Research Article

Topological Indices of the Pent-Heptagonal Nanosheets VC_5C_7 and HC_5C_7

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In this paper, we computed the topological indices of pent-heptagonal nanosheet. Formulas for atom-bond connectivity index, fourth atom-bond connectivity index, Randić connectivity index, sum-connectivity index, first Zagreb index, second Zagreb index, augmented Zagreb index, modified Zagreb index, hyper Zagreb index, geometric-arithmetic index, fifth geometric-arithmetic index, Sanskruti index, forgotten index, and harmonic index of pent-heptagonal nanosheet have been derived.

1. Introduction

Nanostructures, which have scale less than 100 nm, include nanotubes, nanosheets, and nanoparticles. Nanosheets (two-dimensional nanostructures) have large surface and sharp edge, which make them play an important role in many applications such as energy storage [1–3], optoelectronics [4], catalysis [5], and bioelectronics [6]. Graphene [7], silicone [8], and borophene [9] are typical nanosheets. Graphene is the basic nanosheet that exhibits the honeycomb lattice structure. Because of the unique structural, electrical, optical, and mechanical properties, graphene nanosheets drew dramatic attention of academic and industrial research [10].

People found some defects in graphene nanosheets made through experiments. In Figure 1, the 5–7 defect in graphene is illustrated. As nanoscale-level defects introduced into graphene could be extremely useful and exploited to generate novel, innovative, and useful materials and devices [10], the 5–8–5 [11, 12] and 5–7–7–5 defects are obtained by several approaches [13–16]. Naturally, the property of C_5C_7 nanostructures has become an active area of research [17–22].

In chemical graph theory, topological indices are very useful to describe molecules or nanostructures. The vertices of the graph correspond to the atoms of molecules, and the edges correspond to chemical bonds.

In this paper, we only consider undirected graphs without loops and multiple edges. Let G be a graph. We denote $V(G)$ and $E(G)$ as the vertex set and edge set of G , respectively. We denote by $d(v)$ the degree of the vertex v of graph G and $N(v)$ the set of neighbors of v . We denote by $E_{a,b}$ the set of edges with degrees of end vertices a and b , i.e.,

$$E_{a,b} = \{uv \mid \{a, b\} = \{d(u), d(v)\}\}. \quad (1)$$

Let the sum of degrees of neighboring vertices of u be denoted by $s_G(u)$, which is defined as

$$s_G(u) = \sum_{v \in N(u)} d(v). \quad (2)$$

We define

$$E'_{a,b} = \{uv \mid \{a, b\} = \{s_G(u), s_G(v)\}\}. \quad (3)$$

In 1998, Estrada et al. [23] defined the atom-bond connectivity index as

FIGURE 1: The C_5C_7 defect.

$$ABC(G) = \sum_{e=uv \in E(G)} \sqrt{\frac{d(u) + d(v) - 2}{d(u)d(v)}}. \quad (4)$$

The ABC_4 index is defined as

$$ABC_4(G) = \sum_{e=uv \in E(G)} \sqrt{\frac{s_G(u) + s_G(v) - 2}{s_G(u)s_G(v)}}. \quad (5)$$

The Randić connectivity index (see [24]) is defined on the ground of vertex degrees as

$$R(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{d(u)d(v)}}. \quad (6)$$

The sum-connectivity index is defined (see [25]) as

$$X(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{d(u) + d(v)}}. \quad (7)$$

The first Zagreb index $M_1(G)$ is defined as the sum of squares of the vertex degrees $d(u)$ and $d(v)$ of vertices u and v in graph G (see [26]). The mathematical formula of first Zagreb index is

$$M_1(G) = \sum_{v \in V(G)} (d(v))^2 = \sum_{e=uv \in E(G)} (d(u) + d(v)). \quad (8)$$

The second Zagreb index $M_2(G)$ is defined as the sum of squares of the vertex degrees $d(u)$ and $d(v)$ of vertices u and v in graph G (see [26]). The mathematical formula of second Zagreb index is

$$M_2(G) = \sum_{e=uv \in E(G)} (d(u)d(v)). \quad (9)$$

Augmented Zagreb index $AZI(G)$ (see [27]) is defined as

$$AZI(G) = \sum_{e=uv \in E(G)} \left(\frac{d(u)d(v)}{d(u) + d(v) - 2} \right)^3. \quad (10)$$

Modified Zagreb index $M_2^*(G)$ (see [28]) is defined as

$$M_2^*(G) = \sum_{e=uv \in E(G)} \frac{1}{d(u)d(v)}. \quad (11)$$

In 2013, Shirdel et al. [28] introduced hyper Zagreb index as

$$HM(G) = \sum_{e=uv \in E(G)} (d(u)^d(v))^2. \quad (12)$$

The geometric-arithmetic index (see [29]) is defined as

$$GA(G) = \sum_{e=uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \quad (13)$$

The GA_5 is Defined as

$$GA_5(G) = \sum_{e=uv \in E(G)} \frac{2\sqrt{s_G(u)s_G(v)}}{s_G(u) + s_G(v)}. \quad (14)$$

For a detailed survey about topological indices and applications of topological indices, we refer [30–38] and references therein. Recently, motivated by the previous research on topological descriptors and their applications, Hosamani [39] proposed the Sanskruti index which can be utilized to guess the bioactivity of chemical compounds and shows good correlation with entropy of octane isomers.

The Sanskruti index of graph G is defined as

$$S(G) = \sum_{uv \in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3. \quad (15)$$

The forgotten index $F(G)$ is defined as the sum of cubic of the vertex degrees in G . Mathematical formula of forgotten index is [40]

$$F(G) = \sum_{v \in V(G)} (d(v)^3) = \sum_{uv \in E(G)} (d(u)^2 + d(v)^2). \quad (16)$$

The harmonic index $H(G)$ (see [41]) is defined as

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}. \quad (17)$$

2. C_5C_7 Nanosheets and Their Structures

A C_5C_7 net is a trivalent decoration made by alternating C_5 and C_7 . It can cover either a cylinder or a torus and certainly able to cover a sheet.

