SHORT COMMUNICATION

An Accurate Single Section Model of Nonuniform R–C–NR Structures

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In recent years considerable advances have been made in the development of thin film microcircuits. Their properties and characteristics have been intensively studied. The advent of integrated electronic devices has aroused keen interest in the study of the properties of multilayer thin film structures. In particular RC distributed circuit elements have been attracting attention as miniaturization of electronic equipment advances. The use of multiterminal distributed RC networks is in keeping with the trend in modern network theory. The capability of a 4 terminal distributed parameter network to yield various sub-networks with different frequency and pulse characteristics is higher than that of its 3 terminal counterpart, the latter being a special case of the former. It has been shown¹ that for the simplest case of 3 layer R–C–NR networks it is possible to obtain a whole complex of subcircuits such as low and high frequency filters, band filters, single element notch filters, phase shifters, delay lines and other functional units.

Network response can be controlled by shaping the morphology.² Nonuniformity in distributed parameters causes an enhancement in desired characteristics due to tapering. Such structures possess extremely valuable properties and in view of their considerable importance and utility, the need for a thorough study is apparent. Of the various forms of tapering that have been extensively studied in the literature, the exponential one has been pointed out by several authors as the most effective one in achieving a desired set of characteristics. Also it is one of the simplest forms of tapered lines, most readily amenable to analysis.

Woo and Bartlemay³ were the first to present an analytical treatment of exponentially tapered R–C–NR structure (Figure 1). However, modelling of this structure for analysis and design of circuits incorporating this structure has not been considered

FIGURE 1

in the literature. A one dimensional transmission line is the existing mathematical model of the structure.⁴ The use of the general transmission line model poses a formidable problem and involves considerable mathematical intricacy and complexity. This arises from the fact that an infinite number of sections of elements are required for achieving tolerable accuracy. Simulation of the model necessitates prohibitive computational labour.

The continued fraction expansion and truncation technique has been expounded⁵ to arrive at a rational approximation to the exact two-port admittance parameters. This has been applied to obtain a very accurate model of a class of nonuniformly distributed RC lines. The present communication is an extension and generalisation of that work.

The indefinite admittance matrix of the exponentially tapered R–C–NR structure is:³

$$[y] = \frac{1}{R[N+1]}$$

$$\begin{matrix} \alpha + N - \delta & -\beta - N & \beta - 1 & -\alpha + 1 + \delta \\ y_{12} & \gamma + N + \phi & -\gamma + 1 - \phi & y_{13} \\ y_{13} & y_{23} & \gamma + \frac{1}{N} + \phi & -\gamma - \frac{1}{N} \\ y_{14} & y_{13} & y_{34} & \alpha + \frac{1}{N} - \delta \end{matrix} \quad (1)$$

FIGURE 2

Where $\alpha = \theta \cdot \coth \theta \cdot (e^{2B} - 1) \cdot / 2B$

$\beta = \theta \cdot \operatorname{cosech} \theta \cdot e^B (1 - e^{-2B}) \cdot / 2B$

$\gamma = \theta \cdot \coth \theta \cdot (1 - e^{-2B}) \cdot / 2B$

$\delta = (e^{2B} - 1)/2$

$\phi = (1 - e^{-2B})/2$

and $B = kd$, a constant specifying the taper, with k being the taper factor and d the length of the line. B , N and θ are related by:

$$\theta = [B^2 + SR_0 C_0 (1 + N)]^{1/2} \quad (2)$$

$$\text{with } R_0 = \leq_0 d \text{ and } C_0 = c_0 d \quad (3)$$

Constants r_0 and c_0 describe the exponential tapering (for a positive taper factor) as:

$$r(x) = r_0 e^{2kx}, \quad c(x) = c_0 e^{-2kx} \quad (4)$$

and

$$R = \int_0^d r(x) dx = R_0 (e^{2B} - 1) / 2B \quad (5)$$

The admittance parameters given in Eq. (1) involve hyperbolic functions of a complex irrational argument. Synthesis of a single section, lumped, two-port model with simple elements is impossible from these parameters in such a form.

The method of Galitskiy¹ is used to construct a

FIGURE 3

new and accurate lattice equivalent of the exponentially tapered R-C-NR structure (Figures 2 and 3). The model is deemed to be extremely versatile, powerful and useful. It may be noted that for $k = 0$ the model reduces to that given earlier for the uniform R-C-NR structure,⁶ and for $k = 0, N = 0$ reduces to the model postulated earlier for uniform RC structure. This conforms to expectations and serves as verification.

REFERENCES

1. V. V. Galitskiy, *Radio Eng. and Electron. Phys.*, **11**, 249 (1966).
2. W. W. Happ, *Proc. IRE*, **50**, 483 (1962).
3. B. B. Woo and J. M. Bartlemay, Characteristics and applications of a tapered thin film distributed parameter structure, *IEEE International Convention Record, New York*, **11**, Part II, 56 (1963).
4. M. S. Ghausi and J. J. Kelly, *Introduction to Distributed Parameter Networks with Application to Integrated Circuits* (Holt, Rinehart and Winston, New York, 1968).
5. U. Kumar and S. C. Dutta Roy, *Int. J. Circuit Theory and Applications*, **5**, 355 (1977).
6. U. Kumar and K. Pal, *Electron. Lett.*, **14**, 384 (1978).