Research Article

Magnetodielectric and Metalomagnetic 1D Photonic Crystals Homogenization: ϵ - μ Local Behavior

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A theory for calculating the effective optic response of photonic crystals with metallic and magnetic inclusions is reported, for the case when the wavelength of the electromagnetic fields is much larger than the lattice constant. The theory is valid for any type of Bravais lattice and arbitrary form of inclusions in the unitary cell. An equations system is obtained for macroscopic magnetic field and magnetic induction components expanding microscopic electromagnetic fields in Bloch waves. Permittivity and permeability effective tensors are obtained comparing the equations system with an anisotropic nonlocal homogenous medium. In comparison with other homogenization theories, this work uses only two tensors: nonlocal permeability and permittivity. The proposal showed here is based on the use of permeability equations, which are exact and very simple. We present the explicit form of these tensors in the case of binary 1D photonic crystals.

1. Introduction

The material properties and their potential applications research are very important in the industry and scientific areas. However, all these impacts could not be possible without the great advances developed in the last 100 years in materials science. Thanks to metallurgy, ceramics, and plastics advances, engineers have been able to modify a wide range of mechanical and electrical properties in materials. Semiconductor physics advances have allowed adjusting the conductive properties of certain materials, initiating the electronics revolution. Scientists have discovered high-temperature superconductors with new alloys and ceramics. It is impossible to cite all applications derived from materials science. In recent years, scientists have a new challenge, to study artificial materials and their optical properties. A similar challenge is to study the mechanical properties of artificial materials [1, 2]. A great advance in this area is known as nanophotonics,

which tries to solve the information transmission lossless problem through light. Continuous search for new technologies has brought surprising properties to a new materials class known as photonic crystals (PC) [3]. These artificial materials can serve as the new raw material for photonic circuits age. These applications include both submicron sized laser sources as conducting channels (optical waveguides) and logical components (amplifiers, transistors photonic, etc.). Essentially, a photonic crystal is a material with a periodic arrangement composed of multiple elements distributed which scatters light in a coherent and cooperative way. This produces, a similar mechanism that have the electrons in semiconductors, a forbidden energy range for photons propagation, in this case a photonic forbidden gap (gap), which allows control light propagation. By combination and structuration of these elements, photonic circuits can be designed similar to electronics with cables, switches, splitters, and so forth.

FIGURE 1: Superlattice or 1D binary photonic crystal. Periodicity in Z direction.

If the wavelength of the light is greater than the photonic crystal lattice constant, the properties can be described by means of homogeneous medium effective parameters. These parameters can explain the optical properties of photonic crystals in a simple and clear manner. The calculation of these parameters is one of the great interesting theoretical problems.

Homogenization theory of photonic crystals with metallic and magnetic inclusions is presented. Explicit formulas for permittivity and not local permeability tensors calculation are obtained and the application of these relations to calculate the effective parameters of a dielectric-magnetic 1D periodicity photonic crystal is illustrated. Finally, the theoretical results are compared with respect to theories based on bianisotropic response.

2. Material Characterization

2.1. Lattice Homogenization with Metalomagnetic Inclusions. Let us consider a 1D PC composed of metallic and magnetic inclusions. A 1D binary crystal (composed of two materials) representation is shown in Figure 1.

The metalomagnetic isotropic inclusions properties of the photonic crystal will be fully described by two material parameters: the permittivity and permeability, which are position functions; therefore,

$$\begin{aligned}\varepsilon(z) &= \varepsilon_0 + \frac{i}{\omega} \hat{\sigma}(z), \mu(z) \\ \hat{\sigma}(z) &= \sigma(z) - i\omega\varepsilon_0\chi(z),\end{aligned}\quad (1)$$

having the same periodicity as the 1D PC. Here, $\sigma(z)$ and $\chi(z)$ are the usual conductivity and electric susceptibility for isotropic materials in the unit cell. Note that, for metals, $\hat{\sigma} = \sigma$, for dielectrics, $\hat{\sigma} = -i\omega\chi$, and, for air, $\hat{\sigma} = 0$ [4].

