A Nonlinear Strength Criterion for Frozen Sulfate Saline Silty Clay with Different Salt Contents

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1.Introduction

The essential difference between frozen saline soils and unfrozen soils is the ice and salt existing in frozen saline soils, which has an important influence on cohesive strength of the soil particles. The effect of salt content on the mechanical properties of frozen saline silty clay is considerable. Research results suggest that sulfate soil is widely distributed in the western China [1]. With the implementation of the Belt and Road Initiative, infrastructure constructions in the western China, such as metro engineering, tunnel construction, bridges, and other engineering activities, will be of top priority. The strength of frozen soils has great design guidance for construction in the western China. So far, research on strength characteristics of frozen saline silty clay is rarely reported. The strength characteristics of frozen saline soils are urgently needed.

The Mohr–Coulomb strength criterion, which has linear strength envelopes, is one of the widely used strength criteria nowadays. However, a majority of experimental results show that the strength envelope for most of the geomaterials is nonlinear. The linear Mohr–Coulomb strength criterion is not suitable for most of the geomaterials, such as frozen soils and sand. Using the nonlinear strength criterion to reflect strength characteristics of most of the geomaterials is more reasonable. There are some nonlinear strength criteria for geomaterials. So far, for sandy soils, there are two popular strength criteria, MN strength criterion [2] and LD strength criterion [3], respectively. Later, many researchers [4, 5] proposed new strength criteria on the basis of them. Yao et al. [6] proposed a unified hardening (UH) model for overconsolidated clays based on the MN criterion and the corresponding transforming stress method. Frozen soils are made up of mineral particles, ice inclusions, liquid water, and gaseous inclusions [7]. Compared with unfrozen soils, the strength and deformation characteristics of frozen soils are more complex owing to the existence of ice, so these
strength criteria are unreasonable for frozen soils. A series of studies [8–23] on the strength and deformation characteristics of frozen soils were carried out. The experimental results showed that the strength of frozen soils increases with increasing confining pressures at first, but with a further increase in confining pressures, the strength decreases for pressure melting and crushing phenomena under high confining pressures [21, 24, 25]. And many strength criteria were presented for frozen soils. Fish [26] and Ma et al. [27] presented parabolic yield criteria for frozen soils, respectively. Lai et al. [21] presented a nonlinear Mohr criterion which can describe the pressure melting and crushing phenomena of frozen soils. Yang et al. [28, 29] established a new strength criterion for frozen soils based on MN and LD criteria, and then, Yang et al. [30] presented a better one based on the Hoek–Brown criterion.

Compared with general frozen soils, the strength criterion of frozen saline silty clay is more complex than that of general frozen soils for the existence of salt crystals. Liao et al. [31] found that the existence of salt lowers the freezing temperature and salt content can influence the strength of frozen soils. It was proven that the content of sodium sulfate has an important influence on the mechanical properties of frozen soils [32], so strength criteria for general frozen soils and unfrozen soils are not available for frozen saline silty clay. Liao et al. [31] established a strength criterion for frozen sodium sulfate saline soils, which can reflect the influence of salt content on strength. The studies mentioned above are the latest developments in strength criterion and represent a great advance in the investigation on this domain.

In this paper, a series of conventional triaxial compression tests were carried out for frozen saline silty clay with Na₂SO₄ contents 0.0, 0.5, 1.5, and 2.5% under confining pressure from 0 MPa to 18 MPa at −6°C, respectively. Based on the experimental results, a new strength criterion was proposed to describe strength characteristics of frozen saline silty clay, which can well reflect the nonlinear strength characteristic of frozen saline silty clay and the influence of salt content on strength.

2. Test Conditions and Results

2.1. Description of the Test and Test Conditions. The soil studied was taken from Qinghai–Tibet plateau, and its grain size composition is given in Table 1. The soil sample is approximately 22%; (2) mix the mixture of soil and salt; (3) samples were made by testing mold. The experimental apparatus is the MTS-810 material testing system. The cylinder's soil samples with 61.8 mm in diameter and 125 mm in height were prepared. The test procedure was as follows: (1) the soil samples were saturated with the vacuum method so that the saturated water content of the soil sample is approximately 22%; (2) the soil samples were frozen at −6°C for 24 h using the cooling equipment; (3) the soil samples were put into a pressure cell and were kept for 2 h at −6°C using the incubator; (4) the soil samples were loaded under the confining pressures of 0–18 MPa; and (5) triaxial stress was applied. The loading rate during the testing process is 1.25 mm/min according to [33].

