = 5$, and $c = 2$ in the following analyses. In order to investigate the effect of the initial pore pressure on the consolidation of the double-layered foundation, three distributions of the initial pore pressure with depth were selected, namely, $p_1 : p_2 : p_3 = 1:1:1$, $0:2:5$, and $5:2:0$, which represent the uniform, gradually increasing, and decreasing distributions with depth, respectively.

Figure 7 shows the curves of degree of consolidation under different drainage conditions. It is evident from Figure 7 that the initial excess pore pressure distributions have a similar influence on the consolidation of the double-layered foundation under different drainage conditions. Comparing with the consolidation curve of a uniform initial excess pore pressure with depth ($p_1 : p_2 : p_3 = 1:1:1$), an increasing initial excess pore pressure with depth ($p_1 : p_2 : p_3 = 0:2:5$) slows down the consolidation process while the consolidation rate increases with a decreasing initial excess pore pressure with depth ($p_1 : p_2 : p_3 = 5:2:0$). At the same time, the maximum differences of degree of consolidation between the uniform and increasing and decreasing initial excess pore pressure with depth under the single-drained condition (Curve 1 and Curve 2 in Figure 7(a)) are −9.9% and 20.8%, respectively, while the corresponding differences under the double-drained condition (Curve 1 and Curve 2 in Figure 7(b)) are −1.8% and 6.5%. Therefore, the initial excess pore pressure has a greater influence on the consolidation rate under the single-drained condition than that with the double-drained condition. For a practical project, the variation of initial excess pore pressure with depth should be considered, especially for the project with the single-drained condition.

5.3. Effect of Depth-Dependent Additional Stress. Typically, P_3 is not equal to zero and 10% of the gravity stress is always used in settlement calculations. In order to compare the effect of different additional stress, two kinds of distributions were selected, namely, $P_1 : P_2 : P_3 = 1:1:1$ and $1:0.4:0$. In order to investigate the effect of loading rate on the consolidation of the soft foundation with an upper crust, different values of t_c (i.e., T_c) were used.

The consolidation curves of different additional stress with a pervious top of the upper layer are shown in Figure 8. It is evident from Figure 8 that the additional stress distributions have a significant influence on the consolidation of the double-layered foundation. The consolidation with the decreasing additional stress with depth ($P_1 : P_2 : P_3 = 1:0.4:0$) is quicker than that with the uniform additional stress ($P_1 : P_2 : P_3 = 1:1:1$) for different T_c, especially at the middle stage. When $T_c = 0.1, 0.5, 1$, and 5, the maximum differences of degree of consolidation, which appear nearly at the middle stage, are 21%, 21%, 20%, and 17%, respectively. They are large enough to be a crucial factor to the success or failure of a road or building project. In addition, the differences of the degree of consolidation for different T_c indicate that the loading rate also affects the consolidation process. The larger the loading rate is (i.e., with a smaller value of T_c), the greater the effect of the additional stress on the consolidation of the soft foundation with an upper crust is and the quicker the consolidation process is.

