Research Article

Stability of Axially Moving Piezolaminated Viscoelastic Plate Subjected to Follower Force

Yan Wang,¹ Tao Jing,² Jimei Wu,² and Min Xie¹

¹ School of Civil Engineering and Architecture, Xi'an University of Technology, Xi'an 710048, China

² Institute of Printing and Packing Engineering, Xi'an University of Technology, Xi'an 710048, China

Correspondence should be addressed to Yan Wang; wyaiah@163.com

Received 8 August 2014; Accepted 15 September 2014

Academic Editor: Yan Yang

Copyright © 2015 Yan Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The stability of the moving viscoelastic plate with the piezoelectric layer subjected to uniformly distributed tangential follower force is investigated. The force excited by the piezoelectric layer due to external voltage is modeled as the follower tensile force. The differential equation of the axially moving viscoelastic rectangular plate with piezoelectric layer subjected to uniformly distributed tangential follower force is formulated on the basis of the Kirchhoff thin plate theory and the two-dimensional viscoelastic differential constitutive relation. The complex eigenvalue equations are established by the differential quadrature method. Via numerical calculation, the curves of real parts and imaginary parts of dimensionless complex frequencies versus uniformly distributed tangential follower force and dimensionless moving speed are obtained. The effects of nonconservative force, dimensionless axially moving speed, and dimensionless applied voltages on the stability of axially moving nonconservative viscoelastic plate with piezoelectric layer are analyzed.

1. Introduction

Piezoelectric materials are used in the smart structure as sensors and/or actuators in the form of patches or layers. A plate with distributed, surface-bonded piezoelectric sheets has been widely used in active vibration, in structural health monitoring, in acoustic control, in buckling control, and among other applications. The behavior of laminated piezoelectric plates has gained much attention by researchers [1–3]. For example, numerous scholars have studied the piezoelectric effects on the dynamic behavior of composite structures. The influence of using smart materials on the free vibration and natural frequencies of laminated piezoelectric plates has also been addressed [4–8]. Structural systems lose their stability due to divergence, and such systems under partial follower compressive loading were investigated both qualitatively and quantitatively [9]. Chandrashekhara and Bhatia [10] investigated the active buckling control of smart composite plates based on finite element analysis. In [11], both cross force and voltage, together with the bending of

the plate, were analyzed, and the bending of the plate was controlled through the application of different voltages to the actuators. In accordance with the inverse piezoelectric effect, the equivalent action of the actuators was obtained. Using Hamilton's principle, the finite element formula of the bending deformation of the piezoelectric structure was derived. The bending of the annular plate made of piezoelectric material with four boundary conditions was also calculated. Chase and Bhashayam [12] used the piezoelectric sheet to increase the critical load of laminated beams and performed optimal stabilization on plate buckling. Wang [13] and Wang and Quek [14] studied the buckling of column structures with a pair of piezoelectric layers and improve the flutter and buckling capacity of the column using the piezoelectric layers. Ha et al. [15] examined a composite structure containing distributed piezoceramic sensors and actuators using finite element analysis.

Stability and transverse vibrations of axially moving systems are present in various industrial applications, such as the paper webs and plastic sheets, steel strip in a thin

steel sheet production line, band saw blade, conveyor belts and chain in power transmission lines, and aerial cableways. The axially moving considerations have been studied widely, and dynamics and stability have been reviewed by Wickert and Mote [16]. The research on two-dimensional axially moving plate can be dated back to Ulsoy and Mote Jr. [17], who analyzed the coupled transverse and torsional vibration of band saw blade. The stability and vibration characteristics of an axially moving plate with two simple supported edges and two free edges under homogeneous tension have been investigated by Lin [18, 19]. A few researches on transverse vibrations and stability of axially moving viscoelastic plates have also been done. Zhou et al. [20, 21] studied transverse vibration characteristics of axially moving viscoelastic rectangular plates and parabolically varying thickness plate. Marynowski [22] compared a three-parameter Zener model and a two-parameter Kelvin-Voigt model for the viscoelasticity. Saksa et al. [23] investigated stability and dynamic behaviour of axially moving viscoelastic panels with the help of the classical modal analysis. Tang and Chen [24] studied stability in parametric resonance of moving viscoelastic plates with time-dependent travelling speed.

It is well known that instability may be induced in structures systems because of nonconservative forces. Indeed, much of the research in this field has focused on the stability of the structures subjected to different types of nonconservative forces. However, few papers have focused on the stability of axially moving viscoelastic plates with piezoelectric layers under nonconservative forces.

Here, a relatively novel solution technique, the differential quadrature method (DQM), is used to analyze dynamic stability behavior of axially moving viscoelastic plates with piezoelectric layers subjected to nonconservative force. The current work focuses on the stability improvement of the axially moving nonconservative viscoelastic plate that is surface-bonded by piezoelectric layers. An analytical model that fully embodies the piezoelectric effects is first obtained based on the behavior of the follower force imposed by the external voltage on the piezoelectric layer. This paper aims to present the differential equation of the axially moving viscoelastic plate with piezoelectric layers subjected to uniformly distributed tangential follower force. Based on the 2D viscoelastic differential constitutive relation and Kirchhoff thin plate theory, the differential equation of the axially moving viscoelastic rectangular plate with piezoelectric layer subjected to uniformly distributed tangential follower force in the Laplace domain is formulated. And carrying out the Laplace inverse transformation, the differential equation of motion of the axially moving viscoelastic rectangular plate with piezoelectric layer subjected to uniformly distributed tangential follower force in time domain is yielded. The complex eigenvalue equations are established by the differential quadrature method. Via numerical calculation, the complex frequency and the instability type of the axially moving nonconservative viscoelastic plate constituted by elastic behavior in dilatation and the Kelvin-Voigt laws in distortion are obtained. The effects of nonconservative force, dimensionless axially moving speed, and dimensionless

FIGURE 1: Configuration of axially moving viscoelastic plate with piezoelectric layers subjected to uniformly distributed tangential follower force.

applied electric potential difference on the stability of axially moving nonconservative viscoelastic plate with piezoelectric layer are analyzed.

