

Review Article

Functionally Graded Materials: An Overview of Stability, Buckling, and Free Vibration Analysis

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Functionally graded materials (FGMs) are novel materials whose properties change gradually with respect to their dimensions. It is the advanced development of formerly used composite materials and consists of two or more materials in order to achieve the desired properties according to the application where an FGM is used. FGMs have obtained a great attention of researchers in the past decade due to their graded properties at every single point in various dimensions. The properties of an FGM are not identical to the materials that constitute it. This paper aims to present an overview of the existing literature on stability, buckling, and free vibration analysis of FGM carried out by numerous authors in the past decade. Moreover, the analyses of mathematical models adopted for the aforementioned analyses are not the core purpose of this paper. At the end, future work is also suggested in this review paper.

1. Introduction

Materials have been playing an important role in the life of human beings since the first man on Earth. In different eras, man has used different materials or made composites for the sake of their ease in numerous applications. Initially, bronze was frequently used which is actually an alloy of tin and copper. Bronze was first invented in 3700 BC, the era known as the Bronze Age [1]. In 1200 BC, iron was also discovered and remained of interest for the people to yield different objects in the era known as the Iron Age. After that, a number of different alloys of metals and nonmetals were engineered for multiple purposes. Composite materials then attained great attention from researchers due to their wide range of application. Composite materials are lighter and stronger and can also provide design flexibility. They provide resistance to corrosion as well as wear. The disadvantage of composite materials is a sharp transition of properties at the

junction of materials which leads to component failure by the process of delamination. To overcome the drawback of conventional composite materials, a new breed of composite materials named functionally graded materials (FGMs) was first invented in 1984 by Japanese researchers for the core purpose of their aerospace project [2] that required thermal barrier with the outside temperature of 2000 k and inside 1000 k within 10 mm thickness. A decade before, Shen and Bever [3] also worked on graded structure composite materials, but it was delayed due to unsophisticated fabrication equipment [4]. So far, it has been used almost in every field, for example, biomedical, chemical, nuclear, mining, and power plant. FGMs occur in nature as bones, teeth, bamboo trees, human skin, and so on to meet the specified requirement of human beings and environment.

The number of research publications has increased significantly in past two decades [5]. FGMs replace the sharp transition of properties with smooth and continuous varying

properties of the material such as physical, chemical, and mechanical like Young's Modulus, Poisson's ratio, Shear Modulus, density, and coefficient of thermal expansion in a desired spatial direction [6–9] (Figure 1). The gradual changes in volume fraction of constituent and nonidentical structure at preferred direction give continuous graded properties like thermal conductivity, corrosion resistivity, specific heat, hardness, and stiffness ratio [11]. All these advantages made FGMs far better than homogenous composite material to use in multiple applications. Due to prominent characteristics of FGMs, several efforts have been put from time to time by researchers to enhance the properties of FGMs. Several types of FGMs have been introduced up till now based on size and structure. Moreover, a number of fabrication processes can be adopted to manufacture FGMs like gas based method, liquid process method, and solid process method.

2. Evolution of FGM

The term functionally graded material was introduced by a scientist of Japan in 1984 while working on a material being capable of withstanding high temperature. Soon, the importance of FGMs was realized, and to promote research in this area, a five year research based national project with a cost of \$11 Million was started as "Research on the basic Technology for the development of FGM for relaxation of thermal stress" (FGM PART 1) [12]. At the end of this project, researchers were able to develop 300 mm square shell and 50 mm hemispherical bowl for SiC-C FGM nose cones [13]. Another 5-year-project that was a consequence of FGM PART 1 was started in 1992 with a cost of \$9 Million called "Research on Energy Conversion Materials with Functionally Graded Structures" (FGM part 2). This project was focused to enhance energy conversion efficiency using functionally graded structure technology [14]. Furthermore, in April 1996, the New Energy and Industrial Technology Department Organization (NEDO) funded a project with a budget of \$2.5 million known as "Precompetitive Processing and characterization of Functionally Graded Materials." The project was continued until March 2000. The purpose of the project was to develop metal-ceramic FGM on an industrial level using spark plasma sintering (SPS) technique. Polyamide/Cu was one of the FGMs successfully manufactured by SPS technique [13]. Most of the research was conducted on the grading of mechanical and thermal properties. However, it was needed to work on basic properties like physical and chemical. In order to fill this gap, the Ministries of Education, Science, Sports and Culture granted a research program in April 1996 entitled, "Physics and Chemistry of FGMs" that was continued for the next three years until 1999. Physics, Chemistry, Biology, and Agriculture, etc., were the fields investigated in this project [15]. Figure 2 represents the hierarchy of modern material.

3. Fabrication Process of FGM

The fabrication process is one of the most crucial fields in FGM research. A number of research papers have been

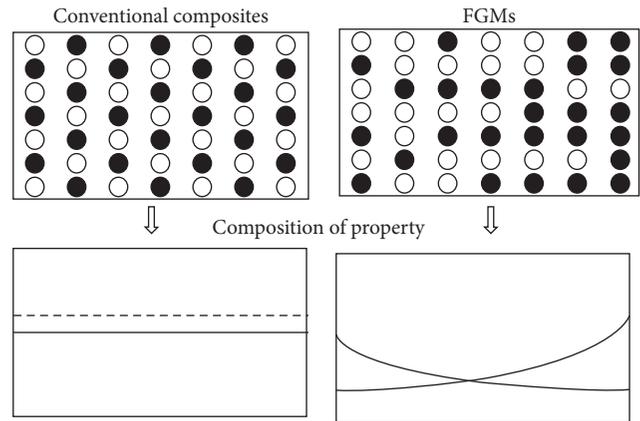


FIGURE 1: Variation of properties in conventional composites and FGM [10].

published till to date on the process techniques of FGM yielding new methods of FGM manufacturing. Based on constructive processing and mass transport processing techniques, FGM can be divided into two major categories [17]. In constructive processing, the FGM is made layer by layer starting with an appropriate distribution in which the gradients are literally fabricated in space, while in mass transport, the gradients within a component are dependent on natural transport phenomena, such as heat conduction, diffusion of atomic species, and flow of fluid [10]. However, advancement in automation technology in the past two decades has made constitutive gradation process both technically and economically more feasible. Table 1 shows fabrication methods while Table 2 shows comparison of processing processes of FGM. The most updated techniques of FGM processing are explained below.

3.1. Vapor Deposition Technique. A number of vapor deposition techniques are now adopted by manufacturers including sputter deposition, chemical vapor deposition, physical vapor deposition, plasma-enhanced chemical vapor deposition, and so on. Using the vapor deposition method, the material is used to condense in a vapor phase through chemical reaction, condensation, or conversion to form a solid material [17]. The aforementioned techniques are fruitful to change the material properties like electrical, mechanical, optical, and thermal. Using these methods, the functionally graded surface coatings are deposited which in turn can supply marvelous microstructure for thin surface coatings. Using vapor deposition techniques, poisonous gases are yielded as a by-product [21].

