Improved Analysis for Squeezing of Newtonian Material between Two Circular Plates

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This article presents a scheme for the analysis of an unsteady axisymmetric flow of incompressible Newtonian material in the form of liquid squeezed between two circular plates. The scheme combines traditional perturbation technique with homotopy using an adaptation of the Laplace Transform. The proposed method is tested against other schemes such as the Regular Perturbation Method (RPM), Homotopy Perturbation Method (HPM), Optimal Homotopy Asymptotic Method (OHAM), and the fourth-order Explicit Runge-Kutta Method (ERK4). Comparison of the solutions along with absolute residual errors confirms that the proposed scheme surpasses HPM, OHAM, RPM, and ERK4 in terms of accuracy. The article also investigates the effect of Reynolds number on the velocity profile and pressure variation graphically.

1. Introduction

The study of squeezing flows has significant applications in the areas of engineering, physics, biology, and material sciences. In the past few years, the study of rheometric properties of fluids has garnered significant attention due to its vast industrial applications. Examples include modelling of lubrication systems involved in squeezing of fluids [1–3], compression moulding processes of metals and polymers [4], injection moulding processes, polymer processes [5], hydrodynamical tools and machines, modelling of chewing and eating [6], and modelling of the functions of heart valves and blood vessels. Many of these applications involve the adjustment of rheometric properties using external stimuli such as electric and magnetic fields. For instance, electrorheological fluids (micron size polymer particles in silicon) may solidify or become extremely viscous under an electric field. The same can be said about magnetorheological fluids involving magnetic particles. Under an applied external field, these particles remain suspended due to which fluid particles are not able to exhibit Brownian motion. As a result, the fluid can adopt viscous properties.

Traditional approaches to study flow patterns involve the configuration of two plates of radius α that are separated by a narrow gap ̃h(t). Three modes of operations on the plates are commonly used; stationary plates resulting in Poiseuille flow, shear mode resulting in Couette flow, and squeeze mode resulting in compressed flow. Some properties of the flow such as mass and momentum are not affected by deformations due to these operations and they remain conserved. The resulting set of properties such as velocity and pressure can be modelled as various boundary value problems. Some configurations may also focus on the interaction between the samples and the plates. The interactions can result in different types of stresses identified as slip, no-slip, or partial slip. Other configurations may also focus on types of fluid such as viscous, plastics (or viscoplastics), and elastic (or viscoelastic) fluids.

The solutions to these different configurations and boundary value problems can be obtained using well known analytical [7–14] and numerical schemes [15, 16]. The most common approach involves the usage of perturbation techniques that assume small or large parameters, which may affect the solutions in different scenarios. To overcome this
limitation, a method known as the Homotopy Perturbation Method was introduced that combined traditional perturbations with homotopy and was applied to various nonlinear boundary value problems [7, 8, 17–19]. Some modifications and extensions to the method have also appeared. Examples include that of dynamical systems of rotating machines [20], nonlinear differential equations [21], singular lane-embed equations [22], Cauchy reaction diffusion equation [23], nonlinear undamped oscillators [24], and fractional differential equations [25].

In this article, we present a detailed analysis of squeezing flow of Newtonian fluids using an improved alteration of the Homotopy Perturbation Method with Laplace Transform. We refer to this alteration as HPLM, that is, Homotopy Perturbation Laplace Method. To check the effectiveness of the proposed scheme, a comparison is performed with Regular Perturbation Method (RPM), Homotopy Perturbation Method (HPM), Runge-Kutta (Explicit) method of fourth-order, and Optimal Homotopy Asymptotic Method (OHAM). Moreover, the effect of Reynolds number on the velocity profile and pressure variation is studied graphically.