2. Formulation

2.1. Differential Equation of Motion. The axially moving viscoelastic thin plate with surface-bonded piezoelectric layers subjected to uniformly distributed tangential follower force q_0 is shown in Figure 1. The piezoelectric layers are symmetrical to the midplane of the structure. The thin plate has a thickness h_b in the z -direction, a material density ρ_b , and Poisson's ratio μ_b . The piezoelectric layers have a thickness h_p , a layer density ρ_p , and Poisson's ratio μ_p .

Using Kirchhoff's hypothesis of classical thin plates, the strain and displacement relations for the viscoelastic plate can be written as

$$\epsilon_x^b = -z \frac{\partial^2 w^*}{\partial x^2}, \quad \epsilon_y^b = -z \frac{\partial^2 w^*}{\partial y^2}, \quad \gamma_{xy}^b = -2z \frac{\partial^2 w^*}{\partial x \partial y}, \quad (1)$$

where w^* is the transverse displacement of the plate or deflection and z is the distance of the arbitrary point of the plate from the neutral plane.

The strain of the piezoelectric bonding layers pertains to the strains of both surfaces of the viscoelastic plate, expressed as

$$\epsilon_x^p = \mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial x^2}, \quad \epsilon_y^p = \mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial y^2}, \quad \gamma_{xy}^p = \mp h_b \frac{\partial^2 w^*}{\partial x \partial y}. \quad (2)$$

Considering mechanical and electrical behavior of piezoelectric materials. Proper care is taken when constitutive equations for piezoelectric materials are used to model induced strain actuation, as they are poling direction dependent. The planar isotropy of poled ceramics is expressed by their piezoelectric strain constants, such that $d_{31} = d_{32}$

[26, 27]. The constitutive equations of the piezoelectric layers for plane stress are

$$\begin{Bmatrix} \varepsilon_x^p \\ \varepsilon_y^p \\ \gamma_{xy}^p \end{Bmatrix} = \frac{1}{E^p} \begin{bmatrix} 1 & -\mu^p & 0 \\ -\mu^p & 1 & 0 \\ 0 & 0 & 2(1 + \mu^p) \end{bmatrix} \begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix} + \begin{Bmatrix} \frac{d_{31}}{h_p} V \\ \frac{d_{31}}{h_p} V \\ 0 \end{Bmatrix}, \quad (3)$$

where V denotes applied electric potential difference.

Substituting (2) into (3) yields

$$\begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix} = \frac{E^p}{1 - (\mu^p)^2} \times \begin{Bmatrix} \left(\mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial x^2} - \frac{d_{31}}{h_p} V \right) + \mu^p \left(\mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial y^2} - \frac{d_{31}}{h_p} V \right) \\ \left(\mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial y^2} - \frac{d_{31}}{h_p} V \right) + \mu^p \left(\mp \frac{h_b}{2} \frac{\partial^2 w^*}{\partial x^2} - \frac{d_{31}}{h_p} V \right) \\ \mp \frac{h_b}{2} (1 - \mu^p) \frac{\partial^2 w^*}{\partial x \partial y} \end{Bmatrix}. \quad (4)$$

The constitutive relations of linear viscoelastic materials in the Laplace domain are

$$\begin{aligned} \bar{P}_0 \bar{\sigma}_x^b &= \bar{Q}_0 \bar{\varepsilon}_x^b + \bar{Q}_1 \bar{\varepsilon}_y^b, \\ \bar{P}_0 \bar{\sigma}_y^b &= \bar{Q}_1 \bar{\varepsilon}_x^b + \bar{Q}_0 \bar{\varepsilon}_y^b, \\ \bar{Q}' \bar{\varepsilon}_{xy}^b &= \bar{P}' \bar{\tau}_{xy}^b. \end{aligned} \quad (5)$$

In the Laplace domain, the constitutive relations of linear viscoelastic materials using Laplace transformation $\bar{w}$ of deflection w^* are expressed as

$$\begin{aligned} \bar{P}_0 \bar{\sigma}_x^b &= \bar{Q}_0 \left(-z \frac{\partial^2 \bar{w}}{\partial x^2} \right) + \bar{Q}_1 \left(-z \frac{\partial^2 \bar{w}}{\partial y^2} \right), \\ \bar{P}_0 \bar{\sigma}_y^b &= \bar{Q}_1 \left(-z \frac{\partial^2 \bar{w}}{\partial x^2} \right) + \bar{Q}_0 \left(-z \frac{\partial^2 \bar{w}}{\partial y^2} \right), \\ \bar{P}' \bar{\tau}_{xy}^b &= \bar{Q}' \left(-z \frac{\partial^2 \bar{w}}{\partial x \partial y} \right), \end{aligned} \quad (6)$$

where the differential operators $P' = \sum_{k=0}^l p'_k (d^k/dt^k)$, $Q' = \sum_{k=0}^r q'_k (d^k/dt^k)$, $P'' = \sum_{k=0}^{l_1} p''_k (d^k/dt^k)$, and $Q'' = \sum_{k=0}^{r_1} q''_k (d^k/dt^k)$; p'_k, q'_k, p''_k , and q''_k depend on the properties of the material; $\bar{P}', \bar{Q}', \bar{P}'',$ and $\bar{Q}''$ are the Laplace transform of the differential operators $P', Q', P'',$ and Q'' . For convenience, let $\bar{P}_0 = \bar{P}'(\bar{P}'\bar{Q}'' + 2\bar{Q}'\bar{P}'')$, $\bar{Q}_0 = \bar{Q}'(2\bar{P}'\bar{Q}'' + \bar{Q}'\bar{P}'')$, $\bar{Q}_1 = \bar{Q}'(\bar{P}'\bar{Q}'' - \bar{Q}'\bar{P}'')$, and the polynomials $\bar{P}_0, \bar{Q}_0$, and $\bar{Q}_1$ about Laplace variable s are independent of spatial coordinates.