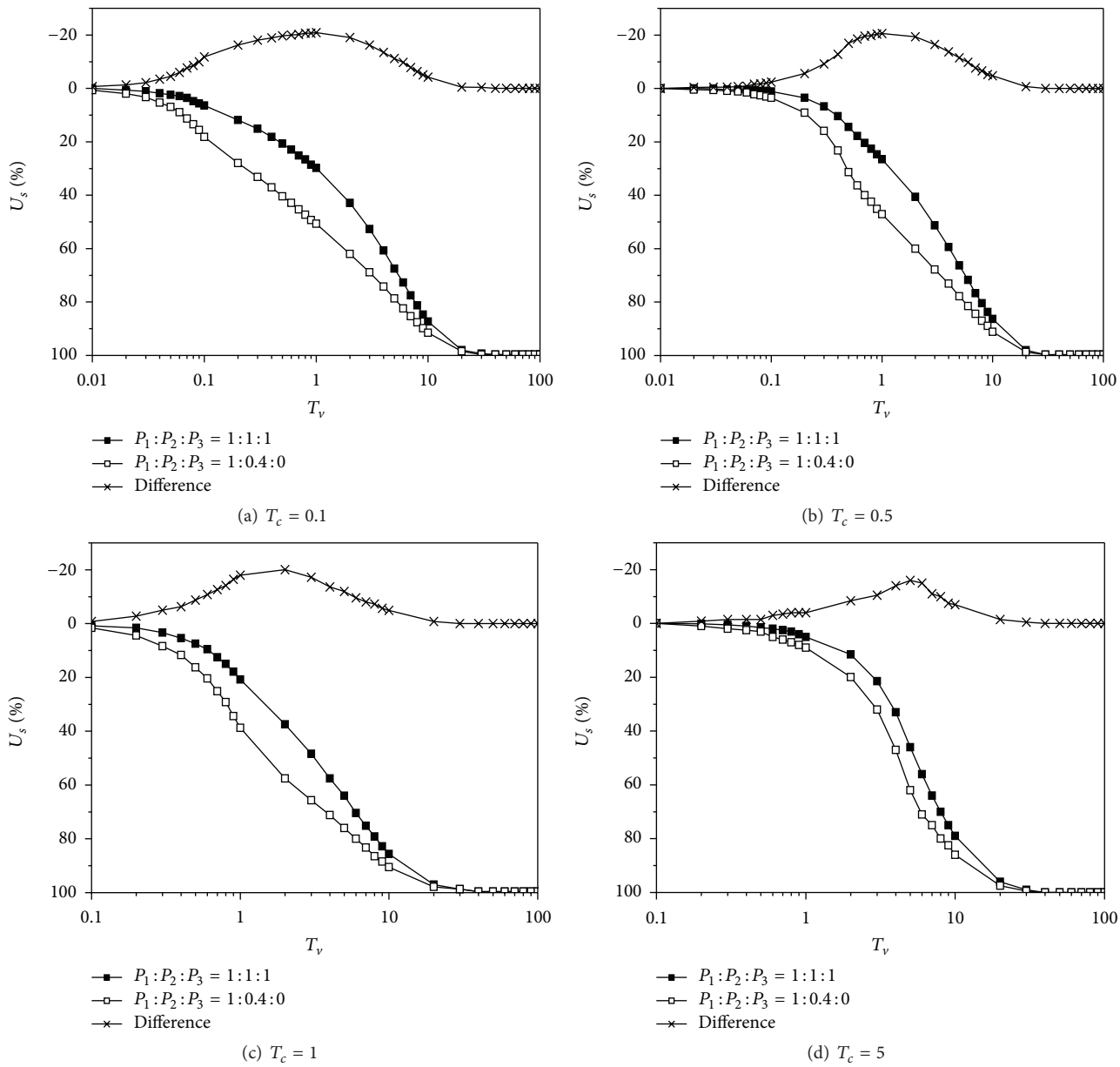


Figure 8: Consolidation curves with a single-drained condition.

Figure 9 gives the consolidation curves of different additional stresses with the double-drained conditions. It is clear from Figure 9 that the additional stress also affects the consolidation of the soft foundation with an upper crust under a double-drained condition and the maximum differences of the consolidation degree are 6%, 6%, 5%, and 2% for different T_c. Comparing with the curves in Figure 8, it indicates that the effect of additional stress on the consolidation degree with the single-drained condition is much greater than that with the double-drained condition. At the same time, the effect of additional stress on the degree of consolidation with a double-drained condition also gradually weakens with the increase of T_c.

6. Conclusions

(1) The 1D consolidation model of the double-layered foundation proposed in this study can account for the depth-dependent initial excess pore pressure and additional stress and time-dependent loading under different drainage conditions, which can be utilized to comprehensively investigate the consolidation behavior of the double-layered foundation. The general solutions of the model under different drainage conditions were gained.

