Hindawi
Advances in Materials Science and Engineering
Volume 2019, Article ID 2980548, 9 pages
https://doi.org/10.1155/2019/2980548

*Research Article*

# Model of Consecutive, Steady-State Underflow for Vertical Tailing Silos

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Received 26 November 2018; Accepted 28 February 2019; Published 24 March 2019

Academic Editor: Luigi Nicolais



A continuous tailing discharge model is proposed for solving the problems of high fluctuation in underflow concentration and low, unstable actual discharge concentration in vertical tailing silos. On the basis of the mass balance equation, a mathematical model for continuous tailing discharge is derived, and a partial differential equation related to the height of the tailing silo in the compression area and tailing slurry concentration is obtained. The effective solid stress equation and the solid flux equation can be obtained from the results of centrifuge tests and intermittent sedimentation tests. The volume concentrations of underflow corresponding to various heights of tailing surfaces are simulated by using fluid mechanics software under dynamic balancing conditions. The results of the simulations are highly similar to those calculated by the derived mathematical model of continuous tailing discharge, and the results of the industrial test are also closely related to the results of the differential equation, thereby verifying the accuracy of the proposed discharge model.

## 1. Introduction

To date, numerous achievements have been made in research on tailings sedimentation, particularly in the following aspects: (1) the study of the settling velocity of tailings with various influencing factors [1–3], (2) the study of the settling velocity of tailings in multifactor conditions [4–6], (3) the study of accelerating the settling velocity of fine-grained tailings by adding different types of flocculant and coagulant aids [7, 8], and (4) the study of the dynamic settling of tailings in thickeners [9–12]. However, few studies have been conducted on discharge model vertical tailing silos.

Vertical tailing silos are critical structures used for hydraulic filling. These silos are designed using cylinders with diameters of 8–10 m, heights of 18–20 m, and half-spherical or conical bases with certain angles. In mines, the mortar volume concentration obtained through the secondary recovery of mortar overflow ranges from 7% to 12% of the overflow. The drastic fluctuations in underflow concentration limit the increase in filling slurry concentration and increase filling cement consumption. Thus, high-cemented filling quality and cost cannot be guaranteed.

Continuous concentration can be achieved through the use of high-density tailings and vertical tailing silos with low-diameter settlement containers. A continuous high-density discharge model is proposed to resolve the problems encountered in the use of the current gravity sedimentation model of vertical tailing silos. These problems include the need for the repeated pulping of alternative tailing discharges through the simultaneous action of multiple silos and tailing discharge from bottom flow, as well as unmanageable and drastic fluctuations in density during tailing discharge.

## 2. Consecutive Underflow Model

Tail slurries present three zones when a vertical tailing silo is in the process of consecutive underflow and steady state, as shown in Figure 1, namely, clear liquid region ($\phi = 0$), hindered settling region ($0 < \phi < \phi_c$), and compression region ($\phi > \phi_c$) [13, 14]. Note that $q_L$ is the volumetric velocity in the overflow zone, $q_R$ is the volumetric velocity in the discharge zone, and $q_F$ is the feed flux.

A consecutive and stable underflow model was established under two basic assumptions: the unit is continuously fed by a singular feed source located at the interface between the clear liquid region and the hindered settling region, and the overflow has no tailings; it only has water.

Vertical tailing silos maintain their steady state when they experience consecutive underflow. Dynamic equilibrium would then exist on the condition of the temporal affecting factor and would meet the following condition: feeding (tailings + water) = overflow (water) + underflow (tailings + water).