2.2. Test Results. Figure 1 shows the typical stress-strain curves of frozen saline silty clay with Na₂SO₄ content 1.5% under confining pressures from 0 MPa to 18 MPa at −6°C, respectively.

From Figure 1, it is found that the confining pressures have a large influence on the deviator stress-axial strain relationship of frozen saline silty clay. The frozen saline silty clay presents strain hardening, and the stress-strain curves have no peak values for ε ≤ 15%, when confining pressures range from 0 MPa to 18 MPa at −6°C. The frozen saline silty clay produces volume contraction at first and then produces volume expansion when confining pressure < 5 MPa. But with a further increase in confining pressures, the frozen saline silty clay only produces volume contraction. The strength of frozen saline silty clay increases with increasing confining pressures, when confining pressure < 10 MPa. The strength of frozen saline silty clay decreases with increasing confining pressure, when confining pressure > 10 MPa. According to the research of Lai et al. [22], in this paper, the values of σ₁ - σ₃ at ε₃ = 15% are taken as the strength of frozen saline silty clay when confining pressures range from 0 MPa to 18 MPa. Figure 2 shows the stress-strain curves of frozen saline silty clay with Na₂SO₄ contents 0, 0.5, 1.5, and 2.5% under the confining pressure of 8 MPa at −6°C, respectively. From Figure 2, it can be seen that the salt contents have an important influence on the strain-strain relationship.

3. Strength Theory of Frozen Saline Silty Clay

Generally, the failure condition can be given by the general state of stress:

\[ F(\sigma_{ij}, k_{ij}) = 0, \]

where \( F \) is the strength function, \( \sigma_{ij} \) is the second-rank symmetric stress tensor, and \( k_{ij} \) is the strength parameter.
Deviator stress $\sigma_{1} - \sigma_{3}$ (MPa) (3) can be expressed by principal stress invariants, respectively. According to the research of Liao et al. [31], Equation (3) can be expressed as

$$q = f_{p-q}g(\theta),$$  \hspace{1cm} (4)

where $f_{p-q}$ stands for the critical state curve on the $p-q$ plane, $g(\theta)$ represents the shape function on the deviatoric plane, and $\theta$ is Lode’s angle.

3.1. Critical Strength Curves of Frozen Saline Silty Clay in $p-q$ Plane. Referring to the research of Chu [34], the critical strength curves of frozen saline silty clay with different salt contents are shown in Figure 3. From Figure 3, the following can be observed. (1) The CSL of frozen saline silty clay with different salt contents passes through the different isotropic tensile points. The reason is that the existence of salt has an important influence on the cohesive strength of frozen soils. (2) The CSL of frozen saline silty clay with different salt contents is nonlinear, and the slope of the CSLs is positive when hydrostatic pressure is relatively small. When hydrostatic pressure reaches a certain value, the CSL is gradually bended downward with the increase of hydrostatic pressure, and the slope of the CSL is negative accordingly. (3) The slope $k$ of the CSL of frozen saline silty clay under different salt contents is different when the hydrostatic pressure $p<8$ MPa, in detail $k_{0.5}<k_{0}<k_{1.5}<k_{2.5}$. And the relationship between shear strength and hydrostatic pressure can be expressed by

$$f(p, q) = \frac{a(s)(p/p_s)^{2} + b(s)(p/p_s) + c(s)}{(p/9.8687p_s) + d(s)}p_s - q = 0,$$  \hspace{1cm} (5)

where $p_s$ is the standard atmospheric pressure, $p_s = 0.10133$ MPa. The parameters $a(s)$, $b(s)$, $c(s)$, and $d(s)$ were gotten by the fitting curves. $a(s)$, $b(s)$, $c(s)$, and $d(s)$ are the material parameters related to salt contents at $-6^\circ$C.
Based on the Mohr strength theorem, the failure envelope can be written as

\[ F(\sigma, \tau, \sigma_1) = \left( \sigma - \frac{\sigma_1 + \sigma_3}{2} \right)^2 + r^2 - \left( \frac{\sigma_1 - \sigma_3}{2} \right)^2 = 0. \quad (7) \]

For the conventional triaxial tests, \( p = \frac{1}{3} (\sigma_1 + 2\sigma_3) \), \( q = \sigma_1 - \sigma_3 \). Substituting \( p, q \) into Equation (7), Equation (7) can be written as

\[ F(p, q) = \left( \sigma - p - \frac{q}{6} \right)^2 + r^2 - \left( \frac{q}{2} \right)^2 = 0. \quad (8) \]

Based on the envelope theory, we can obtain the following formula:

\[ \frac{\partial F}{\partial p} \frac{\partial f}{\partial q} - \frac{\partial F}{\partial q} \frac{\partial f}{\partial p} = 0. \quad (9) \]

