NEW CURRENT-MODE NOTCH AND ALLPASS FILTERS WITH SINGLE CURRENT DIFFERENCE AMPLIFIER

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A new configuration for realization of current-mode notch and allpass filters is presented. It can synthesize second-order notch and allpass filters using a single current difference (Norton) amplifier and at most eight passive RC one port elements. Experimental results obtained from a notch filter realization are presented.

INTRODUCTION

At present, there is a growing interest in designing current-mode signal processing circuits. In these circuits, the current rather than the voltage is used as the active variable either throughout the whole circuit or only in certain critical areas. The use of current as the active parameter can result in circuits operating with higher signal bandwidths, greater linearity, and larger dynamic range than voltage-mode circuits [1]. The current conveyor, a powerful analog building block with current-mode capability, is therefore a strong potential candidate for implementing current-mode circuits, and recently a number of realizations have been presented using first- and second-generation current conveyors [see, for example, [2] and the references cited therein]. However, in many industrial electronic control systems, circuits are designed to operate off only a single power supply voltage. The current conveyor is typically designed for split power supplies and, therefore, current-mode active filters employing conventional current conveyors cannot be used in many industrial control applications. The current difference amplifier, designed to operate from a single power supply, is therefore, a strong candidate for such applications. A literature survey, [see, for example, [3]–[5] and the references cited therein] reveals that while the current-difference amplifier has been extensively used in designing voltage-mode biquadratic transfer functions, no attempt has been reported yet for its use in designing current-mode biquadratic filter circuits.

It is the purpose of this paper to present current-mode biquadratic notch and allpass filter realizations using the current-difference amplifier.

PROPOSED CIRCUITS

Consider the general circuit shown in Fig. 1. Assuming an ideal current difference amplifier defined by $i_+ = i_-$, $v_+ = v_- \cong 0$, routine analysis of the circuit of Fig. 1

FIGURE 1 Proposed general circuit.

yields the current transfer function given by

$$\frac{I_o}{I_i} = \frac{1}{G_4 R_L} \frac{Y_2 - Y_1}{Y_1 + Y_2 + Y_3} \quad (1)$$

where $G_4 = 1/R_4$.

Using (1), it is easy to show that current-mode notch and allpass filters can be realized. For example, if we choose, $Y_1 = 1/(R_1 + 1/sC_1)$, $Y_2 = 1/R_2 + sC_2$, $Y_3 = 0$ (open circuit), shown in Fig. 2(a), then (1) reduces to

$$\frac{I_o}{I_i} = \frac{R_4}{R_L} \frac{1 + s(C_1 R_1 + C_2 R_2 - C_1 R_2) + s^2 C_1 C_2 R_1 R_2}{1 + s(C_1 R_1 + C_2 R_2 + C_1 R_2) + s^2 C_1 C_2 R_1 R_2} \quad (2)$$

Now if

$$C_1 R_1 + C_2 R_2 = C_1 R_2 \quad (3)$$

then (2) reduces to

$$H_1(s) = \frac{R_4}{R_L} \frac{s^2 + \omega_o^2}{s^2 + \frac{\omega_o}{Q_o} s + \omega_o^2} \quad (4)$$

which corresponds to the transfer function of a second-order notch filter with the parameters

$$\omega_o^2 = \frac{1}{C_1 C_2 R_1 R_2} \quad (5)$$

FIGURE 2 Proposed filter realizations (a) Second-order notch (b) Second-order allpass (c) First-order allpass.

and

$$\frac{\omega_o}{Q_o} = \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_2 R_1} \quad (6)$$

Using (3), (5), and (6), the parameter $\frac{\omega_o}{Q_o}$ can be expressed by

$$\frac{\omega_o}{Q_o} = \frac{2}{C_2 R_1} \quad (7)$$

Note that the ratio between the output and input currents of this notch filter is R_4/R_L . Such ratio can be made larger than one and, therefore, this notch realization is not suffering from a constant loss. Note also that this realization requires only six passive elements, including the load resistor R_L , plus one active element.