3.2. Powder Metallurgy. Four steps are involved in powder metallurgy for the production of functionally graded materials [22–24]. These are powder preparation, weighting and mixing of powder, stacking and ramming of premixed powders, and finally sintering [25]. A number of methods are used for preparation of powder like chemical reaction, electrolytic deposition, atomization, solid state reduction, centrifugal disintegration, grinding, pulverization, etc. The

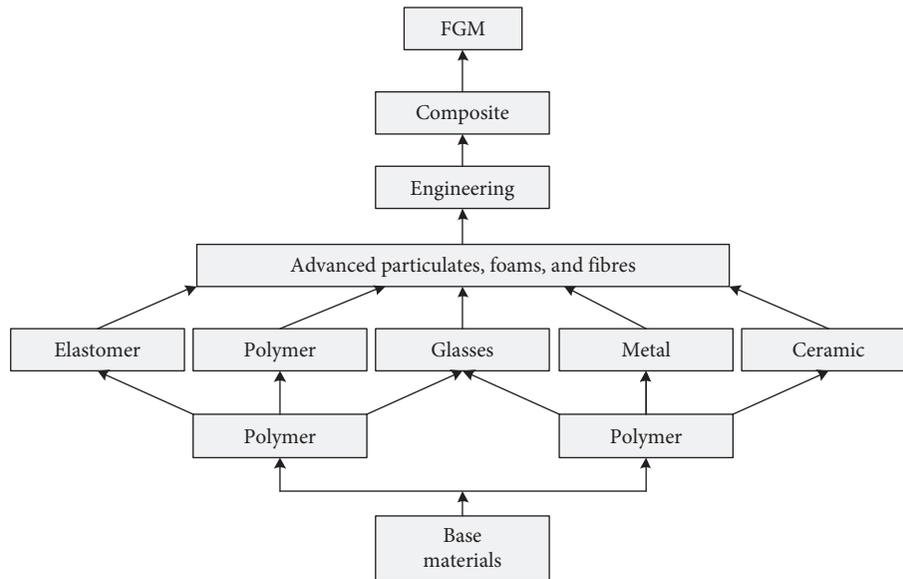


FIGURE 2: Representation of modern material hierarchy [16].

TABLE 1: Fabrication methods of FGM [18].

Liquid-state process	Solid-state process	Deposition process
Settling	Diffusion bonding	Electro deposition
Centrifugal casting		Laser deposition
Infiltration		Vapor deposition
Directional Filling	Powder metallurgy	
Solidification		Spray deposition
Controlled method		

forming process includes compacting of powder into geometric form, and pressing is usually completed in a room temperature [21]. Compatibility insured the strength of pressed and unsintered part [25]. The sintered part is usually made without a particular structure. During the process, some pores may occur which can be removed from secondary process [10].

3.3. Centrifugal Casting. In the centrifugal casting method, the functionally graded material is produced by spinning the mold using gravitational force. Metal in molten state is used to put into spinning mold, and it continues to spin until the metal becomes solidified [10]. Cylindrical parts are usually made through this method. Using this method, the density of metal increased and the mechanical properties of the casting may increase by 10 to 15% [19]. Difference in the centrifugal force which is produced by the density difference in molten and solid particles creates compositional gradient in FGM [4, 26]. From the literature review, it was found that there is limitation on gradient due to its production of natural process (i.e., centrifugal force and density difference).

3.4. Solid Freeform Fabrication Method. The solid freeform fabrication method is one of the most adapted methods for

the production of physical shapes with the help of computer-generated information about the object [10]. This method has an ability to vary the internal composition of materials [27, 28]. This method has many advantages over the other methods such as less energy consumption, higher manufacturing speed, efficient utilization of material, and being capable of producing complex shapes and design [27]. In the solid freeform fabrication method, the laser-based process is widely used for the fabrication of FGM [21].

4. Stability Analysis of FGM

4.1. FGM Shells. Natural frequencies, buckling stress, distribution of displacement, and stress components of FG circular cylindrical shells can be anticipated exactly using 2D higher order deformation theory [29]. The buckling pressure, fundamental cyclic frequencies, and relevant wave number of FG conical shells were obtained using Galerkin Method [30]. The effect of FG composite coatings on critical axial load depends on volume fraction or geometric parameters of FG shells (Deniz et al. [31]). Instability region of FG microshells is inversely proportional to dimensionless length scale parameter and directly proportional to static load factor (Sahmani et al. [32]). Dung et al. [33] worked on the stability of FG truncated conical shells. Results exhibit that critical buckling load and stability both increase when a quantity of stiffeners increases on a conical shell, and it is much affected by foundation parameters. The dynamic stability of a periodic FGM shell conveying fluid for different scope of dimensionless fluid density can be enhanced by increasing the length of a shell, and the main shell structure should adopt periodicity, whereas the dynamic stability varies inversely with the density of a shell (Shen et al. [34]). Anh et al. [35] did stability analysis of FGM shells mounting on elastic foundation. External pressure and elastic foundation play an important role in bifurcation buckling load, temperature resistance ability, and mechanical loading of

TABLE 2: Comparison of different processing processes of FGM [19, 20].

No.	Process	Variability of transition function	Versatility in phase content	Type of FGM	Versatility in components geometry
1	Powder stacking	Very good	Very good	Bulk	Moderate
2	Sheet lamination	Very good	Very good	Bulk	Moderate
3	Wet powder	Very good	Very good	Bulk	Moderate
4	Slurry dipping	Very good	Very good	Coating	Good
5	Jet solidification	Very good	Very good	Bulk	Very good
6	PVD, CVD	Very good	Very good	Bulk	Moderate
7	GMFC process	Very good	Moderate	Bulk	Good
8	Filtration/slip	Very good	Very good	Bulk	Good
9	Laser cladding	Very good	Very good	Bulk, coating	Very good
10	Thermal spraying	Very good	Very good	Bulk, coating	Good
11	Sedimentation	Good	Very good	Bulk	Poor
12	Diffusion	Moderate	Very good	Join bulk	Good
13	Directed solidification	Moderate	Moderate	Bulk	Poor
14	Electrochemical gradation	Moderate	Good	Bulk	Good
15	Foaming of polymer	Moderate	Good	Bulk	Good

FGM shells. Huang and Han [36] studied the elastoplastic buckling analysis of FGM cylindrical shells experiencing external pressure. Based on J2 deformation theory, it was pointed out that the elastic, elastoplastic, and plastic buckling zones of FGM cylindrical shells can be differentiated. Loading capacity, buckling, and postbuckling of ES-FGM elliptical cylindrical shells are enormously affected by geometric parameters, volume fraction, stiffeners, and elastic foundation (Duc et al. [37]). Sofiyev and Kuruoğlu [38] analyzed the stability of FGM-truncated conical shells. The effect of shear deformation and FG profile on axial load and critical and combined hydrostatic pressure was discussed using classical shell theory, shear deformation theory, and geometric parameters. Sofiyev [30] did stability analysis of FGM conical shells. Various results were obtained showing the behavior of both dimensional and nondimensional critical axial load under the impact of numerous parameters like shear stress, volume fraction index, FGM layer, thickness of core, and semivertex angle.