According to the mathematical model for batch, the following formula can be deduced [14]. The volumetric solid concentration is constant across each horizontal cross section, i.e., $\phi = \phi(x, t)$. The conservation of mass equation for solids is then given by equation (1), and the analogue conservation equation for the fluid is as equation (2):

$$S(x)\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}\left(S(x)\phi v_s\right) = 0, \quad (1)$$

$$-S(x)\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}\left(S(x)(1-\phi)v_f\right) = 0, \quad (2)$$

where $t$ is the time, $v_s$ is the solid-phase velocity, and $v_f$ is the fluid-phase velocity. The continuity equation of the mixture is $(\partial/\partial x)Q(x,t) = 0$, and it implies that $Q(\cdot, t)$ is constant as a function of $x$. The equation $Q(x,t) = Q_D(t)$ is obtained. The solid-fluid relative velocity or slip velocity $v_r = v_s - v_f$ for a constitutive equation will be formulated. As such,

$$\begin{aligned} S(x)\phi v_s &= S(x)\left[\left(\phi v_s + (1-\phi)v_f\right)\phi + \phi(1-\phi)\left(v_s - v_f\right)\right] \\ &= Q(t)\phi + S(x)\phi(1-\phi)v_r. \end{aligned} \quad (3)$$

Kynch's kinematic sedimentation theory is based on the assumption that the solid-fluid relative velocity or slip velocity $v_r$ is $v_r = v_r(\phi)$ [15]. Slip velocity is commonly expressed in terms of the Kynch batch flux density function $f_{bk}$. Slip velocity is expressed as $v_r = (f_{bk}(\phi)/\phi(1-\phi))$. Thus, equation (3) can take the form as follows:

$$S(x)\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}\left[Q(t)\phi + S(x)f_{bk}(\phi)\right] = 0. \quad (4)$$

Based on the phenomenological theory of sedimentation [16], we derived the following equation for relative velocity $v_r$, as follows:

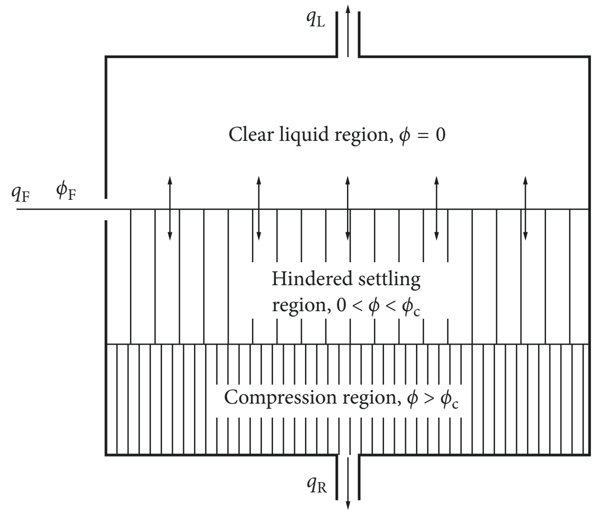


Figure 1: Vertical tailing silo operating at steady state in a consecutive underflow model.

$$v_r = \frac{f_{bk}(\phi)}{\phi(1-\phi)}\left(1 + \frac{\sigma_e'(\phi)}{\Delta\rho g \phi}\phi_x\right), \quad (5)$$

where $\Delta\rho > 0$ denotes the solid-fluid density difference, $g$ is the acceleration of gravity, and $\sigma_e'(\phi)$ is the derivative of the effective solid stress function $\sigma_e(\phi)$ [17–19], and

$$\begin{aligned} a(\phi) &= \frac{f_{bk}(\phi)\sigma_e'(\phi)}{\Delta\rho g \phi}, \\ A(\phi) &= \int_0^{\phi} a(s)ds, \end{aligned} \quad (6)$$

through which the field equation can be obtained, as follows:

$$S(x)\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}\left(Q(t)\phi + S(x)f_{bk}(\phi)\right) = \frac{\partial}{\partial x}\left[S(x)\frac{\partial A(\phi)}{\partial x}\right]. \quad (7)$$

These solutions satisfy the ordinary differential equation, which equation (7) indicates as stationary, i.e., independent of time:

$$Q_D\phi(x) + S(x)f_{bk}(\phi(x)) = S(x)\frac{dA(\phi)}{dx} + C, \quad (8)$$

where $C$ is a constant of integration, and $C = Q_D\phi_D$.

