Application of M Sequence Family Measurement Matrix in Streak Camera Imaging

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1. Introduction

In the inertial confinement fusion (ICF) research, the characteristics of the main diagnostic object are the following: the spatial scale is small (approximately 100 μm), evolution is fast (approximately 100 ps), and the state is extreme (high temperature of approximately 108 K, high density of approximately 200 g/cm³). The X-ray streak camera, as a scientific instrument with high spatial and temporal resolution, is the main diagnostic tool in the ICF research. The core part of the streak tube is composed of a photocathode, accelerating grid, electrostatic focusing system, deflection system, and fluorescent screen. The basic principle is to map the time information of the X-ray radiation into the spatial information of visible light on the screen. Time resolution, spatial resolution, and dynamic range are three main performance indicators of a streak camera. Academician Niu Hanben [1] proved that the accumulation of the space charge effect restricts the improvement of the spatial and temporal resolution of a streak camera. Some progress has been made in improving the spatial and temporal resolution of a streak camera. The Xi’an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences [2], uses a traveling wave deflector to implement the short magnetic focusing technology, which overcomes the deceleration field in the electrostatic focusing mode. The flat electrode structure technology is adopted by the France Photonis Company [3]; this technology can reduce the dispersion time of the electron bunch. The Institute of Optics and Electronics of Shenzhen University [4] has designed a large format tube; in addition, streak tubes with various structures, such as accelerating grid and accelerating slit channel with combined plate electrodes, also exist. The above methods can improve the spatial and temporal resolutions of the streak camera.

The position of the electron beam emitted from different heights of the streak tube is not on the same plane, but on a parabola. An ordinary streak camera uses a flat screen or a spherical fluorescent screen; however, the actual surface of the electron beam is not a complete sphere. As a result, the image and the screen are not completely fitted; therefore,
defocus will occur when the nontangent position of the electron beam is projected on the screen. In actual testing, when adjusting the focus voltage to achieve the best resolution at the center of the cathode, the other off-axis electron beams are not on the optimal image plane. This behavior is similar to that of the optical mirror field curve and, thus, the central axis is the focus area, high static spatial resolution can be obtained, the off-axis area is a defocus area, and the spatial resolution is reduced.

Compressed sensing (CS) can recover from undersampled sparse data to achieve optimal real target images. When the imaging system reduces the intensity of the light source and reduces the exposure time of a charge-coupled device (CCD), the image is reconstructed by CS, and a target image similar to the original condition can be obtained. The Shenzhen advanced technology research institute of CAS applied CS to solve the defocus problem caused by the midfield curvature of acousto-optic imaging [5]; inspired by this, CS is used to improve streak camera spatial resolution.

2. Methods

2.1. Imaging System. The X-ray streak camera mainly consists of the following core parts: micro-channel plate enhancement module, high-voltage and low-voltage power supply modules, scanning circuit control module, image acquisition module (CCD), and image recording system (computer). The imaging principle is that the diagnostic information, such as time, time interval, intensity distribution, and other diagnostic information of the X-ray incident pulse are linearly transmitted to the electronic pulse by the photoelectric effect of the photocathode. Next, the electron beam is accelerated and focused by an electro-optical imaging system consisting of electrostatic focusing or magnetic focusing. After the deflection of the scanning circuit, the time information of the electron beam is projected onto the space dimension, the electro-optical conversion is conducted through the screen, and the location and intensity information are recorded through the CCD to the computer [4, 6]. The streak camera system is shown in Figure 1(a).

By combining a streak camera with 2D detectors such as a CCD, it can provide all types of ultrafast time information, spatial information, and intensity (or spectral) information; correspondingly, the main parameters are the time resolution, spatial resolution, and dynamic range [7, 8]. The time resolution and spatial resolution of the streak camera used in this study has minimum time interval and the smallest space details that a streak camera can record, respectively. The resolution is an important indicator of the streak camera.

2.2. Space Modulation Transfer Function and Spatial Resolution. For a practical electronic optical system, because of aberration, the image formed by a point passing through the system is not an ideal point. Its intensity extends outward from the center and spreads into a bright spot, which is darkened all around. The distribution of space can be represented by a point spread function (PSF) [9]. The distribution of the PSF reflects the imaging properties of the electron optical system. The electron optical system is a linear translation invariant transmission system. The electron beam before and after transmission is the object and image of the system, respectively. Csorba [10] proposed a formula to represent the spatial modulation function of the electron optical imaging system. In the formula, \( \rho_c \) is the focusing error coefficient.

\[
\text{MTF}(f) = \exp \left[-\left(\frac{\pi \rho_c}{2}\right)^2 f^2\right] \tag{1}
\]