Now if a parallel RC combination is used for Y_3 , that is $Y_3 = 1/R_3 + sC_3$ while Y_1 and Y_2 remain as before, shown in Fig. 2(b), then (1) reduces to

$$\frac{I_o}{I_i} = \frac{R_4}{R_L} \frac{R_3}{R_2 + R_3} \frac{1 + s(C_2 R_2 + C_1 R_1 - C_1 R_2) + s^2 C_1 C_2 R_1 R_2}{1 + s(C_1 R_1 + (C_1 + C_2 + C_3) R_p) + s^2 (C_2 + C_3) C_1 R_1 R_p} = \frac{R_4}{R_L} \frac{R_3}{R_2 + R_3} \frac{1 + \alpha s + \beta s^2}{1 + \gamma s + \delta s^2} \quad (8)$$

where $R_p = \frac{R_2 R_3}{R_2 + R_3}$

Now if $\alpha = -\gamma$, i.e.

$$C_2 R_2 + (C_1 + C_2 + C_3) R_p + 2 C_1 R_1 = C_1 R_2 \quad (9)$$

and if $\beta = \delta$, i.e.

$$C_2 R_2 = C_3 R_3 \quad (10)$$

then (8) reduces to

$$H_2(s) = \frac{R_4}{R_L} \frac{R_3}{R_2 + R_3} \frac{s^2 - \frac{\omega_o}{Q_o} s + \omega_o^2}{s^2 + \frac{\omega_o}{Q_o} s + \omega_o^2} \quad (11)$$

which corresponds to the transfer function of a second-order allpass filter with the parameters

$$\omega_o^2 = \frac{1}{C_1 C_2 R_1 R_2} \quad (12)$$

and

$$\frac{\omega_o}{Q_o} = \frac{C_1 R_1 + (C_1 + C_2 + C_3) R_p}{C_1 C_2 R_1 R_2} \quad (13)$$

Using (9), (12) and (13), the parameter $\frac{\omega_o}{Q_o}$ can be expressed by

$$\frac{\omega_o}{Q_o} = \frac{1}{C_2 R_1} - \frac{1}{C_1 R_1} - \frac{1}{C_2 R_2} \quad (14)$$

Finally, if we choose $Y_1 = 1/C_1$, $Y_2 = 1/R_2$ while $Y_3 = 0$, shown in Fig. 2(c), then

$$\frac{I_o}{I_i} = \frac{-R_4}{R_L} \frac{sC_1R_2 - 1}{sC_1R_2 + 1} \quad (15)$$

Equation (15) corresponds to the transfer function of a first-order allpass filter. Note that the ratio between the output and input currents of this allpass filter is R_4/R_L . Such ratio can be made larger than one. Note also that this realization requires only four passive elements, including the load resistor, R_L , plus one active element.

SENSITIVITY ANALYSIS

By defining the sensitivity of a parameter F to the element of variation x_i by

$$S_{xi}^F = \frac{x_i}{F} \frac{dF}{dx_i}$$

the passive sensitivities of the parameters ω_o and $\frac{\omega_o}{Q_o}$ of the proposed circuits of Fig. 2(a) and (b) have been calculated using equations (5), (7), (12), and (14) and their values are given in Table I. From Table I, one can easily see that all the passive ω_o -sensitivities and $\frac{\omega_o}{Q_o}$ -sensitivities of the second-order notch filters are ≤ 1 . One can also see that while the passive ω_o -sensitivities of the second-order allpass filter are ≤ 1 , the $\frac{\omega_o}{Q_o}$ -sensitivities may be appreciably high due to the presence of the

TABLE I
The passive sensitivities for the notch and allpass realizations of Figs. 2(a) and (b)