For the corresponding concentration profile $\phi = \phi(x)$, concentration with respect to the independent variable $x$ is represented as the following equation:

$$\phi'(x) = \frac{d\phi}{dx} = \frac{\Delta\rho g \phi\left(q\left(\phi - \phi_D\right) + f_{bk}(\phi)\right)}{f_{bk}(\phi)\sigma_e'(\phi)}. \quad (9)$$

## 3. Materials and Methods

*3.1. Materials.* Tailings from the Dahongshan copper mine in Yunnan, China, were used as the experimental material. The density of the tailings was 2.897 g/cm$^3$, as measured with a pycnometer. As listed in Table 1, the average particle size of

Table 1: Physical property parameters of tailings.

| Porosity (%) | Average particle size (mm) | Permeability coefficient (cm/h) | Specific gravity (g/cm$^3$) |
|---|---|---|---|
| 43.04 | 0.1165 | 0.9 | 2.897 |

the tailings was 0.1165 mm, as measured through sieving or elution. The permeability coefficient of the tailings was 0.9 cm/h.

*3.2. Methods.* The solid flux density function and effective solid stress are important components of the model of consecutive steady-state underflow for vertical tailing silos. Parameters $\sigma_0$ and $k$ in the effective solid stress function ($\sigma_e(\phi)$) are obtained through centrifuge tests. Parameters $v_\infty$ and $n$ in the solid flux density function ($f_{bk}$) are obtained through sedimentation experiments. The different properties of materials result in different experimental parameters [20].

*3.2.1. Effective Solid Stress Measurement.* Effective solid stress was measured in the compression zone. The effective solid stress can be obtained using the following equation [5, 21, 22]:

$$\sigma_e(\phi) = \begin{cases} 0, & \phi \le \phi_c, \\ \sigma_0\left(\left(\frac{\phi}{\phi_c}\right)^k - 1\right), & \phi > \phi_c, \end{cases} \quad (10)$$

where $\phi_c$ is the gel point and $\sigma_0$ and $k$ are constant parameters. Gel point ($\phi_c$) is the solid volume fraction at the beginning of the compression zone, and it was estimated in accordance with the following equation [23]:

$$\phi_c = \frac{\phi_1 h_1}{h_c}. \quad (11)$$

We estimated the gel point ($\phi_c$) as 0.296 by using equation (11), where $\phi_1$ and $h_1$ are the initial solid volume fraction and initial height of the suspension and $h_c$ is the equilibrium height of the sediment bed.

Effective solid stress was estimated through batch centrifuge experiments that were performed by using a centrifuge (HENGNUO). The centrifuge tubes had a volume of 15 ml and a diameter of 12 mm. The centrifugal acceleration ranges from 500 to 2,000 rpm [24, 25].

Batch centrifuge experiments were performed for the calculation of effective solid stress [26]. The details of the experiments and calculations are listed in Table 2.

The experimental data were fitted to equation (10), and the fitted curve is shown in Figure 2.

The parameter values are $\sigma_0 = 8.26$ and $k = 9.06$. Thus, equation (10) can be expressed as follows:

$$\sigma_e(\phi) = 8.26\left(\left(\frac{\phi}{0.296}\right)^{9.06} - 1\right)(\phi > \phi_c). \quad (12)$$

*3.2.2. Measurement of Solid Flux Density Function.* Michaels and Bolger presented a typical example, as follows [27, 28]:

$$f_{bk}(\phi) = \begin{cases} v_\infty \phi\left(1 - \frac{\phi}{\phi_{max}}\right)^n, & 0 < \phi < \phi_{max}, \\ 0, & \text{others}. \end{cases} \quad (13)$$

The mass concentrations of six groups of mortars ranged from 15% to 40%. Tailings were subjected to batch experiments. The results are presented in Figure 3 and Table 3.

We plotted the solid flux density function curves in accordance with the batch experiment data. In the curves, the abscissa represents volume concentration, and the ordinate represents solid flux density. The experimental data were fitted to equation (13). The fitted curve is shown in Figure 4.