4.2. FGM Plates. The investigation has been made to study elastic buckling of FG rectangular plates, and it is found that the stability boosts as the geometric parameters increase under uniform and linear loading. Moreover, critical buckling temperature difference of FG thick plates is appreciably affected by transverse shear deformation (Bouazza et al. [39]). Jalali et al. [40] investigated laminated, functionally graded circular plates having different thicknesses and constant temperatures using FSDT. It was found that thermal buckling factor increases with increasing volume fraction index and decreases in sheet thickness ratio. Jerysiak and Michalak [41] proposed the model for stability problems in thin plates FG structures. Naderi and Saidi [42] came up with the exact solution of stability analysis of FG sector plates mounted on an elastic foundation. Critical buckling load can be decreased by increasing the Winkler parameter and power law index. The elastic foundation and thickness of

plate can greatly affect the critical buckling load and stability of FG plates having free circular edges. Bateni et al. [43] did a comprehensive study on the stability of FG plates and came to know the significance of in-plane boundary conditions for buckling analysis. Nabian et al. [44] suggested the acceptable pull-in voltage and hydrostatic pressure for FG microplates to be in the stable region. Results were claimed to be useful in the designing of MEMS. Zhang et al. [45] analyzed stability and bifurcation of FG plates and found the numerical solution that meets with the analytical prediction using the fourth-order Runge–Kutta method. Kiani and Eslami [46] worked on the nonlinear thermoinertial stability of FG plates. It was concluded that the rotation of FG plates can stabilize it from an unstable region under thermal loading. During rotation, a snap-through phenomenon can take place. Swaminathan and Naveenkumar [47] proposed the computational model for the stability analysis of FGM plates. Different computational models with varying degree of freedom that acknowledge the consequences of transverse and shear deformation were examined and concluded that the critical buckling load of FGM plates can be achieved by higher-order deformation theory. Dynamic stability analysis of S-FGM using four-variable refined plate theory was studied by Han et al. [48]. Results show that under dynamic load, nondimensional frequency remains constant regardless of variation in stiffness of S-FGM. However, under static load, nondimensional excitation frequencies may get reduced proportionally by thickness ratio. Furthermore, the instability region of S-FGM plates is directly proportional to static load factor and elastic medium parameters. Critical buckling load and temperature of FGM microplates under mechanical and thermal loading were analyzed by Mirsalehi et al. [49]. It was noticed that both critical load and temperature vary with length-scale parameter except the case in which plate thickness is high enough as compared to length. However, critical load and temperature vary inversely with plate length, and volume fraction provided that volume is constant for a specified length. Rezaee and Jahangiri [50]

worked on chaotic vibration and stability of aeroelastic piezoelectric FG plates. Useful results were obtained for resonance amplitude, bifurcation point, and width of resonance region under different excitation (forcing, parametric, dynamic pressure, supersonic aerodynamic, and piezoelectric). In addition, the amplitude of system response is directly proportional to hysteric behavior. Stability and snap-through analysis of FGM plates considering thermal load is done by Ashoori and Sadough Vanini [51]. Results exhibit that thermal preloading causes snap-through behavior in microstructure-dependent and size-dependent FGM plates. Thermal preloading causes bifurcation instability in FGM plates provided that temperature rises uniformly.

4.3. FGM Beams. Ke and Wang [52] showed that the effect of the size of materials on dynamic stability of FG microbeams can only be considered when the length scale parameter has the same value as that of beam thickness. Piovan and Machado [53] suggested that dynamically unstable regions of thin wall FG beams vary inversely with elastic stiffness. Buckling and postbuckling of FG beams resting on nonlinear elastic foundation depend on a temperature of its constituent. Furthermore, critical buckling temperature is affected by coefficients of elastic foundation, when the thermal load is subjected to either uniform temperature rise or heat conduction (Esfahani et al. [54]). Linear and nonlinear parameters of a foundation are responsible for the postbuckling resistance of FG beams (Komijani et al. [55]). Azizi et al. [56] did stability analysis on FG piezoelectric MEMS (micro electromechanical system) and came to the conclusion that for FG piezoelectric microbeams to be in the stable region, an appropriate excitation frequency and amount of AC voltage is needed. For the static analysis, the stability of capacitive FG microbeams does not change as the source temperature changes provided that applied voltage remains constant. On the other hand, in case of dynamic analysis, temperature does affect the stability. It was also suggested that for mechanical behavior analysis, material length scale must be taken into consideration (Zamanzadeh et al. [57]). Kolakowski [58] did inspection about the dynamic stability of trapezoidal FGM beams. The relation between static and dynamic bucklings of structure and primary and secondary local bucklings was given. Nguyen et al. [59] worked on the flexural-torsional stability of FG beams. It was observed that long beam is not favorable for flexural mode, and in case of torsional mode, short beam is not ideal. Fazzolari [60] examined vibration and stability of FG beams. Using different mathematical theories, various material parameters were taken into account to study frequency and buckling load of FG beams.

4.4. FGM Panels. Duc and Tung [61, 62] studied the buckling and postbuckling behaviors of FG cylindrical panels and concluded that materials and geometric parameters both can affect the postbuckling behavior of FG cylindrical panels. Stability analysis of supersonic FGM panels with porous was studied by Barati and Shahverdi [63]. It was found that the

stability of FGM panels depends on the nature of porosity and rate of moisture in FG panels.

Few studies found on stability analysis of FGM shallow arch, pipes, ring, etc., are presented as follows.

FG shallow arches can follow equilibrium track and become unstable depending on the critical load limit of internal forces (Batani and Eslami [64]). Sedighi et al. [65] investigated dynamic stability analysis of asymmetric FGM-NEMS (nano electromechanical structure). Results reveal that pull-in voltage of nanobridges varies proportionally with surface stress and varies inversely with nonlocal parameters. Pull-in voltage and amplitude of nanobridges cannot be examined without finite conductivity of FGM. Deng et al. [66] evaluated the stability of multispan FGM pipes. The stability of FGM pipes varies proportionally with the volume fraction exponent, whereas natural frequencies and velocities vary proportionally with volume fraction exponent and vary inversely with nonlocal parameter. Volume fraction and radius to thickness play an important role in critical buckling hydrostatic pressure and the elastoplastic buckling of FGM circular rings (Huang et al. [67]).

The literature is abundant on stability analysis of FG shells, panel, and beams. Numerous mathematical theories including FSĐT, HODT, J_2 DT, CST, four variable RPT, etc., were used by a number of authors to investigate static, dynamic, and flexural-torsional stability of FGM. It is concluded from the literature that among various other factors, geometric parameters, elastic foundation, and temperature play a crucial role in the stability of FGM. Few investigations were made on the stability of FGM-MEMS and FGM-NEMS as well yielding useful results.