Assume that the piezoelectric sheets are of infinitesimal thickness and disregard its influence on the bending stiffness of the structure. The following terms σ_x^p and σ_y^p result in uniform stress in the piezoelectric layer. The force excited by the piezoelectric layer due to external electric potential difference is modeled as a follower tensile force, and the follower tensile force is a kind of nonconservative force. The tension force caused by the piezoelectric layer can be written as

$$\begin{aligned} T_x &= \int_{h_b/2}^{h_b/2+h_p} \sigma_x^p dz = -\frac{E_p}{1 - \mu_p} d_{31} V, \\ T_y &= \int_{h_b/2}^{h_b/2+h_p} \sigma_y^p dz = -\frac{E_p}{1 - \mu_p} d_{31} V. \end{aligned} \quad (7)$$

In the Laplace domain, the relations between internal torque and Laplace transformation $\bar{w}$ of deflection w^* are given by

$$\begin{aligned} \bar{P}_0 (\bar{M}_x^b) &= -\frac{h_b^3}{12} \left(\bar{Q}_0 \frac{\partial^2 \bar{w}}{\partial x^2} + \bar{Q}_1 \frac{\partial^2 \bar{w}}{\partial y^2} \right), \\ \bar{P}_0 (\bar{M}_y^b) &= -\frac{h_b^3}{12} \left(\bar{Q}_1 \frac{\partial^2 \bar{w}}{\partial x^2} + \bar{Q}_0 \frac{\partial^2 \bar{w}}{\partial y^2} \right), \\ \bar{P}' (\bar{M}_{xy}^b) &= \bar{P}' (\bar{M}_{yx}^b) = -\frac{h_b^3}{12} \bar{Q}' \frac{\partial^2 \bar{w}}{\partial x \partial y}. \end{aligned} \quad (8)$$

The transverse speed of moving plate is

$$\bar{v}(t) = \frac{dw^*}{dt} = \frac{\partial w^*}{\partial t} + v \frac{\partial w^*}{\partial x}. \quad (9)$$

The differential operator is given by $d/dt = (\partial/\partial t) + v(\partial/\partial x)$.

The equilibrium equation of axially moving viscoelastic plate with piezoelectric layer subjected to uniformly distributed tangential follower force is given by

$$\begin{aligned} \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - N_x \frac{\partial^2 w^*}{\partial x^2} - N_y \frac{\partial^2 w^*}{\partial y^2} \\ - 2N_{xy} \frac{\partial^2 w^*}{\partial x \partial y} - (\rho_b h_b + 2\rho_p h_p) \\ \times \left(\frac{\partial^2 w^*}{\partial t^2} + 2v \frac{\partial^2 w^*}{\partial x \partial t} + v^2 \frac{\partial^2 w^*}{\partial x^2} \right) = 0, \end{aligned} \quad (10)$$

where

$$N_x = 2T_x + q_0(a - x), \quad N_y = 2T_y, \quad N_{xy} = 0. \quad (11)$$

Carrying out the Laplace transformation of (10) and (11) and multiplying it by $\bar{P}_0 \bar{P}'$. When the partial derivative is continuous, in the Laplace domain the differential equation of motion of the axially moving viscoelastic rectangular plate with piezoelectric layers subjected to uniformly distributed tangential follower force is obtained:

$$\begin{aligned} & \frac{h_b^3}{12} \bar{Q}' (2\bar{P}' \bar{Q}'' + \bar{Q}' \bar{P}'') \nabla^4 \bar{w}^* - \bar{P}' (\bar{P}' \bar{Q}'' + 2\bar{Q}' \bar{P}'') \\ & \times \frac{2E^p d_{31} V}{1 - \mu^p} \left(\frac{\partial^2 \bar{w}^*}{\partial x^2} + \frac{\partial^2 \bar{w}^*}{\partial y^2} \right) \\ & + \bar{P}' (\bar{P}' \bar{Q}'' + 2\bar{Q}' \bar{P}'') q_0 (a - x) \frac{\partial^2 \bar{w}^*}{\partial x^2} \\ & + \bar{P}' (\bar{P}' \bar{Q}'' + 2\bar{Q}' \bar{P}'') (\rho_b h_b + 2\rho_p h_p) \\ & \times \left(s_1^2 \bar{w}^* + 2\nu s_1 \frac{\partial \bar{w}^*}{\partial x} + \nu^2 \frac{\partial^2 \bar{w}^*}{\partial x^2} \right) = 0, \end{aligned} \quad (12)$$

where $\nabla^4 \bar{w}^* = (\partial^4 \bar{w}^* / \partial x^4) + 2(\partial^4 \bar{w}^* / \partial x^2 \partial y^2) + (\partial^4 \bar{w}^* / \partial y^4)$.

Equation (11) is suitable for various viscoelastic differential models.

Here, we assume that the material of the plate obeys elastic behavior in dilatation and the Kelvin-Voigt laws in distortion. Substituting the polynomials $\bar{P}' = 1$, $\bar{Q}' = 2G + 2\eta s$, $\bar{P}'' = 1$, and $\bar{Q}'' = 3K$ into (11) and carrying out the Laplace inverse transformation yield the differential equation of motion of axially moving nonconservative viscoelastic plate with elastic dilatation and Kelvin-Voigt distortion with piezoelectric layers in the time domain

$$\begin{aligned} & \frac{h_b^3}{12} \left(A_3 + A_4 \frac{\partial}{\partial t} + A_5 \frac{\partial^2}{\partial t^2} \right) \nabla^4 w^* \\ & - \left(A_1 + A_2 \frac{\partial}{\partial t} \right) \frac{2E^p d_{31} V}{1 - \mu^p} \left(\frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^*}{\partial y^2} \right) \\ & + \left(A_1 + A_2 \frac{\partial}{\partial t} \right) q_0 (a - x) \frac{\partial^2 w^*}{\partial x^2} \\ & + \left(A_1 + A_2 \frac{\partial}{\partial t} \right) (\rho_b h_b + 2\rho_p h_p) \\ & \times \left(\frac{\partial^2 w^*}{\partial t^2} + 2\nu \frac{\partial^2 w^*}{\partial t \partial x} + \nu^2 \frac{\partial^2 w^*}{\partial x^2} \right) = 0, \end{aligned} \quad (13)$$

where $A_1 = 3K + 4G$, $A_2 = 4\eta$, $A_3 = 2G(6K + 2G)$, $A_4 = 8G\eta + 12K\eta$, $A_5 = 4\eta^2$

$$\begin{aligned} K &= \frac{E_b}{3(1 - 2\mu_b)}, \quad G = \frac{E_b}{2(1 + \mu_b)}, \\ \nabla^4 w^* &= \frac{\partial^4 w^*}{\partial x^4} + 2 \frac{\partial^4 w^*}{\partial x^2 \partial y^2} + \frac{\partial^4 w^*}{\partial y^4}. \end{aligned} \quad (14)$$