The parameter values were $v_\infty = 2.61$ and $n = 10.86$. Hence, equation (13) can be expressed as follows:

$$f_{bk}(\phi) = 2.61\phi(1 - \phi)^{10.86}. \quad (14)$$

## 4. Results and Analyses

*4.1. Simulation Results.* Four interface levels were selected randomly between tailings and water (8.8, 9.3, 10.6, and 11.3 m), and Fluent software was utilized to simulate the underflow volume concentration as a dynamic balance. The mixture model and standard k-epsilon model were chosen for the calculation and solution, respectively. The mesh used was of double-symmetry plane to reduce the amount of calculation and reduce computing resources. The volume distribution contours of the water phase could be drawn from the results of numerical simulation and are shown in Figure 5. The simulations achieved accurate underflow volume concentration on the four interface levels between tailings and water via the monitoring curve (i.e., 0.5271, 0.5421, 0.5582, and 0.5703, respectively).

*4.2. Results of Consecutive Underflow Model.* Four interface levels between tailings and water were selected at the gel point ($\phi_c$) of 0.296 when the underflow volume concentration (0.5271, 0.5421, 0.5582, and 0.5703) was calculated using the Runge–Kutta method to verify the accuracy of the underflow consecutive models. These can be obtained from the curve of change between volume concentration of the slurry and height of the tailing silo at dynamic sedimentation, as shown in Figure 6.

Four interface levels between tailings and water from different underflow concentrations, i.e., 8.53, 9.34, 10.31, and

Table 2: Results of centrifuge tests.

| Rotation speed (pm) | RCF ($g$) | $h_{eq}$ (m) | $\Delta\rho$ | $\phi_0$ | $h_0$ (m) | $R$ (m) | $\sigma(\phi_0)$ (kPa) | $\phi$ (%) |
|---|---|---|---|---|---|---|---|---|
| 500 | 33.54 | 0.035125 | 1897 | 0.13 | 0.1 | 0.12 | 706.08 | 0.47 |
| 700 | 65.74 | 0.0335 | 1897 | 0.13 | 0.1 | 0.12 | 1394.89 | 0.523 |
| 900 | 108.7 | 0.03175 | 1897 | 0.13 | 0.1 | 0.12 | 2325.37 | 0.551 |
| 1100 | 162.3 | 0.02975 | 1897 | 0.13 | 0.1 | 0.12 | 3507.07 | 0.576 |
| 1300 | 226.7 | 0.02875 | 1897 | 0.13 | 0.1 | 0.12 | 4921.6 | 0.593 |
| 1500 | 301.9 | 0.028 | 1897 | 0.13 | 0.1 | 0.12 | 6575.68 | 0.612 |
| 1700 | 387.7 | 0.0275 | 1897 | 0.13 | 0.1 | 0.12 | 8466.02 | 0.632 |
| 2000 | 536.6 | 0.027 | 1897 | 0.13 | 0.1 | 0.12 | 11,745.25 | 0.655 |

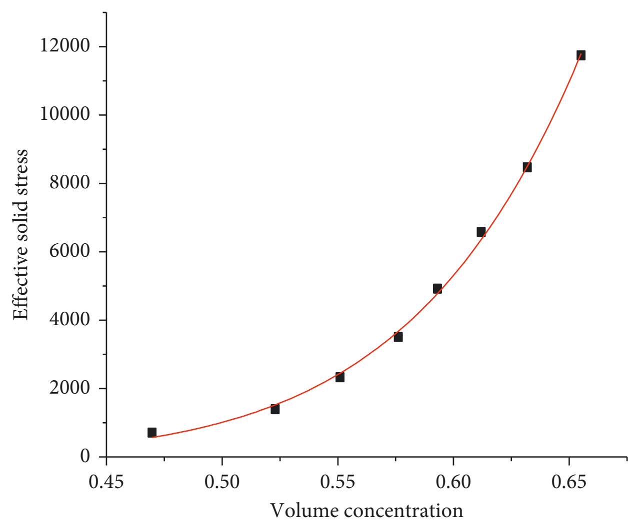


Figure 2: Experimental data obtained from batch centrifuge experiments and curve of equation.