The topological indices of nanostructures have been studied extensively. Graovac et al. [42] discussed the GA index of $TUC_4C_8(S)$ nanotube in 2009. Graovac et al. [42] computed fifth geometric-arithmetic index for nanostar dendrimers in 2011. Asadpour et al. [43] computed Randić, Zagreb, and ABC indices of $TUC_4C_8(S)$ and $TUC_4C_8(R)$ nanotubes and V-phenylenic nanotorus in 2011. Al-Fozan et al. [44] computed the Szeged index of $C_4C_8(S)$ and H-naphtalenic nanosheets $(2n, 2m)$ in 2014. Azari and Iranmanesh [45] computed in Harary index of $Pm \times Pn$ (nanosheets), C_4 nanotubes, and C_4 nanotori in 2014. Ashrafi and Loghman [46] discussed the Padmakar–Ivan (PI) index of $TUC_4C_8(S)$ nanotubes in 2006.

The topological indices of C_5C_7 nanostructures have also been studied. Iranmanesh et al. [47] introduced a GAP program to compute the Szeged and PI indices of $VC_5C_7[p,q]$ and $HC_5C_7[p,q]$ nanotubes in 2008. Farahani and Zhang [48–53] discussed Randić connectivity index and sum connectivity index of the pent-heptagonal nanotube $VAC_5C_7(S)$ in 2013. For the nanosheet $VC_5C_7(m, n)$, we denote the number of pentagons in the first row by n ; in this nanotube, the first four rows of vertices and edges are repeated alternatively and we denote the number of this repetition by m (see, e.g., Figure 2(a) with $(m, n) = (2, 4)$).

For the nanosheet $HC_5C_7(m, n)$, we denote the number of heptagons in the first row by n . In this nanotube, the first four rows of vertices and edges are repeated alternatively and we denote the number of this repetition by m (see, e.g., Figure 2(b) with $(m, n) = (2, 4)$).

The nanosheet $VC_5C_7(2, 4)$ has $16mn + 2m + 5n$ vertices and $24mn + 4n$ edges. Moreover, it has $6m + 7n$ vertices with degree 2 and $16mn - 4m - 2n$ vertices with degree 3. The edge partitions according to different $E_{s,t}$ and $E'_{s,t}$ are presented in Tables 1 and 2, respectively. We computed some topological indices according to different types of $E_{s,t}$ and $E'_{s,t}$. As an example, Figures 3(a) and 3(b) show the edge partition of the nanosheet $VC_5C_7(2, 4)$ by different $E_{s,t}$ and $E'_{s,t}$, respectively.

The nanosheet $HC_5C_7(2, 4)$ has $16mn + 2m + 4n$ vertices and $24mn + 3n$ edges. Moreover, it has $6m + 6n$ vertices with degree 2 and $16mn - 4m - 2n$ vertices with degree 3. We computed some topological indices according to different types of $E_{s,t}$ and $E'_{s,t}$. The edge partitions according to different $E_{s,t}$ and $E'_{s,t}$ are presented in Tables 3 and 4, respectively. As an example, Figures 4(a) and 4(b) show the edge partition of the nanosheet $HC_5C_7(2, 4)$ by different $E_{s,t}$ and $E'_{s,t}$, respectively.

3. Main Results

Theorem 1.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$ABC(G) = 16mn + \left(6\sqrt{2} - \frac{16}{3}\right)n + \left(5\sqrt{2} - \frac{20}{3}\right)m - 2\sqrt{2} + \frac{8}{3}. \quad (18)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$ABC(G) = 5\sqrt{2}m - 4n - \frac{20m}{3} + \frac{9\sqrt{2}}{2}n + 16mn - \sqrt{2} + \frac{4}{3}. \quad (19)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of ABC index, we have

FIGURE 2: The nanosheets (a) $VC_5C_7(2, 4)$ and (b) $HC_5C_7(2, 4)$.

TABLE 1: Partition of the edge set of $VC_5C_7(m, n)$ into $E_{s,t}$.

(s, t)	(2, 3)	(3, 3)	(2, 2)
$E'_{s,t}$	$8m + 10n - 8$	$24mn - 10m - 8n + 4$	$2m + 2n + 4$

TABLE 2: Partition of the edge set of $VC_5C_7(m, n)$ into $E'_{s,t}$.

(s, t)	(6, 7)	(7, 9)	(5, 7)
$E'_{s,t}$	$4m + 6n - 8$	$4m + 4n - 3$	$4m + 2n + 2$
(s, t)	(9, 9)	(4, 5)	(5, 5)
$E'_{s,t}$	$24mn - 14m - 15n + 10$	8	$2m + 2n - 4$
(s, t)	(8, 9)	(5, 8)	(8, 8)
$E'_{s,t}$	$2n - 2$	$2n - 2$	$n - 2$

$$\begin{aligned} ABC(G) &= |E_{2,3}| \sqrt{\frac{3}{6}} + |E_{3,3}| \sqrt{\frac{4}{9}} + |E_{2,2}| \sqrt{\frac{2}{4}} \\ &= (8m + 10n - 8) \sqrt{\frac{3}{6}} + (24mn - 10m - 8n + 4) \sqrt{\frac{4}{9}} \\ &\quad + (2m + 2n + 4) \sqrt{\frac{2}{4}} \\ &= 16mn + \left(6\sqrt{2} - \frac{16}{3}\right)n + \left(5\sqrt{2} - \frac{20}{3}\right)m - 2\sqrt{2} + \frac{8}{3}. \end{aligned} \quad (20)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of ABC index, we have

FIGURE 3: The edge partition of the nanosheets (a) $VC_5C_7(2, 4)$ by different $E_{s,t}$ and (b) $VC_5C_7(2, 4)$ by different $E'_{s,t}$.

TABLE 3: Partition of the edge set of $HC_5C_7(m, n)$ into $E_{s,t}$.

