AN IMPROVED ANALYSIS FOR THE HARMONIC DISTORTION OF MOS VOLTAGE-CONTROLLED-RESISTORS

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In this paper, a fourth-order polynomial expression is obtained for the nonlinear current-voltage characteristic of a MOS transistor operating in the triode region. Using this expression, closed-form expressions are obtained for the second-, third- and fourth-harmonic distortion of a MOS voltage-controlled-resistors. The analytical expressions obtained in this paper can be used for a quantitative study of the effect of different parameters of the performance of MOS voltage-controlled-resistors.

INTRODUCTION

Voltage-controlled-resistors employing MOS transistors operating in their linear region are essential elements in realizing continuous time filters [1-3]. These resistors, however, exhibit nonlinearities that give rise to harmonic distortion. A circuit designer must, therefore, be able to accurately predict the harmonic performance of these resistors. An essential prerequisite for reliable prediction of the harmonic distortion is the accurate modelling of the MOS current-voltage characteristics in the linear region. While the model proposed by Merckel et al. [4] enjoys high accuracy, it is too complicated and cannot yield closed-form expressions for the harmonic distortion. A simplified version of this model was used in a modified version of SPICE to predict the harmonic performance of MOS voltage-controlled resistor circuits [5]. No attempt has been reported, however, to use this simplified model to obtain closed-form expressions for the harmonic distortion. Although, the model proposed by White et al. [6] is simpler, and was successfully used to investigate the effectiveness of a range of linearization strategies of MOS resistors [7], it cannot yield closed-form expressions for the harmonic distortion. The simplified model proposed by Shoucair et al. [8] was successfully used to obtain closed-form expressions for the harmonic distortion of body driven MOSFET circuits [8,9]. At relatively low gate voltages, however, the accuracy of the model deteriorates [8].

In this paper, it will be shown that the simplified model proposed by Ryan [5] can be used to obtain closed-form expressions for the harmonic performance of MOS voltage-controlled resistors operating in the linear region. These expressions are valid over a wide range of gate-source and drain-source voltages.

ANALYSIS

Assuming that the mobility degradation due to the lateral field in the MOS channel is zero, the current-voltage characteristic of the MOS transistor can be expressed by [5]

$$I_{DS} = \beta \int_{V_s}^{V_D} \left(\frac{V_G - V_{FB} - \phi_B - V - \gamma(V + V_B + \phi_B)^{1/2}}{1 + \theta(V_G - V_{FB} - \phi_B - V + \gamma(V + V_B + \phi_B)^{1/2})} \right) dV \quad (1)$$

where β , θ , γ , and ϕ_B are the transistor current gain, mobility degradation factor, body factor, and surface potential and V , V_G , V_D , V_S , V_B , and V_{FB} are the transistor channel, gate, drain, source and bulk voltages, and the flatband potential, respectively. Equation (1) can be rewritten as

$$I_{DS} = \beta \int_{x_2}^{x_1} \left(\frac{(a_o + a_1x + a_2x^2)x}{b_o + b_1x + b_2x^2} \right) dx \quad (2)$$

where

$$a_o = 2(V_G - V_{To} + \gamma\sqrt{2\phi_B} - V_{BS} + 2\phi_B)$$

$$a_1 = -2\gamma$$

$$a_2 = -2$$

$$b_o = 1 + \theta(V_G - V_{To} + \gamma\sqrt{2\phi_B} - V_{BS} + 2\phi_B)$$

$$b_1 = \theta\gamma$$

$$b_2 = -\theta$$

$$V_{To} = V_{FB} + \phi_B + \gamma\sqrt{2\phi_B}$$

$$V_{BS} = \phi_B - V_B$$

$$x = (V - V_{BS} + 2\phi_B)^{1/2}$$

$$x_1 = (V_D - V_{BS} + 2\phi_B)^{1/2}$$

and

$$x_2 = (V_S - V_{BS} + 2\phi_B)^{1/2}$$

By standard integral tables [10], (2) yields

$$I_{DS} = \beta(d_o(x_1^2 - x_2^2) + d_1(x_1 - x_2) + d_2 \ln(\frac{h_1 x_1^2 + h_2 x_1 + 1}{h_1 x_2^2 + h_2 x_2 + 1}) + h_3 \ln(\frac{(h_4 x_1 + 1)(h_5 x_2 + 1)}{(h_5 x_1 + 1)(h_4 x_2 + 1)})) \quad (3)$$

where

$$d_o = \frac{a_2}{2b_2}$$

$$d_1 = \frac{a_1 b_2 - a_2 b_1}{b_2^2}$$

$$d_2 = \frac{a_o}{2b_2} - \frac{a_1 b_1}{2b_2^2} + a_2 \left(\frac{b_1^2 - b_2 b_o}{2b_2^3} \right)$$

$$d_3 = -\frac{a_o b_1}{2b_2} + \frac{a_1(b_1^2 - 2b_2 b_o)}{2b_2^2} + \frac{a_2(3b_o b_1 b_2 - b_1^3)}{2b_2^3}$$

$$h_1 = \frac{b_2}{b_o}$$

$$h_2 = \frac{b_1}{b_o}$$

$$h_3 = \frac{d_3}{\sqrt{b_1^2 - 4b_o b_2}}$$

$$h_4 = \frac{2b_2}{b_1 - \sqrt{b_1^2 - 4b_o b_2}}$$

$$h_5 = \frac{2b_2}{b_1 + \sqrt{b_1^2 - 4b_o b_2}}$$

and

$$b_1^2 > 4b_o b_2$$

Table I shows calculated values for d 's and h 's for

TABLE I
Values of the d 's and h 's.