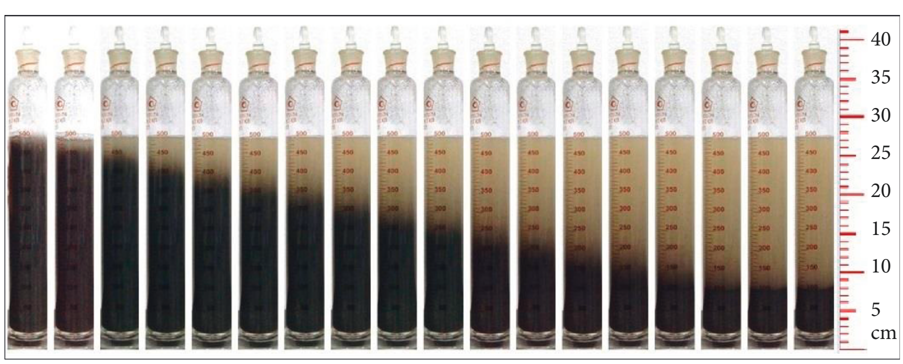


Figure 3: Schematic of tailing settling experiment.

Table 3: Results of tailing settling experiment.

| Mass concentration | Volume concentration | Settling velocity (m/s) | Suspension concentration (kg/m$^3$) | Solid flux (kg/m$^2$·s) |
|---|---|---|---|---|
| 0.15 | 0.06 | 0.00048913 | 166.09 | 0.08124 |
| 0.2 | 0.08 | 0.00037371 | 229.67 | 0.08583 |
| 0.25 | 0.1 | 0.0002698 | 298.14 | 0.08044 |
| 0.3 | 0.13 | 0.00019538 | 372.1 | 0.0727 |
| 0.35 | 0.16 | 0.00013995 | 452.23 | 0.06329 |
| 0.4 | 0.19 | 0.00009677 | 539.34 | 0.05219 |

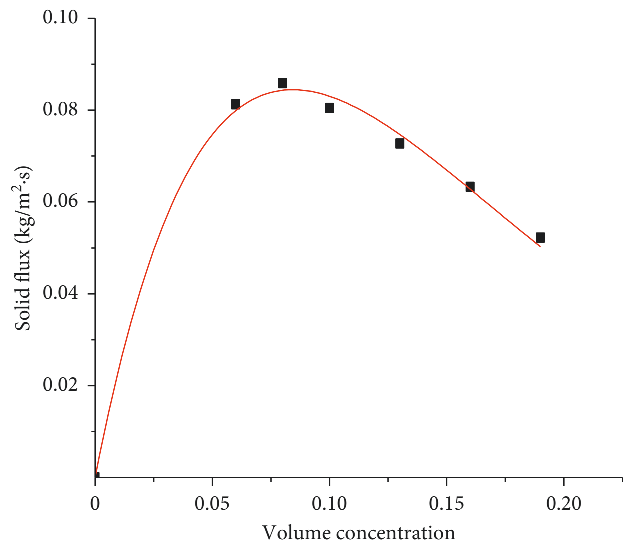


Figure 4: Fitting formula curve of solid flux density.