5. Buckling Analysis of FGM

5.1. FGM Shells. Sofiyev et al. [68] did buckling analysis of FGM shells under hydrostatic pressure and came to the conclusion that material gradation over a volume has an enormous effect on buckling pressure. Buckling analysis of two-layered FG cylindrical shells was done by Sepiani et al. [69]. It was concluded that fundamental frequency of FG cylindrical shells under static and periodic forces is greatly affected by transverse shear, rotary inertia, material composition, and deformation mode. Sofiyev [70] discussed the effect of critical combined load and compositional profiles on FGM circular shells with and without Winkler and Pasternak foundation with respect to semivertex angle and length to radius ratio of FGM circular shells. Compositional profiles, semivertex angle, length to radius and radius to height ratios, and an elastic foundation has a considerable effect on critical axial and combined loads of FGM truncated conical shells (Sofiyev [71, 72]). Huang et al. [73] did buckling analysis of FGM cylindrical shells under bending load. Results indicate that buckling critical moment of a shell has a direct relation with shell thickness, whereas it has an inverse relation with uniform temperature. Satouri et al. [74] applied third-order shear deformation theory to analyze buckling of two-dimensional FG cylindrical shells. Results revealed that critical buckling load varies directly with thickness to radius ratio. Stiffness at the outer side of the

shell makes it capable to withstand high buckling as compared to stiffness inside of the shell. Furthermore, the thickness of cylinder also has considerable effect on a shell to bear the buckling load. Sofeyev [75] accomplished a closed form solution for a freely supported FG truncated conical shell under both pressures (i.e., hydrostatic and critical lateral), using shear deformation theory. Sofeyev and Kur-uoglu [38] evaluated effect of an FG truncated conical shell on critical lateral and hydrostatic pressure under various boundary conditions. Sun et al. [76] investigated the influence of transverse shear deformation and imperfect sensitivity on buckling of FGM cylindrical shells for different boundary conditions. Zhang et al. [77] analyzed the buckling of elastoplastic FG shells subjected to compression and pressure. Results show that lateral pressure and critical axial compression load both encounter their effects if one of them is present. Buckling analysis of FG microshells subjected to axial and radial load was done by Lou et al. [78]. Results present that the existence of radial external pressure causes critical buckling load to decrease. Furthermore, critical buckling load is higher without considering prebuckling deformation. Consequences of internal pressure on buckling of FG cylinder were studied by Seifi and Avatefi [79]. It is concluded that buckling moment is directly proportional with an internal pressure and thickness of FG perfect shells. In addition, defect in shells causes critical buckling moment to decrease. Buckling of FG shells reinforced with graphene platelets was analyzed by Wang et al. [80]. Findings illustrate that buckling load has a direct relation with weight function and length to thickness ratio of graphene platelets. In case cutout is needed in FG shell, geometry of cutout at the edges of shell is supposed to be square or rectangular for better performance of buckling.

5.2. FGM Plates. Boghadi and Saidi [81] studied buckling analysis of FG rectangular plates. Results show that critical buckling load has an inverse relation to aspect ratio, whereas buckling load increases as the thickness of FGM plate increases. El Meiche et al. [82] investigated the buckling load of FG Sandwich plate using hyperbolic shear deformation theory having four known. The results are in good agreement with other higher deformation theories having five unknowns. Ghannadpour et al. [83] carried out experiments on critical buckling temperature of FG plates. It is reported that the critical buckling temperature is directly proportional to the aspect ratio and inversely proportional to width to the thickness ratio. Thai and Choi [84] proposed a simple refined theory for buckling analysis of FG plates. Results reveal that nondimensional critical buckling load decreases with the increase of power index [78, 85–90]. In addition, nondimensional critical buckling load increases with the increase of modulus ratio, aspect ratio [86, 87], and thickness ratio [88, 91–93] of FG plates. Under shear loads, buckling load decreases by increasing the area of rectangular plate (Asemi et al. [94]). The buckling load factor of FG plates on elastic foundation is directly proportional to power law index and foundation parameter. On the other hand, it is inversely proportional

to the aspect ratio of FG plates. Asemi et al. [95] analyzed the buckling of FGM annular plate with, without, and partially mounted on an elastic foundation. Results present that the buckling of FGM annular plate delay by elastic foundation. In addition, elastic buckling creates buckling wave, and it is dependent on the way the plate partially mounted on elastic foundation. Buckling analysis of FG circular porous plate subjected to transverse magnetic field was carried out by Jabari et al. [96]. Conclusion of the analysis reveals that the critical magnetic field varies inversely with porosity in the plate and fluid compression in the pores of materials. However, critical buckling load has direct relation with thickness of plate [97]. Effects of cracks and cutouts on the buckling behavior of FGM plates under thermal and mechanical load are examined by Natarajan et al. [98]. It is concluded that critical buckling load has an inverse relation with number of cracks, the length of a crack, and the gradient index of a plate. Buckling analysis of cracked FG plates was done by Panahandeh-Shahraki and Amiri [99]. Results show that increase in crack to width ratio decreases critical buckling load. However, increase in stiffness of elastic foundation and crack angle causes critical buckling load to increase provided that crack to width ratio is not large enough for uniaxial loading. Kulkarni et al. [87] proposed a new solution for buckling analysis of FG plates with the help of inverse trigonometric deformation theory (ITSDT). Results obtained from ITSDT were matched with the results of other theories. Ceramic isotropic plates are more useful as compared to FGM plates to achieve critical buckling load. Furthermore, the critical buckling load in clamped FGM is greater than that in simply supported FGM plates provided that volume fraction index is same Lal and Ahlawat [100]. In-plan material inhomogeneity plays a vital role to avoid buckling in FG thin plates (Lanc et al. [101]). Mantari and Monge [88] suggested buckling optimization to examine buckling of FG Sandwich plates. The critical buckling load using shear deformation theory is lesser than the values obtained by first-order shear deformation theory. Critical buckling load is more dominant in FG rectangular thin plates as compared to thick plates with respect to aspect ratio (Dong and Li [97]). Existence of crack in FGM microplates decreases critical buckling temperature. Moreover, thermal buckling load has a direct relation with the thickness of cracked FG microplates (Joshi et al. [102]).