2. Mathematical Formulation

The formulation is based on an unsteady squeezing flow of an incompressible Newtonian fluid bearing kinematic viscosity $\nu$, density $\rho$, and viscosity $\mu$. The fluid is squeezed between a pair of circular plates that are at a distance of $2h(t)$. The plates operate in a squeezing mode where the plate velocity is $\epsilon(t)$. A 2D configuration of squeezing flow in $(z, r)$ plane is in Figure 1, where the plates move perpendicular to the central axis $z = 0$. With this operation, an axisymmetric behaviour is observed in the flow about $r = 0$. The velocity components (normal and longitudinal) in axial and radial directions are $w_r(z, r, t)$ and $w_z(z, r, t)$. The governing equations are

$$
\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z} = 0, 
$$

$$
\frac{\partial P}{\partial r} + \rho \left( \frac{\partial w_r}{\partial t} - w_z \Omega \right) = - \frac{\mu}{h} \frac{\partial \Omega}{\partial z}, 
$$

$$
\frac{\partial P}{\partial z} + \rho \left( \frac{\partial w_z}{\partial t} + w_r \Omega \right) = \frac{\mu}{r} \frac{\partial}{\partial r} (r \Omega),
$$

where the boundary conditions are

$$
w_r = 0, \\
w_z = \epsilon(t), \\
\frac{\partial w_r}{\partial \eta} = 0, \\
w_z = 0,
$$

for $\eta = 1$,

$$
\frac{\partial w_r}{\partial \eta} = 0,
$$

for $\eta = 0$.

Two kinds of boundary conditions are placed for the velocity components of the fluid. The first is due to no-slip at the upper plate, while the second is due to symmetry.

$$
w_r(z, r, t) = 0, \\
w_z(z, r, t) = v_w(t),
$$

for $z = h$,

$$
\frac{\partial}{\partial z} w_r(z, r, t) = 0, \\
w_z(z, r, t) = 0,
$$

for $z = 0$.

By introducing the dimensionless parameter $\eta = z/h(t)$, (1), (2), and (3) are transformed to

$$
\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{1}{h} \frac{\partial w_z}{\partial \eta} = 0, 
$$

$$
\frac{\partial P}{\partial r} + \rho \left( \frac{\partial w_r}{\partial t} - w_z \Omega \right) = - \frac{\mu}{h} \frac{\partial \Omega}{\partial \eta}, 
$$

$$
1 \frac{\partial P}{\partial \eta} + \rho \left( \frac{\partial w_z}{\partial t} + w_r \Omega \right) = \frac{\mu}{r} \frac{\partial}{\partial r} (r \Omega),
$$

where the boundary conditions are

$$
w_r = 0, \\
w_z = \epsilon(t), \\
\frac{\partial w_r}{\partial \eta} = 0, \\
w_z = 0,
$$

for $\eta = 1$,

$$
\frac{\partial w_r}{\partial \eta} = 0,
$$

for $\eta = 0$.

After eliminating the generalized pressure between (6) and (7), the following is obtained:

$$
\rho \left[ \frac{\partial \Omega}{\partial t} + w_r \frac{\partial \Omega}{\partial r} + \frac{w_z}{h} \frac{\partial \Omega}{\partial \eta} - \frac{w_r}{r} \Omega \right] = \mu \left[ \nabla^2 \Omega - \frac{\Omega}{r^2} \right], 
$$

where $\nabla^2$ is the Laplacian operator.

Defining velocity components as [4]

$$
w_r = - \frac{r \epsilon(t)}{2h(t)} G'(\eta), \\
w_z = \epsilon(t) G(\eta),
$$

(5) is identically satisfied, while (9) becomes

$$
\frac{d^4 G}{d \eta^4} + R \left( \eta - G \right) \frac{d^3 G}{d \eta^3} + 2 \frac{d^2 G}{d \eta^2} - Q \frac{d^2 G}{d \eta^2} = 0,
$$
where
\[ R = \frac{h c}{\eta}, \] (12)
\[ Q = \frac{h^2 d c}{\eta d}. \] Integrating first equation of (12) gives
\[ h(t) = \sqrt{c t + d}, \] (13)
where \( c \) and \( d \) are constants affecting the directional movement of the plates. From (12) and (13), it follows that \( Q = -R \).

Now, (11) becomes
\[ d^4 G d\eta^4 + R \left[ (\eta - G) d^3 G d\eta^3 + 3 d^2 G d\eta^2 \right] = 0. \] (14)

Using (8) and (10), the boundary conditions in case of no-slip at the upper plate are
\[ G(1) = 1, \]
\[ G'(1) = 0, \]
\[ G(0) = 0, \]
\[ G''(0) = 0. \] (15)
Furthermore, using (10) in (3) and integrating with respect to \( \eta \) while keeping \( r \) fixed, the variation of pressure is
\[ P(\eta, t) - P_c = \rho c^2 \left[ \eta G - \frac{G^2}{2} + \frac{1}{R} \left( G'(\eta) - G'(0) \right) \right], \] (16)
where \( P_c \) is the pressure at \( \eta = 0 \). Similarly, using (10) in (2) and integrating with respect to \( r \) while keeping \( \eta \) fixed lead to the pressure distribution.