The contrast transfer function of the system corresponds to the square wave image. The modulation transfer function of the system corresponds to the sinusoidal image, and it can be obtained from the Fourier transform.

\[
\text{MTF}(f) = \frac{1}{2} \text{CTF}(f) \tag{2}
\]

2.3. Compressed Sensing. The CS method can use undersampling to recover sparse and compressed signals. Compressed observation is denoted as \( y = \Phi x \), where \( y \) is the observed vector (M×1), and \( x \) is the original signal (N×1) (M<<N). \( x \) is generally not sparse; however, it is sparse in a transformation domain \( \Psi \), such as the cosine transform, Fourier transform, and wavelet transform, which have a transformation in the form of \( x = \Psi \theta \). In a streak camera, if the measurement data obtained from the CCD are \( y \) and the measurement matrix related to the imaging system is \( \Phi \), then \( y = \Phi \Psi \theta \). At this point, CS becomes a part of the X-ray scanning imaging system. The reconstruction of sparse transformation \( \Psi \) can be obtained by solving the optimization problem under the condition of bundle [11, 12].

\[
\begin{align*}
\min & \quad \|x\|_1 \\
\text{s.t.} & \quad y = \Phi \Psi^{-1} x
\end{align*} \tag{3}
\]

To reconstruct the streak images formed in the defocus region of a streak camera, a reconstruction model based on CS is constructed [5], given as (5):

\[
\arg \min F = \|\Phi \Psi^{-1} x - y\| + \alpha \|x\|_1 + \beta \text{TV}(\Psi^{-1} x) \tag{4}
\]

Equations (3) and (4) can also be written as

\[
\arg \min F = \|\Phi \Psi^{-1} x - y\| + \alpha \|x\|_1 + \beta \text{TV}(\theta) \tag{5}
\]

The cosine transformation is used in the formula of \( \Psi \). In addition to the \( \ell_1 \) norm of the sparse domain, the total signal variation penalty term is also incorporated into the objective function to improve the accuracy of reconstruction. The first one on the right-hand side of (5) is to reconstruct the mean square error between the image and the measured value, and the second item is the \( \ell_1 \) norms for \( x \); the third item TV is the signal variation penalty factor [13]. In addition, \( \alpha \) and \( \beta \) are the equilibrium parameters that determine the balance between data compatibility and sparsity. Overbalance may result in reconstruction distortion, with only the proportion being close to the objective value; to achieve the
3. Experiment

3.1. Streak Camera Static Test Subsection. The fluorescent screen (Φ52 mm) is coupled to a reduced cone (1:3:1) of light; thus, the PI2048 CCD of size $27.6 \times 27.6 \text{mm}^2$ cannot cover the entire cone of light on the side of full cover the fluorescent. The streak contrast value obtained from the test image is given in the first column. From 1.2 mm near the off-center axis, the fluorescent screen was sampled at intervals of 3.5 mm and values of 4.7 mm, 8.2 mm, and 11.7 mm, with contrast of 21.6%, 30.2%, 45.1%, and 48.4%, respectively; the spatial resolution is the limit resolution when the contrast transfer function CTF is 0.05. The central resolution of the image tube can be calculated using formula (1), which resulted in 31 lp/mm, edge resolution is 20 lp/mm, and the resolution of the delimit streak decreases from the center to the edge. The experimental results are consistent with the analysis results of the field curvature defocus phenomenon.

3.2. Compressed Sensing Algorithm Implementation. The objective function of CS is solved using the orthogonal matching method [14, 15]; the algorithm implementation flow diagram is shown in Figure 1(b). In the flowchart, $r_t$ represents residual value, and $t$ represents the number of iterations, $\Lambda_t$ is an index set for the $t$ iterations (column ordinal), $\lambda_t$ represents the index found in the $t$ iterations, $a_j$ represents the $j$ column of the matrix $A$, $A_t$ represents a set of columns of matrix $A$ selected by index $\Lambda_t$, $\theta_t$ is a column vector of $T \times 1$, $\cup$ represents set merge, and $\prec$ represents the vector inner product. The random Gaussian measurement matrix and the Bernoulli measurement matrix are selected for the measurement matrix, and the effects of three types of measurement matrices on the defocus problem caused by the field curvature of the streak camera are compared.

3.3. Construction of a Measurement Matrix of M Sequence Family (MSMM)

3.3.1. Trace Function. Assuming that $\beta$ is the original domain element of the finite field $\text{GF}(q)$, then all domain elements of $\text{GF}(q)$ can be generated by the power of 0 and $\beta$, namely, $0, \beta^0 = 1, \beta, \ldots, \beta^{q-1}$. Among them, the post $q-1$ nonzero elements constitute the multiplicative group $\text{GF}(q) \{0\}$, which can also be denoted as $\text{GF}(q) \ast \{1\}$.