(s, t)	$(3, 3)$	$(2, 3)$	$(2, 2)$
$E'_{s,t}$	$24mn - 10m - 6n + 10$	$8m + 6n - 4$	$2m + 3n + 2$

TABLE 4: Partition of the edge set of $HC_5C_7(m, n)$ into $E'_{s,t}$.

(s, t)	$(8, 9)$	$(6, 8)$	$(8, 8)$
$E'_{s,t}$	$8m + 6n - 12$	$4m + 2n - 4$	$4m + n$
(s, t)	$(9, 9)$	$(7, 9)$	$(6, 7)$
$E'_{s,t}$	$24mn - 22m - 14n + 13$	$n + 1$	$2n$
(s, t)	$(5, 7)$	$(4, 4)$	—
$E'_{s,t}$	2	n	—

$$\begin{aligned}
 ABC(G) &= |E_{3,3}| \sqrt{\frac{4}{9}} + |E_{2,3}| \sqrt{\frac{3}{6}} + |E_{2,2}| \sqrt{\frac{2}{4}} \\
 &= (24mn - 10m - 6n + 2) \sqrt{\frac{4}{9}} + (8m + 6n - 4) \sqrt{\frac{3}{6}} \\
 &\quad + (2m + 3n + 2) \sqrt{\frac{2}{4}} \\
 &= 5\sqrt{2}m - 4n - \frac{20m}{3} + \frac{9\sqrt{2}}{2}n + 16mn - \sqrt{2} + \frac{4}{3}.
 \end{aligned} \tag{21}$$

Theorem 2.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned}
 ABC_4(G) &= \frac{32\sqrt{2}m}{15} - \frac{20n}{3} - \frac{56m}{9} + \frac{4\sqrt{14}m}{7} + \frac{2\sqrt{462}m}{21} \\
 &\quad + \frac{32\sqrt{2}n}{15} + \frac{23\sqrt{14}n}{56} + \frac{\sqrt{30}n}{6} + \frac{\sqrt{110}n}{10} \\
 &\quad + \frac{\sqrt{462}n}{7} + \frac{32mn}{3} - \frac{13\sqrt{2}}{5} + \frac{9\sqrt{14}}{56} - \frac{\sqrt{30}}{6} \\
 &\quad + \frac{4\sqrt{35}}{5} - \frac{\sqrt{110}}{10} - \frac{4\sqrt{462}}{21} + \frac{40}{9}.
 \end{aligned} \tag{22}$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned}
 ABC_4(G) &= \frac{2\sqrt{14}}{7} - \frac{47n}{9} - \frac{70m}{9} + \frac{32mn}{3} + \frac{\sqrt{2}(n+1)}{3} \\
 &\quad + \frac{2\sqrt{2}(2m-2)}{5} + \frac{\sqrt{110}(2m+n-1)}{10} \\
 &\quad + \frac{\sqrt{30}(4m+3n-6)}{6} + \frac{\sqrt{35}(n+2)}{5} \\
 &\quad + \frac{\sqrt{6}n}{4} + \frac{\sqrt{462}n}{21} + \frac{\sqrt{14}(4m+n)}{8} + \frac{34}{9}.
 \end{aligned} \tag{23}$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 2. By the definition of ABC_4 index, we have

$$\begin{aligned}
 ABC_4(G) &= \sqrt{\frac{11}{42}}|E'_{6,7}| + \sqrt{\frac{14}{63}}|E'_{7,9}| + \sqrt{\frac{10}{35}}|E'_{5,7}| + \sqrt{\frac{16}{81}}|E'_{9,9}| \\
 &\quad + \sqrt{\frac{7}{20}}|E'_{4,5}| + \sqrt{\frac{8}{25}}|E'_{5,5}| + \sqrt{\frac{15}{72}}|E'_{8,9}| \\
 &\quad + \sqrt{\frac{11}{40}}|E'_{5,8}| + \sqrt{\frac{14}{64}}|E'_{8,8}| \\
 &= \sqrt{\frac{11}{42}}(4m + 6n - 8) + \sqrt{\frac{14}{63}}(4m + 4n - 3) \\
 &\quad + \sqrt{\frac{10}{35}}(4m + 2n + 2) \\
 &\quad + \sqrt{\frac{16}{81}}(24mn - 14m - 15n + 10) + \sqrt{\frac{7}{20}}(8) \\
 &\quad + \sqrt{\frac{8}{25}}(2m + 2n - 4) + \sqrt{\frac{15}{72}}(2n - 2) \\
 &\quad + \sqrt{\frac{11}{40}}(2n - 2) + \sqrt{\frac{14}{64}}(n - 1) \\
 &= \frac{32\sqrt{2}m}{15} - \frac{20n}{3} - \frac{56m}{9} + \frac{4\sqrt{14}m}{7} + \frac{2\sqrt{462}m}{21} \\
 &\quad + \frac{32\sqrt{2}n}{15} + \frac{23\sqrt{14}n}{56} + \frac{\sqrt{30}n}{6} + \frac{\sqrt{110}n}{10} \\
 &\quad + \frac{\sqrt{462}n}{7} + \frac{32mn}{3} - \frac{13\sqrt{2}}{5} + \frac{9\sqrt{14}}{56} - \frac{\sqrt{30}}{6} \\
 &\quad + \frac{4\sqrt{35}}{5} - \frac{\sqrt{110}}{10} - \frac{4\sqrt{462}}{21} + \frac{40}{9}.
 \end{aligned} \tag{24}$$

FIGURE 4: The edge partition of the nanosheets (a) HC₅C₇(2, 4) by different $E_{s,t}$ (b) and HC₅C₇(2, 4) by different $E'_{s,t}$.