Function	Passive Sensitivities
Notch of Fig. 2(a)	$S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = -\frac{1}{2}$
	$S_{R_1}^{\omega_o/Q_o} = S_{C_2}^{\omega_o/Q_o} = -\frac{1}{2}$
Allpass of Fig. 2(b)	$S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = -\frac{1}{2}$
	$S_{R_1}^{\omega_o/Q_o} = \frac{-1}{1 + \frac{R_1}{R_2} \frac{1}{C_2/C_1 - 1}}$
	$S_{R_2}^{\omega_o/Q_o} = \frac{-1}{1 + \frac{R_2}{R_1} \left(\frac{C_2}{C_1} - 1 \right)}$
	$S_{C_1}^{\omega_o/Q_o} = \frac{-1}{1 + \frac{C_1}{C_2} \left(\frac{R_1}{R_2} - 1 \right)}$
	$S_{C_2}^{\omega_o/Q_o} = \frac{-1}{1 + \frac{C_2}{C_1} \frac{1}{R_1/R_2 - 1}}$

difference terms in the denominators. However, the sensitivity figures may be made low, in the range of the notch filter realization, by careful design.

The effect of the finite gain of the current-difference amplifier on the performance of the proposed circuits can be studied by replacing G_4 in equation (1) by $G_4 + \frac{G_4 + Y_1}{A}$ where A is the finite gain of the amplifier [4]. Thus, (1) reduces to

$$\frac{I_o}{I_i} = \frac{1}{\left(G_4 + \frac{G_4 + Y_1}{A}\right)R_L} \frac{Y_2 - Y_1}{Y_1 + Y_2 + Y_3} \quad (16)$$

For the second-order notch-filter realization with $Y_1 = 1/(R_1 + 1/sC_1)$, $Y_2 = 1/R_2 + sC_2$, $Y_3 = 0$, equation (16) reduces to

$$\begin{aligned} \frac{I_o}{I_i} &= \frac{R_4}{R_L(1 + 1/A)} \frac{1 + sC_1R_1}{1 + s(C_1R_1 + C_1R_4/(1 + A))} \\ &\frac{1 + s(C_1R_1 + C_2R_2 - C_1R_2) + s^2C_1C_2R_1R_2}{1 + s(C_1R_1 + C_2R_2 + C_1R_2) + s^2C_1C_2R_1R_2} \end{aligned} \quad (17)$$

The transfer function of (17) can be decomposed into two cascaded transfer functions $T_1(s)$ and $T_2(s)$. The first transfer function, $T_1(s)$, is expressed by

$$T_1(s) = \frac{1 + sC_1R_1}{1 + sC_1(R_1 + R_4/(1 + A))} \quad (18)$$

This transfer function corresponds to a lowpass filter with high-frequency gain determined by the ratio $\frac{R_4}{R_1} \frac{1}{1 + A}$. Both the pole $\omega_p = 1/C_1(R_1 + R_4/(1 + A))$ and the zero $\omega_z = 1/C_1R_1$ in this transfer function are adjustable through C_1 , but the ratio is held constant. The second transfer function, $T_2(s)$, is expressed by

$$T_2(s) = \frac{R_4}{R_L(1 + 1/A)} \frac{1 + s(C_1R_1 + C_2R_2 - C_1R_2) + s^2C_1C_2R_1R_2}{1 + s(C_1R_1 + C_2R_2 + C_1R_2) + s^2C_1C_2R_1R_2} \quad (19)$$

This transfer function is the same as the transfer function of (2) with the gain slightly modified to $R_4/R_L(1 + 1/A)$ rather than R_4/R_L . The notch filter parameters ω_o and ω_o/Q_o will remain the same inspite of the finite gain of the amplifier. Thus, by selecting $\frac{R_4}{1 + 1/A} \ll R_1$ the notch characteristics of the circuit of Fig. 2(a) will preserve its shape.

In a similar way, if we take into consideration the effect of the finite gain of the amplifier, the transfer function of the allpass circuit of Fig. 2(b) will be

$$\begin{aligned} \frac{I_o}{I_i} &= \frac{R_4}{(1 + 1/A)R_L} \frac{R_3}{R_2 + R_3} \frac{1 + sC_1R_1}{1 + sC_1(R_1 + R_4/(1 + A))} \\ &\frac{1 + s(C_2R_2 + C_1R_1 - C_1R_2) + s^2C_1C_2R_1R_2}{1 + s(C_1R_1 + (C_1 + C_2 + C_3)R_p) + s^2(C_2 + C_3)C_1R_1R_p} \end{aligned} \quad (20)$$

