Research Article

Semiconductor-Superlattice Parametric Oscillator as a Subterahertz and Possible Terahertz Radiation Source

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We describe the operation of a semiconductor-superlattice parametric oscillator (SPO) at a subterahertz frequency (near 300 GHz). The oscillator is driven by a microwave source (frequency near 100 GHz). We also present an analysis indicating that operation at frequencies above 1 THz should be possible. The SPO is based on the ability of conduction electrons in a superlattice to perform Bloch oscillations. Broadband tunability as well as the monochromacy of a driving microwave field are transferred to the SPO.

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1. INTRODUCTION

It is of great interest with respect to applications to develop monochromatic radiation sources for the terahertz (THz) frequency range of 1 THz to 10 THz or for the sub-THz range from 1 THz down to 0.1 THz (by some experts also attributed to the THz range). For frequencies up to about 100 GHz, well-developed current-driven semiconductor sources (e.g., the Gunn diode) are available. There is a lack of current-driven sources at higher frequencies, especially in the wavelength range around 1 mm (frequency 300 GHz) and in the submillimeter wavelength range (frequencies above 300 GHz).

Applications of sub-THz or THz radiation concern [1, 2], for example, the sensing of gas molecules or radicals (such as OH) in the atmosphere, the study of spectral properties of small and large molecules, of materials or of drugs, the imaging of materials, especially biological materials, and the development of communication systems. An important application may become the security sector: imaging and sensing with THz or sub-THz radiation. Presently, systems operating with radiation sources based on femtosecond optical lasers are tested [1, 2]; THz radiation can also be produced by mixing of radiation at two different frequencies produced with visible or near-infrared cw laser radiation [3].

Regarding simplicity and costs, current-driven semiconductor sources emitting monochromatic sub-THz or THz radiation would be most desirable. An actual development towards a current-driven monochromatic THz radiation source is the quantum cascade laser. Quantum cascade lasers are operated at frequencies between 1.8 THz and 4 THz [4–7]. In a quantum cascade laser, it is made use of a semiconductor heterostructure which is the basis of appropriate energy levels suitable for laser action. A laser is operated at a fixed frequency that is determined by a specific heterostructure. The THz-quantum cascade laser has to be cooled to temperatures near liquid nitrogen temperature.

Another approach to realize a monochromatic sub-THz, and possibly, THz radiation source is the superlattice parametric oscillator (SPO). First experiments have shown that an SPO, pumped with monochromatic radiation near 100 GHz, is suitable to produce monochromatic radiation near 300 GHz and that it is tunable [8]. The SPO is operated at room temperature.

Here, we report on an SPO using new superlattice material. We demonstrate SPO action with the new material for generation of radiation near 300 GHz and we present a theoretical study indicating that the new material should be suitable as the basic nonlinear medium of an SPO operating at frequencies above 1 THz.

2. PRINCIPLE OF THE SUPERLATTICE PARAMETRIC OSCILLATOR

The principle of the SPO is illustrated in Figure 1. A semiconductor superlattice is coupled to a pump field (frequency ω)
and to a third harmonic field (frequency $3\omega$). A resonator for the $3\omega$ radiation delivers feedback to the superlattice. A partial reflector allows to couple third-harmonic radiation out of the resonator. The SPO is, effectively, a frequency tripler. However, due to the feedback for the third-harmonic radiation, a threshold-like onset of an oscillation occurs—the superlattice acts as an active element for the $3\omega$ radiation. Due to the feedback, a much higher efficiency for conversion of pump to third harmonic radiation is expected than for a conventional frequency tripler.

### 3. NONLINEAR SEMICONDUCTOR-SUPERLATTICE NANOMATERIAL

In our experiment, we have used a GaAs/AlAs superlattice (Figure 2(a)) with GaAs layers and AlAs layers in turn. Each GaAs layer consisted of 14 monolayers GaAs (monolayer thickness 0.283 nm) and each AlAs layer of 2 monolayers AlAs (monolayer thickness 0.283 nm). The length of a superlattice period, $a$, was thus equal to the thickness of 16 monolayers ($a = 4.52$ nm). The superlattice had a length of 110 periods ($\sim 0.5 \mu$m) and was homogeneously doped with silicon. The free carrier concentration was about $10^{17}$ cm$^{-3}$.