- (ii) The proof is calculated based on the edge partition given in Table 4. By the definition of ABC₄ index, we have

$$\begin{aligned}
 ABC_4(G) &= \sqrt{\frac{15}{72}}|E'_{8,9}| + \sqrt{\frac{12}{48}}|E'_{6,8}| + \sqrt{\frac{14}{64}}|E'_{8,8}| + \sqrt{\frac{16}{81}}|E'_{9,9}| \\
 &+ \sqrt{\frac{14}{63}}|E'_{7,9}| + \sqrt{\frac{11}{42}}|E'_{6,7}| + \sqrt{\frac{11}{40}}|E'_{5,8}| \\
 &+ \sqrt{\frac{7}{20}}|E'_{4,5}| + \sqrt{\frac{8}{25}}|E'_{5,5}| \\
 &+ \sqrt{\frac{10}{35}}|E'_{5,7}| + \sqrt{\frac{6}{16}}|E'_{4,4}| \\
 &= \sqrt{\frac{15}{72}}(8m + 6n - 12) + \sqrt{\frac{12}{48}}(4m + 2n - 4) \\
 &+ \sqrt{\frac{14}{64}}(4m + n) + \sqrt{\frac{16}{81}}(24mn - 22m - 14n + 13) \\
 &+ \sqrt{\frac{14}{63}}(n + 1) + \sqrt{\frac{11}{42}}(2n) + \sqrt{\frac{11}{40}}(4m + 2n - 2) \quad (25) \\
 &+ \sqrt{\frac{7}{20}}(2n + 4) + \sqrt{\frac{8}{25}}(2m - 2) \\
 &+ \sqrt{\frac{10}{35}} \times 2 + \sqrt{\frac{6}{16}}n \\
 &= \frac{2\sqrt{14}}{7} - \frac{47n}{9} - \frac{70m}{9} + \frac{32mn}{3} + \frac{\sqrt{2}(n+1)}{3} \\
 &+ \frac{2\sqrt{2}(2m-2)}{5} + \frac{\sqrt{110}(2m+n-1)}{10} \\
 &+ \frac{\sqrt{30}(4m+3n-6)}{6} + \frac{\sqrt{35}(n+2)}{5} + \frac{\sqrt{6}n}{4} \\
 &+ \frac{\sqrt{462}n}{21} + \frac{\sqrt{14}(4m+n)}{8} + \frac{34}{9}.
 \end{aligned}$$

Theorem 3.

- (i) Let G be a VC₅C₇(m, n) nanosheet, then

$$R(G) = 8mn - \frac{5n}{3} - \frac{7m}{2} + \frac{\sqrt{6}}{6}(8m + 10n - 8) + \frac{10}{3}. \quad (26)$$

- (ii) Let G be a HC₅C₇(m, n) nanosheet, then

$$R(G) = 8mn - \frac{n}{2} - \frac{7m}{3} + \frac{\sqrt{6}}{3}(4m + 3n - 2) + \frac{5}{3}. \quad (27)$$

Proof.

- (i) The proof is calculated based on the edge partition given in Table 1. By the definition of the Randić connectivity index, we have

$$\begin{aligned}
 R(G) &= |E_{2,3}| \frac{1}{\sqrt{6}} + |E_{3,3}| \frac{1}{\sqrt{9}} + |E_{2,2}| \frac{1}{\sqrt{4}} \\
 &= (8m + 10n - 8) \frac{1}{\sqrt{6}} + (24mn - 10m - 8n + 4) \frac{1}{\sqrt{9}} \\
 &+ (2m + 2n + 4) \frac{1}{\sqrt{4}} \\
 &= 8mn - \frac{5n}{3} - \frac{7m}{3} + \frac{\sqrt{6}}{3}(4m + 5n - 4) + \frac{10}{3}. \quad (28)
 \end{aligned}$$

- (ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the Randić connectivity index, we have

$$\begin{aligned}
 R(G) &= |E_{3,3}| \frac{1}{\sqrt{9}} + |E_{2,3}| \frac{1}{\sqrt{6}} + |E_{2,2}| \frac{1}{\sqrt{4}} \\
 &= (24mn - 10m - 6n + 2) \frac{1}{\sqrt{9}} + (8m + 6n - 4) \frac{1}{\sqrt{6}} \\
 &+ (2m + 3n + 2) \frac{1}{\sqrt{4}} \\
 &= 8mn - \frac{n}{2} - \frac{7m}{3} + \frac{\sqrt{6}}{3}(4m + 3n - 2) + \frac{5}{3}. \quad (29)
 \end{aligned}$$

Theorem 4.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$X(G) = m + n + \frac{\sqrt{5}(8m + 10n - 8)}{5} - \frac{\sqrt{6}(5m + 4n - 12mn - 2)}{3} + 2. \quad (30)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$X(G) = m + \frac{3n}{2} + \frac{\sqrt{5}(8m + 6n - 4)}{5} - \frac{\sqrt{6}(5m + 3n - 12mn - 1)}{3} + 1. \quad (31)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the sum connectivity index, we have

$$\begin{aligned} X(G) &= |E_{2,3}| \frac{1}{\sqrt{5}} + |E_{3,3}| \frac{1}{\sqrt{6}} + |E_{2,2}| \frac{1}{\sqrt{4}} \\ &= (8m + 10n - 8) \frac{1}{\sqrt{5}} + (24mn - 10m - 8n + 4) \frac{1}{\sqrt{6}} \\ &\quad + (2m + 2n + 4) \frac{1}{\sqrt{4}} \\ &= m + n + \frac{\sqrt{5}(8m + 10n - 8)}{5} \\ &\quad - \frac{\sqrt{6}(5m + 4n - 12mn - 2)}{3} + 2. \end{aligned} \quad (32)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the sum connectivity index, we have

$$\begin{aligned} X(G) &= |E_{3,3}| \frac{1}{\sqrt{6}} + |E_{2,3}| \frac{1}{\sqrt{5}} + |E_{2,2}| \frac{1}{\sqrt{4}} \\ &= (24mn - 10m - 6n + 2) \frac{1}{\sqrt{6}} + (8m + 6n - 4) \frac{1}{\sqrt{5}} \\ &\quad + (2m + 3n + 2) \frac{1}{\sqrt{4}} \\ &= m + \frac{3n}{2} + \frac{\sqrt{5}(8m + 6n - 4)}{5} \\ &\quad - \frac{\sqrt{6}(5m + 3n - 12mn - 1)}{3} + 1. \end{aligned} \quad (33)$$