According to these assumptions, you can study electromagnetic fields behavior in PC at structural level using “microscopic” Maxwell’s equations:

$$\nabla \times \mathbf{h}(\mathbf{r}) = \mathbf{j}(\mathbf{r}) - i\omega\varepsilon_0\mathbf{e}(\mathbf{r}), \quad (2)$$

$$\nabla \cdot \mathbf{e}(\mathbf{r}) = -\frac{i}{\varepsilon_0\omega} \nabla \cdot \mathbf{j}(\mathbf{r}), \quad (3)$$

$$\nabla \times \mathbf{e}(\mathbf{r}) = i\omega\mathbf{b}(\mathbf{r}), \quad (4)$$

$$\nabla \cdot \mathbf{b}(\mathbf{r}) = 0. \quad (5)$$

Furthermore, current density $\mathbf{j}$ and magnetic induction $\mathbf{b}$ are related to magnetic field $\mathbf{h}$ and electric field $\mathbf{e}$ by materials equations:

$$\begin{aligned}\mathbf{j}(\mathbf{r}) &= \hat{\sigma}(z)\mathbf{e}(\mathbf{r}), \\ \mathbf{b}(\mathbf{r}) &= \mu(z)\mathbf{h}(\mathbf{r}).\end{aligned}\quad (6)$$

Given the 1D PC periodicity, the generalized conductivity and magnetic permeability can be expanded in Fourier series:

$$\begin{aligned}\hat{\sigma}(z) &= \sum_{G_z} \hat{\sigma}(G_z) e^{iG_z z}, \\ \mu(z) &= \sum_{G_z} \mu(G_z) e^{iG_z z},\end{aligned}\quad (7)$$

G_z being the 1D lattice reciprocal vectors.

3. Average Fields

The $\mathbf{e}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ “microscopic” fields within the periodic structure obey the Bloch theorem. Therefore, fields are obtained in Bloch wave form:

$$\begin{aligned}\mathbf{e}(\mathbf{r}) &= e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{G_z} \mathbf{e}(G_z) e^{iG_z z} \\ &= e^{i\mathbf{k}\cdot\mathbf{r}} \mathbf{e}(G_z = 0) + e^{i\mathbf{k}\cdot\mathbf{r}} \sum'_{G_z} \mathbf{e}(G_z) e^{iG_z z}, \\ \mathbf{b}(\mathbf{r}) &= e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{G_z} \mathbf{b}(G_z) e^{iG_z z} \\ &= e^{i\mathbf{k}\cdot\mathbf{r}} \mathbf{b}(G_z = 0) + e^{i\mathbf{k}\cdot\mathbf{r}} \sum'_{G_z} \mathbf{b}(G_z) e^{iG_z z},\end{aligned}\quad (8)$$

where $\sum'$ indicates that $G_z = 0$ term is excluded from the sum.

For macroscopic electromagnetic fields, the corresponding microscopic fields (8) are averaged over a length L much greater than the lattice constant a but much smaller than the wavelength $2\pi/k$ ($a \ll L \ll 2\pi/k$). This averaging procedure smooths the rapid oscillations of the microscopic fields within the interval of length L . Thus, in the case of wavelengths much larger than the lattice constant a ,

$$ka \ll 1, \quad (9)$$

and macroscopic fields are described as plane waves:

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &\equiv \langle \mathbf{e}(\mathbf{r}) \rangle = \mathbf{e}(G_z = 0) e^{i\mathbf{k} \cdot \mathbf{r}} = \mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r}}, \\ \mathbf{B}(\mathbf{r}) &\equiv \langle \mathbf{b}(\mathbf{r}) \rangle = \mathbf{b}(G_z = 0) e^{i\mathbf{k} \cdot \mathbf{r}} = \mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{r}}.\end{aligned}\quad (10)$$

Another equivalent way to define the macroscopic fields is to eliminate the $G_z \neq 0$ terms in (8) expansions. This definition is correct when inequality (9) is accomplished and the $G_z = 0$ component in such expansions proves to be a smooth function in variations over distances much larger than the lattice constant.

4. Permittivity and Permeability Effective

In a homogeneous magnetic medium, the constitutive equations that describe the relationship of the displacement $\mathbf{D}$ and the magnetic induction $\mathbf{B}$ vectors to the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ can be written as

$$\begin{aligned}\mathbf{D} &= \overleftrightarrow{\epsilon} \cdot \mathbf{E}, \\ \mathbf{B} &= \overleftrightarrow{\mu} \cdot \mathbf{H},\end{aligned}\quad (11)$$

where $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$ are the permittivity and permeability tensors. Maxwell's equations in this case are written in the form

$$\nabla \times \mathbf{H} = -i\omega \overleftrightarrow{\epsilon} \cdot \mathbf{E}, \quad (12)$$

$$\nabla \cdot \mathbf{D} = 0, \quad (13)$$

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}, \quad (14)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (15)$$