The superlattice had been grown, by molecular beam epitaxy, on an n-GaAs layer (carrier concentration $2 \cdot 10^{18}$ cm$^{-3}$, thickness 600 nm) that itself had been grown on a seminsulating GaAs substrate. On the top of the superlattice, there was an n-GaAs layer (doping $2 \cdot 10^{18}$ cm$^{-3}$, thickness 200 nm). By use of photolithography and reactive ion etching, we prepared superlattice mesas. A single mesa consisted of a superlattice that was covered on top with a metal alloy (AuGeNi) which served as one of the two electric contacts. The other contact was realized via the n-GaAs layer. For our experiment, we used a chip (size $150 \mu$m $\times$ $150 \mu$m) carrying about 200 superlattice mesas at distances of about $10 \mu$m from each other. Each mesa had a diameter of about $4 \mu$m. Electric connection of a single mesa to a voltage source was obtained via a whisker (a thin gold wire) which we pressed on top of the mesa. The other connection was obtained via the n-GaAs layer and a large-area superlattice mesa.

The current-voltage curve (Figure 2(b)) was almost antisymmetric according to the symmetry of the superlattice. The curve was strongly nonlinear at voltages higher than a critical voltage (1 V). At voltages above the critical voltage, the current decreased slightly with increasing voltage. This indicates a negative differential resistance of the superlattice. A state of negative differential resistance corresponds to an active state of the superlattice. The ohmic resistance around zero voltage was partly due to the ohmic resistance $R_0$ of the superlattice and partly due to a series resistance $R_s$. The analysis of the current-voltage curve delivered $R_0 \approx 10 \Omega$ and $R_s \approx 40 \Omega$. The series resistance was mainly due to a contact resistance of the contact between the metal alloy layer and the n-GaAs layer on top of the superlattice.
We now illustrate the mechanism of the SPO action. We describe the current-voltage ($I-U$) characteristic by an idealized curve (Figure 3(a)). The curve (Esaki-Tsu curve [8]) shows a peak current $I_p$ at a critical voltage $U_c$ and a negative differential conductance above $U_c$. The SPO mechanism is indicated in Figure 3(b). A high-frequency pump voltage $U_1$ transfers the superlattice, twice per period, into an active superlattice state. An additionally applied third-harmonic voltage $U_3$ (as realized by the feedback with the third-harmonic resonator) results in an energy transfer from the pump field to the third-harmonic field. This is seen by analyzing the current, $I$, produced at the total voltage $U_1 + U_3$, which contains a third-harmonic component $I_3$ with opposite phase in comparison to $U_3$. The time average $\langle U_3 I_3 \rangle$ is negative, indicating gain. The high-frequency resistance $R_3 = U_3/I_3$ is also negative: the superlattice represents a negative resistance for the third-harmonic voltage and can therefore serve as an active element in a resonator.

4. THE SUPERLATTICE PARAMETRIC OSCILLATOR

Our SPO (Figure 4) consisted of a pump waveguide and a third-harmonic resonator. A chip carrying a large number of superlattice mesas was glued on one wall of the third-harmonic resonator. A whisker antenna (a metal wire of 20 μm diameter) was pressed on one of the superlattice mesas and delivered electromagnetic coupling between the superlattice mesa and the third-harmonic resonator. A fin line (hatched) and a stamp together with the whisker served for electromagnetic coupling of the superlattice to the pump waveguide. The connection between the pump waveguide and the third-harmonic resonator was designed as a quarter-wavelength coaxial line for the third harmonic to minimize third-harmonic radiation loss to the pump waveguide. By use of a coaxial filter, blocking pump radiation, a static voltage ($U$) source is used to measure the current-voltage curve of the superlattice; SPO action is obtained for $U = 0$. The pump waveguide is electromagnetically closed towards the bias circuit by use of a coaxial filter (marked by a cross).
was coupled out from the third-harmonic resonator through a horn antenna. A mismatch between the output port of the third-harmonic resonator and the horn acted as partial reflector. The pump waveguide was designed for the W band (cutoff frequency 60 GHz) and had a width of 2.54 mm and a height of 1.27 mm. The third-harmonic waveguide (1.09 mm × 0.54 mm) had a cutoff frequency of 131 GHz. The outer dimensions of the waveguide block were 2 cm × 5 cm × 5 cm (height × width × length). Attached to the block were a horn and three micrometer screws (not shown in Figure 4). One of the screws was used to press the whisker onto a superlattice mesa and the two others for shifting the backshorts.

For pumping the SPO, we used radiation of a synthesizer, amplified with a microwave amplifier (tuning range 90–94 GHz, maximum power 100 mW). Third-harmonic radiation was monitored with a thermal detector (Golay cell).