Theorem 5.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$M_1(G) = 10n - 12m + 144mn. \quad (34)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$M_1(G) = 6n - 12m + 144n. \quad (35)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the first Zagreb index, we have

$$\begin{aligned} M_1(G) &= 5|E_{2,3}| + 6|E_{3,3}| + 4|E_{2,2}| \\ &= 5(8m + 10n - 8) + 6(24mn - 10m - 8n + 4) \\ &\quad + 4(2m + 2n + 4) \\ &= 10n - 12m + 144mn. \end{aligned} \quad (36)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the first Zagreb index, we have

$$\begin{aligned} M_1(G) &= 6|E_{3,3}| + 5|E_{2,3}| + 4|E_{2,2}| \\ &= 6(24mn - 10m - 6n + 2) + 5(8m + 6n - 4) \\ &\quad + 4(2m + 3n + 2) \\ &= 6n - 12m + 144n. \end{aligned} \quad (37)$$

Theorem 6.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$M_2(G) = 216mn - 4n - 34m + 4. \quad (38)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$M_2(G) = 216mn - 6n - 34m + 2. \quad (39)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the second Zagreb index, we have

$$\begin{aligned} M_2(G) &= 6|E_{2,3}| + 9|E_{3,3}| + 4|E_{2,2}| \\ &= 6(8m + 10n - 8) + 9(24mn - 10m - 8n + 4) \\ &\quad + 4(2m + 2n + 4) \\ &= 216mn - 4n - 34m + 4. \end{aligned} \quad (40)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the second Zagreb index, we have

$$\begin{aligned} M_2(G) &= 9|E_{3,3}| + 6|E_{2,3}| + 4|E_{2,2}| \\ &= 9(24mn - 10m - 6n + 2) + 6(8m + 6n - 4) \\ &\quad + 4(2m + 3n + 2) \\ &= 216mn - 6n - 34m + 2. \end{aligned} \quad (41)$$

Theorem 7.(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$AZI(G) = \frac{39n}{8} - \frac{1085m}{32} + \frac{2187mn}{8} + \frac{217}{16}. \quad (42)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$AZI(G) = \frac{117n}{32} - \frac{1085m}{32} + \frac{2187mn}{8} + \frac{217}{32}. \quad (43)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the augmented Zagreb index, we have

$$\begin{aligned} AZI(G) &= \left(\frac{6}{3}\right)^3 |E_{2,3}| + \left(\frac{9}{4}\right)^3 |E_{3,3}| + \left(\frac{4}{2}\right)^3 |E_{2,2}| \\ &= \left(\frac{6}{3}\right)^3 (8m + 10n - 8) + \left(\frac{9}{4}\right)^3 \\ &\quad \cdot (24mn - 10m - 8n + 4) + \left(\frac{4}{2}\right)^3 (2m + 2n + 4) \\ &= \frac{39n}{8} - \frac{1085m}{32} + \frac{2187mn}{8} + \frac{217}{16}. \end{aligned} \quad (44)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the augmented Zagreb index, we have

$$\begin{aligned} AZI(G) &= \left(\frac{9}{4}\right)^3 |E_{3,3}| + \left(\frac{6}{3}\right)^3 |E_{2,3}| + \left(\frac{4}{2}\right)^3 |E_{2,2}| \\ &= \left(\frac{9}{4}\right)^3 (24mn - 10m - 6n + 2) + \left(\frac{6}{3}\right)^3 \\ &\quad \cdot (8m + 6n - 4) + \left(\frac{4}{2}\right)^3 (2m + 3n + 2) \\ &= \frac{117n}{32} - \frac{1085m}{32} + \frac{2187mn}{8} + \frac{217}{32}. \end{aligned} \quad (45)$$

Theorem 8.(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$M_2^*(G) = \frac{13m}{18} + \frac{23n}{18} + \frac{8mn}{3} + \frac{1}{9}. \quad (46)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$M_2^*(G) = \frac{13m}{18} + \frac{13n}{12} + \frac{8mn}{3} + \frac{1}{18}. \quad (47)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the modified Zagreb index, we have

$$\begin{aligned} M_2^*(G) &= \frac{1}{2 \times 3} |E_{2,3}| + \frac{1}{3 \times 3} |E_{3,3}| + \frac{1}{2 \times 2} |E_{2,2}| \\ &= \frac{1}{2 \times 3} (8m + 10n - 8) + \frac{1}{3 \times 3} \\ &\quad \cdot (24mn - 10m - 8n + 4) + \frac{1}{2 \times 2} (2m + 2n + 4) \\ &= \frac{13m}{18} + \frac{23n}{18} + \frac{8mn}{3} + \frac{1}{9}. \end{aligned} \quad (48)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the modified Zagreb index, we have

$$\begin{aligned} M_2^*(G) &= \frac{1}{3 \times 3} |E_{3,3}| + \frac{1}{2 \times 3} |E_{2,3}| + \frac{1}{2 \times 2} |E_{2,2}| \\ &= \frac{1}{3 \times 3} (24mn - 10m - 6n + 2) + \frac{1}{2 \times 3} \\ &\quad \cdot (8m + 6n - 4) + \frac{1}{2 \times 2} (2m + 3n + 2) \\ &= \frac{13m}{18} + \frac{13n}{12} + \frac{8mn}{3} + \frac{1}{18}. \end{aligned} \quad (49)$$