3. Basic Theory of HPLM
The basic concept of HPLM can be understood by applying it to the following differential equation:
\[ \mathfrak{F}[u(x)] + \mathfrak{N}[u(x)] - f(x) = 0, \] (17)
where \( \mathfrak{F} \) and \( \mathfrak{N} \) are linear and nonlinear operators, while \( f(x) \) and \( u(x) \) are known and unknown functions. According to HPLM, a homotopy can be constructed as \( u(x, p) : \mathbb{R} \times [0, 1] \rightarrow \mathbb{R} \) such that it satisfies
\[ (p - 1) [f(x) - \mathfrak{F}(u(x, p))] - p [f(x) - \mathfrak{F}(u(x, p))] - \mathfrak{N}(u(x, p)) = 0, \] (18)
where \( p \in [0, 1] \) is an embedding parameter while \( x \in \mathbb{R} \). Expanding \( u(x, p) \) using a Taylor series about \( p \), we obtain an approximate solution:
\[ u(x, p) = u_0(x) + \sum_{k=1}^{m} u_k p^k. \] (19)
Various order problems can be obtained by substituting (19) into (18) and equating with the coefficients of \( p \). The zeroth-order problem would be
\[ \mathfrak{F}[u_0(x)] - f(x) = 0. \] (20)
Applying Laplace Transform to (20) gives
\[ s^n Lu_0(x) - s^{n-1} u_0(\alpha) - s^{n-2} u_0'(\alpha) - \cdots - u_0^{n-1}(\alpha) - L[f(x)] = 0. \] (21)
Application of the inverse Laplace Transform to (21) gives
\[ u_0(x) = L^{-1} \left[ \frac{1}{s^n} s^{n-1} u_0(\alpha) + s^{n-2} u_0'(\alpha) + \cdots + u_0^{n-1}(\alpha) + L[f(x)] \right]. \] (22)
The general \( k \)th order problem would be
\[ \mathfrak{F}[u_k(x)] - \mathfrak{N}[u_0, u_1, \ldots, u_{k-1}] = 0, \]
\[ k = 1, 2, 3, \ldots. \] (23)
Applying Laplace Transform to (23) gives
\[ s^n L u_k(x) - s^{n-1} u_k(\alpha) - s^{n-2} u_k'(\alpha) - \cdots - u_k^{n-1}(\alpha) - L[\mathfrak{N}[u_0, u_1, \ldots, u_{k-1}]] = 0. \] (24)
Application of the inverse Laplace Transform to (24) gives
\[ u_k(x) = L^{-1} \left[ \frac{1}{s^n} s^{n-1} u_k(\alpha) + s^{n-2} u_k'(\alpha) + \cdots + u_k^{n-1}(\alpha) + L[\mathfrak{N}[u_0, u_1, \ldots, u_{k-1}]] \right]. \] (25)
Using the boundary conditions, the approximate solution will be
\[ \bar{U} = \lim_{p \rightarrow 1} u = u_0 + u_1 + u_2 + \cdots. \] (26)
Substituting (26) into (17), the expression for residual can be obtained as
\[ \text{Residual error} = \mathfrak{F}[\bar{U}(x)] + \mathfrak{N}[\bar{U}(x)] - f(x). \] (27)
The approach defined in this section minimizes the limitations of the ordinary perturbation methods and, in contrast, can take full advantage of the traditional perturbation techniques.

4. Application of HPLM
Using (11) and (15), various order problems are presented with their solutions in this section. The zeroth-order problem is
\[ u_0^{iv}(\eta) = 0. \] (28)
with boundary conditions \( u_k(0) = 0, u_k'(0) = 0, u_k''(0) = A, \) and \( u_k'''(0) = B. \) The solution to (28) is given as

\[
u_0(\eta) = A\eta + \frac{B_1}{6}. \tag{29}\]

The first-order problem is

\[
u_1^{iv}(\eta) = -3Ru_0''(\eta) - Rxu_0'''(\eta) + R_0u_0(\eta)u_0''(\eta). \tag{30}\]