5. EXPERIMENTAL RESULTS

The third-harmonic signal (Figure 5) showed a jump at a threshold pump power \( P_{th} \), reached a peak immediately at \( P_{th} \) and then decreased. At small pump power \( P < P_{th} \) the signal was due to third-harmonic generation according to the nonlinearity of the Esaki-Tsu current-voltage curve for pump field amplitudes not exceeding the critical voltage. We attribute the threshold to the onset of parametric oscillation.

The SPO signal changed with the third-harmonic resonator length (Figure 6) according to the wavelength of the radiation within the resonator. In the frequency range attainable by our pump source, corresponding to third-harmonic frequencies from 270 GHz to 282 GHz, third-harmonic power varied only slightly with frequency if the backshort of the pump waveguide and the backshort of the third-harmonic resonator were appropriately adjusted.

We estimated from our results that the power of the third-harmonic radiation was of the order of 10 \( \mu \)W corresponding to an efficiency for the conversion of pump to third-harmonic radiation of about \( 10^{-3} \). This value is smaller than a theoretical value (\( \sim 0.1 \)) and also smaller than an experimental value in a different arrangement [9]. Our present SPO device is not yet optimized with respect to the high-frequency properties. However, our results demonstrate that the new superlattice material used in the experiment is well suitable as active material in an SPO.

6. BLOCH OSCILLATIONS AND MINIBAND TRANSPORT

In the superlattice (Figure 2(a)), the AlAs layers present barriers of the potential for electrons moving along the superlattice axis. This has the consequence that the energy, \( \epsilon \), that is connected with the electron motion along the superlattice axis is confined to a miniband. The periodic potential gives rise to dispersion which we describe by the dispersion relation (Figure 7(a))

\[
\epsilon = \Delta \left( \frac{1}{2} - \frac{1}{2} \cos k a \right),
\]

where \( k \) is the wave vector along the superlattice axis and \( \Delta \) the width of the miniband. The energy has a minimum (\( \epsilon = 0 \)) for \( k = 0 \) and maxima at the mini-Brillouin zone boundaries \( k = \pm \pi/a \). The miniband is separated from the next higher miniband by a minigap. An electron has the total energy \( \epsilon + (2m^*)^{-1} \hbar^2 k_{\perp}^2 \), where \( m^* \) is the effective mass and \( k_{\perp} \) the wavevector for the propagation perpendicular to the superlattice axis; the perpendicular motion is similar to that of a conduction electron in a GaAs crystal \( (m^* = 0.07m_0; m_0 = \text{electron mass}) \). A calculation of the miniband width for our superlattice delivered the miniband width \( \Delta \sim 140 \text{ meV} \) and a minigap to the next higher miniband of \( \sim 180 \text{ meV} \).
Under the action of a static field, an electron is accelerated, reaches the mini-Brillouin zone boundary, and is then Bragg reflected and decelerated. Then, it is accelerated again. The electron performs a Bloch oscillation with the Bloch frequency

\[ \omega_B = \frac{e a E}{\hbar}, \]  

where \( e \) is the electron charge and \( E \) the static-field strength (\( \hbar \), Planck's constant). This follows from the acceleration theorem (Newton's law for a crystal electron),

\[ \hbar \dot{k} = eE. \]  

The wave vector increases linearly with time \( t \),

\[ k a = \omega_B t, \]  

and the group velocity

\[ v_g = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k} = \frac{\Delta a}{2\hbar} \sin \omega_B t \]  

oscillates with the Bloch frequency. By integration, the spatial position (trajectory) \( \xi(t) = \int_0^t v_g dt \) is obtained. It follows that the electron performs a Bloch oscillation in space with the trajectory (Figure 7(b))

\[ \xi = \xi_0 \left( \frac{1}{2} - \frac{1}{2} \cos \omega_B t \right), \]  

where

\[ \xi_0 = \frac{\Delta \hbar}{\hbar \omega_B a} \]  

is the length of the trajectory. For \( E = 10 \text{kV/cm} \), the Bloch frequency for an electron in our superlattice is \((1/2\pi)\omega_B \sim 1 \text{THz}\) and the length of the trajectory \( \xi_0 \sim 30a \). The electron oscillates over a large number of superlattice periods.

The description of the Bloch oscillation implies: under the influence of a static electric force along the superlattice axis, a conduction electron at zero energy (\( \varepsilon = 0 \)) gains energy until it reaches the zone boundary (\( \varepsilon = \Delta \)). Then, the electron undergoes Bragg reflection and moves against the electric force until it reaches again zero energy and begins a new oscillation cycle. The matter wavelength of the electron is almost infinitely large at \( k = 0 \) and decreases with increasing energy until the wavelength is equal to twice the period of the superlattice giving rise to Bragg reflection. The Bragg reflection is an extremely nonlinear process and is the origin of the nonlinearity of transport properties of a superlattice. The actual basis of the nonlinear transport is the interplay of Bloch oscillations, that is, Bragg reflections, with intramini-band relaxation.