Theorem 9.(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$HM(G) = 864mn - 6n - 128m + 8. \quad (50)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$HM(G) = 864mn - 18n - 128m + 4. \quad (51)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the hyper Zagreb index, we have

$$\begin{aligned} HM(G) &= (2 + 3)^2 |E_{2,3}| + (3 + 3)^2 |E_{3,3}| + (2 + 2)^2 |E_{2,2}| \\ &= (2 + 3)^2 (8m + 10n - 8) + (3 + 3)^2 \\ &\quad \cdot (24mn - 10m - 8n + 4) + (2 + 2)^2 (2m + 2n + 4) \\ &= 864mn - 6n - 128m + 8. \end{aligned} \quad (52)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the hyper Zagreb index, we have

$$\begin{aligned} HM(G) &= (3 + 3)^2 |E_{3,3}| + (2 + 3)^2 |E_{2,3}| + (2 + 2)^2 |E_{2,2}| \\ &= (3 + 3)^2 (24mn - 10m - 6n + 2) + (2 + 3)^2 \\ &\quad \cdot (8m + 6n - 4) + (2 + 2)^2 (2m + 3n + 2) \\ &= 864mn - 18n - 128m + 4. \end{aligned} \quad (53)$$

Theorem 10.(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$GA(G) = 4m + 4n + \frac{\sqrt{30}(16m + 20n - 16)}{5} - \sqrt{6}(10m + 8n - 24mn - 4) + 8. \quad (54)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$GA(G) = 4m + 6n + \frac{\sqrt{30}(16m + 12n - 8)}{5} - \sqrt{6}(10m + 6n - 24mn - 2) + 4. \quad (55)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the first geometric-arithmetic index, we have

$$\begin{aligned} GA(G) &= \frac{2 \times \sqrt{6}}{5} |E_{2,3}| + \frac{2 \times \sqrt{9}}{6} |E_{3,3}| + \frac{2 \times \sqrt{4}}{4} |E_{2,2}| \\ &= \frac{2 \times \sqrt{6}}{5} (8m + 10n - 8) + \frac{2 \times \sqrt{9}}{6} \\ &\quad \cdot (24mn - 10m - 8n + 4) + \frac{2 \times \sqrt{4}}{4} (2m + 2n + 4) \\ &= 4m + 4n + \frac{\sqrt{30}(16m + 20n - 16)}{5} \\ &\quad - \sqrt{6}(10m + 8n - 24mn - 4) + 8. \end{aligned} \quad (56)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the first geometric-arithmetic index, we have

$$\begin{aligned} GA(G) &= \frac{2 \times \sqrt{9}}{6} |E_{3,3}| + \frac{2 \times \sqrt{6}}{5} |E_{2,3}| + \frac{2 \times \sqrt{4}}{4} |E_{2,2}| \\ &= \frac{2 \times \sqrt{9}}{6} (24mn - 10m - 6n + 2) + \frac{2 \times \sqrt{6}}{5} \\ &\quad \cdot (8m + 6n - 4) + \frac{2 \times \sqrt{4}}{4} (2m + 3n + 2) \\ &= 4m + 6n + \frac{\sqrt{30}(16m + 12n - 8)}{5} \\ &\quad - \sqrt{6}(10m + 6n - 24mn - 2) + 4. \end{aligned} \quad (57)$$

Theorem 11.(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned} GA_5(G) &= 4n + \frac{32\sqrt{5}}{3} + \frac{3\sqrt{7}(4m + 4n - 3)}{2} \\ &\quad - 3\sqrt{2}(14m + 15n - 24mn - 10) \\ &\quad + \frac{\sqrt{10}(4m + 4n - 8)}{2} + \frac{\sqrt{105}(4m + 2n + 2)}{3} \\ &\quad + \frac{\sqrt{546}(8m + 12n - 16)}{13} + \frac{6\sqrt{34}(4n - 4)}{17} \\ &\quad + \frac{2\sqrt{130}(4n - 4)}{13} - 4. \end{aligned} \quad (58)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned} GA_5(G) &= 16m + 4n + \frac{2\sqrt{105}}{3} + 2\sqrt{2}n + \frac{3\sqrt{7}(n + 1)}{2} \\ &\quad + \frac{2\sqrt{5}(4n + 8)}{3} - 3\sqrt{2}(22m + 14n - 24mn - 13) \\ &\quad + \frac{2\sqrt{42}(8m + 4n - 8)}{7} + \frac{2\sqrt{130}(8m + 4n - 4)}{13} \\ &\quad + \frac{6\sqrt{34}(16m + 12n - 24)}{17} + \sqrt{10}(2m - 2) \\ &\quad + \frac{4\sqrt{546}n}{13}. \end{aligned} \quad (59)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 2. By the definition of the fifth geometric-arithmetic index, we have

$$\begin{aligned} GA_5(G) &= \frac{2 \times \sqrt{42}}{13} |E'_{6,7}| + \frac{2 \times \sqrt{63}}{16} |E'_{7,9}| + \frac{2 \times \sqrt{35}}{12} |E'_{5,7}| \\ &\quad + \frac{2 \times \sqrt{81}}{18} |E'_{9,9}| + \frac{2 \times \sqrt{20}}{9} |E'_{4,5}| + \frac{2 \times \sqrt{25}}{10} |E'_{5,5}| \\ &\quad + \frac{2 \times \sqrt{72}}{17} |E'_{8,9}| + \frac{2 \times \sqrt{40}}{13} |E'_{5,8}| + \frac{2 \times \sqrt{64}}{16} |E'_{8,8}| \\ &= \frac{2 \times \sqrt{42}}{13} (4m + 6n - 8) + \frac{2 \times \sqrt{63}}{16} (4m + 4n - 3) \\ &\quad + \frac{2 \times \sqrt{35}}{12} (4m + 2n + 2) + \frac{2 \times \sqrt{81}}{18} \\ &\quad \cdot (24mn - 14m - 15n + 10) + \frac{2 \times \sqrt{20}}{9} (8) \\ &\quad + \frac{2 \times \sqrt{25}}{10} (2m + 2n - 4) + \frac{2 \times \sqrt{72}}{17} (2n - 2) \\ &\quad + \frac{2 \times \sqrt{40}}{13} (2n - 2) + \frac{2 \times \sqrt{64}}{16} (n - 1) \\ &= 4n + \frac{32\sqrt{5}}{3} + \frac{3\sqrt{7}(4m + 4n - 3)}{2} \\ &\quad - 3\sqrt{2}(14m + 15n - 24mn - 10) \\ &\quad + \frac{\sqrt{10}(4m + 4n - 8)}{2} + \frac{\sqrt{105}(4m + 2n + 2)}{3} \\ &\quad + \frac{\sqrt{546}(8m + 12n - 16)}{13} + \frac{6\sqrt{34}(4n - 4)}{17} \\ &\quad + \frac{2\sqrt{130}(4n - 4)}{13} - 4. \end{aligned} \quad (60)$$