2.2. Differential Equation of Vibration Mode. Introducing dimensionless quantities and parameters,

$$\xi = \frac{x}{a}, \quad \psi = \frac{y}{b}, \quad w = \frac{w^*}{a}, \quad r = \frac{a}{b},$$

$$\tau = \frac{th_b}{a^2} \sqrt{\frac{E^b}{12\rho_b(1 - (\mu^b)^2)}},$$

$$C = \frac{a}{h_b} \sqrt{\frac{12\rho_b(1 - (\mu^b)^2)}{E^b}} \nu,$$

$$q = \frac{12q_0 a^3 (1 - (\mu^b)^2)}{E^b h_b^3}, \quad \lambda = \frac{h_p}{h_b}, \quad p = \frac{\rho_p}{\rho_b} \lambda,$$

$$H = \frac{h_b}{a^2} \sqrt{\frac{E^b}{12\rho_b(1 - (\mu^b)^2)}} \frac{\eta}{E^b},$$

$$U = \frac{24E^p d_{31} a^2 V (1 - (\mu^b)^2)}{E^b h_b^3}.$$

Substituting (15) into (13) yields the following dimensionless differential equation:

$$\begin{aligned} & \left(1 + \frac{4}{3} (1 + \mu^b) (2 - \mu^b) H \frac{\partial}{\partial \tau} \right. \\ & \left. + \frac{4}{3} (1 - 2\mu^b) (1 + \mu^b)^2 H^2 \frac{\partial^2}{\partial \tau^2} \right) \nabla^4 w \\ & - \left(\frac{1}{1 - \mu^p} + \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^p)(1 - \mu^b)} H \frac{\partial}{\partial \tau} \right) \\ & \times \left(U \frac{\partial^2 w}{\partial \xi^2} + U \cdot r^2 \frac{\partial^2 w}{\partial \psi^2} \right) \\ & + \left(1 + \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} H \frac{\partial}{\partial \tau} \right) q (1 - \xi) \frac{\partial^2 w}{\partial \xi^2} \\ & + \left(1 + \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} H \frac{\partial}{\partial \tau} \right) \\ & \times (1 + 2p) \left(\frac{\partial^2 w}{\partial \tau^2} + 2C \frac{\partial^2 w}{\partial \xi \partial \tau} + C^2 \frac{\partial^2 w}{\partial \xi^2} \right) \frac{\partial^2 w}{\partial \tau^2} = 0, \end{aligned} \quad (16)$$

where τ is the dimensionless time, H denotes the dimensionless time delay, C is the dimensionless moving speed, and U represents the dimensionless electric potential difference parameter. One has $\nabla^4 w = (\partial^4 w / \partial \xi^4) + 2r^2 (\partial^4 w / \partial \xi^2 \partial \psi^2) + r^4 (\partial^4 w / \partial \psi^4)$.

Suppose that the solution to (16) takes the form

$$w(\xi, \psi, \tau) = W(\xi, \psi) e^{i\omega\tau}, \quad (17)$$

where $i = \sqrt{-1}$ and ω is a dimensionless complex frequency. Substituting (17) into (16) gives

$$\begin{aligned} D_1 \nabla^4 W - D_3 \left(U \frac{\partial^2 W}{\partial \xi^2} + U r^2 \frac{\partial^2 W}{\partial \psi^2} \right) \\ + D_2 q (1 - \xi) \frac{\partial^2 W}{\partial \xi^2} + D_2 (1 + 2p) \\ \times \left(-\omega^2 W + 2C i \omega \frac{\partial W}{\partial \xi} + C^2 \frac{\partial^2 W}{\partial \xi^2} \right) = 0, \end{aligned} \quad (18)$$

where

$$\begin{aligned} D_1 &= 1 + i \frac{4}{3} (1 + \mu^b) (2 - \mu^b) H \omega \\ &\quad - \frac{4}{3} (1 - 2\mu^b) (1 + \mu^b)^2 H^2 \omega^2, \\ D_2 &= 1 + i \frac{4}{3} \frac{(1 - 2\mu^b) (1 + \mu^b)}{(1 - \mu^b)} H \omega, \\ D_3 &= \frac{1}{1 - \mu^p} + i \frac{4}{3} \frac{(1 - 2\mu^b) (1 + \mu^b)}{(1 - \mu^p) (1 - \mu^b)} H \omega. \end{aligned} \quad (19)$$

Boundary conditions for four-edge simply supported (SSSS) rectangular plates are

$$\begin{aligned} \xi = 0, 1: \quad W(\xi, \psi) = \frac{\partial^2 W}{\partial \xi^2} = 0, \\ \psi = 0, 1: \quad W(\xi, \psi) = \frac{\partial^2 W}{\partial \psi^2} = 0. \end{aligned} \quad (20)$$

Boundary conditions for two opposite edges simply supported and other edges clamped (CSCS) are as follows:

$$\begin{aligned} \xi = 0, 1: \quad W(\xi, \psi) = \frac{\partial W}{\partial \xi} = 0, \\ \psi = 0, 1: \quad W(\xi, \psi) = \frac{\partial^2 W}{\partial \psi^2} = 0. \end{aligned} \quad (21)$$

2.3. Complex Eigenvalue Equation. The complex eigenvalue equation of the axially moving viscoelastic rectangular plate with piezoelectric layers subjected to uniformly distributed tangential follower force is derived by the DQM [28–31]. The DQM is used to approximate the partial derivatives of a function with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable.