For (30) and the rest of higher order problems \( u_k^{iv}(\eta), \) the boundary conditions are set as \( u_k(0) = 0, u_k'(0) = 0, u_k''(0) = 0, \) and \( u_k'''(0) = 0, \) where \( k = 1, 2, 3, \ldots \) The solution to (30) is given as

\[
u_1(\eta) = \frac{1}{120}(-4BR + ABR)\eta^5 + \frac{B^2R}{5040}\eta^7. \tag{31}\]

The second-order problem is

\[
u_2^{iv}(\eta) = -3Ru_1''(\eta) + Ru_1(\eta)u_1'''(\eta) - Rxu_1'''(\eta) - R_0u_0(\eta)u_1''(\eta), \tag{32}\]

for which the solution is given as

\[
u_2(\eta) = \frac{1}{1680}(8BR^2 - 6ABR + A^2BR^2)\eta^7 + \frac{1}{90720}(-13B^2R^2 + 4ABR^2)\eta^9 + \frac{B^2R}{1108800}\eta^{11}. \tag{33}\]

The third-order problem is

\[
u_3^{iv}(\eta) = -3Ru_2''(\eta) + Ru_2(\eta)u_2'''(\eta) + Ru_1(\eta)u_1'''(\eta) - Rxu_2'''(\eta) + R_0u_0(\eta)u_2''(\eta), \tag{34}\]

for which the solution is given as

\[
u_3(\eta) = \left[ \frac{BR^3}{1209600}(-64 + 88A - 38A^2 + 5A^3)\right]\eta^9 + \frac{B^2R^3}{39916800}(1720 - 1340A + 241A^2)\eta^{11} \tag{35}\]

\[
+ \frac{B^2R}{3113510400}(-2750 + 923A)\eta^{13} + \frac{1051B^4R^3}{217945728000}\eta^{15} \right].
\]

The fourth-order problem is

\[
u_4^{iv}(\eta) = -3Ru_3''(\eta) + Ru_3(\eta)u_3'''(\eta) + Ru_2(\eta)u_2'''(\eta) + Ru_1(\eta)u_1'''(\eta) - Rxu_3'''(\eta) + R_0u_0(\eta)u_3''(\eta), \tag{36}\]

for which the solution is given as

\[
u_4(\eta) = \left[ \frac{BR^4}{13305600}(640 - 132888A + 996A^2 - 316A^3 + 35A^4)\eta^{11} + \frac{B^2R^4}{1037836800}(-8752 + 11592A - 4917A^2 + 665A^3)\eta^{13} + \frac{B^3R^4}{326918592000}(121474 - 95159A + 17641A^2)\eta^{15} + \frac{21919B^4R^4}{844757641728000}(-83432 + 29159A)\eta^{17} \right].
\]

In a similar way, higher order problems solutions can be obtained. Considering the fourth-order solution,

\[
\tilde{U}(\eta) = u_4(\eta) + u_3(\eta) + u_2(\eta) + u_1(\eta). \tag{38}\]

The use of boundary condition gives the values of unknown constants \( A \) and \( B \) for fixed values of \( R \) in (38). For \( R = 0.5, A = 1.5357, \) and \( B = -3.42755, \) the solution is represented as follows:

\[
\tilde{U}(\eta) = \left[ 1.5357\eta - 0.57125\eta^5 + 0.0351938\eta^7 + 5.81905 \times 10^{-4}\eta^9 - 2.02697 \times 10^{-4}\eta^9 - 5.93588 \times 10^{-7}\eta^{11} + 2.03112 \times 10^{-6}\eta^{13} - 4.72306 \times 10^{-8}\eta^{15} - 2.2477 \times 10^{-8}\eta^{17} - 7.67169 \times 10^{-10}\eta^{19} \right].
\]

The residual error of the problem is

\[
\text{Res. Error} = \frac{d^5\tilde{U}(\eta)}{d\eta^5} + R \left[ (\eta - \tilde{U}(\eta)) \frac{d^5\tilde{U}(\eta)}{d\eta^5} + 3 \frac{d^2\tilde{U}(\eta)}{d\eta^2} \right]. \tag{40}\]

5. Results and Discussion

In this article, an unsteady axisymmetric flow of incompressible Newtonian fluid squeezed between two circular plates is considered. The resulting nonlinear boundary value problem is solved analytically with HPLM and HPM and numerically with ERK4.