Definition 1. Let $n$ be positive integers; thus, the trace function from $\text{GF}(2^n)$ to $\text{GF}(2)$ is

$$
\text{Tr} (x) = x + x^2 + \cdots + x^{2^{(n-1)}}, \quad x \in \text{GF}(2^n)
$$

(6)

3.3.2. Construction of the Measurement Matrix. The measurement matrix of M sequence family is a bipolar matrix composed of $+1$ and $-1$. The size is fixed to $(2^n - 1) \times 2^{n+1}$ ($n \geq 3$), and the concrete implementation is as follows [17].

Step 1. According to the length of information $N = 2^{n+1}$ ($n \geq 3$), when $n$ is odd, select the trace representation function shown in (6). When $n$ is even, select the trace representation function shown in (7).

$$
\hat{f}_{\lambda_0, \lambda_1}(x) = \text{Tr}(\lambda_0 x) + \text{Tr}(\lambda_1 x^2) + \sum_{i=2}^{l} \text{Tr}(x^{1+2^i}),
$$

(7)

$x \in \text{GF}(2^n) \ast, \lambda_0, \lambda_1 \in \text{GF}(2^n)$
Step two. Select the original domain elements \( \beta \) on GF \((2^n)\). Order \( b_{\lambda_0,\lambda_1} = f_{\lambda_0,\lambda_1}(\beta^t) \); here, \( t \in \{0,1,\ldots,2^n-2\} \), \( \lambda_0 \in \text{GF}(2^n) \). Order \( b_{\lambda_0} = \{b_{\lambda_0,0},b_{\lambda_0,1},\ldots,b_{\lambda_0,2^{n-2}}\} \), \( b_{\lambda_0} \) is a binary sequence with a period of \( 2^n-1 \). When the value of \( \lambda_0 \) is fixed, the binary sequence, m sequence \( b_{\lambda_0} \), can be obtained. The input and output transformations of the elements of \( \lambda_0 \) are conducted by successive passes (8), and a bipolar sequence is obtained. For iterative values of parameters \( \lambda_0 \) in a finite field GF\((2^n)\), a bipolar sequence family \( \{c_{\lambda_0}^t | \lambda_0 \in \text{GF}(2^n)\} \) of \( 2^n \) can be obtained. Arranging elements as column vectors can form matrices \( A_1 \). Another primitive domain element on GF\((2^n)\) is selected, and the corresponding matrix \( A_2 \) can be obtained.

Step three. In the two matrices obtained above, the line extension is in the form of \( A = [A_1,A_2] \). Thus, the measurement matrix of this study can be obtained.

### 3.4. Analysis of the Experimental Results

#### 3.4.1. One-Dimensional Signal Compression Sensing Reconstruction

To generate a sampled \( K \) sparse signal \( x \), the support position is selected randomly, and the Gauss distribution with the support value obeys the standard; its length is \( 2^n+1 \). For each rare sparsity \( K \), which is generated 1000 times with Matlab, the Orthogonal Matching Pursuit (OMP) algorithm is selected for the reconstruction algorithm; if the resulting \( X_\text{R} \) of the reconstruction is satisfied with \( \|X - X_\text{R}\| < 10^{-5} \), then it is believed that the reconstruction experiment is successful and the probability of reconstruction is the ratio of exact reconstruction to the total number. Under two conditions of no noise and noise. Regarding the reconstruction performance of the measurement matrix MSMM, the Gaussian random measurement matrix and the Bernoulli random measurement matrix are compared. For order \( n = 8 \), after generating a measurement matrix with a size of \( 255 \times 512 \), sparsity \( k = 1 - 85 \). The comparison curves of the reconstruction probability under different sparsities are shown in Figure 2(a). For the sampled signal with a signal-to-noise ratio of 30dB, comparison curves of the reconstruction probability under different sparsities are shown in Figure 2(b). According to graphs (a) and (b), in a noisy environment and a no-noise environment, the signal recovery effect of the matrix MSMM is better than that of the Gaussian matrix and the Bernoulli matrix of the same size.