5.3. FGM Beams. Buckling analysis of FG microbeams using modified couple stress theory was carried out by Nateghi et al. [103]. It was found that deviation in buckling load may be obtained by modified couple stress theory and other classical theories. Moreover, Poisson's ratio plays a significant role in the buckling of FG microbeams. Sahmani and Ansari [104] did buckling analysis of FG microbeams subjected to thermal effect. It was revealed that critical buckling load of FG microbeams in elastic medium decreases with an increase of temperature provided that slenderness ratio is high. The buckling of FG microbeams with the help of modified couple stress theory was analyzed

by Şimşek and Reddy [105]. Results declared that critical buckling load can be increased by including elastic medium constant. Moreover, critical buckling load varies directly with the slenderness ratio and varies inversely with gradient index [106]. Lanc et al. [101] attempted buckling analysis of FG Sandwich box beams considering different boundary conditions. It was inferred that deceleration in critical buckling in all boundary conditions has a direct relation with skin-core-skin ratio of the box beam. Moreover, material distribution over a volume of box beam plays an important role in critical buckling load. Nguyen et al. [107] introduced a new shear deformation theory that distributes transverse shear stress in FG Sandwich beams in a hyperbolic manner. The results gained from this new theory accounts for critical buckling load considering parameters like power law index, length to depth ratio, and skin-core-skin thickness and match with other existing theories. Huang et al. [108] examined the buckling of axial FG beams using Timoshenko theory. It was concluded that gradient and geometric parameter plays significant role in finding critical buckling load. Nguyen et al. [109] used quasishear deformation theory to analyze the buckling of FG Sandwich beams. Various parameters like power law index, skin-core-skin thickness ratio, and span to depth ratio were studied for critical buckling load. Results seem to be coinciding with the previous results reported in the literature. By adjusting gradient index, the buckling of two-dimensional FGM Timoshenko beams can be controlled (Simsek et al. and Deng et al. [106, 110]). Taati [111] analyzed the buckling of FG microbeams. Findings reveal that length scale parameter has the most significant effect on critical buckling of FG microbeams. The effect of power index of FG porous beam is more prominent if it is varying in axis direction in contrast to thickness direction. Furthermore, nondimensional buckling load decreases as volume fraction increases (Hydri et al. and Shafiei and Kazemi [89, 112]). Critical buckling load has a direct relation with length scale parameter and an inverse relation with nonlocal parameter. In addition, on the basis of size-dependent parameter, stiffness softening and hardening effect may be produced by axial FG beams on the critical buckling force (Li et al. [113]). Nguyen et al. [114] analyzed the buckling of FG open sections beams. Findings reveal that buckling parameters rely on the variation of volume fraction index. Also, it was explained that the angle of beam, end moment ratio, and ceramic core have considerable effect on buckling capacity. Chen et al. [115] used shear deformation theory to evaluate free vibration of FGM shells having a stretching effect. Results reveal that dimensionless fundamental frequency is directly proportional to the thickness of a core, aspect ratio, and length to thickness ratio. Furthermore, stretching has a significant effect on free vibration of FGM shells. Symmetric porosity distributed in FGM shells has less pronounced effect on natural frequency as compared to nonsymmetric porosity distribution (Wang et al. [80]).

5.4. Other Structures. Other than the abovementioned structures (i.e., FGM shells, plates, and beams), buckling analysis was also carried out on FGM structures.

Oyeka et al. [116] suggested the optimized design criteria for FG composite structure to enhance critical buckling load. Singh and Li [117] proposed a low-dimensional mathematical model comprising of Newton's eigenvalue iteration method (NEIM) to calculate the buckling load of FG column in an adequate way. Huang and Li [118] introduced a new model considering shear deformation to analyze the buckling load of FG circular columns. The method was found to be simple and results were matched with other existing theories like Timoshenko, Reddy-Bickford, and Euler-Bernoulli. Bich et al. [119] examined the buckling of FGM conical panel subjected to a mechanical load. Results present that geometric parameter and gradation of material significantly affect the buckling behavior of FGM conical panel. Semivertex and subtended angles have no considerable effect on critical buckling load. Thai and Wu [84, 120] went through buckling analysis of FG circular cylinder. Useful results were obtained showing changes in lowest critical load under different parameters like aspect ratio, gradient index, and load intensity. The buckling stress of P-FGM has an inverse relation with the power law. In addition, increase in radius to thickness ratio leads buckling stress to decrease (Hajlaoui et al. [86]).

Buckling analysis of FGM has been reviewed thoroughly in this section. Various interesting investigations were found to be the benchmark for further research in the field of FGM. Moreover, few researchers proposed a new mathematical model or theory for buckling analysis of FGM [71, 72, 74, 75, 85, 107, 109]. Numerous investigations were made for buckling analysis of FGM subjected to different loads, (mechanical, thermal, shear, axial, and radial) pressures (uniaxial, biaxial, and hydrostatic), thermal effect, transverse magnetic field, etc., with the help of extensive mathematical theories, i.e., HOSDT, SDT, MCST, ITSDT, QSDT, hyperbolic SDT, and Timoshenko beam theory. It is derived from the literature that among various other factors, material gradation over a volume, compositional profile, and geometric parameters of FGM play a vital role in buckling. Furthermore, relation between various indexes, ratios (power law index, gradient index, slenderness ratio, modulus ratio, and aspect ratio), and buckling loads were examined by most of the researchers. Few studies were also found investigating buckling analysis with respect to cracks in FGM [104, 105, 119].

6. Free Vibration Analysis

6.1. FGM Shells. Cinefera et al. [121] proposed variable kinematics model which is a further extension of Carrera's unified solution, to study free vibration of multilayered FGM shells. It was reported that the presented model can be used to analyze multilayered shells due to its high accuracy. Fadaee et al. [122] used Donnell's and Sander's shell theories to obtain the closed form solution of Levy type FGM spherical shells under different boundary conditions. Results indicate that the frequency parameter has a direct relation with curvature ratio. Neves et al. [123] analyzed free vibration of FG shells using the Carrera unified formula merged with the radial basis function collocation method.

Results show that the fundamental frequency decreases with an increase of radii of curvature and power law exponent. Furthermore, FGM simply supported shell has a lower value than clamped one. To obtain the desired result, the natural frequency of FG cylindrical shells plays an important role by considering the volume fraction of the constituent Ebrahemi and Najafizadeh [124]. Tornabene et al. [125] used different mathematical models to analyze the free vibration of FGM doubly curved shells. Among various other conclusions, it was reported that accurate results for natural frequencies do not necessarily be obtained by increasing number of higher-order theories. Xie et al. [126] used the Haar Wavelet method to examine free vibration of FGM shells and plates. It was reported that frequencies of FGM shells and plates have an inverse relation with material exponent, length to radius ratio, and semivertex angle, whereas the frequencies have a direct relation to the thickness of FGM shells and plates [127, 128]. Furthermore, circumferential wave number also plays an important role in natural frequencies of FGM shells [129]. Bahadori and Najafizadeh [130] evaluated free vibration of 2D cylindrical shells mounted on Winkler–Pasternak elastic foundation with the help of FSDT and DQM. It was concluded that natural frequencies of the 2D cylindrical shells increase with the increase of power law index and shear modulus of foundation. In addition, a greater value of height to radius ratio increases natural frequencies, while a greater value of length to radius ratio decreases natural frequency of FGM cylindrical shells [129]. The Fourier Ritz method was adopted by Jin et al. [131] to study free vibration of laminated FG shells. It was reported that thickness and material of shell [132] greatly affect the fundamental frequency of FGM shells. Kim [133] evaluated free vibration of FGM shells mounted on elastic foundation having an oblique edge with the help of FSDT. It was found that the frequency of FGM shells decreases with an increase of oblique angle. Furthermore, natural frequency can be adjusted by changing material profile. Greater value of stiffness leads to increase frequency of FGM shells and microplates (Lou and He [134]). Greater value of length scale parameter increases the natural frequency of FG shells (Tadi Beni et al. [127]). The Haar Wavelet discretization method was adopted by Xie et al. [126] to evaluate FGM spherical and parabolic shells. Natural frequencies of FGM shells increase with elastic restraint and decrease with volume fraction. Punera and Kant [135] applied different higher theories to evaluate the effect of geometrical and material parameters on the frequency of FGM open cylindrical shells.