Postulating smooth function $f(x, y)$ in the region $0 \leq x \leq a$, $0 \leq y \leq b$, the r th order partial derivative of $f(x, y)$ with respect to x , the s th order partial derivative of $f(x, y)$ with respect to y , and the mixed partial derivative of the s th

order with respect to y and the r th order with respect to x are, respectively, approximated as

$$\begin{aligned} \frac{\partial^r f(x_i, y_j)}{\partial x^r} &= \sum_{k=1}^N A_{ik}^{(r)} f(x_k, y_j), \\ (i &= 1, 2, \dots, N; \quad r = 1, 2, \dots, N-1), \\ \frac{\partial^s f(x_i, y_j)}{\partial y^s} &= \sum_{m=1}^M A_{jm}^{(s)} f(x_i, y_m), \\ (j &= 1, 2, \dots, M; \quad s = 1, 2, \dots, M-1), \\ \frac{\partial^{r+s} f(x_i, y_j)}{\partial x^r \partial y^s} &= \sum_{k=1}^N A_{ik}^{(r)} \sum_{m=1}^M A_{jm}^{(s)} f(x_k, y_m), \end{aligned} \quad (22)$$

where N and M are the number of grid points in the x and y direction, respectively, $A_{ik}^{(r)}$ and $A_{jm}^{(s)}$ are the weighted coefficients, and they are defined by

$$\begin{aligned} A_{ik}^{(1)} &= \begin{cases} \frac{\prod_{u=1, u \neq i, k}^N (x_i - x_u)}{\prod_{u=1, u \neq k}^N (x_k - x_u)} & (i, k = 1, 2, \dots, N; \quad k \neq i), \\ \sum_{u=1, u \neq k}^N \frac{1}{(x_i - x_u)} & (i, k = 1, 2, \dots, N; \quad k = i), \end{cases} \\ A_{jm}^{(1)} &= \begin{cases} \frac{\prod_{u=1, u \neq j, m}^M (y_j - y_u)}{\prod_{u=1, u \neq m}^M (y_m - y_u)} & (j, m = 1, 2, \dots, M; \quad m \neq j), \\ \sum_{u=1, u \neq m}^M \frac{1}{(y_j - y_u)} & (j, m = 1, 2, \dots, M; \quad m = j). \end{cases} \end{aligned} \quad (23)$$

In the case of $r = 2, 3, \dots, N-1$, $s = 2, 3, \dots, M-1$,

$$\begin{aligned} A_{ik}^{(r)} &= \begin{cases} r \left(A_{ii}^{(r-1)} A_{ik}^{(1)} - \frac{A_{ik}^{(r-1)}}{x_i - x_k} \right) & (i, k = 1, 2, \dots, N; \quad k \neq i), \\ - \sum_{u=1, u \neq i}^N A_{iu}^{(r)} & (i = 1, 2, \dots, N; \quad 1 \leq r \leq (N-1); \quad k = i), \end{cases} \end{aligned}$$

FIGURE 2: Dimensionless complex frequencies versus dimensionless axially moving speed C ($H = 10^{-5}$, $q = 50$, $r = 1$, $U = 0$, and $\lambda = 0.05$).

FIGURE 3: Dimensionless complex frequencies versus dimensionless axially moving speed C ($H = 10^{-5}$, $q = 50$, $r = 1$, $U = 10$, and $\lambda = 0.05$).

$$A_{jm}^{(s)} = \begin{cases} s \left(A_{jj}^{(s-1)} A_{jm}^{(1)} - \frac{A_{jm}^{(s-1)}}{y_j - y_m} \right) & (j, m = 1, 2, \dots, M; m \neq j), \\ -\sum_{\substack{u=1 \\ u \neq j}}^M A_{ju}^{(s)} & (j = 1, 2, \dots, M; 1 \leq s \leq (M-1); m = j). \end{cases} \quad (24)$$

In this paper, $N = M$. The distribution forms of the grid points are nonuniform; the distribution forms of the grid points are

$$\begin{aligned} \xi_1 &= 0, & \xi_N &= 1, \\ \xi_i &= \frac{1}{2} \left[1 - \cos \left(\frac{2i-3}{2N-4} \pi \right) \right], & (i = 2, 3, \dots, N-1) \\ \eta_1 &= 0, & \eta_N &= 1, \\ \eta_i &= \frac{1}{2} \left[1 - \cos \left(\frac{2i-3}{2N-4} \pi \right) \right], & (i = 2, 3, \dots, N-1). \end{aligned} \quad (25)$$

FIGURE 4: Dimensionless complex frequencies versus dimensionless axially moving speed C ($H = 10^{-5}$, $q = 50$, $r = 1$, $U = 20$, and $\lambda = 0.05$).

FIGURE 5: The first-order dimensionless complex frequencies versus dimensionless follower force q ($H = 10^{-3}$, $r = 1$, $U = 10$, and $\lambda = 0.05$).

According to the DQM procedures, (18) can be discretized into the following forms:

$$\begin{aligned} & \sum_{k=1}^N A_{ik}^{(4)} W_{kj} + 2r^2 \sum_{m=1}^M B_{jm}^{(2)} \sum_{k=1}^N A_{ik}^{(2)} W_{km} + r^4 \sum_{m=1}^M B_{jm}^{(4)} W_{im} \\ & - \frac{1}{1 - \mu^p} \left(U \sum_{k=1}^N A_{ik}^{(2)} W_{kj} + U \cdot r^2 \sum_{m=1}^M B_{jm}^{(2)} W_{im} \right) \\ & + q(1 - \xi_i) \sum_{k=1}^N A_{ik}^{(2)} W_{kj} + (1 + 2p) C^2 \sum_{k=1}^N A_{ik}^{(2)} W_{kj} \end{aligned}$$

$$\begin{aligned} & + \left[i \frac{4}{3} (1 + \mu^b) (2 - \mu^b) H \right. \\ & \times \left(\sum_{k=1}^N A_{ik}^{(4)} W_{kj} + 2r^2 \sum_{m=1}^M B_{jm}^{(2)} \sum_{k=1}^M A_{ik}^{(2)} W_{km} \right. \\ & \quad \left. \left. + r^4 \sum_{k=1}^M B_{jm}^{(4)} W_{ik} \right) \right. \\ & \left. - i \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^p)(1 - \mu^b)} H \right] \end{aligned}$$