#### 3.4.2. Image Reconstruction of the Streak Camera

In the observable cathode range, the background noise of the image should be deducted in practical application. The formula for calculating the contrast in actual calculation is as follows [18]:

\[
C = \frac{(I_{\text{max}} - I_{\text{nos}}) - (I_{\text{min}} - I_{\text{nos}})}{(I_{\text{max}} - I_{\text{nos}}) + (I_{\text{min}} - I_{\text{nos}})} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}} - 2I_{\text{nos}}} \tag{9}
\]

In the formula, \( I_{\text{max}}, I_{\text{min}}, \) and \( I_{\text{nos}} \) are the intensity values of the streak, intensity of the adjacent dark lines, and background noise of the images, respectively. The contrast between the three matrix reconstruction diagrams and the original diagram is given in Table 1. The limit static spatial resolution is calculated from the spherical fluorescent screen, and the corresponding limit resolution is obtained as shown in Table 2. The following results are obtained: (1) the image restoration effect of the MSMM matrix is better than that of the Gaussian matrix and the Bernoulli matrix of the same.

**Table 1**: Comparison of the static spatial resolution of the streak camera in the experiment.

<table>
<thead>
<tr>
<th>SSR (lp/mm)</th>
<th>DCA (mm)</th>
<th>OI CTF (%)</th>
<th>OMP-G CTF (%)</th>
<th>OMP-B CTF (%)</th>
<th>OMP-MSMM CTF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>1.2</td>
<td>21.6%</td>
<td>23.7%</td>
<td>24.2%</td>
<td>25.2%</td>
</tr>
<tr>
<td>18</td>
<td>4.7</td>
<td>30.2%</td>
<td>35.2%</td>
<td>38.1%</td>
<td>40.5%</td>
</tr>
<tr>
<td>12</td>
<td>8.2</td>
<td>45.1%</td>
<td>50.1%</td>
<td>54.2%</td>
<td>60.4%</td>
</tr>
<tr>
<td>10</td>
<td>11.7</td>
<td>48.4%</td>
<td>55.8%</td>
<td>65.8%</td>
<td>68.1%</td>
</tr>
</tbody>
</table>

SSR: streak spatial resolution; DCA: distance from the centre of the axis; OR: original image; GM: Gaussian measurement; BM: Bernoulli measurement; MSMM: M sequence family measurement matrix.

**Table 2**: Comparison of the limit spatial resolution of the streak camera in the experiment.

<table>
<thead>
<tr>
<th>SSR (lp/mm)</th>
<th>DCA (mm)</th>
<th>LSR (lp/mm)</th>
<th>OMP-G LSR (lp/mm)</th>
<th>OMP-B LSR (lp/mm)</th>
<th>OMP-M LSR (lp/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
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<td>31</td>
<td>32</td>
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<tr>
<td>18</td>
<td>4.7</td>
<td>28</td>
<td>30</td>
<td>32</td>
<td>33</td>
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<tr>
<td>12</td>
<td>8.2</td>
<td>23</td>
<td>25</td>
<td>27</td>
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</tr>
<tr>
<td>10</td>
<td>11.7</td>
<td>20</td>
<td>23</td>
<td>26</td>
<td>28</td>
</tr>
</tbody>
</table>

SSR: streak spatial resolution; DCA: distance from the center of the axis; LSR: limit spatial resolution.
Figure 2: Comparison of the effect of measurement matrix reconstruction. (a) Reconstruction probability under different degrees of sparsity. (b) Output signal-to-noise ratio under different degrees of sparsity.

Figure 3: Comparison of the effect of measurement matrix reconstruction. (a) The original image; (b) reconstruction on the Gaussian matrix; (c) reconstruction on the Bernoulli matrix; (d) reconstruction on the MSMM.

4. Conclusion
The position of an electron beam emitting at different heights on the streak cathode is not on the same plane, but is on
a parabola. Through analysis of the static spatial resolution test data of the streak image tube, the off-center axis area is the defocus area, and the CS algorithm is applied to improve the imaging quality of the defocus region of the streak tube camera. To achieve the best image restoration effect of a streak tube camera, a reconstruction model based on CS was constructed. The optimization of the reconfiguration parameters can be achieved by examining different combinations of equilibrium parameters. The experimental results showed the following: the static image contrast increased by 12.2%, and the limit resolution of the static test image increased by 5 lp/mm on average. At a distance of 1.2 mm from the center to the center of the axis, the contrast increase is 3.6%, limit resolution is increased by 1 lp/mm, contrast is raised by 19.7% at the center distance of the off-axis, and the limit resolution is improved by 8 lp/mm. This result showed that the contrast improvement of the edge is more obvious and the M sequence family measurement matrix compression sensing method has a certain compensation effect on the edge defocus caused by the field curve, thereby providing a new approach for improving the spatial resolution of the streak tube camera.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare no conflicts of interest.

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Figure 5: Comparison of time resolved channel in the streak camera dynamic image. (a) Streak camera dynamic image. (b) The line strength graph of single time resolved channel after reconstructed and original image.

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