Differentiating Equations (5) and (8), we can get

\[ \frac{\partial f}{\partial q} = -1, \]

\[ \frac{\partial f}{\partial p} = \left[ \left( \frac{p}{9.8687 \sigma_3} \right) + d(s) \right] \left[ a(s) \left( \frac{p}{\sigma_3} \right)^2 + b(s) \left( \frac{p}{\sigma_3} \right) + c(s) \right] \frac{1}{\left[ \left( \frac{p}{9.8687 \sigma_3} \right) + d(s) \right]^2} \]

\[ \frac{\partial F}{\partial q} = -\frac{1}{3} \left( \sigma - p - \frac{q}{6} \right) - \frac{q}{2}, \]

\[ \frac{\partial F}{\partial p} = -2 \left( \sigma - p - \frac{q}{6} \right). \]

By substituting Equations (10), (11), (12), and (13) into Equation (9), we can get the following relationship
Liao et al. [31]. (4) Under any salt content, when the phenomenon so that its strength is gradually reduced, and the greater the confining pressure, the more obvious the phenomenon under high confining pressures. xV_ The angles obtained by Equation (16) will become negative. xV_ The curves between friction angle and hydrostatic pressure with different salt contents 0.0, 0.5, 1.5, and 2.5%, shown in Figure 4. From Figure 4, the following can be observed. (1) Under any same hydrostatic pressure, the friction angles reached a minimum value when salt content is equal to 0.5%. (2) Under each kind of salt content, the friction angles decrease with the increase of hydrostatic pressure. (3) Salt content has an important influence on the friction angles when the hydrostatic pressure $p < 18$ MPa. The influence of salt content on the friction angles is less obvious when $p > 18$ MPa. This phenomenon is contrary to the finding of Liao et al. [31]. (4) Under any salt content, when the hydrostatic pressure $p$ is beyond a certain value, the friction angles obtained by Equation (16) will become negative. The main reason to result in this phenomenon is the pressure melting phenomenon under high confining pressures. The greater the confining pressure, the more obvious the phenomenon so that its strength is gradually reduced, and the strength envelope bends downwards gradually, so the calculated friction angles is negative.

Using Equations (14) and (15), we can get the shear stress $\tau$ as follows:

$$\tau = \left[\frac{2}{3} \left(\frac{(p + q/6) - q/2}{(p/9.8687p_s + d/s) + b(s) + c(s)} \right)^{1/3} \right] \frac{\sqrt{3}}{2} \left(\frac{8 + \sin^2 \varphi_0 - \sin \varphi_0}{2 + \sin^2 \varphi_0 \cos \theta} \right)^{1/2} \left[\frac{1}{(3/2) \cos \theta} \left(\frac{3}{2 + \sin^2 \varphi_0} \right)^{3/2} \sin \varphi_0 \cos 3\theta \right]$$

(15)

The friction angle $\varphi$ of frozen saline silty clay can be calculated by

$$\varphi = \arctan \left( \frac{d\tau}{d\delta} \right) = \arctan \left( -\frac{\partial f/\partial \sigma}{\partial f/\partial \tau} \right)$$

(16)

From Equations (14), (15), and (16), it can be seen that the friction angle $\varphi$ of frozen saline silty clay is related to salt content. Using Equations (14), (15), and (16), we can get the curves between friction angle $\varphi$ and hydrostatic pressure with different salt contents 0.0, 0.5, 1.5, and 2.5%, shown in Figure 4. From Figure 4, the following can be observed. (1) Under any same hydrostatic pressure, the friction angles reached a minimum value when salt content is equal to 0.5%. (2) Under each kind of salt content, the friction angles decrease with the increase of hydrostatic pressure. (3) Salt content has an important influence on the friction angles when the hydrostatic pressure $p < 18$ MPa. The influence of salt content on the friction angles is less obvious when $p > 18$ MPa. This phenomenon is contrary to the finding of Liao et al. [31]. (4) Under any salt content, when the hydrostatic pressure $p$ is beyond a certain value, the friction angles obtained by Equation (16) will become negative. The main reason to result in this phenomenon is the pressure melting phenomenon under high confining pressures. The greater the confining pressure, the more obvious the phenomenon so that its strength is gradually reduced, and the strength envelope bends downwards gradually, so the calculated friction angles is negative.

Using Equations (14) and (15), we can get the shear stress envelope of frozen saline silty clay in the $\sigma - \tau$ space, shown in Figure 5. From Figure 5, it can be seen that the strength envelope is able to reflect the strength of frozen saline silty clay considering the influence of confining pressures well.