6.2. FGM Plates. Free vibration of thick FG plates was analyzed using three-dimensional elastic theory by Malekzadeh [136]. It was concluded that natural frequency parameters are greatly affected by shearing layer elastic coefficient provided that Winkler elastic coefficient has moderate value. Furthermore, higher value of length to thickness ratio [33, 34, 46, 47, 55], power law index [35–38, 48–51, 58,65], and material property graded indexes lead natural frequency parameters to be reduced [46, 63, 137]. Zhao et al. [138] used the element-free Kp-Ritz

method to analyze free vibration of FGM plates. Results present that volume fraction exponent [131, 133] and length to thickness ratio have considerable effect on the frequency of FGM plates [139] with letter one influencing frequency free from the effect of former one. Moreover, the frequency of FG skew plates also increases with the increase of skew angle above 30°. Hashemi-Hashemi et al. [140] used FSDT to analyze the free vibration of FGM rectangular plates. It was revealed that the frequency parameter increases with the increase of Winkler and Pasternak foundation [122, 136, 138, 141, 142], stiffness parameter, and aspect ratio. However, it decreases with the thickness of a plate. Moreover, normalized eigen frequency parameter has a direct relation with foundation stiffness parameter up to the value of critical gradient index. With the increasing value of aspect ratio [37] and thickness to length ratio, normalized eigen frequency decreases. Liu et al. [143] explained the consequences of in-plane material inhomogeneity on the fundamental frequency of FGM plates [144]. Frequencies of a homogenous plate under different boundary conditions, i.e., clamped-free, clamped-simply supported, and free simply supported, were found to be the same. Benachour et al. [145] used the four-variable plate theory to analyze the free vibration of FGM plates. Effects of various parameters like aspect ratio, length to thickness ratio, and gradient index on free vibration with the help of examples were given. Results show good agreement with other existing theories. Hashemi-Hashemi et al. [146] studied free vibration of FGM rectangular plates using Reddy's third-order shear deformation plate theory. The presented approach can be used to forecast both in-plane and out-plane modes of FGM plates. Moreover, frequency parameter decreases by increasing aspect ratio of FG plates [147]. Jodaei et al. [147] used the artificial neural network (ANN) method and the state-space-based differential quadrature method (SSDQM) to study free vibration of FGM annular plate, and the results were compared with the existing literature. Findings reveal that ANN is a useful method to predict natural frequency while SSDQM has fast convergence speed. It was also revealed that natural frequency is directly proportional to circumferential wave number [148, 149]. Nondimensional frequency of FGM plates on Winkler foundation is reduced by increasing power law index [150], and it has no effect on FGM plates mounted on the Pasternak foundation (Thai and Choi [151]). Dozio [152] on the basis of results of conducted experiment suggested the use of higher-order theories for FGM plates. It was reported that higher-order theories are favorable to use when length to thickness ratio is less than 10 and one or two clamped edges are included in FGM plates. Furthermore, the exact frequency of many FGM plates with different boundary conditions was also presented. Jedrysiak [153] used asymptotic tolerance, asymptotic, and tolerance model to analyze the frequency of microstructure FGM plates. It was suggested that all presented models can be applied to analyze lower free vibration frequencies. Moreover, both lower and higher free-vibration frequencies have an inverse relation to Young's modulus ratio. Quasi 2D and 3D SDT were used by Akavci and Tanrikulu [154] for the analysis of free vibration of FG plates. It is reported that

transverse normal strain plays a significant role in a free vibration of FGM plates as that of transverse shear strain. Chen et al. [115] applied the meshless local natural neighbor interpolation method to study free vibration of FG plates. It was suggested that the method is not useful for the analysis of very thin plates. Pandey and Pradyumna [155] applied Love's and Donnell's shell theories to obtain natural frequencies of FG Sandwich plates. Results were found to be the same for both Love's and Donnell's theories. By increasing sector angle, natural frequencies of FGM sector plates tend to decrease (Su et al. [156]). Li and Zhang [157] examined free vibration of rotary FGM plates by means of dynamic model considering the dynamic stiffening effect. It was reported that frequency crossing phenomenon does not exhibit in both rotating cantilever plate and FGM plate. This phenomenon is due to sudden change in the mode of a plate. Fundamental frequency of S-FG plates increases by increasing the number of transverse and longitudinal stiffeners (Thang and Lee [128]). Zur [158] analyzed free vibration of FGM circular plates with elastically supported using Quasi-Green's function. Dimensionless frequencies of FGM plates were found to be less than ceramic plates. Moreover, FGM plates are considerably affected by stiffness and position of ring support.

6.3. FGM Beams. Rahmani et al. [159] carried out an experiment on the free vibration of a Sandwich structure having FG syntactic core with the help of high-order Sandwich panel theory. Findings reveal that the inhomogeneity of the material plays an important role in the eigen modes of a beam. Moreover, eigen frequencies have an inverse relation with span to thickness ratio. Simsek and Kocatürk [160] investigated the free vibration of FG beams subjected to concentrated moving harmonic load. It was concluded that power law exponent plays a key role in analyzing free vibration taking in to account Euler-Bernoulli beam theory. Dimensionless frequencies increase by increasing Young's modulus ratio [161] of upper to lower surfaces (Eratio) of the beam until the value of the power law index is small. In addition, dimensionless frequency has a direct relation with power law exponent (when Eratio is less than one) [162] and normalized dynamic deflections. Sina et al. [163] applied a new beam theory to study free vibration of FG beams. A comprehensive analysis was presented regarding mode shapes of FG beams using first-order shear deformation beam theory (FSDBT1 and FSDBT2) and classical beam theory. It was illustrated that power law exponent [131, 133, 135], power law distribution, mode of vibration, geometry of structure, and thickness greatly affect the free vibration of FG beams (Tornabene and Viola [141, 164]). Huang and Li [165] proposed a new method based on Fredholm integral equations and evaluated the natural frequency of FGM beams having nonuniform cross section taking into account flexural rigidity, mass density, and axial gradient parameter. Results were claimed to be useful for designing inhomogeneous beam structure. Alshorbagy et al. [162] used the finite element method to elaborate free vibration of FGM beams. It was shown that the