A Bloch oscillation is interrupted by relaxation. The electron looses energy by emission of a phonon and starts a Bloch oscillation with a new phase at a lower energy. This leads to a drift in the direction of the static force. The average drift velocity is

\[ v = \frac{1}{\tau} \int_0^\infty \exp \left( -\frac{t}{\tau} \right) v_g(t) dt, \]  

where \( \tau \) is the intramini-band relaxation time. By integration, the Esaki-Tsu characteristic [8] is obtained:

\[ v = v_p \frac{2E/E_c}{1 + E^2/E_c^2} = v_p \frac{2\omega_B \tau}{1 + \omega_B^2 \tau^2}, \]  

where

\[ v_p = \frac{\Delta a}{4\hbar} \]  

is the peak-drift velocity occurring at a critical field \( E_c = \hbar(ect)^{-1} \).

The current, \( I \), through the superlattice is given by

\[ I = N e A v, \]  

Figure 7: (a) Dispersion relation for a miniband electron propagating along the superlattice axis (\( \varepsilon = \) energy and \( k = \) wave vector of an electron propagating along the superlattice axis; \( a = \) superlattice period; \( \Delta = \) miniband width). (b) Trajectory (horizontal line with arrows; length \( \xi_0 \)) for the Bloch oscillation of a miniband electron under the influence of a static electric force directed along the superlattice axis (\( x \)). The lower of the two parallel lines symbolizes the bottom of the miniband (\( \varepsilon = 0 \)) and the upper line the upper boundary (\( \varepsilon = \Delta \)). The tilt of the miniband accounts for a decrease of the potential energy along the direction of the static electric force. The electron propagates from the bottom to the upper boundary of the miniband, where it undergoes a Bragg reflection and moves back to the bottom to start a new oscillation cycle. On a trajectory, the sum of potential and kinetic energy of an electron is constant.
where \( N \) is the carrier concentration and \( A \) the area of the superlattice. Equation (9) delivers, with \( U = E/L \) and \( U_e = E_e/L \), where \( L \) is the length of the superlattice, the Esaki-Tsu current-voltage curve (Figure 3(a)). The superlattice has an ohmic resistance \( R_0 = (1/2)U_e/I_p \) around zero voltage and shows a negative differential resistance for voltages larger than \( U_e \). We obtained the theoretical current-voltage curve (dotted in Figure 2(b)) that is in accordance with the experimental curve for voltages up to the critical voltage, taking into account a series resistance. The voltage of maximum current corresponded to a critical field \( E_c \), where

\[
E_c = \frac{k_BT}{eA}
\]

is not small compared to 1, we have \( 3\omega\tau \approx 0.3 \). For this frequency, the quasistatic theory is at its limits.

In the general case that \( \omega \tau \) is not small compared to 1, we cannot apply the Esaki-Tsu characteristic for the description of the electron motion. Instead, we derive the time dependent drift velocity for a miniband electron in a time dependent electric field \( E(t) \) directly from the acceleration theorem, (3). It follows that

\[
k(t, t_0) = \frac{e}{\hbar} \int_{t_0}^{t} E(t') dt',
\]

where \( t_0 \) is the time at which the acceleration of an electron starts, at a field \( E(t_0) \). If the electron moves unscattered until the time \( t \), its group velocity is

\[
v_g(t, t_0) = \frac{\Delta a}{2\hbar} \sin \left[ \int_{t_0}^{t} \omega_B(t') dt' \right],
\]

where

\[
\omega_B(t) = \frac{1}{\hbar} eaE(t) = \frac{1}{\hbar} ea(\hat{E}_1 \cos \omega t + \hat{E}_3 \cos 3\omega t)
\]

is the instantaneous Bloch frequency. We can interpret the instantaneous Bloch frequency as an instantaneous Bragg scattering frequency. The maximum instantaneous Bloch (Bragg scattering) frequency is obtained twice per period of the pump field and has the value

\[
\omega_{B,\text{max}} = \frac{1}{\hbar} ea(\hat{E}_1 + \hat{E}_3).
\]

The maximum Bloch frequency corresponds to a minimum length of an instantaneous trajectory

\[
\xi_{0,\text{min}} = \frac{\Delta}{\hbar\omega_{B,\text{max}}} a.
\]