3.2. Strength Formulation in $\pi$ Plane. The experiments show that failure curves of some geomaterials in $\pi$ plane are close to the SMP criterion [35], so we take the shape function of the SMP criterion in $\pi$ plane as the shape function of frozen saline silty clay in the $\pi$ plane. Matsuioka et al. [36] gave the shape function for the SMP criterion in $\pi$ plane as follows:

$$g(\theta) = \frac{\sqrt{3}}{4} \left( \frac{8 + \sin^2 \varphi_0 - \sin \varphi_0}{2 + \sin^2 \varphi_0 \cos \theta} \right)^{1/2} \left[\frac{1}{(3/2) \cos \theta} \left(\frac{3}{2 + \sin^2 \varphi_0} \right)^{3/2} \sin \varphi_0 \cos 3\theta \right]$$

(17)

$$\theta = \frac{1}{3} \cos^{-1} \left[ -\left(\frac{3}{2 + \sin^2 \varphi_0} \right)^{3/2} \sin \varphi_0 \cos 3\theta \right]$$

(18)

$$\tan \varphi_0 = \frac{2\sqrt{2}}{3} \tan \varphi,$$

(19)

where $\varphi_0$ is the friction angle in the SMP.

In this paper, the shape function for frozen saline silty clay in $\pi$ plane can be written as

$$g(\theta) = \frac{\sqrt{3}}{4} \left( \frac{8 + \sin^2 \varphi_0 - \sin \varphi_0}{2 + \sin^2 \varphi_0 \cos \theta} \right)^{1/2} \left[\frac{1}{(3/2) \cos \theta} \left(\frac{3}{2 + \sin^2 \varphi_0} \right)^{3/2} \sin \varphi_0 \cos 3\theta \right]$$

(18)

Finally, using Equations (4), (5), and (20), the entire strength criterion for frozen saline silty clay can be expressed as

$$F = \frac{a(s)(p/p_0)^2 + b(s)(p/p_0 + c(s)}{(p/9.8687p_s + d/s)P_s}$$

(21)

$$\times \left[ \frac{\sqrt{3}}{4} \left( \frac{8 + \sin^2 \varphi_0 - \sin \varphi_0}{2 + \sin^2 \varphi_0 \cos \theta} \right)^{1/2} \right] - q = 0.$$
Figure 4: Relationship between hydrostatic pressure and friction angle of frozen saline silty clay with different salt contents.

Figure 5: The shear strength envelope of frozen saline silty clay.

Figure 6: The failure surface and the shape function curves for frozen saline silty clay with salt content 1.5%: (a) principal stress space; (b) π plane.
plane is a curved triangle when the hydrostatic pressure $p = 1$ MPa; with the increase of the hydrostatic pressure, the shape function changes from curved triangle to circle, and the shape function is approximately a circle when the hydrostatic pressure $p = 18$ MPa. The shape function curves with different salt contents in the $\pi$ plane are shown in Figure 7. From Figures 7(a) and 7(b), it can be seen that the shape function changes versus salt content variation from 0.0% to 2.5% when the hydrostatic pressure $p$ is a constant. The innermost curve is the shape function curve of frozen saline silty with salt content 0.5%, while the outermost curve is the shape function curve of frozen saline silty with salt content 2.5%. Therefore, the strength reached a minimum value when salt content is equal to 0.5%, while it reached a maximum value when salt content is equal to 2.5%.

4. Conclusions

A series of conventional triaxial compression tests were carried out for frozen saline silty with Na$_2$SO$_4$ contents 0.0, 0.5, 1.5, and 2.5% under confining pressures ranging from 0 MPa to 18 MPa at $-6^\circ$C, respectively. From the test results, the following conclusions can be made:

1. The strength of frozen saline silty clay presents obvious nonlinearity under each salt content, and the strength of frozen saline silty clay increases with increasing confining pressures under low confining pressures, but the strength decreases with a further increase in confining pressures.

2. The salt contents have an important influence on strength of frozen saline silty clay. The strength reaches a minimum value when salt content is equal to 0.5%, while it reaches a maximum value when salt content is equal to 2.5%.

3. The slope $k$ of the CSL of frozen saline silty clay under different salt contents is different when the hydrostatic pressure $p < 8$ MPa. In this study, it is $k_{0.5} < k_0 < k_{1.5} < k_{2.5}$.

4. The strength criterion proposed in this paper could well reflect the effects of confining pressures and salt contents on strength properties of frozen saline silty clay. The shape function in the $\pi$ plane changes from curved triangle to circle with the increases of the hydrostatic pressure.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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