(2) The consolidation solutions of special cases, which consider two loading modes, that is, instantaneous

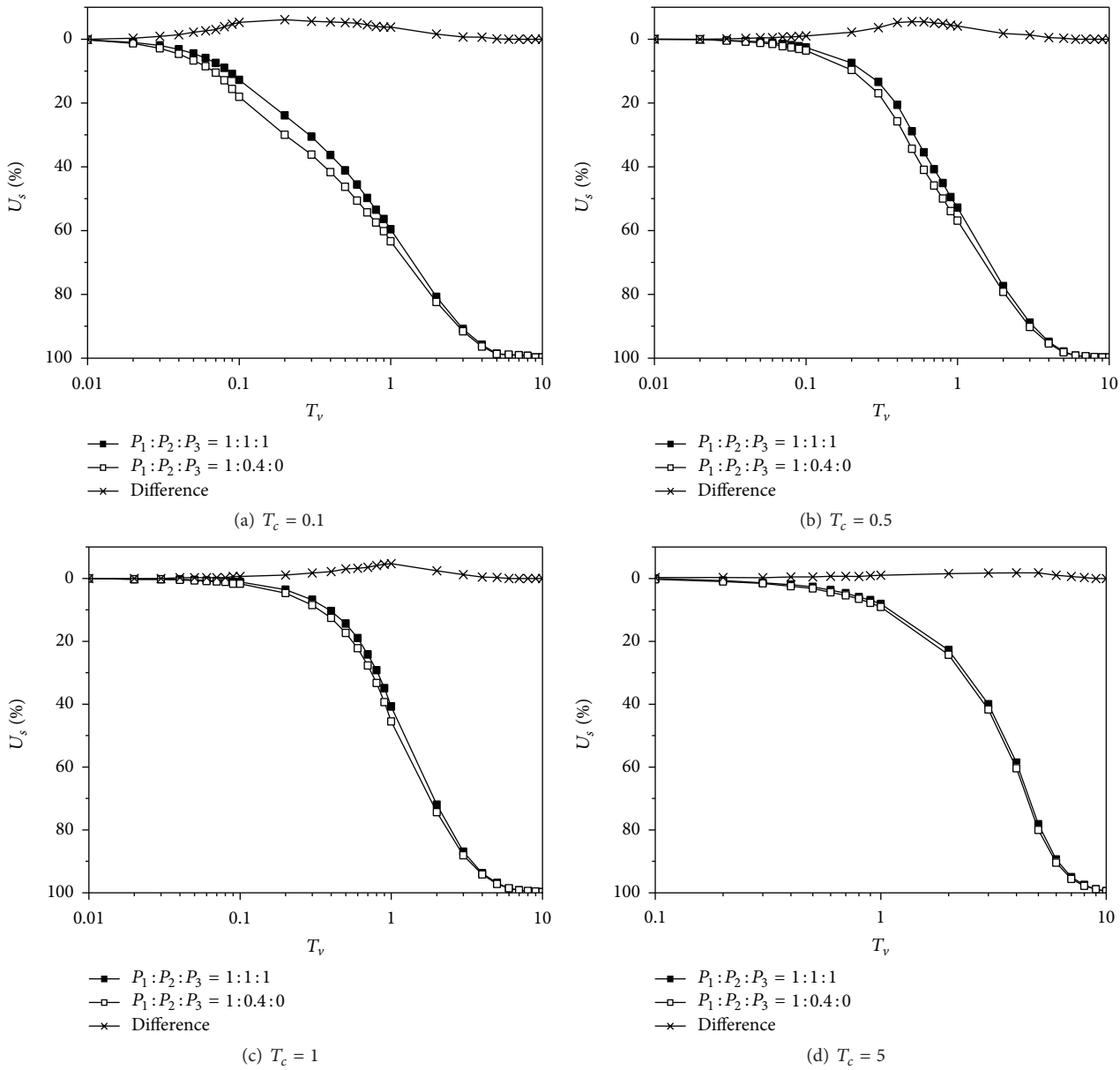


Figure 9: Consolidation curves with a double-drained condition.

loading and single-level uniform loading, and the bilinear distributions with depth of the initial excess pore pressure and additional stress, were derived. Then the average degree of consolidation defined by settlement was deduced and verified.

(3) Regardless of the drainage conditions, both the distribution of initial excess pore pressure and additional stress with depth and the loading rate have a great influence on the consolidation process of the soft foundation with an upper crust. This influence is larger with the single-drained condition than that with the double-drained condition. With the decrease of the initial excess pore pressure or the additional stress with depth, the consolidation rate increases. The larger the loading rate is, the quicker the consolidation process of the soft foundation with an upper crust is.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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