FIGURE 6: The first-order dimensionless complex frequencies versus dimensionless follower force q ($H = 10^{-3}$, $r = 1$, $U = 20$, and $\lambda = 0.05$).

$$\begin{aligned}
 & \times \left(U \sum_{k=1}^N A_{ik}^{(2)} W_{kj} + U r^2 \sum_{m=1}^M B_{jm}^{(2)} W_{im} \right) \\
 & + i \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} H q (1 - \xi_i) \sum_{k=1}^N A_{ik}^{(2)} W_{kj} \\
 & + i(1 + 2p) 2C \sum_{k=1}^N A_{ik}^{(1)} W_{kj} + i(1 + 2p) \\
 & \times \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} H C^2 \sum_{k=1}^N A_{ik}^{(2)} W_{kj} \Big] \omega \\
 & - \left[\frac{4}{3} (1 - 2\mu^b)(1 + \mu^b)^2 H^2 \right. \\
 & \times \left(\sum_{k=1}^N A_{ik}^{(4)} W_{kj} + 2r^2 \sum_{m=1}^N B_{jm}^{(2)} \sum_{k=1}^N A_{ik}^{(2)} W_{km} \right. \\
 & \quad \left. + r^4 \sum_{k=1}^N B_{jm}^{(4)} W_{ik} \right) \\
 & + \frac{4}{3} (1 + 2p) \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} 2CH \sum_{k=1}^N A_{ik} W_{kj} \\
 & \left. + (1 + 2p) W_{ij} \right] \omega^2 \\
 & - i(1 + 2p) \frac{4}{3} \frac{(1 - 2\mu^b)(1 + \mu^b)}{(1 - \mu^b)} H W_{ij} \omega^3 = 0.
 \end{aligned} \tag{26}$$

The differential quadrature forms of the boundary conditions (20) are

$$\begin{aligned}
 W_{1j} = W_{Nj} = W_{i1} = W_{iN} = 0, \quad i, j = 1, 2, \dots, N, \\
 \sum_{k=1}^N A_{ik}^{(2)} W_{kj} = 0, \quad i = 1, N, \quad j = 1, 2, \dots, N, \\
 \sum_{k=1}^N B_{jk}^{(2)} W_{ik} = 0, \quad j = 1, N, \quad i = 1, 2, \dots, N.
 \end{aligned} \tag{27}$$

The differential quadrature forms of the boundary conditions (21) are

$$\begin{aligned}
 W_{1j} = W_{Nj} = W_{i1} = W_{iN} = 0, \quad i, j = 1, 2, \dots, N, \\
 \sum_{k=1}^N A_{ik}^{(1)} W_{kj} = 0, \quad i = 2, N - 1, \quad j = 2, 3, \dots, N - 2, \\
 \sum_{k=1}^N B_{jk}^{(2)} W_{ik} = 0, \quad j = 1, N, \quad i = 1, 2, \dots, N.
 \end{aligned} \tag{28}$$

After eliminating the boundary degrees of freedom from (22) using the boundary conditions, the equation can be written in the matrix form as

$$(\omega^3 \mathbf{Q} + \omega^2 \mathbf{R} + \omega \mathbf{G} + \mathbf{K}) \{\mathbf{W}_{kj}\} = \{\mathbf{0}\}. \tag{29}$$

The complex eigenvalue equation of the axially moving viscoelastic rectangular plate with piezoelectric layers subjected to uniformly distributed tangential follower force is that coefficient determinant equal to zero; that is,

$$|\omega^3 \mathbf{Q} + \omega^2 \mathbf{R} + \omega \mathbf{G} + \mathbf{K}| = 0. \tag{30}$$

In (30), ω is a generalized complex eigenvalue, where the matrices $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{G}$, and $\mathbf{K}$ involve such parameters as dimensionless moving speed C , the dimensionless follower force q , the dimensionless time delay H , the dimensionless time τ , the dimensionless electric potential difference parameter U , the dimensionless electric potential difference geometrical parameter λ , and aspect ratio r of the plate.

3. Results and Discussion

In the case of $H = 0$, $C = 0$, $U = 0$, and $p = 0$, (18) is reduced to the differential equation of motion of the elastic plate. Equation (30) is reduced to the complex eigenvalue equation of the elastic plate. In order to verify the DQM program, the first three-order natural frequencies of the transverse free vibration of elastic plate with four-edge simply supported boundary conditions are calculated firstly, and the results are in good agreement with those exhibited in [31], which can be seen from Table 1.

In the numerical examples, the number of grid points is $N = M = 9$. Numerical studies have been conducted to investigate the effects of several key parameters on the dynamics stabilities of axially moving viscoelastic plates with the piezoelectric layer subjected to uniformly distributed tangential follower force. The critical moving speed of axially moving plates for $H = 10^{-5}$, $q = 0$, $r = 1$, $U = 0$, and $p = 0$ with different boundary conditions is seen from Table 2.

3.1. The Influence of the Dimensionless Electric Potential Difference. Figure 2 displays the variation of the curve for the cases $H = 10^{-5}$, $q = 50$, $r = 1$, $U = 0$, and $\lambda = 0.05$. It can be seen that, with increase of axially moving speed C , the real part $\text{Re}(\omega)$ of complex frequencies in the first mode becomes zero and the imaginary part $\text{Im}(\omega)$ of complex frequencies has two branches. This shows that the first mode behaves unstably by the divergence instability when the axially moving speed becomes equal to or larger than the critical speed $C = 3.13$. When the moving speed increases further, the piezolaminated plate regains stability in the first-order mode. By maintaining an increase in the moving velocity, the plate undergoes divergence instability in the second mode. When the speed of motion is $C = 8.78$, the first mode behaves unstably by the divergence instability again. When $C = 10.85$, the second mode behaves divergence instability again.