(ii) The proof is calculated based on the edge partition given in Table 4. By the definition of the fifth geometric-arithmetic index, we have

$$\begin{aligned}
GA_5(G) &= \frac{2 \times \sqrt{72}}{17} |E'_{8,9}| + \frac{2 \times \sqrt{48}}{14} |E'_{6,8}| + \frac{2 \times \sqrt{64}}{16} |E'_{8,8}| \\
&\quad + \frac{2 \times \sqrt{81}}{18} |E'_{9,9}| + \frac{2 \times \sqrt{63}}{16} |E'_{7,9}| + \frac{2 \times \sqrt{42}}{13} |E'_{6,7}| \\
&\quad + \frac{2 \times \sqrt{40}}{13} |E'_{5,8}| + \frac{2 \times \sqrt{20}}{9} |E'_{4,5}| + \frac{2 \times \sqrt{25}}{10} |E'_{5,5}| \\
&\quad + \frac{2 \times \sqrt{35}}{12} |E'_{5,7}| + \frac{2 \times \sqrt{16}}{8} |E'_{4,4}| \\
&= \frac{2 \times \sqrt{72}}{17} (8m + 6n - 12) + \frac{2 \times \sqrt{48}}{14} (4m + 2n - 4) \\
&\quad + \frac{2 \times \sqrt{64}}{16} (4m + n) + \frac{2 \times \sqrt{81}}{18} \\
&\quad \cdot (24mn - 22m - 14n + 13) + \frac{2 \times \sqrt{63}}{16} (n + 1) \\
&\quad + \frac{2 \times \sqrt{42}}{13} (2n) + \frac{2 \times \sqrt{40}}{13} (4m + 2n - 2) \\
&\quad + \frac{2 \times \sqrt{20}}{9} (2n + 4) + \frac{2 \times \sqrt{25}}{10} (2m - 2) \\
&\quad + \frac{2 \times \sqrt{35}}{12} \times 2 + \frac{2 \times \sqrt{16}}{8} n \\
&= 16m + 4n + \frac{2\sqrt{105}}{3} + 2\sqrt{2}n + \frac{3\sqrt{7}(n+1)}{2} \\
&\quad + \frac{2\sqrt{5}(4n+8)}{3} - 3\sqrt{2}(22m+14n-24mn-13) \\
&\quad + \frac{2\sqrt{42}(8m+4n-8)}{7} + \frac{2\sqrt{130}(8m+4n-4)}{13} \\
&\quad + \frac{6\sqrt{34}(16m+12n-24)}{17} + \sqrt{10}(2m-2) \\
&\quad + \frac{4\sqrt{546}n}{13}.
\end{aligned} \tag{61}$$

Theorem 12.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned}
S(G) &= \frac{1594323mn}{512} - \frac{14620607759501n}{21249536000} - \frac{2717055933m}{2725888} \\
&\quad + \frac{36950846358693}{116872448000}.
\end{aligned} \tag{62}$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$\begin{aligned}
S(G) &= \frac{1594323mn}{512} - \frac{1783308545665133n}{3155556096000} \\
&\quad - \frac{126009252864147m}{116872448000} + \frac{50620823728941}{233744896000}.
\end{aligned} \tag{63}$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 2. By the definition of the Sanskruti index, we have

$$\begin{aligned}
S(G) &= \left(\frac{42}{11}\right)^3 |E'_{6,7}| + \left(\frac{63}{14}\right)^3 |E'_{7,9}| + \left(\frac{35}{10}\right)^3 |E'_{5,7}| + \left(\frac{81}{16}\right)^3 |E'_{9,9}| \\
&\quad + \left(\frac{20}{7}\right)^3 |E'_{4,5}| + \left(\frac{25}{8}\right)^3 |E'_{5,5}| + \left(\frac{72}{15}\right)^3 |E'_{8,9}| \\
&\quad + \left(\frac{40}{11}\right)^3 |E'_{5,8}| + \left(\frac{64}{14}\right)^3 |E'_{8,8}| \\
&= \left(\frac{42}{11}\right)^3 (4m + 6n - 8) + \left(\frac{63}{14}\right)^3 (4m + 4n - 3) + \left(\frac{35}{10}\right)^3 \\
&\quad \cdot (4m + 2n + 2) + \left(\frac{81}{16}\right)^3 (24mn - 14m - 15n + 10) \\
&\quad + \left(\frac{20}{7}\right)^3 \times 8 + \left(\frac{25}{8}\right)^3 (2m + 2n - 4) + \left(\frac{72}{15}\right)^3 (2n - 2) \\
&\quad + \left(\frac{40}{11}\right)^3 (2n - 2) + \left(\frac{64}{14}\right)^3 (n - 1) \\
&= \frac{1594323mn}{512} - \frac{14620607759501n}{21249536000} - \frac{2717055933m}{2725888} \\
&\quad + \frac{36950846358693}{116872448000}.
\end{aligned} \tag{64}$$