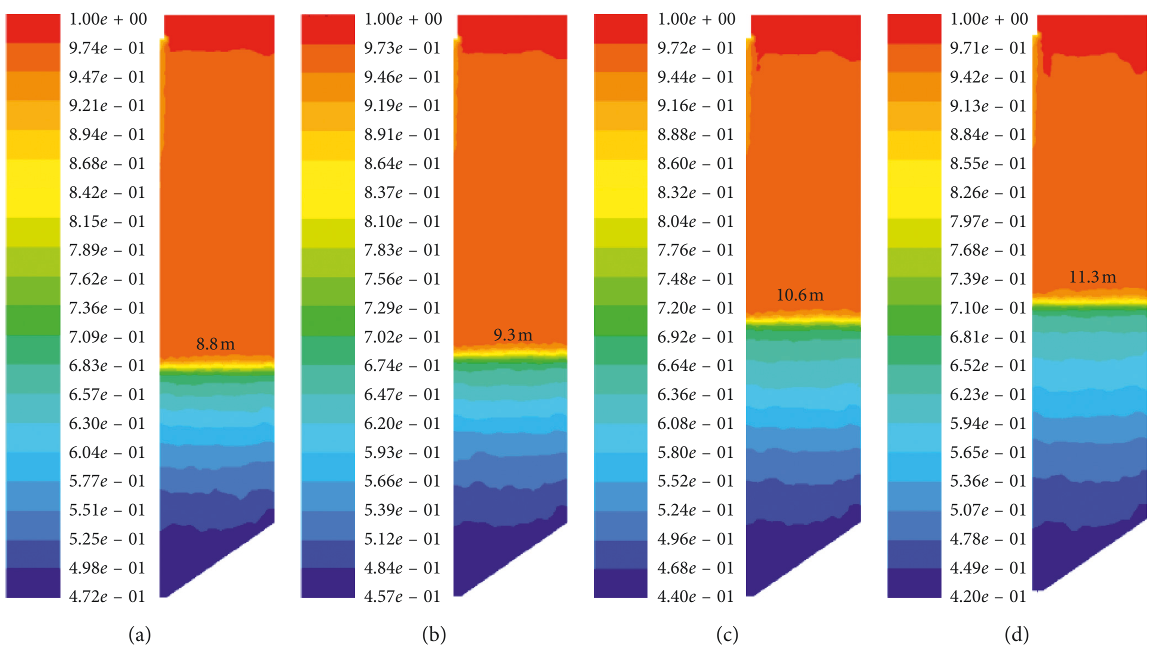


Figure 5: Volume fraction contours of the water phase, with interface levels of (a) 8.8 m, (b) 9.3 m, (c) 10.6 m, and (d) 11.3 m.

11.12 m, respectively, were calculated using the partial differential equation (9).

*4.3. Industrial Test.* An industrial test was carried out by the first filling station from the Dahongshan copper mine in Yunnan, China. The photographs are presented in Figure 7. The volume concentration of the tailing slurry is 0.13, and the flow is 300 $m^3$/h at feeding. Underflow volume is 0.5189, 0.5371, 0.5512, and 0.5693, respectively, when interface levels between tailings and water are 8.5, 9.3, 10.3, and 11.1 m of the industrial test result.

*4.4. Analyses.* The simulation results of the interface level between tailings and water according to Fluent software were compared with the calculation results using partial differential equation (9) when the volume concentrations of underflow were 0.5274, 0.5421, 0.5582, and 0.5703 for the tailing silo, as presented in Table 4 and Figure 8. The industry test results of underflow volume compared with the results of the differential equation are presented in Table 5 and Figure 8.

Solving the differential equation shows that the concentrations of underflow volume (0.5271, 0.5421, 0.5582, and 0.5703) corresponding to different heights of tailing surfaces are closely related to the results simulated using the data, and