In Figure 3, the case of $H = 10^{-5}$, $q = 50$, $r = 1$, $U = 10$, and $\lambda = 0.05$ is shown. The first-order mode exhibits divergence instability when the moving speed $C = 5.77$; at $C = 7.41$ the first-order divergence instability is over. When the moving speed increases, the plate undergoes divergence instability in the second mode. By maintaining an increase in the moving velocity, the first mode behaves unstably by the divergence instability again. Then, the second mode behaves divergence instability again. In comparison with Figure 2, with the dimensionless electric potential difference increase, the critical speed increases, while the types of instability of the moving piezolaminated plate are in agreement.

Figure 4 shows the variation of the first three-order dimensionless complex frequencies of the plate with dimensionless axially moving speed C for $H = 10^{-5}$, $q = 50$, $r = 1$, $U = 20$, and $\lambda = 0.05$. It can be seen that the influence of the dimensionless electric potential difference on the critical speed is obvious. With the dimensionless electric potential difference increase, the critical speed increases, while the types of instability of the moving piezolaminated plate subjected to uniformly distributed tangential follower force have not been changed.

3.2. The Influence of the Dimensionless Moving Speed on Stability. Figures 5 and 6 display the variation of the curve of the first-order dimensionless complex frequencies versus dimensionless follower force q for the cases $U = 10$ and $U = 20$. It can be seen that the viscoelastic plate exhibits divergence instability in the first-order mode under different moving speed C . This result indicates that increases in the dimensionless speed of moving piezolaminated viscoelastic plate subjected to uniformly distributed tangential follower force reduce the critical load but will not change the type of instability experienced by the moving plate.

3.3. The Influence of the Dimensionless Uniformly Distributed Tangential Follower Force on Stability. Figures 7 and 8 indicate the variation of the first-order dimensionless complex frequencies with the dimensionless moving speed for the different values of U , respectively. In Figure 7, for dimensionless follower force $q = 0$, it shows that the first-order mode behaves divergence instability; then the plate undergoes single-mode flutter. In the case of $q = 50$ and $q = 100$, it shows that the first-order mode behaves divergence instability first, when the dimensionless moving speed increases, the plate regains stability in the first-order mode. By maintaining an increase in the moving speed, the first mode behaves divergence instability again. It can be seen that the real part of the dimensionless complex frequencies decreases with the increase of the dimensionless follower force q . The critical speed of the first mode decreases when the dimensionless follower force q increases.

In comparison with Figures 7 and 8, respectively, it is observed that the same dimensionless follower force q , the real parts of the dimensionless complex frequencies under $U = 20$, is greater than that in the case of $U = 10$. The critical speed of the first mode increases with the increase of electric potential difference.

4. Conclusions

The formulation and results for the stability of the moving viscoelastic plate with the piezoelectric layer subjected to uniformly distributed tangential follower force are presented. The general complex frequencies are calculated. This paper analyzes stabilities of the moving viscoelastic plate with the piezoelectric layer subjected to follower force. The results of the analysis of the present study are summarized below.

- (1) With the dimensionless electric potential difference increase, the critical speed increases, while the types

TABLE 1: The first three-order natural frequencies of the uniform thickness elastic plate with different boundary conditions.

Aspect ratio r		0.5			1		
Present solutions	SSSS	12.3370	19.7401	31.8919	19.7392	49.3519	78.9647
	SCSC	23.8184	28.9567	38.9379	28.9559	54.7609	70.1555
Results in [25]	SSSS	12.33	19.73	32.07	19.73	49.35	78.96
	SCSC	23.82	28.95	39.09	28.95	54.74	69.33

TABLE 2: Critical moving speed of axially moving plates with different boundary conditions.

Boundary conditions	Divergence speed in first order	Regaining stability in the first order	Coupled-mode flutter speed
SSSS	6.31	7.81	8.22 (1st and 2nd couple)
CSCS	8.21	10.31	11.79 (1st and 3rd couple)

FIGURE 7: Dimensionless complex frequencies versus dimensionless axially moving speed C ($H = 10^{-3}$, $r = 1$, $U = 10$, and $\lambda = 0.05$). FIGURE 8: Dimensionless complex frequencies versus dimensionless axially moving speed C ($H = 10^{-3}$, $r = 1$, $U = 20$, and $\lambda = 0.05$).

of instability of the moving piezolaminated plate subjected to uniformly distributed tangential follower force have not been changed. The moving piezolaminated plate subjected to follower force behaves unstably by the divergence instability.

- (2) By increasing in the dimensionless speed of moving piezolaminated viscoelastic plate subjected to follower force, the critical load (i.e., critical follower force) decreases but will not change the type of divergence instability experienced by the moving plate.
- (3) The influence of the dimensionless uniformly distributed tangential follower force on stability of moving piezolaminated viscoelastic plate is obvious. By maintaining an increase in the dimensionless follower force, the critical speed of the first mode decreases.

This research demonstrates the potential of using piezoelectric materials in enhancing the stability of axially moving viscoelastic plates subjected to uniformly distributed tangential follower force. The conclusions provide a theoretical basis and effective approach for improving structure designing and working stability.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors thank the reviewer for careful reading of the paper. The authors gratefully acknowledges the support of the National Natural Science Foundation of China (no. 11202159, no. 11272253, and no. 11101330) and the Natural Science Foundation of Education Department of Shaanxi Province of China (no. 11JK0535).