(ii) The proof is calculated based on the edge partition given in Table 4. By the definition of the Sanskruti index, we have

$$\begin{aligned}
S(G) &= \left(\frac{72}{15}\right)^3 |E'_{8,9}| + \left(\frac{48}{12}\right)^3 |E'_{6,8}| + \left(\frac{64}{14}\right)^3 |E'_{8,8}| + \left(\frac{81}{16}\right)^3 |E'_{9,9}| \\
&\quad + \left(\frac{63}{14}\right)^3 |E'_{7,9}| + \left(\frac{42}{11}\right)^3 |E'_{6,7}| + \left(\frac{40}{11}\right)^3 |E'_{5,8}| + \left(\frac{20}{7}\right)^3 |E'_{4,5}| \\
&\quad + \left(\frac{25}{8}\right)^3 |E'_{5,5}| + \left(\frac{35}{10}\right)^3 |E'_{5,7}| + \left(\frac{16}{6}\right)^3 |E'_{4,4}| \\
&= \left(\frac{72}{15}\right)^3 (8m + 6n - 12) + \left(\frac{48}{12}\right)^3 (4m + 2n - 4) \\
&\quad + \left(\frac{64}{14}\right)^3 (4m + n) + \left(\frac{81}{16}\right)^3 (24mn - 22m - 14n + 13) \\
&\quad + \left(\frac{63}{14}\right)^3 (n + 1) + \left(\frac{42}{11}\right)^3 (2n) + \left(\frac{40}{11}\right)^3 (4m + 2n - 2) \\
&\quad + \left(\frac{20}{7}\right)^3 (2n + 4) + \left(\frac{25}{8}\right)^3 (2m - 2) + \left(\frac{35}{10}\right)^3 \\
&\quad \times 2 + \left(\frac{16}{6}\right)^3 n \\
&= \frac{1594323mn}{512} - \frac{1783308545665133n}{3155556096000} \\
&\quad - \frac{126009252864147m}{116872448000} + \frac{50620823728941}{233744896000}.
\end{aligned} \tag{65}$$

Theorem 13.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$F(G) = 432mn + 2n - 60m. \quad (66)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$F(G) = 432mn - 6n - 60m. \quad (67)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the forgotten topological index, we have

$$\begin{aligned} F(G) &= (2^2 + 3^2)|E_{2,3}| + (3^2 + 3^2)|E_{3,3}| + (2^2 + 2^2)|E_{2,2}| \\ &= (2^2 + 3^2)(8m + 10n - 8) + (3^2 + 3^2) \\ &\quad \cdot (24mn - 10m - 8n + 4) + (2^2 + 2^2)(2m + 2n + 4) \\ &= 432mn + 2n - 60m. \end{aligned} \quad (68)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the forgotten topological index, we have

$$\begin{aligned} F(G) &= (3^2 + 3^2)|E_{3,3}| + (2^2 + 3^2)|E_{2,3}| + (2^2 + 2^2)|E_{2,2}| \\ &= (3^2 + 3^2)(24mn - 10m - 6n + 2) + (2^2 + 3^2) \\ &\quad \cdot (8m + 6n - 4) + (2^2 + 2^2)(2m + 3n + 2) \\ &= 432mn - 6n - 60m. \end{aligned} \quad (69)$$

Theorem 14.

(i) Let G be a $VC_5C_7(m, n)$ nanosheet, then

$$H(G) = \frac{13m}{15} + \frac{7n}{3} + 8mn + \frac{2}{15}. \quad (70)$$

(ii) Let G be a $HC_5C_7(m, n)$ nanosheet, then

$$H(G) = \frac{13m}{15} + \frac{19n}{10} + 8mn + \frac{1}{15}. \quad (71)$$

Proof.

(i) The proof is calculated based on the edge partition given in Table 1. By the definition of the harmonic index, we have

$$\begin{aligned} H(G) &= \frac{2}{2+3}|E_{2,3}| + \frac{2}{3+3}|E_{3,3}| + \frac{2}{2+2}|E_{2,2}| \\ &= \frac{2}{2+3}(8m + 10n - 8) + \frac{2}{3+3} \\ &\quad \cdot (24mn - 10m - 8n + 4) + \frac{2}{2+2}(2m + 2n + 4) \\ &= \frac{13m}{15} + \frac{7n}{3} + 8mn + \frac{2}{15}. \end{aligned} \quad (72)$$

(ii) The proof is calculated based on the edge partition given in Table 3. By the definition of the harmonic index, we have

$$\begin{aligned} H(G) &= \frac{2}{3+3}|E_{3,3}| + \frac{2}{2+3}|E_{2,3}| + \frac{2}{2+2}|E_{2,2}| \\ &= \frac{2}{3+3}(24mn - 10m - 6n + 2) + \frac{2}{2+3}(8m + 6n - 4) \\ &\quad + \frac{2}{2+2}(2m + 3n + 2) \\ &= \frac{13m}{15} + \frac{19n}{10} + 8mn + \frac{1}{15}. \end{aligned} \quad (73)$$

4. Conclusions

QSARs represent predictive models derived from the application of statistical tools correlating to the biological activity (including desirable therapeutic effects and undesirable side effects) of chemicals (drugs/toxicants/environmental pollutants) with descriptors representative of molecular structure and/or properties. QSARs are being applied in many disciplines like risk assessment, toxicity prediction, and regulatory decisions in addition to drug discovery and lead optimization. In this paper, some important topological indices of the pent-heptagonal nanosheet that is covered by pentagons and heptagons are calculated, and formulas for their atom-bond connectivity index, fourth atom-bond connectivity index, Randić connectivity index, sum-connectivity index, first Zagreb index, second Zagreb index, augmented Zagreb index, modified Zagreb index, hyper Zagreb index, geometric-arithmetic index, fifth geometric-arithmetic index, Sanskruti index, forgotten index, and harmonic index of two types of pent-heptagonal nanosheets are derived. Our calculated results help in understanding topology of understudy nanotubes. For instance, the first Zagreb index is used to be known as pi-electronic energy. The augmented Zagreb index can be used to investigate the heat of formation. Randić index is the mostly applied and investigated degree-based topological index.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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