parameter	value	parameter	value
d_0	8.3333	h_1	-0.054158
d_1	28.3333	h_2	0.046034
d_2	81.4861	h_3	-68.5993
d_3	-71.0908	h_4	0.256871
		h_5	-0.210837

$$V_{To} = 1.28V, V_G = 5V, V_S = 0, V_{BS} = -5V, 2\phi_B = 0.7V, \theta = 0.12V^{-1}, \gamma = 0.85V^{1/2}$$

Although (3) is exact, it cannot yield closed-form expressions for the harmonic distortion components of the drain current, I_{DS} , resulting from a sinusoidal drain-to-source voltage V_{DS} . However, by using the following approximations

$$\ln(1+x) \cong x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$$

and

$$(1+x)^{1/2} \cong 1 + 0.5x - 0.125x^2$$

it is easy, though tedious, to express the drain current as

$$I_{DS} = \beta(\alpha_1 V_{DS} + \alpha_2 V_{DS}^2 + \alpha_3 V_{DS}^3 + \alpha_4 V_{DS}^4) \quad (4)$$

where

$$\alpha_1 = T_1 + 0.5T_2\psi^{-1/2} + 2T_3\psi + 1.5T_4\psi^{1/2} + 2.5T_5\psi^{3/2}$$

$$\alpha_2 = -0.125T_2\psi^{-3/2} + T_3 + 0.375T_4\psi^{-1/2} + 1.875T_5\psi^{1/2}$$

$$\alpha_3 = -0.125T_4\psi^{-3/2} - 1.5T_5\psi^{-1/2}$$

$$\alpha_4 = -0.125T_5\psi^{-3/2}$$

$$\psi = \sqrt{-V_{BS} + 2\phi_B}$$

and

$$T_1 = d_0 + d_2h_1 - \frac{1}{2}h_2^2d_2 - \frac{1}{2}h_3h_4^2 + \frac{1}{2}h_3h_5^2$$

$$T_2 = d_1 + d_2h_2 + h_3h_4 - h_3h_5$$

$$T_3 = -\frac{1}{2}d_2h_1^2 - \frac{1}{4}h_3(h_4^4 - h_5^4)$$

$$T_4 = -d_2h_1h_2 + \frac{1}{3}h_3(h_4^3 - h_5^3)$$

$$T_5 = \frac{1}{5}h_3(h_4^5 - h_5^5)$$

Table II shows calculated values of T 's and α 's using the values of the parameters in Table I.

Using (4) the drain current I_{DS} resulting from a drain-to-source voltage

$$V_{DS} = V_p \sin \omega t \quad (5)$$

can be expressed as

$$I_{DS} = \beta(K_o + K_1 \sin \omega t + K_2 \cos 2\omega t + K_3 \sin 3\omega t + K_4 \cos 4\omega t) \quad (6)$$

where

$$K_o = \frac{1}{2}\alpha_2 V_p^2 + \frac{3}{8}\alpha_4 V_p^4 \quad (7)$$

$$K_1 = \alpha_1 V_p + \frac{3}{4}\alpha_3 V_p^3 \quad (8)$$

$$K_2 = -\frac{1}{2}\alpha_2 V_p^2 - \frac{1}{2}\alpha_4 V_p^4 \quad (9)$$

$$K_3 = -\frac{1}{4}\alpha_3 V_p^3 \quad (10)$$

and

TABLE II
Values of the T 's and α 's.

parameter	value	parameter	value
T_1	4.57236	α_1	1.530526
T_2	0.1526E-04	α_2	-0.235625
T_3	-0.078725	α_3	0.0168936
T_4	-0.398719	α_4	0.19344E-03
T_5	-0.0210595		

$$K_4 = \frac{1}{8} \alpha_4 V_p^4 \quad (11)$$

Using (8–11) the second, third, and fourth harmonic distortions can be expressed as

$$HD_2 = \frac{K_2}{K_1} \quad (12)$$

$$HD_3 = \frac{K_3}{K_1} \quad (13)$$

and

$$HD_4 = \frac{K_4}{K_1} \quad (14)$$

RESULTS

Using (12)–(14) the harmonic performance of the fully-differential MOS voltage-controlled resistor circuit shown in Fig. 1 [11] can be calculated. However, because of the fully-differential nature of the circuit, the second- and fourth-harmonic components will cancel and only the third-harmonic distortion will dominate. The results obtained, using the parameters of Tables I and II are shown in Fig. 2. From Fig. 2, it is obvious that for input voltages between 0.2 V and 4 V peak-to-peak, the third-harmonic distortion levels relative to the fundamental varies from –91 dB to –40 dB. These trends are similar to the previously reported results [5,11].

FIGURE 1 MOS Voltage-Controlled Resistor Circuit [11].

FIGURE 2 Third Harmonic Distortion for the Circuit of Fig. 1.

CONCLUSION

In this paper, a simple fourth-order polynomial expression has been presented for the dependence of the drain current of a MOS transistor on the drain-to-source voltage and other physical and dimensional parameters. Using this expression, closed-form expressions are obtained for predicting the harmonic performance of a MOS transistor operating in the triode region.

The closed-form expressions obtained in this paper can be used for quantitative analysis of the effect of different MOS transistor parameters on the harmonic distortion performance of MOS voltage-controlled-resistors and for selecting the values of the transistor parameters that will yield a prespecified distortion performance.

The results obtained in this paper are in fairly good agreement with previously published simulation results obtained using a modified version of SPICE [5].

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