modal shape and frequency of FGM beams both have an effect of material gradation varying along the axial direction rather than spatial direction. Due to limitations of Euler's beam theory used in the analysis, effect of slenderness ratio cannot be determined. Moreover, it was suggested to use Timoshenko or Reddy theories to study the effect of slenderness ratio. Based on the Timoshenko beam theory, FGM microbeams were analyzed by Ansari et al. [166]. Dimensionless natural frequency was examined against various parameters like gradient index, slenderness ratio, beam mode, and beam thickness using classical theory, modified couple stress theory (MCST), and strain gradient theory (SGT). It was concluded that FGM microbeams have a larger value of dimensionless natural frequency as compared to other microbeams and less than SiC microbeams. Moreover, FGM microbeams have frequencies intermediate in metal and ceramic microbeams. Giunta et al. [167] proposed the one-dimensional beam model to analyze free vibration of FGM beams. Frequencies like flexural, torsional, and axial were determined and verified with those of three-dimensional finite modal solutions. Hein and Feklistova [149] used Haar wavelet approach to find frequencies of FGM beams using different geometries, mass density, and boundary coefficient. Results show that the approach requires less computation time with accurate results. The method can be easily implemented on any system. Shahba et al. [168] analyzed the free vibration of FGM tapered Timoshenko beam. Results show that natural frequency decreases with taper ratio [161] and attached mass to beams [161, 169, 170]. Using the Ritz method, free vibration of FGM spatial beam was examined by Yousefi and Rastgoo [171]. It was reported that by increasing number of turns and angle of helix, frequency parameter gets increased. Shahba and Rajasekaran [172] used the differential transform element method (DTEM) and differential quadrature element method (DQEM) of lower order to find out the longitudinal transverse frequencies of FGM beams. Results illustrate that DTEM is fast over DTM, and obtained results are more accurate. Shear deformation effects reduce the natural frequencies of FG beams (Thai and Vo [173]). Based on improved third-order shear deformation theory, it was concluded that the position of the mass added to the beam has a substantial effect on frequencies (Wattanasakulpong et al. [169]). Wei et al. [174] used transform matrix method to study the effect of number and location of cracks, rotary inertia, and shear deformation on the frequencies of Euler-Bernoulli and Timoshenko beams. Finding illustrates that the existence of cracks in FGM beams decreases the frequencies [175] and alter the vibration mode. In addition, rotary inertia has negligible effect, and shear deformation has a significant effect on the free vibration of FGM beams. Aydin [175] proposed the rotational spring method having a third-order determinant to solve the frequency of FGM beams with different number of cracks. The proposed method can easily be employed in short time. Results show that frequency gets decreased when cracks develop at a point where bending moment is concentrated. Huang et al. [176] examined free vibration of FG Timoshenko beam having a nonuniform cross section and proposed an approach to

obtain higher- and lower-order natural frequencies in an efficient way. Free vibration analysis of axially loaded FG beams was done by Nguyen et al. [177]. It was concluded that changing the mode of axial force from tension to compression, natural frequencies get vanished. In addition, natural frequency increases by the impact of poison's ratio. The Rayleigh–Ritz method was used by Paradhan and Chakraverty [178] to examine free vibration of Euler and Timoshenko FG beams. Results were obtained for the effect of volume, length to thickness ratio [179], and various boundary conditions against natural frequency. Ziane et al. [180] used FSDT to calculate the natural frequencies of thick- and thin-walled FGM box beams. It was reported that torsional natural frequencies are directly proportional to thickness to side ratio. Aghazadeh et al. [181] used three beam theories, i.e., Euler-Bernoulli theory, Timoshenko beam theory, and TSDT to investigate the free vibration of FGM beams having variable length scale parameter. Results show that by increasing length scale parameter, transverse deformation mode frequency increases. It was also declared that the presented method can be beneficial to analyze and design small-scale FGM beams. Li et al. [182] investigated rotating hub FGM beams using rigid flexible coupled dynamics theory. A two-mode model was also developed to study the frequency-varying behavior of critical veering angular velocities. It was also examined that natural frequencies of FGM beams decrease with increasing gradient index [106], whereas it has an increasing trend with the hub angular velocity. Liu and Shu [183] studied the impact of delamination on exponentially FGB beam's frequencies using Euler–Bernoulli hypothesis, the “free mode,” and “constrained mode” assumptions. It was concluded that the constrained-mode and free-mode frequencies increase by increasing Young's Modulus ratio up to a unity provided that delamination effect does not exist. Delamination causes natural frequencies of FGM to increase and this effect becomes more prominent by increasing Young's Modulus ratio and decreasing material properties [184]. Mashat et al. [185] used Carrera Unified Formulation along with other theories to analyze free vibration of FGM-layered beams. It is reported that in order to determine the flexural and torsional frequencies of thick- and thin-walled FGM beams accurately, higher-order theories must be used. In addition, CUF is useful to obtain various one-dimensional models. Yang et al. [186] examined the free vibration of 2D-FGM structure and FGM Sandwich beams [187] with the help of mesh-free boundary domain integral equation method. The material gradient was found to be an important parameter which plays a vital role in natural and fundamental frequencies of FGM structure [188]. Increasing the stiffness of the layer of FGM beams, the thickness stretching phenomenon gets enhanced. It was also concluded that the method is efficient and fast, and the results obtained are accurate. Jin and wang [189] evaluated the frequencies of FGM beams using weak the form quadrature element method. Results were found to be in good agreement with those in the existing literature. Şimşek [170] studied free vibration of bi-directional FGM Timoshenko beam using Timoshenko beam theory. It was reported that in order to