Removing the magnetic field $\mathbf{H}$ in (12) and (14) and using (11), the wave equation is obtained in electric field $\mathbf{E}$ terms:

$$\mathbf{k} \times \overleftrightarrow{\mu}^{-1} \cdot (\mathbf{k} \times \mathbf{E}_0) + \omega^2 \overleftrightarrow{\epsilon} \cdot \mathbf{E}_0 = 0. \quad (16)$$

Equivalently, the electric field $\mathbf{E}$ can be eliminated from (12) and (14), and write the wave equation for magnetic field $\mathbf{H}$:

$$\mathbf{k} \times \overleftrightarrow{\epsilon}^{-1} \cdot (\mathbf{k} \times \mathbf{H}_0) + \omega^2 \overleftrightarrow{\mu} \cdot \mathbf{H}_0 = 0. \quad (17)$$

To obtain photonic crystal effective tensors, (16) and (17) must be compared with macroscopic fields exact equations that satisfy $\mathbf{E}_0 = \mathbf{e}(0)$ and $\mathbf{H}_0 = \mathbf{h}(0)$ amplitudes. Equations (6)–(8) are substituted in (2) and (4) and expansions are used:

$$\frac{1}{\epsilon(z)} = \sum_{G_z} \eta(G_z) e^{-iG_z z}, \quad (18)$$

$$\frac{1}{\mu(z)} = \sum_{G_z} \xi(G_z) e^{-iG_z z}.$$

Removing the magnetic field components, an electric field components equations system is obtained:

$$\sum_{G'_z} \overleftrightarrow{\mathbf{D}}_{\mathbf{E}}(\mathbf{k}; G_z, G'_z) \cdot \mathbf{e}(G'_z) = 0, \quad (19)$$

where

$$\begin{aligned}\overleftrightarrow{\mathbf{D}}_{\mathbf{E}}(\mathbf{k}; G_z, G'_z) &= (\mathbf{k} + G_z \hat{z}) \\ &\times \xi(G_z - G'_z) (\mathbf{k} + G'_z \hat{z}) \times \overleftrightarrow{\mathbf{I}} \\ &+ (i\omega \hat{\sigma}(G_z - G'_z) + \epsilon_0 \omega^2) \overleftrightarrow{\mathbf{I}}.\end{aligned}\quad (20)$$

Here $\overleftrightarrow{\mathbf{I}}$ is the unitary dyadic. From equations with $G_z \neq 0$ (19), $\mathbf{e}(G_z)$ components can be expressed in $\mathbf{e}(0)$ component terms. Substituting in the $G_z = 0$ equation, equations system (19) takes the form

$$\left\{ \overleftrightarrow{\mathbf{D}}_{\mathbf{E}}^{-1}(\mathbf{k}; G_z = 0, G'_z = 0) \right\}^{-1} \cdot \mathbf{e}(G_z = 0) = 0. \quad (21)$$

Here, $\overleftrightarrow{\mathbf{D}}_{\mathbf{E}}^{-1}(0, 0)$ is a 3×3 block, obtained from the inverse of the matrix (20). Similarly, the magnetic field $\mathbf{e}(G_z)$ components equations system is obtained:

$$\sum_{G'_z} \overleftrightarrow{\mathbf{D}}_{\mathbf{H}}(\mathbf{k}; G_z, G'_z) \cdot \mathbf{h}(G'_z) = 0, \quad (22)$$

where

$$\begin{aligned}\overleftrightarrow{\mathbf{D}}_{\mathbf{H}}(\mathbf{k}; G_z, G'_z) &= (\mathbf{k} + G_z \hat{z}) \\ &\times \eta(G_z - G'_z) (\mathbf{k} + G'_z \hat{z}) \times \overleftrightarrow{\mathbf{I}} \\ &+ \omega^2 \mu(G_z - G'_z) \overleftrightarrow{\mathbf{I}}.\end{aligned}\quad (23)$$

Eliminating G_z nonzero components in (22), an $\mathbf{h}(0)$ homogeneous algebraic equation is obtained:

$$\left\{ \overleftrightarrow{\mathbf{D}}_{\mathbf{H}}^{-1}(\mathbf{k}; G_z = 0, G'_z = 0) \right\}^{-1} \cdot \mathbf{h}(G_z = 0) = 0, \quad (24)$$

where $\overleftrightarrow{\mathbf{D}}_{\mathbf{H}}^{-1}(0, 0)$ is a 3×3 block, from the inverse of the matrix (23).