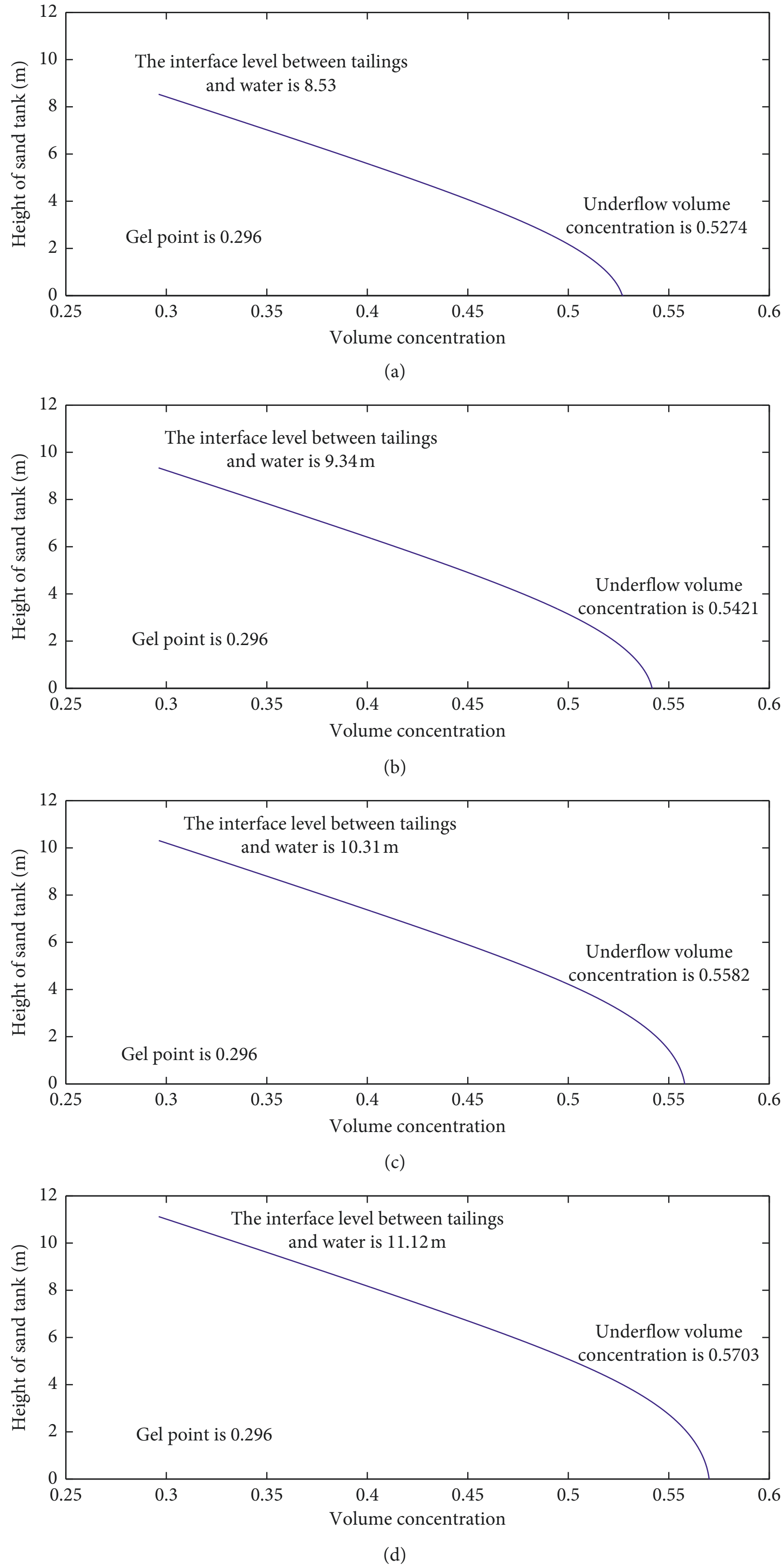


FIGURE 6: Curve of changing between volume concentration of the slurry and height of the tailing silo, with underflow volume concentrations of (a) 0.5274, (b) 0.5421, (c) 0.5582, and (d) 0.5703.

(a) (b)

Figure 7: Photographs of industrial test.

Table 4: Interface level results of numerical simulation and calculation of differential equation.

| Underflow concentration | Numerical simulation by Fluent software (m) | Calculation of differential equation (m) |
|---|---|---|
| 0.5274 | 8.8 | 8.53 |
| 0.5421 | 9.3 | 9.34 |
| 0.5582 | 10.6 | 10.31 |
| 0.5703 | 11.3 | 11.12 |

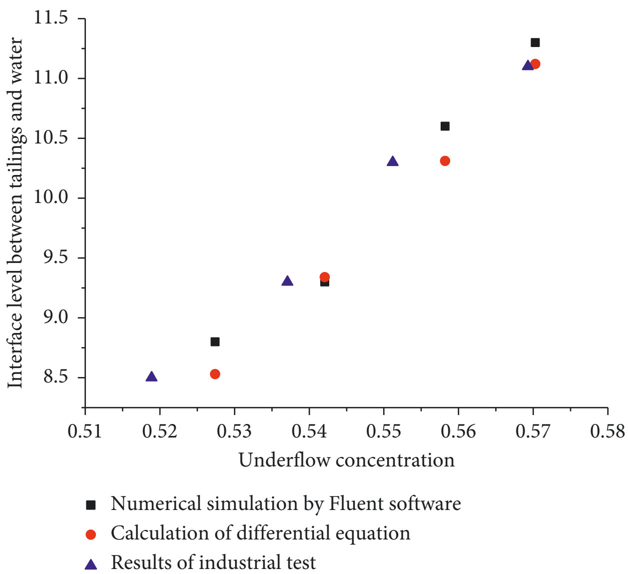


Figure 8: Relationship of underflow volume concentration and interface levels between tailings and water via numerical simulation by Fluent software, calculation of differential equation, and industrial test.

the results of the industrial test are closely related to the results of the differential equation too, thus verifying the accuracy of the differential equation (9).