This transfer function can be decomposed into two cascaded transfer functions, $T_1(s)$, of equation (18), followed by the transfer function $T_3(s)$ given by

$$T_3(s) = \frac{R_4}{(1 + 1/A)R_L} \frac{R_3}{R_2 + R_3} \frac{1 + s(C_2R_2 + C_1R_1 - C_1R_2) + s^2C_1C_2R_1R_2}{1 + s(C_1R_1 + (C_1 + C_2 + C_3)R_p) + s^2(C_2 + C_3)C_1R_1R_p} \quad (21)$$

Equation (21) is the same as (8) with a slight modification in the gain due to the effect of finite gain of the amplifier. The allpass filter parameters ω_o and $\omega_o Q_o$ are not affected by the finite gain of the amplifier. Thus by selecting $R_4/(1 + A) \ll R_1$, the effect of the finite gain of the amplifier can be minimized, and the allpass characteristic of the circuit of Fig. 2(b) will preserve its shape.

DESIGN PROCEDURE

Design procedure is given for the notch and allpass filter circuits of Figs. 2(a) and (b). For the notch filter circuit of Fig. 2(a), the parameters ω_o^2 and $\frac{\omega_o}{Q_o}$ are given by equations (5) and (7). Combining (5) and (7), then

$$C_1R_2 = \frac{1}{2Q_o\omega_o} \quad (22)$$

Thus, for given values of ω_o and Q_o , design values for C_1, C_2, R_1 and R_2 can be obtained that satisfy equations (3), (7), and (22) with $Q_o \leq 0.25$. For example, $R_2 = 2R_1 = 2R$, $C_1 = 2C_2 = 2C$ gives $\omega_o = \frac{1}{2CR}$ and $Q_o = 0.25$. Values of C and R can be selected to yield the required ω_o .

Similarly, for the allpass circuit of Fig. 2(b), the parameters ω_o^2 and $\frac{\omega_o}{Q_o}$ are given by equations (12) and (14). From (14), one can see that values of C_1, C_2, R_1 and R_2 must satisfy the condition

$$\frac{1}{C_2R_1} > \frac{1}{C_1R_1} + \frac{1}{C_2R_2} \quad (23)$$

Thus using (10), (12), (14), and (23), design values for C_1, C_2, C_3, R_1, R_2 and R_3 for given values of ω_o and Q_o with $Q_o \leq 1$ are obtained. For example, $R_2 = 3R_1 = R_3 = 3R$ and $C_1 = 3C_2 = 3C_3 = 3C$ gives $\omega_o = 1/3CR$ and $Q_o = 1$.

EXPERIMENTAL RESULTS

The proposed notch and allpass filters were realized using the LM3900 current-difference amplifier. In the laboratory, the input current I_i was obtained using the Howland voltage-to-current converter [6]. The results obtained for the notch filter with $2R_1 = 2R_4 = R_2 = R_B = 2R_L = 3.2K$, $C_1 = 2C_2 = 2nF$ and $V_B = 9V$ are shown in Fig. 3. From Fig. 3, it is easy to see that the notch occurs at 49 KHz while the predicted value using eqn (5) is 49.736 KHz. Thus, the experimental results are in good agreement with the theoretical analysis presented here.

FIGURE 3 Measured notch filter characteristic using $2R_1 = 2R_4 = R_2 = R_B = 2R_L = 3.2k\Omega$, $C_1 = 2C_2 = 2nF$, $V_B = 9V$.

CONCLUSION

A new circuit for realizing current-mode, second-order notch and allpass filters has been presented. The proposed circuit uses a single current difference (Norton) amplifier and at most eight passive RC one-port elements, including the load resistor. The circuit can also realize a first-order allpass filter using a single current difference amplifier and four passive elements only including the load resistor. The current-difference amplifier requires a single dc power supply and, therefore, the proposed realizations are very useful for many industrial electronic control systems designed to operate off only a single power supply voltage. The proposed realizations enjoy low active and passive sensitivities.

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