meet the desired requirement of designing BDFGM, material gradient index and properties need to be considered. Variation in material gradient index affects the vibration period and displacement of FGM Timoshenko beam (Calim [188]). On the basis of Timoshenko beam theory, Chen et al. [179] evaluated free vibration of FGM beams having porosity. It was reported that increasing the porosity of FGM leads to increase in the fundamental frequency of beam having 10 porosity layers but decreases for the beams with 20 porosity layers. Jing et al. [190] used Timoshenko beam theory together with the finite element method to study FGM beams. It was reported that natural frequencies decrease with the increase of volume fraction exponent and increase with the increase of span to depth ratio. Natural frequencies of FGM beams can be controlled by grading the material through thickness and power law index (Li et al. [191]). Shear deformation has more considerable impact on higher-order frequencies than lower order ones. In addition, FGM beams may exert stiffness hardening and softening impact depending on the comparative value of material characteristics parameter and nonlocal parameter. Useful results of flexural, torsional, and flexural-torsional vibration of FG beams were obtained by Nguyen et al. [192]. Rezaiee-Pajand and Hozhabrossadati [193] studied the effect of spring's stiffness, suspended mass, and gradient parameter on double-axial FGM beams. Frequency of FGM Sandwich beams is directly proportional to the spring constant factor and inversely proportional to the thickness of a beam Tossapanon and Wattanasakulpong [194]. Increasing value of power law index ultimately increases the natural frequency of FG cantilever beam, whereas it decreases the natural frequency of a simple beam. Moreover, above critical frequency, sudden change in natural frequency of two directional FGM was found (Wang et al. [195]). Timoshenko beam theory causes large number of natural frequencies in beams due to shear effect as compared to Euler–Bernoulli beam theory. Shear deformation makes the beam more flexible (Simsek and Al-shujairi [196]). Axial dominated frequencies of FG beams have an inverse relation with length to thickness ratio Lee and Lee [197]. In addition, length to height ratio causes exchange of mode within axial and bending dominated frequencies. Zhao et al. [198] applied the chebyshev polynomial method to obtain the natural frequencies and mode shapes of axial FGM beams. The adopted method was found to be convenient, and results obtained are matched with other methods. Length scale parameter and variation in material gradient play a vital role in the deformation of size-dependent rotating FGM microbeams (Fang et al. [199]).

6.4. FGM Panels. Sobhani Aragh and Yas [200] have obtained useful results for normalized natural frequency of FGM fiber orientation and volume fraction cylindrical panel using the differential quadrature method (DQM). Zahedi-nejad et al. [148] proposed three-dimensional free vibration analysis of FGM curved panels using DQM. It was proposed that frequency parameter has an inverse relation with material property exponent, panel length, and angle. Zhao and

Liew [201] used a meshless method to analyze free vibration of FGM conical panels. Volume fraction exponent and semivertex angle both have significant effect on frequency parameter if one of them is kept constant. Circumferential mode number of fundamental frequency parameter has a direct relation with flexure of support and the opening of FGM lavy conical panel (Akbari et al. [202]). Fantuzzi et al. [203] proposed 2D and 3D shell models to investigate the free vibration of FGM cylindrical and spherical panel. It was concluded that use of dimensional generalized differential quadrature (2D-GDQ) is an utmost need for the evaluation of free vibration of FGM cylindrical and spherical panel in an efficient way. Moreover, 3D exact frequencies can be obtained by 2D-GDQ.

In the current section, vast literature has been discussed for vibration of FGM. Investigation mainly focused on natural, axial, fundamental, and flexural-torsional frequencies of FGM using various mathematical theories. Some of them include TDET, HOSPT, FSDPT, DQM, FEM, MCST, SGT, FVPT, SDQM, DTEM, DQEM, and so on. The main bulletin from the present review of vibration analysis of FGM can be presented as follows:

- (1) Variable kinematic model was proved to be highly accurate to study multilayered FGM shell
- (2) Timoshenko and Reddy theories were found to be more effective than Euler's beam theory to study the effect of slenderness ratio on free vibration of FGM beams
- (3) Haar Wavelet approach takes less computation time to find frequencies of FGM with high accuracy
- (4) SSDQM has fast convergence speed in predicting natural frequencies of FGM plates
- (5) In order to find the longitudinal transverse frequency of FGM beams, DTEM was supposed to be preferred over DTM due to its high accuracy and fast result
- (6) Higher-order theories are useful in obtaining flexural and torsional frequencies of thin- and thick-walled FGM beams
- (7) CUF is useful to obtain various one-dimensional models
- (8) For efficient investigation of free vibration of FGM shells, 2D-GDQ must be used

7. Concluding Remarks and Future Work

The present paper shows an overview of stability, buckling, and free vibration analysis of FGM evaluated by different authors worldwide in the past few decades. The research conducted on FGM analysis are either purely analytical or numerical method based. Admirable work has been done on various aspects of FGMs and several mathematical models adopted for the various analysis of FGM proved to be very efficient and of fast convergence. However, there are still some gaps that need to be filled to take more in depth advantages of FGM.

- (1) To save computation time and cost, few researchers preferred 2D theories with some modification. However, to give a more precise and accurate analysis on FGM (stability, buckling, free vibration, etc.), it is necessary to develop some more 3D theoretical or numerical methods. Moreover, in the existing studies, 3D analysis of FGM is mainly focused on linear buckling and free vibration analysis. The nonlinear 3D models need to be explored as well need to broaden the application of the 3D theories.
- (2) FSDT has been extensively used in a numerical solution of FGM. However, HSDT is supposed to be employed for more accurate results.
- (3) Most of the researches have focused on the properties of FGM without taking into account the environmental effect (deformation, temperature, etc.). Moreover, few researchers discussed only the simple cases, like the transverse shear or the transverse normal deformation. The real situation, however, is usually a complex case. The more general and complex cases should be concerned as well.
- (4) Specific geometries of FGM beams, i.e., unsymmetric, antisymmetric, and arbitrary lay ups are not widely investigated in the literature. However, no remarkable efforts have been made on buckling of beams in terms of exact elasticity problems. The aforementioned issues need to be addressed to validate different refined theories.
- (5) In the literature, higher-order beam theories have not been applied to the laminated FGM taking in to account the consequences of transverse normal deformation on buckling and vibration response. Thus, refined higher-order beam theories are supposed to apply while tackling transverse normal deformation.
- (6) Besides the extensive literature available independently on analysis of FGM's performance, representation methods of FGM's parts, and its fabrication techniques, they are hardly investigated together. Therefore, a comprehensive design system is needed to be accomplished that makes the researchers able to design models, analyze and fabricate complex geometry of FGM.
- (7) Among various other manufacturing techniques of FGM, powder metallurgy is one of the most frequently used techniques. However, desired dissemination of material properties all over the structure with more perfection still needs some improvement by means of modification in fabrication techniques.
- (8) Although numerous application of FGM exists in aerospace, defense, nuclear, automobile, and other industries, FGM has a vital role in medical field as well. Keeping a wake glance of this noble application for the betterment of human beings, more in-depth investigation needs to be done on FGM with respect

to health care such as bone implantation, dentistry, etc.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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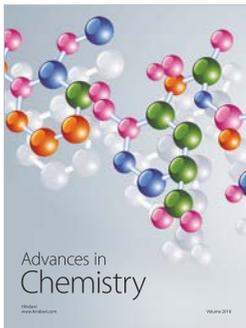
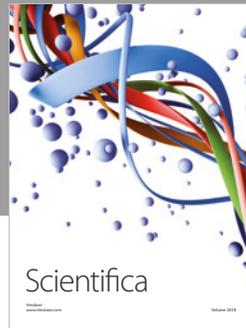
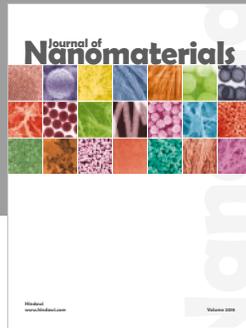
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