References

- [1] C. K. Lee, "Theory of laminated piezoelectric plates for the design of distributed sensors/actuators. Part I: governing equations and reciprocal relationships," *Journal of the Acoustical Society of America*, vol. 87, no. 3, pp. 1144–1158, 1990.
- [2] Y.-Y. Yu, "On the ordinary, generalized, and pseudo-variational equations of motion in nonlinear elasticity, piezoelectricity, and classical plate theories," *Journal of Applied Mechanics, Transactions ASME*, vol. 62, no. 2, pp. 471–478, 1995.
- [3] Y. Y. Tang, A. K. Noor, and K. Xu, "Assessment of computational models for thermoelectroelastic multilayered plates," *Computers & Structures*, vol. 61, no. 5, pp. 915–933, 1996.
- [4] R. C. Batra and T. S. Geng, "Enhancement of the dynamic buckling load for a plate by using piezoceramic actuators," *Smart Materials and Structures*, vol. 10, no. 5, pp. 925–933, 2001.
- [5] M. Krommer, "Piezoelastic vibrations of composite Reissner-Mindlin-type plates," *Journal of Sound and Vibration*, vol. 263, no. 4, pp. 871–891, 2003.
- [6] X. Shu, "Free vibration of laminated piezoelectric composite plates based on an accurate theory," *Composite Structures*, vol. 67, no. 4, pp. 375–382, 2005.
- [7] A. Robaldo, E. Carrera, and A. Benjeddou, "A unified formulation for finite element analysis of piezoelectric adaptive plates," *Computers and Structures*, vol. 84, no. 22–23, pp. 1494–1505, 2006.
- [8] Z. Zhang, C. Feng, and K. M. Liew, "Three-dimensional vibration analysis of multilayered piezoelectric composite plates," *International Journal of Engineering Science*, vol. 44, no. 7, pp. 397–408, 2006.
- [9] A. N. Kounadis, C. J. Gantes, and V. V. Bolotin, "An improved energy criterion for dynamic buckling of imperfection sensitive nonconservation systems," *International Journal of Solids and Structures*, vol. 38, no. 42–43, pp. 7487–7500, 2001.
- [10] K. Chandrashekhara and K. Bhatia, "Active buckling control of smart composite plates-finite-element analysis," *Smart Materials and Structures*, vol. 2, no. 1, article 5, pp. 31–39, 1993.
- [11] L. Q. Yao and H. R. Yu, "Finite element method on bending control of thin plate with piezoelectric material," *Chinese Journal of Computational Mechanics*, vol. 16, no. 3, pp. 330–333, 1999.
- [12] J. G. Chase and S. Bhashyam, "Optimal stabilization of plate buckling," *Smart Materials and Structures*, vol. 8, no. 2, pp. 204–211, 1999.
- [13] Q. Wang, "On buckling of column structures with a pair of piezoelectric layers," *Engineering Structures*, vol. 24, no. 2, pp. 199–205, 2002.
- [14] Q. Wang and S. T. Quek, "Enhancing flutter and buckling capacity of column by piezoelectric layers," *International Journal of Solids and Structures*, vol. 39, no. 16, pp. 4167–4180, 2002.
- [15] S. K. Ha, C. Keilers, and F.-K. Chang, "Finite element analysis of composite structures containing distributed piezoceramic sensors and actuators," *AIAA Journal*, vol. 30, no. 3, pp. 772–780, 1992.
- [16] J. A. Wickert and C. D. Mote, "Current research on the vibration and stability of axially-moving materials," *Shock and Vibration Digest*, vol. 20, pp. 3–13, 1988.
- [17] A. G. Ulsoy and C. D. Mote Jr., "Vibration of wide band saw blades," *ASME Journal of Engineering for Industry*, vol. 104, no. 1, pp. 71–78, 1982.
- [18] C. C. Lin, "Stability and vibration characteristics of axially moving plates," *International Journal of Solids and Structures*, vol. 34, no. 24, pp. 3179–3190, 1997.
- [19] C. C. Lin, "Finite width effects on the critical speed of axially moving materials," *Journal of Vibration and Acoustics*, vol. 120, no. 2, pp. 633–634, 1998.
- [20] Y.-F. Zhou and Z.-M. Wang, "Transverse vibration characteristics of axially moving viscoelastic plate," *Applied Mathematics and Mechanics*, vol. 28, no. 2, pp. 209–218, 2007.
- [21] Z. Yin-feng and W. Zhong-min, "Vibrations of axially moving viscoelastic plate with parabolically varying thickness," *Journal of Sound and Vibration*, vol. 316, no. 1–5, pp. 198–210, 2008.
- [22] K. Marynowski, "Free vibration analysis of the axially moving Levy-type viscoelastic plate," *European Journal of Mechanics—A/Solids*, vol. 29, no. 5, pp. 879–886, 2010.
- [23] T. Saksa, N. Banichuk, J. Jeronen, M. Kurki, and T. Tuovinen, "Dynamic analysis for axially moving viscoelastic panels," *International Journal of Solids and Structures*, vol. 49, no. 23–24, pp. 3355–3366, 2012.
- [24] Y.-Q. Tang and L.-Q. Chen, "Stability analysis and numerical confirmation in parametric resonance of axially moving viscoelastic plates with time-dependent speed," *European Journal of Mechanics A: Solids*, vol. 37, pp. 106–121, 2013.

- [25] D. J. Gorman, *Free Vibration Analysis of Rectangular Plates*, Elsevier, New York, NY, USA, 1982.
- [26] X. Cao, F. Jin, I. Jeon, and T. J. Lu, "Propagation of Love waves in a functionally graded piezoelectric material (FGPM) layered composite system," *International Journal of Solids and Structures*, vol. 46, no. 22-23, pp. 4123–4132, 2009.
- [27] X. Cao, F. Jin, and Z. Wang, "On dispersion relations of Rayleigh waves in a functionally graded piezoelectric material (FGPM) half-space," *Acta Mechanica*, vol. 200, no. 3-4, pp. 247–261, 2008.
- [28] T. M. Teo and K. M. Liew, "A differential quadrature procedure for three-dimensional buckling analysis of rectangular plates," *International Journal of Solids and Structures*, vol. 36, no. 8, pp. 1149–1168, 1999.
- [29] C. W. Bert and M. Malik, "Differential quadrature: a powerful new technique for analysis of composite structures," *Composite Structures*, vol. 39, no. 3-4, pp. 179–189, 1997.
- [30] X. Wu and Y.-E. Ren, "Differential quadrature method based on the highest derivative and its applications," *Journal of Computational and Applied Mathematics*, vol. 205, no. 1, pp. 239–250, 2007.
- [31] Z.-M. Wang, Y.-F. Zhou, and Y. Wang, "Dynamic stability of a non-conservative viscoelastic rectangular plate," *Journal of Sound and Vibration*, vol. 307, no. 1-2, pp. 250–264, 2007.