5. Effective Parameters

Equating (16) and (17) with (21) and (24), respectively, a system of two algebraic equations that define the effective tensors $\overleftrightarrow{\epsilon}_{\text{eff}}$ and $\overleftrightarrow{\mu}_{\text{eff}}$ is obtained:

$$\begin{aligned}\overleftrightarrow{\epsilon}_{\text{eff}}(\mathbf{k}) &= \frac{1}{\omega^2} \left\{ \left[\overleftrightarrow{\mathbf{D}}_{\mathbf{E}}^{-1}(\mathbf{k}; 0, 0) \right]^{-1} - \mathbf{k} \times \overleftrightarrow{\mu}_{\text{eff}} \cdot \mathbf{k} \times \overleftrightarrow{\mathbf{I}} \right\}, \\ \overleftrightarrow{\mu}_{\text{eff}}(\mathbf{k}) &= \frac{1}{\omega^2} \left\{ \left[\overleftrightarrow{\mathbf{D}}_{\mathbf{H}}^{-1}(\mathbf{k}; 0, 0) \right]^{-1} - \mathbf{k} \times \overleftrightarrow{\epsilon}_{\text{eff}} \cdot \mathbf{k} \times \overleftrightarrow{\mathbf{I}} \right\}.\end{aligned}\quad (25)$$

FIGURE 2: Effective permittivity versus filling fraction.

FIGURE 3: Effective permeability versus filling fraction graphic representation.

In the long wavelengths limit case ($k = 0$), the effective tensors are given explicitly by

$$\begin{aligned}\overleftrightarrow{\epsilon}_{\text{eff}}(0) &= \frac{1}{\omega^2} \left[\overleftrightarrow{\mathbf{D}}_{\text{E}}^{-1}(0; 0, 0) \right]^{-1}, \\ \overleftrightarrow{\mu}_{\text{eff}}(0) &= \frac{1}{\omega^2} \left[\overleftrightarrow{\mathbf{D}}_{\text{H}}^{-1}(0; 0, 0) \right]^{-1}.\end{aligned}\quad (26)$$

6. Results

The results obtained with the theory developed in the previous section are described in this section. The effective permittivity and permeability for a 1D photonic crystal composed of alternating layers of dielectric and magnetic materials in the direction of z are calculated. The set of parameters for the calculation are: $\epsilon_b = 12.25$ and $\mu_b = 1$ for dielectric matrix (host) and $\epsilon_a = 13$ and $\mu_a = 8$ for isotropic ferrite layer (inclusion) [5].

In Figures 2 and 3, effective permeability and permittivity tensors perpendicular components $\epsilon_{\text{eff}} = \epsilon_{xx} = \epsilon_{yy}$ and $\mu_{\text{eff}} = \mu_{xx} = \mu_{yy}$ behavior is showed versus magnetic inclusion filling fraction ($f = d_a/a$, where d_a is the magnetic layer thickness).

Local approach was used in calculations; that is, $\mathbf{k} = 0$ (see (26) formulas). ϵ_{eff} permittivity and μ_{eff} permeability calculated are filling fraction f functions and are in agreement with classical quasistatic results: $\epsilon_{\text{eff}} = \epsilon_a f + \epsilon_b(1 - f)$, $\mu_{\text{eff}} = \mu_a f + \mu_b(1 - f)$.

The formulas obtained in the previous section are validated with these results and open the possibility for future studies of electromagnetic waves propagation in 1D PC non-local effects. In such case, it will be necessary to include in the formulas obtained in this work (25) the dispersion relation $\mathbf{k}(\omega)$. Considering the condition $ka \ll 1$ (9) gives nonlocal effective permittivity and permeability corrections.

7. Conclusions

The analytical relationships for 1D periodical photonic crystal effective permittivity and permeability tensors with periodicity a were obtained. Demonstrating that effective ϵ - μ material response of photonic crystal is not local (25), thus the photonic crystal homogenized effective parameters depend on wave vector. We point out that theoretical formalism developed in this work uses only two effective tensors: permittivity and permeability (25), unlike the bianisotropic response [4–7], which uses four effective tensors (permittivity, permeability, and two magnetoelectric tensors). Obviously, permittivity and permeability definitions used here and the bianisotropic approach [4–7] are different for finite wave vectors. However, in the local case ($\mathbf{k} = 0$) the definitions are in agreement. Unlike the previous works [5–11], in this limit, our formulas provide the dependence of the effective parameters in photonic crystals upon the filling fraction of the inclusions.

Competing Interests

The authors declare that they have no competing interests.

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