## 5. Conclusions

This study focused on the simulation and solution of dynamic settlement of tailings in a silo and the solution of a continuous, high-concentration, and stable model. In this study, the tailings from the first filling station in the Dahongshan copper mine in Yuxi, located in Yunnan Province, China, were taken as the experimental material. Solid flux function, effective bulk stress, and critical compression ratios were obtained experimentally through the use of batch settling and separation trials run in a centrifuge. Taking a vertical silo with a diameter of 9 m as the geometrical model, the dynamic and static settlements of tailings in the vertical silo were simulated. In the dynamic settlement of the tailings, volume concentrations of the underflow for four different cumulative tailing heights were evaluated, and the variation of slurry concentration with the height of the silo was simulated. Next, the cumulative heights of tailings required for various underflow concentrations and the variation of slurry concentration with soil height were acquired by running the proposed continuous, high-concentration, and stable model. The results were all close to the simulated results, which verified the accuracy of the model, and then, the theoretical results were applied to the tailing filling and discharge system. The industrial testing of the model was carried out in a vertical silo (with a diameter of 9 m). The accuracy was also evinced by comparison of the results of the industrial test and the model. The key conclusions were as follows:

Table 5: Underflow results of industrial test and calculation of differential equation.

| Interface level (m) | Industrial test (m) | Calculation of differential equation (m) |
|---|---|---|
| 8.5 | 0.5189 | 0.5274 |
| 9.3 | 0.5371 | 0.5421 |
| 10.3 | 0.5512 | 0.5582 |
| 11.1 | 0.5693 | 0.5703 |

(1) In existing tailing discharge systems, feeding, settlement, and discharge are alternately conducted in multiple vertical silos. Differing from this mode of operation, a continuous, high-concentration, and stable tailing discharge model is established on the basis of the mass balance between the tailings and water. The dynamic settlement of the tailings in the silo was simulated using the same software. In the simulation, four accumulated heights of tailings were selected to record the corresponding underflow volume concentration and the variation of slurry concentration with changing height of the silo based on the preset monitoring curves.

(2) The equations governing the solid flux and the effective solid stress are derived by carrying out settling experiments and separation experiments using the centrifuge. Then, these factors are substituted into the proposed continuous, high-concentration, and stable tailing discharge model to solve the model using mathematical software. In this way, the underflow volume concentrations corresponding to different accumulation heights and the variation of the volume concentration of the slurry with silo height are obtained. The comparison of the results and the simulated results using Fluent software showed that they were sufficiently similar

to verify the correctness of the model. The application of the model to an industrial test displays favorable effects, which validates the theoretical efficacy of the model.

The application of the proposed continuous, high-concentration, and stable tailing discharge model for vertical silos effectively overcomes the technical problems facing existing tailing discharge systems in mines. These problems include low discharge concentration, large concentration fluctuations, and the necessity of frequently applying high-pressure flow and water. The proposed model significantly improved the work efficiency of vertical silos, as it reduced the number of working vertical silos, omitted the process of completely discharging and charging the silos, and simplified the preparation of slurry materials. With these advantages, the model guaranteed the filling efficiency and quality. Industrial tests showed that the model worked and that the underflow volume concentration met the requirements for mine production. Therefore, it avoided the waste of resources and equipment and saved water and electricity. The research provided a theoretical basis and technical guidance for the design of continuous tailing discharge and filling systems.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Weicheng Ren performed the experiments and wrote the manuscript. Rugao Gao was involved in the data analysis and the manuscript preparation. Deqing Gan and Youzhi Zhang was the tutor and approved the manuscript.

## Acknowledgments

This work was financially supported by the National Natural Science Foundation of China (51774137) and the PhD Research Startup Foundation of North China University of Science and Technology (0088/28413799).

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