The time-dependent drift velocity is obtained by averaging the group velocities over all starting times,

\[
v(t) = \int_{-\infty}^{t} p(t, t_0) v_g(t, t_0) dt_0
\]

\[
= 2v_p \int_{-\infty}^{t} \left\{ p(t, t_0) \sin \left[ \int_{t_0}^{t} \omega_B(t') dt' \right] \right\} dt_0,
\]

where

\[
p(t, t_0) = \frac{1}{T} \exp \left[ - (t - t_0) \tau^{-1} \right]
\]

is the probability that an electron did not undergo a relaxation process in the time interval between \( t_0 \) and \( t \). A derivation of \( v(t) \) by use of a Boltzmann equation [27] leads to the same result. If a finite temperature \( T \) is taken into account, \( v_p \) has to be replaced by \( v_p I(z)/I_0(z) \) where \( I_0 \) and \( I_1 \) are the Bessel functions of the zeroth and first order and

\[
z = \Delta(2k_B T)^{-1}; \text{ for } k_B T \ll \Delta, \text{ we have } I_1/I_0 = 1 \text{ (Boltzmann constant). For our superlattice (} \Delta \sim 140 \text{ meV) at room}
\]
A negative value of \( R \) voltage corresponds to our experiment is shown in Figure 8. We have chosen a pump voltage \( U_1 = 5U_c \). The third-harmonic resistance \( R_3 \) is negative at small amplitude \( U_3 \), its absolute value increases with \( U_3 \). This indicates the possibility of a self oscillation of the SPO, with feedback delivered by the resonator. The efficiency has an optimum of about 10 percent. For too large amplitudes \( U_3 \), \( \eta \) decreases and finally becomes zero.

Another theoretical result (Figure 9) indicates the possibility to operate an SPO at 3 THz. An efficiency of 10 percent may be, in principle, achievable at a pump voltage \( U_1 = 5U_c \). A comparison of our results shows that the condition to reach an efficiency of about 10 percent is different for the 300-GHz and the 3-THz SPO. For the higher frequency, larger voltage amplitudes are necessary. These are joint with lower current amplitudes. By appropriate impedance matching of pump and resonance circuit, the necessary coupling of pump and third-harmonic radiation should be achievable.

As a main result we find, theoretically, that pumping with, for example, a 10 mW source delivers at 300 GHz and 3 THz almost the same third-harmonic power of about 1 mW. By use of another design of the superlattice, it should in principle be possible to increase both pump and third-harmonic power.

8. THE UPPER LIMIT FREQUENCY FOR SPO ACTION

A superlattice is suitable as active medium in an SPO under the condition that instantaneous Bragg reflection processes are possible at the field amplitudes necessary for operating...
an SPO, that is, if

$$\xi_{0,\text{min}} \gg a.$$

(23)

If $\xi_{0,\text{min}} \leq a$, the electron is localized to one superlattice spatial period.

It follows that the upper limit frequency for SPO action is given by the condition

$$\hbar \omega_{\text{max}} \ll \Delta.$$

(24)

For our superlattice, we find for a third-harmonic frequency of 3 THz (with the necessary field amplitudes $U_1 = 5U_c$ and $U_3 \sim U_c$) that $\xi_{0,\text{min}} \approx 5a$. Thus, the condition $\xi_{0,\text{min}} \gg a$ is fulfilled. The frequency of 3 THz represents an upper limit for SPO action of our superlattice.

We estimate for a superlattice used in the first SPO experiment [8], with $\Delta \sim 24$ meV (18 monolayers GaAs and 4 monolayers AlAs, per superlattice period), a limit frequency of about 0.6 THz.

9. DISCUSSION

We have described an SPO with superlattice material that should be suitable to realize a THz SPO. Starting with a 100-GHz or a 200-GHz microwave pump source it should, in principle, be possible to use SPOs in series to produce radiation up to few THz.

However, there are several technical problems to be solved: an SPO of a higher power requires the improvement of the arrangement. It is necessary to develop submillimeter-size waveguides, filters, and antennas of high quality, and it is necessary to appropriately couple a superlattice to the waveguide structure. In our present experiment, we have used a mesa shaped superlattice. This allowed us to use different superlattice geometries (especially of different diameters) in a simple way. However, a stable oscillator requires the use of a quasiplanar superlattice device (as it has been used for the first SPO experiment [9]). Furthermore, more efficient cooling of the superlattice should be introduced allowing for larger area and higher pump power and reducing thermal heating.

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