Tables 1, 2, and 3 present the comparison of ERK4, HPM, OHAM, and HPLM solutions and residual errors for fixed values of RPM and HPLM solutions and residual errors for fixed values of HPN, OHAM, and HPLM solutions along with absolute residual errors for fixed values of RPM and HPLM solutions and residual errors for fixed values of HPN, OHAM, and HPLM solutions along with absolute
**Table 1:** Comparison of solutions along with absolute residual errors for various analytical and numerical schemes at $R = 0.1$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>ERK4</th>
<th>HPM</th>
<th>OHAM [9]</th>
<th>HPLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
<td>1.738 $\times 10^{-5}$</td>
<td>0</td>
<td>0.697548</td>
</tr>
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<td>0.1</td>
<td>0.150158</td>
<td>5.21981 $\times 10^{-8}$</td>
<td>0.150158</td>
<td>3.15139 $\times 10^{-8}$</td>
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<tr>
<td>0.2</td>
<td>0.297237</td>
<td>2.69705 $\times 10^{-9}$</td>
<td>0.297237</td>
<td>7.73447 $\times 10^{-9}$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.43817</td>
<td>6.20969 $\times 10^{-12}$</td>
<td>0.43817</td>
<td>1.43931 $\times 10^{-8}$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5699</td>
<td>2.38437 $\times 10^{-10}$</td>
<td>0.5699</td>
<td>2.22904 $\times 10^{-9}$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.689397</td>
<td>2.77365 $\times 10^{-11}$</td>
<td>0.689397</td>
<td>2.87631 $\times 10^{-9}$</td>
</tr>
<tr>
<td>0.6</td>
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<td>2.2498 $\times 10^{-10}$</td>
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<tr>
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<td>5.10685 $\times 10^{-6}$</td>
<td>1.</td>
<td>3.634 $\times 10^{-8}$</td>
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</table>

**Table 2:** Comparison of solutions along with absolute residual errors for various analytical and numerical schemes at $R = 0.3$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>ERK4</th>
<th>HPM</th>
<th>OHAM [9]</th>
<th>HPLM</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
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<td>0</td>
<td>0.689397</td>
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<td>1.80166 $\times 10^{-7}$</td>
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<tr>
<td>0.3</td>
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**Table 3:** Comparison of solutions along with absolute residual errors for various analytical and numerical schemes at $R = 0.5$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>ERK4</th>
<th>HPM</th>
<th>OHAM [9]</th>
<th>HPLM</th>
</tr>
</thead>
<tbody>
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<td>2.53597 $\times 10^{-4}$</td>
<td>1.</td>
<td>1.13563 $\times 10^{-4}$</td>
</tr>
</tbody>
</table>
to other stated schemes. In addition to the above-mentioned tables, Table 5 shows various order solutions along with absolute residual errors and confirms the convergence of HPLM solution. Moreover, Table 6 indicates the comparison of HPLM solution with numerical (ERK4) solutions. It shows that HPLM results are in very good agreement with ERK4.

Furthermore, Figure 2 indicates the residual errors of HPLM, OHAM, HPM, and ERK4 for $R = 0.5$. In order to capture more details, the figure also shows an inset at a more finer resolution showing the residual errors between HPLM, OHAM, and ERK4. Figure 3 presents the comparison of second-order absolute residual errors between HPLM and RPM. The convergence of HPLM solution is shown in Figure 4, where the average absolute residual errors against various order solutions are presented.

Figure 5 demonstrates the pressure variation for various values of $R$. It is observed that when the plates approach each other the pressure at the plates is higher than that at the centre and vice versa.

Figures 6–9 show the effect of positive and negative values of Reynolds number $R$ on the velocity profiles. The effect of
negative $R$ shows an opposite effect on the velocity profile as compared to positive $R$.

6. Conclusion

A similarity solution for an unsteady axisymmetric squeezing flow is obtained using a modification of the HPM named as Homotopy Perturbation Laplace Method (HPLM). Analysis of the residual errors confirms that HPLM is an efficient method.
scheme as compared to other techniques presented in this article. The convergence and validity of the proposed HPLM scheme is verified by means of residual error and compared with numerical solutions. The analysis of obtained result shows that HPLM can be effectively used in various fields of science and technology as it gives improved results in terms of accuracy.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


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